Studies of Elastic Scattering of Protons and Deuterons by Calcium Isotopes*

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Angular distributions of elastically scattered protons and deuterons from enriched isotopic targets of Ca^{42}, Ca^{44}, and Ca^{48} have been measured at bombarding energies of 9 and 12 MeV, between 30° and 168°. The data are analyzed in terms of the optical model, with particular emphasis on the importance of a symmetry term in the proton potential. The energy dependence of the parameters is also discussed. The results are compared with those obtained from earlier work on isotopes of nickel and copper. An attempt is made to find systematic trends in the deuteron potential and a comparison is made with earlier measurements of deuteron scattering from Ca^{40} and from Ni and Cu isotopes.

INTRODUCTION

THE elastic scattering of charged particles from various nuclei has been studied extensively in recent years. Detailed optical-model analyses of the proton-scattering results have been published recently by Perey,[1] and the deuteron scattering has been similarly analyzed by Perey and Perey[2] and by Halbert.[3] One interesting result of the proton scattering is the dependence of the real potential on a symmetry term proportional to $(N-Z)/A$ as was proposed by Lane.[4] Perey[1] found that the real potential depth in MeV was given by

$$V_S=53.3-0.55E+0.4(Z/A^{1/3})+27(N-Z)/A\,. \quad (1)$$

This formula was derived mostly from data on protons elastically scattered from targets of natural isotopic abundance.[5] One of the main uncertainties in the determination of the coefficient of the symmetry term arises from the fact that $(N-Z)/A$ and $Z/A^{1/3}$ both vary similarly as a function of mass number for stable nuclei. It seems of interest, therefore, to measure the elastic scattering from several isotopes of the same element where the variation of the $Z/A^{1/3}$ term is very much smaller than the variation of the symmetry term.[6] In a previous paper,[7] two of us have reported on the elastic scattering from Cu, Ni, and Fe isotopes. In this paper we report on the scattering from the calcium isotopes in which the changes in the symmetry term are quite large.

The isotopes studied were Ca^{42}, Ca^{44}, and Ca^{48} and the energies of the bombarding protons were 9 and 12 MeV. The threshold for the (p,n) reaction on Ca^{40} is 14.680 MeV and the cross section for proton scattering from this isotope shows very strong and sharp fluctuations with energy.[8] It should not be expected, of course, that the optical-model predictions in such a case would fit the elastic-scattering data. For this reason, the proton scattering from Ca^{40} has not been studied in detail.

The elastic scattering of deuterons is interesting in principle and provides important information useful for distorted-wave calculations of (d,p) and other direct reactions. Perey and Perey[2] analyzed the elastic scattering of deuterons from medium and heavy nuclei at bombarding energies of 10–22 MeV. They found four sets of potential parameters that fit the scattering reasonably well. Optical-model predictions based on any one of these sets could not fit the scattering of deuterons from Ca^{40}. A detailed study of the deuteron scattering from Ca^{40} has been reported by Bassel *et al.*[9] It is of interest to see how the scattering changes from isotope to isotope; one does not expect a symmetry term.

EXPERIMENTAL PROCEDURE

The experimental method was similar to the one described earlier.[7,9] The targets of Ca^{42}, Ca^{44}, and Ca^{48} were bombarded with 9.0- and 12.0-MeV protons and deuterons from the Argonne tandem Van de Graaff accelerator. Calcium targets enriched to about 95% in the isotopes of interest were prepared by evaporation from a tantalum tube which had a small hole in the center of its cylindrical surface and was closed at both ends. The isotopes were in the chemical form of $CaCO_3$ initially, and were converted to calcium metal during the evaporation which was effected at about 1700°C. Carbon foils supported with about 30 $\mu g/cm^2$ of Formvar were found to be the most durable backing. The distance between the target holder and the heated tube was about 2 cm during the evaporation; a tantalum collimator was used to protect the thin foils from the heat radiated by the tube. Targets, about 0.5

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[1] F. G. Perey, Phys. Rev. **131**, 746 (1963).
[2] C. M. Perey and F. G. Perey, Phys. Rev. **132**, 755 (1963).
[3] E. C. Halbert, Nucl. Phys. **50**, 353 (1964).
[4] A. M. Lane, Phys. Rev. Letters **8**, 171 (1962).
[5] The known information about the symmetry term has been summarized by P. E. Hodgson, Phys. Letters **3**, 352 (1963).
[6] J. B. Ball, C. B. Fulmer, and R. H. Bassel, [Phys. Rev. **135**, B706 (1964)] have recently made a similar study by bombarding the zirconium isotopes.
[7] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. **134**, B765 (1964).
[8] R. G. Allas, L. L. Lee, Jr., and J. P. Schiffer (unpublished data).
[9] R. H. Bassel, R. M. Drisko, G. R. Satchler, L. L. Lee, Jr., J. P. Schiffer, and B. Zeidman, Phys. Rev. **136**, B960 (1964).

mg/cm^2 thick, were obtained by evaporation of about 7 mg of $CaCO_3$.

The data were taken in almost the same way as has been described previously.[7,9] The experiments made use of the 18-in. scattering chamber developed by Heinrich and Braid.[10] The scattered particles were detected with a surface-barrier Si counter. The pulses from the detector were recorded in a pulse-height analyzer and the information was punched on cards. The data were analyzed by an IBM 704 computer which summed the number of counts under the elastic peak, subtracted background, converted to center-of-mass yield, and computed the ratio to Rutherford scattering. In obtaining absolute cross sections, the target thicknesses were determined by comparing with the scattering of 4.5- and 5.0-MeV alpha particles at 90° and assuming that these were scattered by the Coulomb field only. The results at 90° for the 4.5- and 5.0-MeV alpha particles are consistent with this assumption. A monitor counter was kept at a fixed angle and all the measurements at the different angles were normalized to it. It is believed that the measured absolute cross sections are accurate to ±5%.

RESULTS AND DISCUSSION

A. Elastic Scattering of Protons

The measured cross sections are tabulated in Table I. In order to allow cross sections for various isotopes to be listed in the same table, laboratory angles are given rather than center-of-mass angles. Figure 1 shows the angular dependence of the measured differential elastic-scattering cross sections expressed as the ratio to the Rutherford cross section. There are pronounced differences between the isotopes. At 9 MeV the cross section for the scattering from Ca^{42} becomes very large at backward angles. This may be due to compound elastic scattering since the (p,n) threshold is at 6.7 MeV. In this case, therefore, we cannot expect that the scattering will be well fitted by the optical model.

Optical-model analyses have been carried out for all the other cases. The potential used in the analysis was of the surface-absorption type and had the form

$$U(r)=-V_S f(r,r_{0S},a_S)+i4a_I W_D (d/dr) f(r,r_{0I},a_I) + \boldsymbol{\sigma}\cdot\mathbf{l}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{V_{SO}}{r}\frac{d}{dr} f(r,r_{0S},a_S)+U_c(r)\,, \quad (2)$$

where V_S is the depth of the real potential, W_D is the depth of the imaginary potential, V_{SO} is the strength of the spin-orbit potential, and m_π is the mass of the charged pion.

