This difficulty does not exist however in most of the calculations which have been made with a pairing force. Indeed in these calculations the self-energy terms have, most of the time, been neglected. Therefore (51) and (52) become equivalent except for a shift in the chemical potential.

To end this section we wish to show a more narrow connection between seniority and quasiparticle number by proving that the physical states of v quasiparticles have seniority v in the degenerate case.

We have seen before for two particles that all the states have the same quasispin, or seniority two, except the ground state, in complete analogy with the case of a single j. If we consider now the BCS state and the states with two quasiparticles we find two states with the same energy as the ground state: the BCS state and a combination of pair of quasiparticles. Both have seniority zero but the combination of two quasiparticles coupled to 0 is clearly unphysical,[21] which proves our statement for the zero- and two-quasiparticle states.

[21] It has the same energy as the ground state and its components on the two quasiparticle states are of the form $(2j_i+1)^{1/2}$ which indicate that this state is nothing but $(N-\langle \mathrm{BCS}|N|\mathrm{BCS}\rangle)|\mathrm{BCS}\rangle$.

This argument can be extended without a difficulty to the case of four or more quasiparticles.

VI. CONCLUSION

We have been able to show in this paper that the surface delta interaction provides a very simple scheme which generalizes for degenerate mixed orbitals the seniority scheme well known for a $(j)^N$ configuration. Moreover, we have seen that the BCS method is very appropriate here since the number of quasiparticles commutes with the Hamiltonian. All these results are of course no longer exactly valid when the single-particle energies are not degenerate.

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Surface Delta Interaction and Single-Closed-Shell Nuclei*

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The surface delta interaction has been applied in a calculation of the energy levels of isotopes of Pb, Sn and Ni, Po^{210}, and nuclei belonging to the 82-neutron shell. The Bogoliubov-Valatin method combined with the Tamm-Dancoff approximation has been utilized. In certain cases, exact shell-model calculations have also been performed. The comparison between the results obtained in these two ways shows that the approximation method works quite well for the two-particle case. For more than two particles outside closed shells, the approximation works relatively well only for the 2_1+ and 4_1+ states. The agreement with experiment is fairly good in general. For Pb^{206} the results are even slightly better than those of True and Ford. For Pb^{204} and Pb^{202} it was found necessary to reduce the strength of the interaction about 10%. In Ni and in the 82-neutron nuclei the results are fairly good, but in the isotopes of Sn and in Pb^{210} the 2+ and 4+ states lie somewhat too high.

I. INTRODUCTION

IN two recent studies[1,2] a new interaction has been proposed to be used in the nuclear-shell theory which has been called a surface delta interaction (SDI). The theoretical basis of this interaction is only partially understood at present, but it has some interesting properties when applied to spherical nuclei. In particular it was shown in Ref. 1 that the SDI provides strong collective effects. In a $(s,d)^2$ configuration for example, the 0+ is lowered much more than in the case of an ordinary delta interaction. Moreover, spectra approximately rotational in nature are expected for $T=0$ states involving several nearly degenerate orbits. In Ref. 2 it has been shown by using the quasispin techniques that the generalized seniority and the number of quasiparticles (defined by the Bogoliubov-Valatin transformation) are good quantum numbers when the single-

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particle states are degenerate. Another interesting property of this interaction is its extreme simplicity and the absence of any radial integrals. Indeed the two-body matrix elements are simple products of two Clebsch-Gordan coefficients. The strength of this interaction is actually the only parameter present. All these features have encouraged us to apply this interaction to the calculation of the spectra of spherical nuclei containing identical particles in the active orbitals, i.e., single-closed-shell nuclei, and primarily to the spectra of even nuclei. In principle, we should consider those nuclei in the (s,d) shell. However, since it has been found necessary to consider deformed states as well as spherical ones[3] to explain the spectra of O^{16} and O^{18}, these nuclei may not be the best examples. Indeed, we have restricted our calculations to spectra of nuclei for which there is actually no evidence that the interplay between deformed states and spherical states has to be taken into account, such as the nuclides of $Z=28$, 50, 82 or of $N=82$ and 126. Several calculations of the spectra of these nuclei have been done in the past years by using either an exact shell-model calculation[4–7] or an approximation which consists essentially in a Bogoliubov-Valatin canonical transformation followed by a diagonalization in a finite subspace of quasiparticle states.[8–12] Several types of interactions have also been used: the pairing-plus-quadrupole interaction,[8,9] finite-range interactions containing spin-exchange[5–7,10–12] terms or even tensor interactions,[6] or more realistic[12] interactions like the Tabakin force.[13] A large part of these calculations have therefore dealt with forces in which radial effects, range effects, and exchange effects have been taken into account explicitly. It is expected, of course, that the surface delta interaction gives only a first approximation and cannot account for all the effects which are given by such complicated forces. However, one of the main aims of our calculation has been to see what features are common to these "realistic" interactions and the surface delta interaction and also whether these different interactions result in drastic changes in the level spectra. It was also our aim to test, using that simple interaction, the validity of the Tamm-Dancoff method in various cases such as the spectrum of nuclei with two particles or two holes outside closed shells. In fact we will also give results concerning the comparison of the Tamm-Dancoff method with exact calculations of nuclei with four and six valence particles. Since the Tamm-Dancoff method gives the exact results for a degenerate system[2] the study of the nondegenerate system is of considerable interest. In the first part of this paper we will give some theoretical properties of the two-body matrix element of a surface delta interaction and we will give the main formulas which are used in a Tamm-Dancoff (TD) calculation. In the second part we will discuss the results of the numerical calculation: the comparison between Tamm-Dancoff approximation solutions and exact solutions, the comparison of our results with experiment and with those of the previous calculations. We will discuss also the convergence of the results when the number of single-particle states included in the calculation is increased.