The function $f(r,r_0,a)$ is the usual Woods-Saxon form factor

$$f(r,r_0,a)=\{1+\exp[(r-r_0A^{1/3})/a]\}^{-1}$$

[10] J. T. Heinrich and T. H. Braid (to be published).

TABLE I. Cross sections for elastic scattering of 9- and 12-MeV protons.

Laboratory angles (degrees)	Center-of-mass differential cross sections (mb/sr)[a] E_p=9 MeV Ca^{42}	E_p=9 MeV Ca^{44}	E_p=9 MeV Ca^{48}	E_p=12 MeV Ca^{42}	E_p=12 MeV Ca^{44}	E_p=12 MeV Ca^{48}
30.0	1359	1085		867	886	907
37.5	735	546	600	362	401	383
45.0	375	330	285	187	202	180
52.5	224	184	150	105	97	79
60.0	145	98	66	64	56	37
67.5	77	48	24.5	34.7	27.9	14.8
75.0	37	19.9	7.9	13.4	10.2	6.2
82.5	14.7	8.0	6.9			
90.0	6.7	5.4	11.3	3.1	3.7	10.3
97.5	5.8	9.9	17.0	7.3	7.7	12.5
105.0	8.6	13.2	19.2	11.8	9.7	12.3
112.5	15.1	13.1	16.9	13.5	11.3	8.4
120.0	17.0	14.5	13.0	12.3	9.0	4.6
127.5	16.9	12.2	9.0	9.2	6.4	2.14
135.0	15.2	11.1	5.4	6.4	3.7	1.15
142.5	16.2	9.0	2.97	4.4	2.05	1.78
150.0	17.1	7.5	1.52	3.2	2.03	3.2
157.5	16.7	6.6	0.77	3.0	2.54	4.5
165.0	18.7	5.9	0.41	3.2	3.02	6.2

[a] The errors in the cross sections are estimated to be ±5%.

and the Coulomb potential U_c arising from a uniform charge of radius R_c is given by

$$U_c=(Ze^2/2R_c)(3-r^2/R_c^2) \quad \text{for} \quad r\leqslant R_c\,,$$
$$=Ze^2/r \quad \text{for} \quad r>R_c\,.$$

In the theoretical calculations, there are eight adjustable parameters: V_S, W_D, V_{SO}, r_{0S}, a_S, r_{0I}, a_I, and R_c. It is usually possible to fit the data very well by allowing all the parameters to vary. However, their values then show large fluctuations from nucleus to nucleus and very little systematic trend as a function of mass number or energy. Since our main interest was in the small variations in V_S that arise from the symmetry term $(N-Z)/A$, we first tried to fit the data by using the same set of parameters for all the cases and varying only V_S. The geometrical and spin-orbit parameters that were used in the analysis were the same

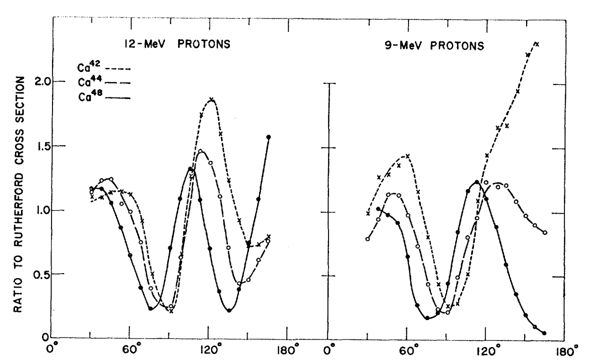

FIG. 1. The ratio of elastic-scattering cross sections to Rutherford cross sections as a function of angle for protons incident on Ca isotopes. The lines were drawn through the experimental points.

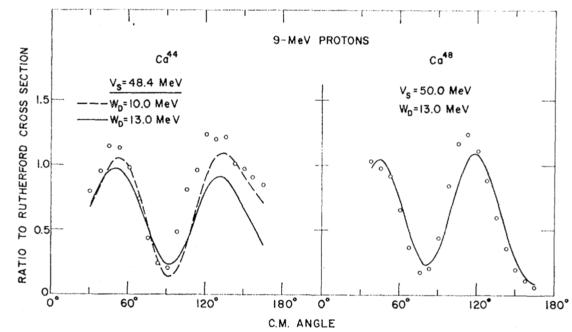

FIG. 2. Comparison of the measured σ/σ_R with predictions of the optical model using the geometrical parameters suggested by Perey. The values of V_S and W_D are given in each case.

as those used by Perey,[1] namely, $r_{0S}=1.25$ F, $a_S=0.65$ F, $r_{0I}=1.25$ F, $a_I=0.47$ F, $R_c=1.25\ A^{1/3}$ F, and $V_{SO}=7.5$ MeV.

The depth of the imaginary potential was also fixed at $W_D=13.0$ MeV. The calculations were done on the IBM 704 computer by use of the ABACUS-2 code.[11] The program adjusts the parameter (V_S in our case) so as to minimize the deviation between experimental and theoretical cross sections. The measure of deviation used is

$$\chi^2=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_{\rm th}(\theta_i)-\sigma_{\rm exp}(\theta_i)}{\Delta\sigma_{\rm exp}(\theta_i)}\right)^2, \tag{3}$$

where N is the number of experimental points and the $\Delta\sigma$ are a set of weighting factors chosen to be equal to the estimated 5% experimental errors. Figures 2 and 3 compare σ/σ_R with optical-model fits for the scattering of 9- and 12-MeV protons, respectively. In addition, these figures give the value of W_D and also the best values of V_S obtained under the above conditions. The scattering of 9-MeV protons from Ca^{48} and of 12-MeV protons from Ca^{44} and Ca^{48} is reproduced reasonably well by the optical model with these parameters. In the case of 9-MeV protons on Ca^{44}, the best fit was obtained with $V_S=48.4$ MeV. The fit at forward angles is reasonable, but it becomes worse at backward angles. In order to see whether the scattering is sensitive to the value of W_D, calculations were carried out with $V_S=48.4$ MeV and all the other parameters as before, while W_D was allowed to vary. This search yielded $W_D=10.0$ MeV and the improved fit is shown by the dashed line in Fig. 2. If one searches for both V_S and W_D together while keeping all the other parameters fixed at their previous values, one gets the best fit for $V_S=50.3$ MeV and $W_D=8.3$ MeV. For the purpose of comparison between the values of the depth V_S of the real potential in the different isotopes, it seems reasonable to use the value 48.4 MeV that was obtained while W_D was 13.0 MeV as in the other cases. In the case of the scattering of 12-MeV protons from Ca^{42} (shown by the full line in Fig. 3), it is impossible to get a reasonable fit by varying only V_S. Both V_S and W_D were allowed to vary during the calculation and the best fit was obtained with $V_S=49.4$ MeV and $W_D=9.5$ MeV. This theoretical distribution is shown by the dashed line in Fig. 3.

In Fig. 4, the values of V_S obtained in our analysis are plotted as a function of $(N-Z)/A$. The errors were estimated approximately from the variation of V_S that causes χ^2 to vary 2% from its minimum value. The slope of the line corresponding to the scattering of 9-MeV protons is 21 ± 10 MeV. A least-squares fit was plotted through the points corresponding to the scattering of 12-MeV protons. The slope of this line is 20 ± 5 MeV. Since the depth of the real potential for 12-MeV protons on Ca^{42} was obtained with a value of W_D different from the one used in all the other cases, one has some difficulty in justifying the use of this value in such a comparison. If one takes into account only the results from the scattering of 12-MeV protons on Ca^{44} and Ca^{48}, the value obtained for the depth of the symmetry potential is 26 ± 8 MeV. The weighted mean of these three results is 22.3 ± 5.0 MeV. The magnitude of the symmetry term found here is in fair agreement with the value obtained by Perey[1] and it is within the range of values summarized by Hodgson.[5]

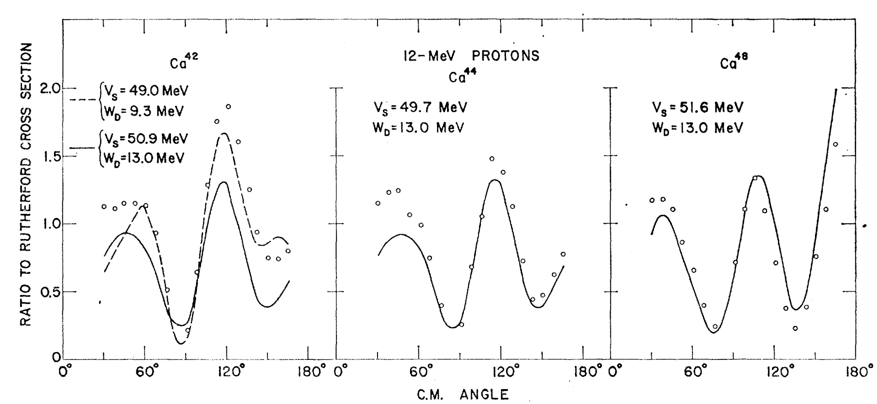

FIG. 3. Comparison of the measured σ/σ_R with predictions of the optical model using the geometrical parameters suggested by Perey. The values of V_S and W_D are given in each case.

[11] E. H. Auerbach, Brookhaven National Laboratory Report BNL-6562, 1962 (unpublished).

As one can see in Fig. 4, the values of V_S obtained for the scattering of 9-MeV protons are smaller than the corresponding values for the scattering of 12-MeV protons. This is not in agreement with Perey's relationship [Eq. (1)], and it also contradicts other experimental and theoretical results.[12] It would be of interest to know whether this energy dependence exists for larger differences in energy. One can compare the values of V_S obtained in our experiments with the values of the proton well depth needed to give agreement with the experimental separation energies of a bound state in the scandium isotopes. In making these calculations for the $1f_{7/2}$ states in the odd-Z, even-A isotopes of scandium, we used the same parameters as in the previous calculations. The binding energy of the $1f_{7/2}$ state in each case was taken as the center of gravity of the binding energies of the ground state and of the isobaric analog state of the $1f_{7/2}$ state in the appropriate calcium isotope. In calculating the position of the analog states, we took account of the Coulomb energy difference between the two isobaric analogs and also the neutron-proton mass difference. In calculating the center of gravity, the analog state was weighted by the factor $S/(2T+1)$, where S is the spectroscopic factor for the (d,p) reaction on Ca^{42}, Ca^{44}, and Ca^{46} leading to the ground state ($f_{7/2}$) in Ca^{43}, Ca^{45}, and Ca^{47}, respectively, and T is the isotopic spin of the even isotopes.[13] (The analog of the ground state of Ca^{41} is the ground state of Sc^{41}; there is no $1f_{7/2}$ state in Ca^{49}.) The results of this calculation are shown in Fig. 4. The depth V_{SO} of the spin-orbit potential is somewhat questionable. It is known that the elastic-scattering cross sections are not very sensitive to the value of V_{SO}, but the values obtained for the depths of the real potential for the bound states might change, depending on the depth of the spin-orbit potential used in the calculation. The

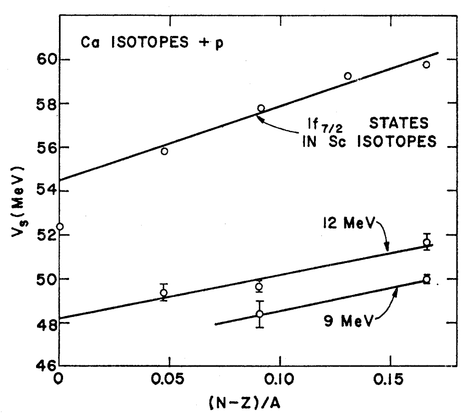

FIG. 4. Plot of the real well depth V_S for the scattering of 9- and 12-MeV protons from Ca isotopes and for the $1f_{7/2}$ states in the Sc isotopes as a function of the symmetry parameter $(N-Z)/A$.

[12] F. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962).
[13] J. B. French and M. H. Macfarlane, Nucl. Phys. **26**, 168 (1961).

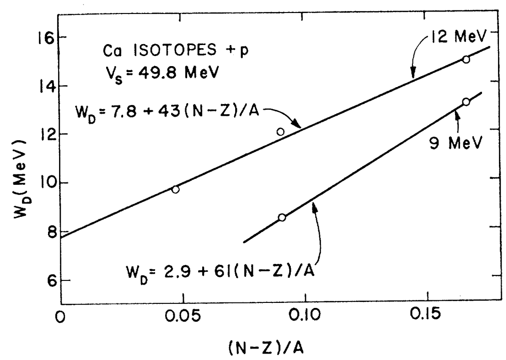

FIG. 5. Surface imaginary potentials W_D as a function of the symmetry parameter $(N-Z)/A$ for the fits to the scattering of 9- and 12-MeV protons by Ca isotopes.

results shown in Fig. 4 have been obtained with the value $V_{SO}=7.5$ MeV used in analyzing the scattering data. However, calculations with $V_{SO}=5.0$ and 10 MeV showed the same dependence of V_S on $(N-Z)/A$ as is shown in Fig. 4. The only difference is in the absolute magnitude of V_S. The lines for V_S obtained with $V_{SO}=5.0$ and 10.0 MeV lie, respectively, about 1.5 MeV above and below the line for $V_{SO}=7.5$ MeV.

As one can see in Fig. 4, the depths of the real potential corresponding to the $f_{7/2}$ states in $Sc^{43,45,47,49}$ fall approximately on a straight line, while the potential corresponding to the $f_{7/2}$ state in Sc^{41} deviates significantly from this line. The slope of this line is 33.4 ± 3.7 MeV. For each isotope, the value of V_S obtained in this calculation is larger than the corresponding values obtained for the scattering of 9- or 12-MeV protons. This is in agreement with Perey's relationship and also with the interpretation of the energy-dependent local potential as an equivalent of a nonlocal potential.[12] The reason for the different energy dependence obtained by comparing the results for 9- and 12-MeV protons is not clear.