II. THEORY

1. Two-Particle Matrix Elements of a Surface Delta Interaction

In this section we want to briefly review some of the properties of the matrix elements of a surface delta interaction. From the practical point of view of calculating matrix elements, the surface delta interaction can be considered as a δ interaction for which one makes the further assumption that all the radial integrals are equal.[1] The matrix elements of a conventional δ interaction have been given many times in the past [see for example Ref. (14)]. However, they have been usually given only for the diagonal case. Since we need the particle-particle and the particle-hole matrix elements in the quasiparticle formalism, we will give expressions for both of these in the general case of mixed orbits. For their derivation the reader is referred to the literature.[14] We assume that the interaction is attractive and of strength G. It can be written as a delta function in the *angular* coordinates of the interacting particles[15]

$$V_{ik}=-4\pi G\delta(\Omega_{ik}). \tag{1}$$

Since the SDI is spin-independent, its matrix elements have a particularly simple form in the L-S representation. For a nucleon pair scattering from configuration $(l_al_b)_L$ to $(l_cl_d)_L$, the particle-particle matrix element is

$$W_L(l_al_b,l_cl_d)=-Gh_L(l_al_b)h_L(l_cl_d), \tag{2a}$$

where

$$h_L(l_al_b)=(-1)^{l_a+L}\left[\frac{(2l_a+1)(2l_b+1)}{(2L+1)}\right]^{1/2}\times\langle l_al_b00|L0\rangle. \tag{2b}$$

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[15] Provided that all the relevant radial wave functions and integrals are arbitrarily set equal.

Of course, the symmetry properties of the Clebsch-Gordan coefficients require that l_a+l_b and l_c+l_d both have the same parity as L.

Equation (2) was already given, in a different notation in Ref. 1. The j-j coupled matrix elements can be readily obtained from the above expressions using LS to jj transformation coefficients. The result is quite similar to (2) but slightly more complicated

$$W_J(j_aj_b,j_cj_d)=-\tfrac{1}{2}Gh_J(j_aj_b)h_J(j_cj_d) \times[1+(-1)^{j_a-j_c+l_a-l_c}\phi_J(j_aj_b)\phi_J(j_cj_d)], \quad (3a)$$

where

$$h_J(j_aj_b)=(-)^{j_a+J-\frac{1}{2}}[(2j_a+1)(2j_b+1)/(2J+1)]^{1/2} \times\langle j_aj_b\tfrac{1}{2}-\tfrac{1}{2}|J0\rangle \quad (3b)$$

and

$$\phi_J(j_aj_b)=[j_a+\tfrac{1}{2}+(-1)^{j_a+j_b+J}(j_b+\tfrac{1}{2})]/[J(J+1)]^{1/2} \quad \text{if} \quad J\neq 0,$$
$$=0 \quad \text{if} \quad J=0. \quad (3c)$$

Incidentally, Eqs. (3) hold independently of the values of l_a etc., provided only that l_a+l_b and l_c+l_d have the same parity (otherwise W vanishes). It is convenient to rewrite Eq. (3a) in the form

$$W_J(j_aj_b,j_cj_d)=G_J{}^{(0)}+G_J{}^{(1)}, \quad (4)$$

where $G^{(0)}$ is the product of G and the two factors h_J and $G^{(1)}$ is the term involving the ϕ's.

So far we have only considered the direct matrix element, i.e., used nonantisymmetrized wave functions. For two identical particles, the properly antisymmetrized wave function is

$$(j_aj_b)_J{}^A=[2(1+\delta_{ab})]^{-1/2}[(j_aj_b)_J+(-1)^\partial(j_bj_a)_J], \quad (5a)$$

where

$$(-1)^\partial=(-1)^{l_a+l_b+j_a-j_b+J} \quad (5b)$$

and $\delta_{ab}=1$ if the particles are in the same orbit $l_a=l_b$, $j_a=j_b$, and 0 otherwise. In the former case, we have

$$(j_a{}^2)_J{}^A=(j_a{}^2)_J \quad \text{if} \quad J=\text{even},$$
$$=0 \quad \text{if} \quad J=\text{odd}. \quad (6)$$

The phase $(-1)^{l_a+l_b}$ in the two-particle wave functions is a consequence of our definition of the two-particle states.