In order to check any possible dependence of W_D on the energy of the scattered particles or on the neutron excess, W_D was also searched while keeping V_S constant. Most of the calculations cited from here on were done on the CDC 3600 computer by use of the ABACUS-2 code with an attached search program due to Perey. Figure 5 shows W_D as a function of $(N-Z)/A$. These results were obtained with $V_S=49.8$ MeV, the average of the values from Fig. 4. Similar results are obtained if one uses different values of V_S for the scattering of 12-MeV protons (average of Ca^{42}, Ca^{44}, and Ca^{48}) and for the scattering of 9-MeV protons (average of Ca^{44} and Ca^{48}). The same characteristic result is obtained if one uses different values of V_S for each isotope (the average corresponding to 9 and 12 MeV in each case). Figure 5 shows a strong dependence of W_D on $(N-Z)/A$ and also on the energy of the protons. A similar dependence on $(N-Z)/A$ was also found by Perey.[1] Such an effect on W_D may perhaps be qualitatively expected from the isotopic-spin splitting of single-particle states.

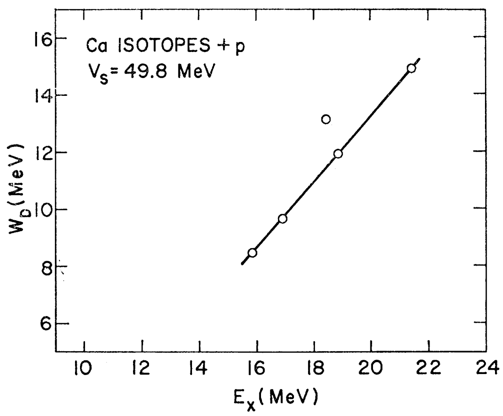

FIG. 6. Surface imaginary potentials W_D from Fig. 5 as a function of the excitation energy of the compound nucleus.

Possibly the dependences of W_D on an $(N-Z)/A$ term and on the energy of the bombarding proton can be explained together as a dependence on the excitation energy of the compound nucleus. It seems reasonable to expect that W_D will increase with the excitation energy because the number of modes of the nucleus that can be excited becomes larger. One can argue that the number of modes of excitation that are available for absorption depends more directly on the excitation energy of the compound nucleus. To check this possibility, the same values of W_D that were shown in Fig. 5 are plotted now in Fig. 6 as a function of the excitation energy E_x of the compound nucleus. A straight line can be drawn through all the points except the one corresponding to the scattering of 9-MeV protons from Ca^{48}. In Sec. B, these results will be compared with the results obtained by analyzing the elastic scattering of protons from nickel and copper isotopes.

Table II gives the results of calculations in which all the parameters (except the depth $V_{SO}=7.5$ MeV of the spin-orbit potential and the charge radius $R_c=1.25$

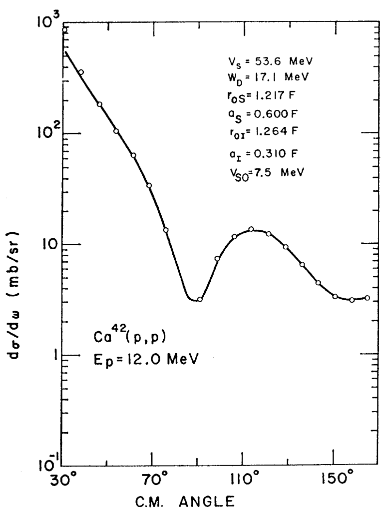

FIG. 7. Comparison of the measured cross sections with predictions of best-fit potentials for the scattering of 12-MeV protons from Ca^{42}.

TABLE II. "Best-fit parameters" of optical-model potentials for protons.[a]

Proton energy (MeV)	Isotope	V_S (MeV)	r_{0S} (F)	a_S (F)	W_D (MeV)	r_{0I} (F)	a_I (F)	σ_R (mb)	χ^2
9[b]	Ca^{42}	52.4	1.209	0.678	6.9	1.363	0.305	638	12
9	Ca^{44}	50.8	1.233	0.720	11.8	1.366	0.290	759	11
9	Ca^{48}	51.4	1.240	0.630	8.6	1.194	0.642	916	26
12	Ca^{42}	53.6	1.217	0.600	17.1	1.264	0.310	701	3.6
12	Ca^{44}	48.9	1.278	0.590	17.3	1.246	0.324	788	6.0
12	Ca^{48}	41.6	1.441	0.456	20.2	1.215	0.364	965	2.7

[a] The charge radius used in the calculations was $R_c=1.25\ A^{1/3}$ F, and the depth of the spin-orbit potential was $V_{SO}=7.5$ MeV.
[b] The contamination from compound-elastic scattering in this case was estimated by means of calculations of the Hauser-Feshbach type.

$A^{1/3}$ F) were allowed to vary. These calculations were done by G. R. Satchler using the IBM 7090 computer in Oak Ridge. Table II also gives the calculated values of the reaction cross section and the value of χ^2 as defined in Eq. (3). The compound elastic effects that exist in the scattering of 9-MeV protons from Ca^{42} were estimated by use of Hauser-Feshbach calculations. As an example, in Fig. 7 the scattering of 12-MeV protons from Ca^{42} is compared with the calculated curve. One can see that by varying all the parameters one obtains very good agreement with the experimental results, but one cannot attach much meaning to the variation in optical-model parameters from such a fit. A fit was also obtained to the data allowing only the real and imaginary potentials to vary. The results are shown in Table III. The trend in W_D is similar to the one found in the one-parameter fits but the trend in V_S seems to have disappeared.

B. Comparison with Elastic Scattering of Protons from Nickel and Copper Isotopes

At this stage of the analysis it was thought that it might be useful to compare the results for calcium isotopes, in particular the dependence of V_S and W_D on the energy and on the symmetry term, with the results obtained in the same way from analyzing the scattering of protons from nickel and copper isotopes. The data from Ref. 7 were used in these calculations, and the

TABLE III. Results of parameter search on V_S and W_D for protons scattering from calcium isotopes. The other parameters were fixed at the values $r_{0S}=0.65$ F, $r_{0I}=1.25$ F, $a_I=0.47$ F, $R_c=1.25\ A^{1/3}$ F, and $V_{SO}=7.5$ MeV.

Proton energy (MeV)	Isotope	V_S (MeV)	W_D (MeV)	σ_R (mb)	χ^2
9[a]	Ca^{42}	49.5	5.0	654	17
9	Ca^{44}	50.3	8.3	778	17
9	Ca^{48}	50.4	13.2	874	33
12	Ca^{42}	49.5	9.5	846	12
12	Ca^{44}	49.3	11.6	898	7.8
12	Ca^{48}	52.4	15.4	989	21

[a] The contamination from compound-elastic scattering in this case was estimated by means of calculations of the Hauser-Feshbach type.