The antisymmetrized matrix element is a linear combination of direct and exchange terms:

$$W_J{}^A(j_aj_b,j_cj_d)=[(1+\delta_{ab})(1+\delta_{cd})]^{-1/2} \times[W_J(j_aj_b,j_cj_d)+(-1)^\partial W_J(j_aj_b,j_dj_c)]. \quad (7)$$

Now since

$$h_J(j_bj_a)=(-1)^{j_a-j_b}h_J(j_aj_b) \quad (8a)$$

and

$$\phi_J(j_bj_a)=(-1)^{j_a+j_b+J}\phi_J(j_aj_b), \quad (8b)$$

we find for the exchange matrix elements

$$W_J(j_aj_b,j_dj_c)=(-1)^{\partial+l_a+l_b+J}G_J{}^{(0)}+(-1)^{\partial+1}G_J{}^{(1)}, \quad (9)$$

and thus

$$W_J{}^A(j_aj_b,j_cj_d) =[(1+\delta_{ab})(1+\delta_{cd})]^{-1/2}[1+(-1)^{l_a+l_b+J}]G_J{}^{(0)}$$
$$=-[(1+\delta_{ab})(1+\delta_{cd})]^2h_J(j_aj_b)h_J(j_cj_d)G \quad (10)$$

if l_a+l_b+J is even and zero otherwise.

In order to use the BCS method, we need also the particle-hole matrix elements of the interaction. We use a notation close to that in previous work[16] by one of us (R. A.). The direct particle-hole matrix element is written here as $H_J(j_a\tilde{j}_b,\tilde{j}_cj_d)$. As before, j_a and j_b (as well as j_c and j_d) are coupled to a resultant J. However, as is indicated by the tilde, j_b and j_c are assumed to be *hole* states, so that we actually have a pair scattering between orbits j_aj_c and j_bj_d. This matrix element is, in fact, just the multipole term of rank J in a Slater expansion. For an SDI, we have

$$H_J(j_a\tilde{j}_b,\tilde{j}_cj_d)=2G_J{}^{(0)} \quad (11a)$$

and

$$H_J(j_a\tilde{j}_b,\tilde{j}_dj_c)=(-1)^{\partial+l_a+l_b+J}2G_J{}^{(0)}. \quad (11b)$$

There is also an exchange particle-hole term which we write as $K_J(j_a\tilde{j}_b,j_c\tilde{j}_d)$. Again, j_a and j_b (and also j_c and j_d) are coupled to a total J. This time j_b and j_d are the hole states. For the SDI, and indeed any spin-independent interaction, it has been shown[16] that

$$K_J(j_a\tilde{j}_b,j_c\tilde{j}_d)=W_J(j_aj_b,j_cj_d). \quad (12)$$

As we have seen, the antisymmetric particle-particle matrix element is equal to the direct particle-hole term and indeed both are separable. This property simplifies greatly the two-particle calculations as shown in Refs. 1 and 2. In general, there is no very simple relation between the nonantisymmetrized particle-particle and particle-hole matrix elements, except for special configurations for which $G_J{}^{(1)}$ vanishes, e.g., if $j_a=j_c$ and J is even. For this case $H_J=2G^{(0)}$, i.e., the direct particle-hole matrix element is twice the nonantisymmetrized particle-particle matrix element, as was remarked by Belyaev.[17]

It should be pointed out that this behavior is quite the opposite of what is found for a long-range interaction

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such as the quadrupole force, for which H_2 is considered but K_2 and G_2 are generally neglected. When $J=0$, the matrix elements G_0 are equivalent to those of a pairing force since, in this case

$$h_0(j_aj_b)=\delta_{ab}(2j_a+1)^{1/2}, \tag{13}$$

$$W_0(j_aj_a,j_cj_c)=(-G/2)[(2j_a+1)(2j_c+1)]^{1/2} \tag{14}$$

$$=H_0(j_a\tilde{j}_a,\tilde{j}_cj_c)-K_0(j_a\tilde{j}_a,j_c\tilde{j}_c). \tag{15}$$

2. Two-Quasiparticle Matrix Elements of a Surface Delta Interaction

In this paper we want to use the surface delta interaction for the calculation of the spectra of single-closed-shell nuclei. For nuclei having two particles outside of (or missing from) closed shells an exact shell-model calculation can be carried out, without much difficulty. In that case the interaction matrix is constructed by just adding to the matrix (10) the single-particle energies. The eigenvalue problem can be solved by a numerical diagonalization or by a graphical procedure since it is a Cooper pair-type problem for each value of the total angular momentum J.

For nuclei having more than two particles outside closed shells, the surface delta interaction does not represent a very big simplification if one still wants to treat that problem exactly since the main problem is the construction of the four-particle energy matrix in terms of the two-body matrix elements. However, there is still a very simple numerical treatment applying the Bogoliubov-Valatin canonical transformation to construct a set of quasiparticle states and diagonalizing the interaction in the two-quasiparticle basis (Tamm-Dancoff-approximation states, as in Refs. 10–12).

First of all the equations which determine the u's and v's, the coefficients of the Bogoliubov-Valatin canonical transformation, are the same as those for a pairing force, except that the self-energy terms, which are generally neglected, have here only the effect of shifting the chemical potential. In other words these terms can be ignored exactly if one is not interested in quantities like the absolute ground-state energy or the separation energy. Indeed, the self-energy terms which have to be added to the single-particle energies have the form

$$\mu_a=(-G/2)\sum_{a,c}[H_0(j_a\tilde{j}_a,j_c\tilde{j}_c)-K_0(j_a\tilde{j}_a,j_c\tilde{j}_c)]\times v_c^2[(2j_c+1)/(2j_a+1)]^{1/2}. \tag{16}$$

By using (14) and (15) and the auxiliary condition fixing the average value of the number of particles in the system,

$$\sum_c v_c^2(2j_c+1)=N, \tag{17}$$

we obtain

$$\mu_a=(-G/2)N \tag{18}$$

independent of the state a. This self-energy term leads to a *total* energy shift of

$$-\tfrac{1}{2}GN\Omega \tag{18a}$$

[$\Omega=\sum_i(j_i+\frac{1}{2})$ is the total pair degeneracy] or $-\frac{1}{2}G\Omega$ per particle. This result was also derived in Ref. 2 by a different method. Since μ_a enters in the combination $\epsilon_a+\mu_a-\lambda$, where ϵ_a is a single-particle energy and λ is the chemical potential, we see that introducing μ has no effect on the u's, the v's, and the quasiparticle energies, since μ can be absorbed in λ by the replacement $\lambda\rightarrow\lambda-\mu$. This is a very reasonable result since even with a finite-range interaction the self-energy terms vary smoothly with the number of particles.[10–12]

The quasiparticle representation having been defined, we can calculate the matrix element of the total interaction between two-quasiparticle states. The general expression for the interaction matrix between two quasiparticles in states ab coupled to J, interacting and going to cd, can be written as follows:

$$\begin{aligned}P_J(j_aj_b,j_cj_d)&=\langle(j_aj_b)_J|H-\lambda N|(j_cj_d)_J\rangle-\langle\mathrm{BCS}|H-\lambda N|\mathrm{BCS}\rangle\\&=(E_a+E_b)\delta_{ac}\delta_{bd}+[(1+\delta_{ab})(1+\delta_{cd})]^{-1/2}\\&\quad\times\{(u_au_bu_cu_d+v_av_bv_cv_d)[W_J(j_aj_b,j_cj_d)+(-1)^\vartheta W_J(j_aj_b,j_dj_c)]\\&\quad+(u_av_bu_cv_d+v_au_bv_cu_d)[H_J(j_a\tilde{j}_b,\tilde{j}_cj_d)-K_J(j_a\tilde{j}_b,j_c\tilde{j}_d)]\\&\quad+(-1)^\vartheta(u_av_bv_cu_d+v_au_bu_cv_d)[H_J(j_a\tilde{j}_b,\tilde{j}_dj_c)-K_J(j_a\tilde{j}_b,j_a\tilde{j}_c)]\}.\end{aligned} \tag{19}$$

This expression simplifies greatly for an SDI. Using Eqs. (4), (9), (11), and (12), we can rewrite the quantity in brackets as follows:

$$\begin{aligned}\{\ \}&=(u_au_c+v_av_c)(u_bu_d+v_bv_d)G_J^{(0)}\\&\quad+(u_bu_c+v_bv_c)(u_au_d+v_av_d)(-1)^{l_a+l_b+J}G_J^{(0)}\\&\quad-(u_av_b-v_au_b)(u_cv_d-v_cu_d)G_J^{(1)}.\end{aligned} \tag{20}$$