TABLE IV. Results of parameter search on V_S and W_D in the Ni and Cu isotopes. The other parameters were held fixed at the values $r_{0S}=1.25$ F, $a_S=0.65$ F, $r_{0I}=1.25$ F, $a_I=0.47$ F, $R_c=1.25$ $A^{1/3}$ F, and $V_{SO}=7.5$ MeV.[a]

Proton energy (MeV)	Isotope	V_S (MeV)	W_D (MeV)	σ_R (mb)	χ^2
7	Cu^{63}	53.1	12.9	546	3.1
7	Cu^{65}	52.4	11.5	552	4.6
8	Cu^{63}	53.3	11.8	636	4.0
8	Cu^{65}	52.3	11.0	641	9.4
9	Ni^{58}	54.4	8.7	672	30
9	Ni^{60}	52.1	13.7	738	9.1
9	Cu^{63}	52.8	11.9	720	2.8
9	Cu^{65}	54.0	12.5	753	8.5
10	Ni^{58}	52.2	6.9	676	23
10	Ni^{60}	51.2	12.2	790	2.3
10	Cu^{63}	52.7	12.5	803	3.9
10	Cu^{65}	53.6	12.4	832	4.4
11	Ni^{58}	49.8	7.7	739	12
11	Ni^{60}	50.8	12.0	843	3.6
11	Cu^{63}	52.5	12.6	866	8.4
11	Cu^{65}	52.4	12.5	886	3.8
12	Ni^{58}	49.6	8.7	810	9.6
12	Ni^{60}	50.4	12.2	891	9.2
12	Ni^{62}	51.5	12.5	922	6.5
12	Ni^{64}	50.9	13.3	946	2.5
12	Cu^{63}	51.5	11.7	899	18
12	Cu^{65}	51.1	12.0	919	4.8

[a] The measured cross sections were taken from Ref. 7.

same geometrical and spin-orbit parameters were used as before. Initially, V_S and W_D were allowed to vary together in order to minimize χ^2. We then used the average value of one parameter and varied the other in order to see how the latter depends on the symmetry term or the bombarding energy.

The values of V_S for the scattering of 12-MeV protons from Ni and Cu isotopes were calculated (with $W_D=11.7$ MeV, the average of the six values at $E_p=12$ MeV in Table IV) and plotted (Fig. 8) as a function of $(N-Z)/A$. The circles of the plot correspond to the four nickel isotopes (Ni^{58}, Ni^{60}, Ni^{62}, and Ni^{64}), and the x's correspond to the two copper isotopes (Cu^{63} and Cu^{65}). The line giving a least-squares fit has a slope of 16 ± 7 MeV, which is smaller than the slope in the case of the calcium isotopes.

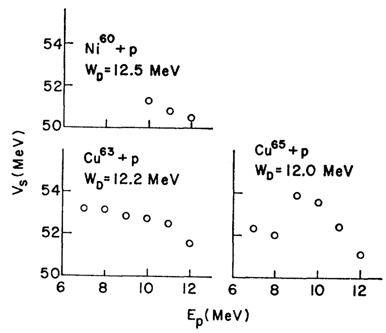

FIG. 9. Plot of the real well depths V_S as a function of the energy of the incident proton for the scattering of protons from Cu^{63}, Cu^{65}, and Ni^{60}.

Figure 9 shows the dependence of V_S on the proton energy E_p for Cu^{63} and Cu^{65} for $E_p=7$–12 MeV, and for Ni^{60} for $E_p=10$–12 MeV. The values of W_D used in each case are also given in the figure. The (p,n) threshold for Ni^{58} is about 9 MeV and for Ni^{60} is about 7 MeV. One can expect that compound-nucleus effects will contribute significantly to the elastic scattering in Ni^{58} up to energies of about 12 MeV and in Ni^{60} up to about 10 MeV. Therefore, the data on Ni^{60} below 10 MeV and all the data on Ni^{58} are omitted in Fig. 9. The real depth V_S tends to become smaller at higher energies, although in Cu^{65} the values of V_S are larger at 9 and 10 MeV than at 7 and 8 MeV. No clear dependence on $(N-Z)/A$ is seen when the values of V_S for Cu^{63} and Cu^{65} are compared at each energy; at 7, 8, and 12 MeV the values of V_S for Cu^{63} are larger than for Cu^{65} at the same energies, while at 9 and 10 MeV V_S is larger for Cu^{65} than for Cu^{63}.

Figure 10 shows the dependence of W_D on $(N-Z)/A$ for the scattering of 12-MeV protons from the nickel and copper isotopes, the slope of the line is 39 ± 9 MeV. The dependence here is similar to that obtained in the calcium isotopes. Figure 11 shows the same results of W_D plotted as function of the excitation energy of the compound nucleus. Figure 12 shows the dependence of

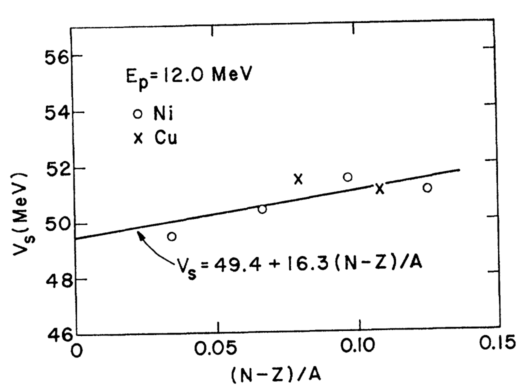

FIG. 8. Plot of the real well depths V_S for the scattering of 12-MeV protons from Ni and Cu isotopes as a function of the symmetry parameter $(N-Z)/A$.

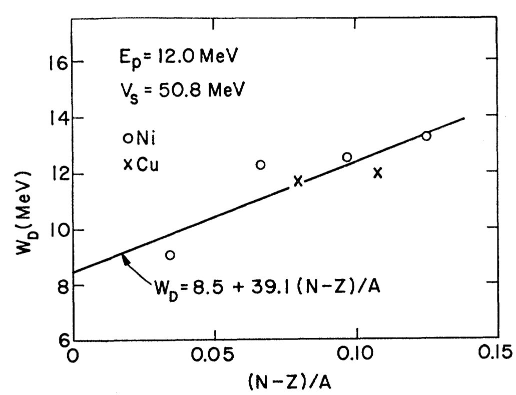

FIG. 10. Surface imaginary potentials W_D as a function of the symmetry parameter $(N-Z)/A$ for the scattering of 12-MeV protons from Ni and Cu isotopes.

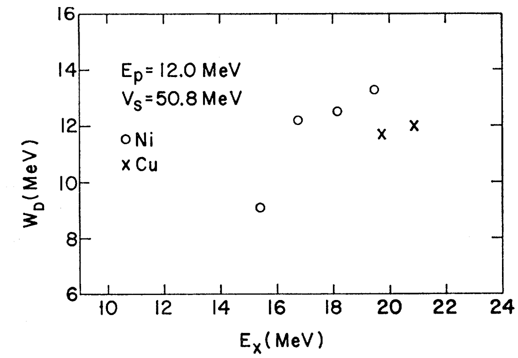

FIG. 11. Surface imaginary potentials W_D from Fig. 10 as a function of the excitation energy of the compound nucleus.

W_D on E_p for Cu^{63} and Cu^{65} at E_p=7–12 MeV, and for Ni^{60} at E_p=10–12 MeV. It is seen that the dependence of W_D on E_p or on the excitation energy of the compound nucleus is much less pronounced in the nickel and copper isotopes than in the calcium isotopes.

C. Elastic Scattering of Deuterons

The measured cross sections for elastic scattering of 9- and 12-MeV deuterons from Ca^{42}, Ca^{44}, and Ca^{48} are tabulated in Table V. Figure 13 shows the results (expressed as the ratio to the Rutherford cross section) plotted as a function of the scattering angle. In a detailed study of the scattering of deuterons by Ca^{40}, Bassel *et al.*[9] found several sets of optical-model parameters that fitted the data equally well. They reasoned that if the potential inside the nucleus has any physical significance, it would be difficult to understand how its depth could be much more than the sum of the optical potentials for the free neutron and proton. On the basis of this argument, they concluded that perhaps the most meaningful potential is the one they called the type-Z potential.

The starting point of our analysis was an average potential of this type (potential Z2 of Ref. 9, whose parameters are V_S=112 MeV, r_{0S}=1.0 F, a_S=0.9 F, W_D=17 MeV, r_{0I}=1.55 F, a_I=0.47 F, and R_c=1.3 $A^{1/3}$ F.) In the first phase of the search,[14] only V_S and W_D were allowed to vary from these values in order to minimize χ^2. The results of these calculations are shown by the solid lines in Fig. 14. One can see that while the fit to the scattering of 9-MeV deuterons from Ca^{42} and Ca^{48} is not bad, it is significantly worse in the case of Ca^{44}. One also can see that no reasonable fit to the scattering of 12-MeV deuterons from all the three isotopes can be obtained by starting with these parameters and varying only V_S and W_D.

TABLE V. Cross sections for elastic scattering of 9- and 12-MeV deuterons.

Laboratory angle (degrees)	Center-of-mass differential cross sections (mb/sr)[a] E_d=9 MeV			E_d=12 MeV		
	Ca^{42}	Ca^{44}	Ca^{48}	Ca^{42}	Ca^{44}	Ca^{48}
20.0	5504	5581				
25.0	2168	2123			1143	
30.0	824	855	1183		330	
35.0	347	372	464		167	
40.0	198.	205	249		140	
45.0	152	169	187		114	
50.0	120	117	140	59.4	74	72
55.0	75	83	104	35.0	42.5	43.4
60.0	52.3	48.5	65	21.5	24.3	25.8
65.0	31.9	29.0	37.4	12.3	15.5	15.2
70.0	18.6	16.5	21.8	8.2	10.5	10.0
75.0	11.7	11.5	14.2	5.4	6.5	6.4
80.0	9.6	9.1	10.7	3.76	4.7	4.8
85.0	8.7	9.0	10.6	3.29	4.4	4.2
90.0	8.7	8.6	10.3	4.0	4.7	4.8
95.0	8.5	9.1	10.3	4.5	5.3	5.3
100.0	8.3	8.6	9.8	4.6	4.7	4.5
105.0	8.0	7.2	8.9	4.0	3.8	3.8
110.0	6.3	5.4	7.8	2.87	2.34	2.68
115.0	5.4	4.5	6.0	1.80	1.49	1.87
120.0	4.4	3.3	4.8	1.25	0.96	1.25
125.0	3.2	2.68	3.20	0.86	0.78	
130.0	2.33	2.09	2.56		1.13	1.03
135.0	1.81	1.77	2.07	0.98	1.19	
140.0	1.69	1.52	2.01	1.17	1.32	1.23
145.0	1.99	1.73	2.35	1.19	1.33	
150.0	2.68	2.13	3.10	1.43	1.32	1.46
155.0	3.27	2.41	4.1	1.46	1.20	
160.0	4.1	2.93	5.0	1.60	1.02	1.54
165.0	4.6	3.43	5.6	1.41	0.83	
168.0	5.1	3.84		1.34	0.66	
169.0			6.0			1.46

[a] The errors in the cross sections are estimated to be ±5%.

TABLE VI. Results of parameter search on V_S and W_D for deuterons scattering from calcium isotopes. The other parameters were kept fixed at the values r_{0S}=1.0 F, a_S=0.9 F, r_{0I}=1.55 F, a_I=0.47 F, and R_c=1.3 $A^{1/3}$ F.

Deuteron energy (MeV)	Isotope	V_S (MeV)	W_D (MeV)	σ_R (mb)	χ^2
9	Ca^{42}	110.1	16.3	1138	7.1
9	Ca^{44}	108.9	16.1	1138	36
9	Ca^{48}	112.4	17.2	1138	8.4
12	Ca^{42}	106.6	16.4	1266	177
12	Ca^{44}	112.9	17.2	1271	212
12	Ca^{48}	109.6	17.1	1268	143

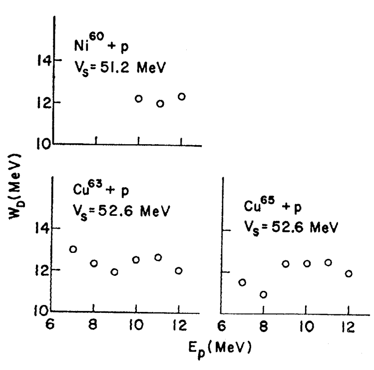

FIG. 12. Surface imaginary potentials W_D as a function of the energy of the incident proton for the scattering of protons from Cu^{63}, Cu^{65}, and Ni^{60}.

[14] These calculations and the searches for best-fit parameters for the Ca isotopes mentioned below were done by Dr. G. R. Satchler at Oak Ridge National Laboratory.

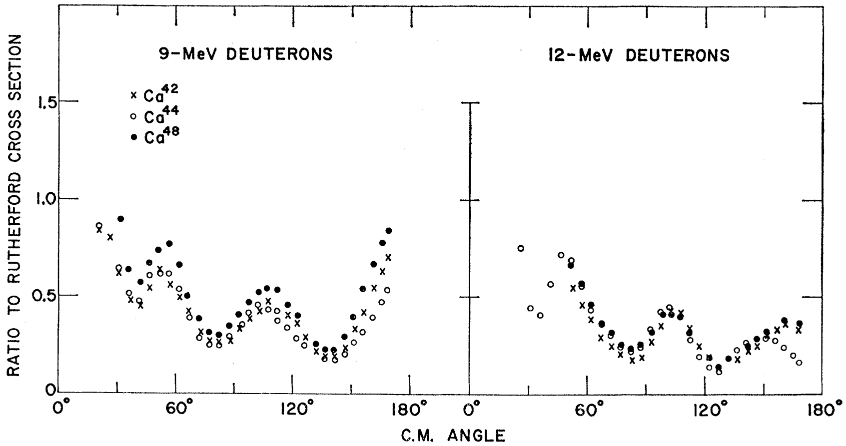

FIG. 13. The ratio of elastic-scattering cross sections to Rutherford cross sections as a function of angle for deuterons incident on Ca isotopes.