In the degenerate case, all u's and thus also all v's are equal. The coefficients of the $G_J^{(0)}$ terms are $u^2+v^2=1$ while the coefficient of $G_J^{(1)}$ vanishes. We obtain the very simple result:

$$P_J(j_aj_bj_cj_d)=(E_a+E_b)\delta_{ac}\delta_{bd}-Gh_J(j_aj_b)h_J(j_cj_d)\times[(1+\delta_{ab})(1+\delta_{cd})]^{-1/2}, \tag{21}$$

which is just the matrix element for two particles increased by the quasiparticle energies (which should be

equal if the u's and v's are). This is an independent check of the result mentioned in Ref. 2 that the two-quasiparticle energy matrix is the same as the two-particle matrix, which coincides with the seniority-two matrix when all the single-particle energies are degenerate. For $J=0$, it is thus found that the matrix (21) has an eigenvalue equal to the energy of the BCS state. The components of this state in the two-quasiparticle basis are simply $h_0(j_aj_a)=2j_a+1$. Clearly, this is the spurious state associated with the nonconservation of the number of particles. It turns out here that this state, which can be written as

$$(N_{\text{op}}-\langle N\rangle)|\text{BCS}\rangle, \tag{22}$$

is an exact eigenstate of $H-\lambda N$ since $|\text{BCS}\rangle$ is, itself, an eigenstate.

Of course, since the number of quasiparticles commutes with $H-\lambda N$, it is readily verified that the random-phase approximation (RPA) is exactly equivalent to the Tamm-Dancoff approach in the degenerate limit. This may be seen if we write the matrix of the RPA in the usual form:

$$\begin{pmatrix} P & Q \\ -Q & -P \end{pmatrix}. \tag{23}$$

The expression for Q is similar to that for P given above except for the absence of single-quasiparticle energies. For an SDI, we obtain

$$\begin{aligned}[(1+\delta_{ab})(1+\delta_{cd})]^{1/2}&Q_J(j_aj_bj_cj_d)\\ &=(u_av_c-v_au_c)(u_bv_d-v_bu_d)G_J{}^{(0)}\\ &\quad+(u_bv_c-v_bu_c)(u_av_d-v_au_d)(-1)^{l_a+l_b+J}G_J{}^{(0)}\\ &\quad-(u_av_b-v_au_b)(u_cv_d-v_cu_d)G_J{}^{(1)},\end{aligned} \tag{24}$$

which vanishes in the degenerate limit.

III. NUMERICAL CALCULATIONS

1. Description of the Calculations

The surface delta interaction has been used to calculate the spectrum of single-closed-shell nuclei. We have made the usual assumption of taking into account only the interaction between the valence particles, protons or neutrons, filling the incomplete shell. Two kinds of calculations have been made.

(A) Exact shell-model calculations have been carried out for nuclei having two particles or two holes outside closed shells (Ni^{58}, Te^{134}, Pb^{206}, Pb^{210}, Po^{210}) and, in two other cases, for nuclei having four and six neutrons (Ni^{60-62}).

(B) The Bogoliubov-Valatin canonical transformation followed by a diagonalization in the subspace of two quasiparticles has been done for the following even nuclei: $Ni^{58-64}(Z=28)$, $Sn^{112-124}(Z=50)$, $Pb^{200-206}(Z=82)$, Xe^{136}, Ba^{138}, Ce^{140}, $Nd^{142}(N=82)$ and also for Pb^{210} and Po^{210}. Therefore, part (A) of this calculation can be used as a test of the validity of the approximations used in (B) for the same nuclei and the result of this comparison can be extrapolated to draw some conclusion about nuclei for which an exact diagonalization has not yet been done, like the Sn isotopes for example. In the calculation made in (B) some pairing properties, viz. even-odd mass differences and the quasiparticle states of odd nuclei, have been calculated on the basis of the pure quasiparticle model. As was pointed out in Sec. II, the u's and v's and the quasiparticle energies, and therefore the above pairing properties, are the same as those calculated with a pairing interaction. However, our results differ from those of Kisslinger and Sorensen (KS)[8] in two respects:

(1) In their first calculation KS did not use a pure pairing interaction but added a quadrupole force which, treated in the lowest order, modifies the one-quasiparticle energies. This effect is completely absent in our work since the pairing part of a SDI is exactly a pairing force. Nevertheless we think that this introduces only minor differences with respect to the first KS calculation.

In their second paper KS used for the odd nuclei a much more involved treatment, since they coupled the one-quasiparticle states to the quadrupole "phonon" of the even core. This method is outside the scope of our work. A coupling between the one-quasiparticle states and the three-quasiparticle states does exist.[12] We believed, however, that it is weaker for single-closed-shell nuclei, in particular for the SDI.

(2) For all the nuclei we have used parameters slightly different from KS, for example, the single-particle energies were extracted from more recent theoretical work or more recent experiments. In view of the similarity of these results with those of KS our results will be given for the sake of completeness but not discussed.

The surface delta interaction contains only a single parameter, the strength of the interaction. This parameter can be adjusted by fitting the odd-even mass differences and the one-quasiparticle spectrum of odd nuclei, and then the spectrum of even nuclei can be calculated. However, for some cases we found it most convenient to adjust the strength by fitting the energy of the first 2+ as in the lead isotopes and to look afterwards after the other properties.