TABLE VII. "Best-fit parameters" of optical-model potentials for deuterons. The search started at the $Z2$ potential of Ref. 9 ($V_S=112$ MeV, $r_{0S}=1.0$ F, $a_S=0.9$ F, $W_D=17$ MeV, $r_{0I}=1.55$ F, and $a_I=0.47$ F).[a]

Deuteron energy (MeV)	Isotope	V_S (MeV)	r_{0S} (F)	a_S (F)	W_D (MeV)	r_{0I} (F)	a_I (F)	σ_R (mb)	χ^2
9	Ca^{42}	119.1	0.952	0.895	15.3	1.511	0.537	1169	1.7
9	Ca^{44}	170.7	0.739	0.909	13.7	1.411	0.637	1158	2.1
9	Ca^{48}	127.6	0.962	0.772	13.9	1.427	0.564	1036	4.4
12	Ca^{42}	114.6	0.995	0.833	16.2	1.410	0.548	1224	7.2
12	Ca^{44}	122.8	0.966	0.719	11.2	1.449	0.732	1385	5.2
12	Ca^{48}	117.0	1.006	0.749	15.6	1.356	0.592	1173	7.8

[a] The charge radius was held fixed at the value $R_c=1.3\ A^{1/3}$ F.

In the next step, all the parameters (except the charge radius) were allowed to vary. The results of these calculations are shown by the dashed lines in Fig. 14. One can see that the fit is reasonably good. Table VI shows the values of V_S and W_D that were obtained with fixed geometrical parameters, while Table VII gives the "best-fit parameters." Especially for the case of 12-MeV deuterons, it is evident that χ^2 becomes much smaller when all the parameters are adjusted. However, no particular trend in the behavior of the parameters can be seen. The 12-MeV data were then re-analyzed by using average sets of parameters obtained from Table VII and including the "best-fit parameter" for Ca^{40} ($V_S=112.8$ MeV, $r_{0S}=1.021$ F, $a_S=0.846$ F, $W_D=19.8$ MeV, $r_{0I}=1.471$ F, and $a_I=0.444$ F).[9] The results of these calculations are shown by the dot-dashed lines in Fig. 14 and the values of χ^2 obtained are given in Table VIII. The fit here is better than the one obtained by using $Z2$ geometrical parameters and varying V_S and W_D (see Table VI and the solid lines in Fig. 14). Further calculations used these average parameters and allowed one parameter to vary at a time.

The results of the one-parameter search (with the value of χ^2 in each case) are given in Table IX. Table X shows the results obtained by using the average geometrical parameters and varying V_S and W_D together. It is seen that V_S becomes smaller as the number of neutrons in the nucleus becomes larger. The values of r_{0S} in Table IX also become smaller at higher A—possibly because of invariances of the type $V_SR_S{}^n=$ constant. Table IX also shows that the values of r_{0I} become smaller and the values of a_S and a_I become larger as the number of neutrons in the isotope becomes larger.

The data from Ref. 7 on the scattering of 12-MeV deuterons by nickel and copper isotopes were similarly analyzed. Table XI gives the results of a six-parameter search starting with the potentials (set 3 of Ref. 2) that correspond to the scattering of 11.8-MeV deuterons from copper. It is seen that the parameters corresponding to Ni^{58}, Ni^{64}, and Cu^{65} are similar, and the same is true of the ones for Ni^{60} and Ni^{62}.

An average potential was obtained by averaging the "best-fit parameters" (Table XI) corresponding to Ni^{58}, Ni^{64}, and Cu^{65}. These parameters give a better fit

TABLE VIII. Values of χ^2 and σ_R for the scattering of 12-MeV deuterons from calcium isotopes. The parameters used in these calculations were the average values $V_S=116.8$ MeV, $r_{0S}=0.997$ F, $a_S=0.787$ F, $W_D=15.7$ MeV, $r_{0I}=1.422$ F, $a_I=0.579$ F, and $R_c=1.3\ A^{1/3}$ F.

Isotope	Ca^{40}	Ca^{42}	Ca^{44}	Ca^{48}
χ^2	34	42	35	129
σ_R(mb)	1180	1204	1228	1281

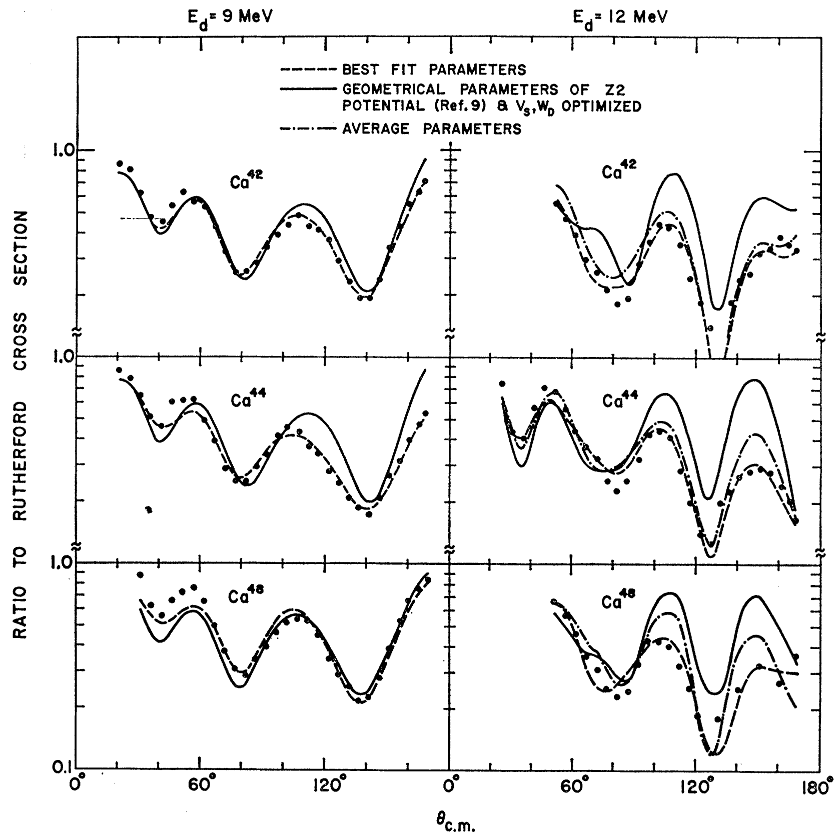

FIG. 14. Comparison of the measured σ/σ_R with predictions of the optical model using different sets of parameters as explained in the text.

TABLE IX. Results of one-parameter searches corresponding to the scattering of 12-MeV deuterons from calcium isotopes. The searches started at the average potentials $V_S=116.8$ MeV, $r_{0S}=0.997$ F, $a_S=0.787$ F, $W_D=15.7$ MeV, $r_{0I}=1.422$ F, and $a_I=0.579$ F.[a]

Parameter varied	Ca^{40} Value of parameter	χ^2	Ca^{42} Value of parameter	χ^2	Ca^{44} Value of parameter	χ^2	Ca^{48} Value of parameter	χ^2
V_S (MeV)	117.5	34	113.7	30	113.0	20	103.7	46
r_{0S} (F)	0.999	34	0.986	36	0.982	26	0.990	79
s_S (F)	0.776	34	0.817	32	0.800	33	0.834	115
W_D (MeV)	15.2	34	13.6	22	13.9	28	16.5	128
r_{0I} (F)	1.435	33	1.372	13.4	1.368	12	1.311	95
a_I (F)	0.532	28	0.546	33	0.565	35	0.646	118

[a] The charge radius was held fixed at the value $R_c=1.3\,A^{1/3}$ F.