As was said in the Introduction, there have been already several calculations on the single-closed-shell nuclei using the quasiparticle model as a basis. One can find in the literature detailed discussions concerning the usefulness of this model for interpreting the experimental results, e.g., ground-state spins of odd nuclei. In the following treatment we will emphasize only what is new in our calculations, what can be said about the approximations which are used, what are the properties of a surface delta interaction considered as an effective interaction, what is the need of using more realistic interactions which contain a radial and a spin dependence. Except in a few cases we will not give references to

TABLE I. Single-particle states, energies and strength of the SDI used in the calculations published here. The strength noted as SDIa was used with a pure SDI. The one noted as SDIb was used in SDI minus pairing (see text). The last column on the right shows the method used in determining the parameters from experiment.

Nuclei	Single-particle states	Single-particle energies (MeV)	Source of single-particle energy	Strength $4\pi G$	Method for determining the strength
$Z=28$, Ni	$2p_{3/2}$	0			
	$2f_{5/2}$	0.78	exp.	0.48a	Odd-even mass differences
	$2p_{1/2}$	1.08			
$Z=50$, Sn	$2d_{5/2}$	0			
	$1g_{7/2}$	0.2	[a]		
	$3s_{1/2}$	2.1		0.23a	Odd-even mass differences
	$2d_{3/2}$	3.2		0.414b	and odd nuclei
	$1h_{11/2}$	3.2			
$Z=82$, Pb	$2f_{7/2}$	0			
	$1i_{13/2}$	−0.72	exp.		
	$3p_{3/2}$	−1.45		0.14a	Best fit for the 2_1+ in
	$2f_{5/2}$	−1.78		(0.154 for	all the isotopes
	$3p_{1/2}$	−2.35		Pb^{206})a	
N=82	$1g_{7/2}$	0			
	$2d_{5/2}$	0.75			
	$2d_{3/2}$	2.9	[b]	0.2 a	Odd-even mass difference
	$3s_{1/2}$	3.4		0.28b	
	$1h_{11/2}$	2.6			
	$1h_{9/2}$	5.4			
Po^{210}	$1h_{9/2}$	0			
	$2f_{7/2}$	0.90	exp.	0.159a	Fit of the 2_1+
	$1i_{13/2}$	1.62			
Pb^{210}	$2g_{9/2}$	0			
	$1i_{11/2}$	0.77			
	$1j_{15/2}$	1.41			
	$3d_{5/2}$	1.56	exp.	0.092a	Fit of the 2_1+
	$4s_{1/2}$	2.03			
	$2g_{7/2}$	2.47			
	$3d_{3/2}$	2.52			

[a] Arvieu *et al.* (Ref. 10).
[b] Reference 11.

the experimental work (the reader is referred to Refs. 8 to 12).

The single-particle states used, the values of the strength of the forces, and the way this strength has been determined, are listed in Table I.

2. Comparison Between the Exact Calculation and the Tamm-Dancoff Approximation

(a) *Two-Particle Problem*

In Fig. 1 are plotted the theoretical excitation energies of the levels of spin 0+, 2+, 4+ up to 4 MeV in Te^{134}, Ni^{58}, and Pb^{206}. In the last case the 5− has also been plotted. The Tamm-Dancoff calculation (Bogoliubov-Valatin method plus diagonalization in the subspace of two quasiparticles) is denoted as TD, and can be compared to the exact calculation. It is seen that there is a very good over-all agreement, especially for the lowest levels and in particular for Te^{134} and Ni^{58}. However, for the highest levels in general and for Pb^{206}, the agreement becomes poorer in absolute value. For example, for the third excited 0+ in Pb^{206} there is a difference of 500 keV between the two calculations. This is an upper limit but differences of the order of 200 keV are frequent. This disagreement is not neglected but it shows that a very accurate fit of the experimental results by a Tamm-Dancoff calculation does not make very much sense for the highest levels. More optimism seems justified for the lowest states such as the first excited 2+.

A similar study has been made for Pb^{210} and Po^{210} and confirms the preceding conclusions (see Fig. 2), also for states of high angular momentum and opposite parity 5−, 7−, 9−.

(b) *Four and Six Particles*

In Fig. 3 are plotted the spectra of Ni^{58}, Ni^{60}, and Ni^{62} up to 3.5 MeV. Tamm-Dancoff and exact calculations have also been performed in these cases.[18] It is seen that again the 2_1+ energy changes only slightly when one goes from the exact to the approximate calculation, and that the first excited 4+ seems also quite well described. However, it does not seem to be possible any more to set up a precise correspondance between the highest states. Indeed large energy differences can

[18] We are grateful to Dr. Edith Halbert and Dr. J. B. McGrory of Oak Ridge National Laboratory for carrying out the exact calculations for Ni^{60} and Ni^{62}.