TABLE X. Results of a parameter search on V_S and W_D corresponding to the scattering of 12-MeV deuterons from calcium isotopes. The other parameters were held fixed at the values $r_{0S}=0.997$ F, $a_S=0.787$ F, $r_{0I}=1.422$ F, $a_I=0.579$ F, and $R_c=1.3\,A^{1/3}$ F.

Isotope	V_S (MeV)	W_D (MeV)	σ_R (mb)	χ^2
Ca^{40}	118.1	14.9	1176	33
Ca^{42}	115.3	13.9	1191	18.6
Ca^{44}	113.9	14.6	1218	17.0
Ca^{48}	104.8	13.3	1244	31

TABLE XI. "Best-fit parameters" of optical-model potentials for 12-MeV deuterons scattered from Ni and Cu isotopes. The search started at "set 3" potentials of Ref. 2, $V_S=90.7$ MeV, MeV, $r_{0S}=1.172$ F, $a_S=0.822$ F, $W_D=18.34$ MeV, $r_{0I}=1.410$ F, and $a_I=0.661$ F.[a]

Isotope	V_S (MeV)	r_{0S} (F)	a_S (F)	W_D (MeV)	r_{0I} (F)	a_I (F)	σ_R (mb)	χ^2
Ni^{58}	121.7	0.928	1.004	19.1	1.438	0.584	1391	3.6
Ni^{60}	82.9	1.217	0.855	25.0	1.427	0.546	1427	3.2
Ni^{62}	87.9	1.162	0.931	21.0	1.480	0.582	1545	5.1
Ni^{64}	126.0	0.910	1.084	20.4	1.482	0.575	1553	5.7
Cu^{63}	100.0	1.077	0.969	19.2	1.483	0.602	1523	3.9
Cu^{65}	118.5	0.954	1.049	19.8	1.480	0.588	1533	5.8

[a] The charge radius was held fixed at the value $R_c=1.3\,A^{1/3}$ F.

TABLE XII. Values of χ^2 and σ_R obtained for the scattering of 12-MeV deuterons from Ni and Cu isotopes by using average parameters having the values $V_S=122.1$ MeV, $r_{0S}=0.931$ F, $a_S=1.046$ F, $W_D=19.76$ MeV, $r_{0I}=1.467$ F, $a_I=0.582$ F, and $R_c=1.3\ A^{1/3}$ F.

Isotope	Ni^{58}	Ni^{60}	Ni^{62}	Ni^{64}	Cu^{63}	Cu^{65}
σ_R(mb)	1458	1479	1499	1520	1478	1498
χ^2	7.4	9.2	16.7	7.5	11.9	8.1

to the data than do the set 3 potentials of Perey and Perey; the values of χ^2 and σ_R obtained with this potential are summarized in Table XII. The values of V_S and W_D in Table XIII were obtained in a search starting with the average potential mentioned above and keeping the geometrical parameters fixed.

Table XIV summarizes the results of searching on one parameter at a time, starting from the same average parameters. In both tables it is seen that V_S is increasing with the number of neutrons in the Ni isotopes while it is slightly decreasing in the Cu isotopes. The decrease in V_S that was found in the Ca isotopes does not appear here. The radius r_{0S} seems to follow the variation of V_S, possibly because of invariances of the type $V_S R_S^n$ = constant; but r_{0I} seems to be almost constant in comparison with the large variation in the Ca isotopes. The diffuseness a_S is increasing in the Ni and Cu isotopes in a way that is somewhat similar to its behavior in the Ca isotopes. In the Ni isotopes a_I seems to increase about as it does in the Ca isotopes, but it appears to decrease in the Cu isotopes.

TABLE XIII. Results of parameter search on V_S and W_D corresponding to the scattering of 12-MeV deuterons from Cu and Ni isotopes. The other parameters were held fixed at the values $r_{0S}=0.931$ F, $a_S=1.046$ F, $r_{0I}=1.467$ F, $a_I=0.582$ F, and $R_c=1.3$ $A^{1/3}$ F.

Isotope	V_S (MeV)	W_D (MeV)	σ_R (mb)	χ^2
Ni^{58}	118.4	20.5	1453	4.5
Ni^{60}	121.6	20.4	1480	8.7
Ni^{62}	126.3	20.5	1512	11.6
Ni^{64}	124.0	19.5	1525	6.2
Cu^{63}	125.9	20.7	1490	7.0
Cu^{65}	124.6	19.8	1505	6.0

TABLE XIV. Results of a one-parameter search corresponding to the scattering of 12-MeV deuterons from Ni and Cu isotopes. The searches started at average potentials $V_S=122.1$ MeV, $r_{0S}=0.931$ F, $s_S=1.046$ F, $W_D=19.76$ MeV, $r_{0I}=1.467$ F, and $a_I=0.582$ F.[a]

Parameter varied	Ni^{58} Value of parameter	χ^2	Ni^{60} Value of parameter	χ^2	Ni^{62} Value of parameter	χ^2	Ni^{64} Value of parameter	χ^2	Cu^{63} Value of parameter	χ^2	Cu^{65} Value of parameter	χ^2
V_S (MeV)	118.2	4.9	120.0	8.9	126.5	12.3	124.1	6.3	126.1	8.0	124.6	6.0
r_{0S} (F)	0.914	4.1	0.931	9.2	0.947	12.7	0.939	6.1	0.944	9.3	0.941	6.1
a_S (F)	1.032	5.4	1.047	9.2	1.055	15.9	1.057	6.3	1.050	11.8	1.055	7.3
W_D (MeV)	20.9	6.5	20.4	8.7	20.6	15.5	19.5	7.5	20.8	10.5	19.9	8.1
r_{0I} (F)	1.478	6.4	1.463	9.1	1.465	16.6	1.459	7.0	1.472	11.8	1.464	8.0
a_I (F)	0.577	7.3	0.613	6.1	0.627	7.0	0.587	7.4	0.623	4.9	0.600	6.7

[a] The charge radius was held fixed at the value $R_c=1.3\ A^{1/3}$ F.

CONCLUSION

The $(N-Z)/A$ dependence of the depth V_S of the real part of the proton potential was found for the calcium isotopes and also for the nickel and copper isotopes, but the dependence of V_S on the proton energy is less definite. Both in the calcium isotopes and in the isotopes of nickel and copper, the depth W_D of the imaginary part of the proton potential was found to depend strongly on $(N-Z)/A$, which may also be described by a dependence of W on excitation energy. The optical-model parameters corresponding to the scattering of deuterons from calcium isotopes show some systematic trends, but not all of them persist for the nickel and copper isotopes.

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