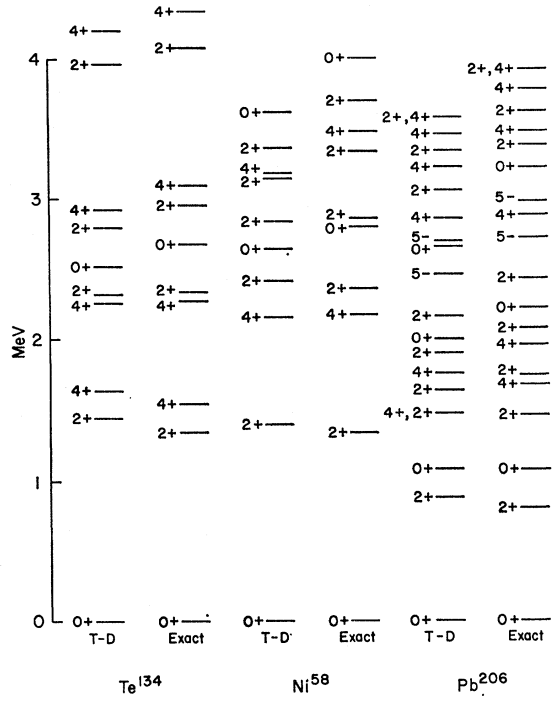

FIG. 1. Spectrum of the SDI for two particles or two holes outside closed shells. An exact diagonalization has been performed as well as a BCS + Tamm-Dancoff approximation (noted as TD). The 0+, 2+, 4+ MeV have been plotted up to 4 MeV. In Pb^{206} the 5− has also been calculated. See Table I for the parameters of these calculations.

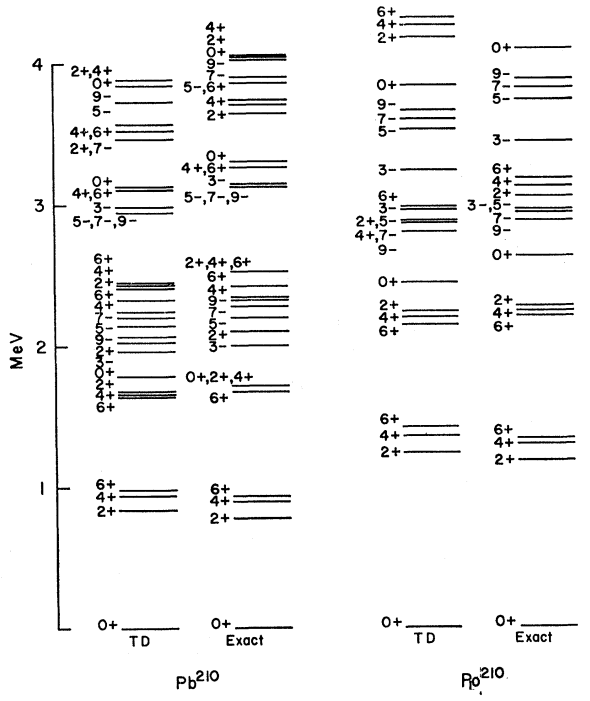

FIG. 2. Same as Fig. 1 but for Pb^{210} and Po^{210}. In that case 3−, 5−, 7−, 9− have been calculated and plotted as well as 0+, 2+, 4+, 6+.

exist. There is, for example, a 900-keV difference in Ni^{62} between the energy of the first excited 0+ in the two calculations, and of 700 keV between the energies of the second 2+. Moreover, there is an important change in the number of levels. In Ni^{60} there are 14 excited states of spin 0+, 2+, or 4+ below 3.3 MeV in the exact calculation, whereas the TDA gives only 9 levels. Therefore, as was already found by the Argonne group, for

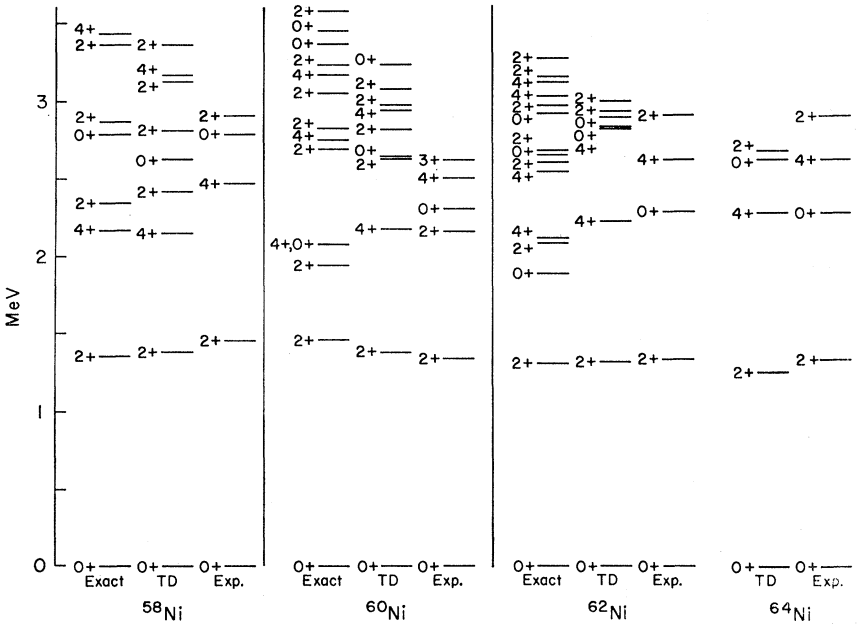

FIG. 3. Spectrum of the SDI for two, four, and six particles outside closed shells in the Ni isotopes from an exact diagonalization and with the Tamm-Dancoff method. The spectrum of Ni^{64} has not been calculated exactly. Note the variation in the number of levels between 1.5 and 2.5 MeV when one goes from the exact calculation to the TD method.

these nuclei, there seems to be an important effect of the seniority four (or if one prefers, of the four quasiparticles) in the region of the spectrum above the 2_1+ and the 4_1+. If the agreement is poor for certain levels, the possibility that it would be better in some cases still remains. For example the 2_3+, 2_4+, 4_2+, 4_3+ have energies which differ by the order of 200 keV from one calculation to the other. It would then be necessary to compare the wave function of these states to see whether this correspondence has sense or not. This comparison has not yet been made. It should be emphasized that the Tamm-Dancoff method gives exact results in the limit of degenerate subshells, and that on this basis, as was said before, it is plausible that the coupling between states which differ in the number of quasiparticles should be weaker for the SDI in the nondegenerate case than for other interactions. This argument leads us to a conclusion which confirms Argonne's: The Tamm-Dancoff method has the greatest validity for the first 2+ or the first 4+, but one has to be very careful in classifying higher states as two-quasiparticle (q.p.) states. At this point one should remark that no attempt has been made yet to justify the calculation of states of *high* angular momentum and of negative parity, such as 5−, 7−, 9− on the basis of a two-q.p. model for more than two particles. There is still a possibility for these states of being well approximated in that way. The interaction energy is weaker in odd-parity states since there are fewer ways of forming such states out of the available orbits.

3. Comparison with Experiment

Two kinds of calculation have been carried out in the present study. Firstly a calculation with a pure delta interaction. In several cases ($Z=28$ and 82 nuclei) this was found sufficient to account for *both* the pairing properties *and* the energies of the first 2+ and of some other states. In the other cases ($Z=50$, and $N=82$) all these properties could not be fitted at the same time by a pure SDI. When the gap was at the right place the 2+ was found to be too high. Therefore we have tried to increase the quadrupole effect in a very simple way. We subtracted from the SDI a pairing force, and adjusted the difference between the SDI strength and the pairing strength so as to get the same pairing properties as before. The strength of the SDI was chosen so as to obtain the first 2+ at the right energy. This procedure is admittedly somewhat arbitrary, but it is interesting to see how much the strength of the SDI needs to be changed to improve the fit to the data. This percentage can indeed be roughly viewed as a measure of all the neglected effects: radial, range, spin exchange, tensor effects, and core polarization. From the numerical point of view this calculation is very simple. Once the Bogoliubov-Valatin canonical transformation has been performed and the quasiparticle energies have been found, the strength between quasiparticles is just taken as a free parameter and several successive diagonalizations for different strengths can be performed by keeping constant the q.p. energies and the u's and v's and by multiplying the other terms of the 2-q.p. matrix. This procedure had of course to be done for each angular momentum and the sensitivity of all the energy levels to this calculation had to be investigated. The calculations with a pure SDI will be denoted either as SDI or SDIa, the calculations with a surface δ interaction minus pairing will be denoted as SDIb.

(a) *Lead Isotopes*

Because of the lack of a precise measurement of odd-even mass differences throughout the lead isotopes we have preferred to adjust the strength using only the even isotopes.

First we studied the level scheme of Pb^{206} by means of an exact calculation. We adjusted the strength of the SDI to fit exactly the excitation energy of the 2_1+. When the other levels 0+, 2+, 4+, 5− were compared to the experimental ones it was found that an excellent agreement existed (Fig. 4), in fact, slightly better than the one obtained by True and Ford[4] (for the second 2+ for example). However, the SDI of the same strength could not fit the energy levels of $Pb^{204-200}$ as well. In order to get a fit for all the even lead isotopes with a single SDI, we decreased slightly the strength of the SDI from 0.154 to 0.140. The fit which we have been able to obtain in this way (Fig. 5) is quite reasonable, the largest disagreement with experiment being of the order of 150 keV. This agreement is therefore of the

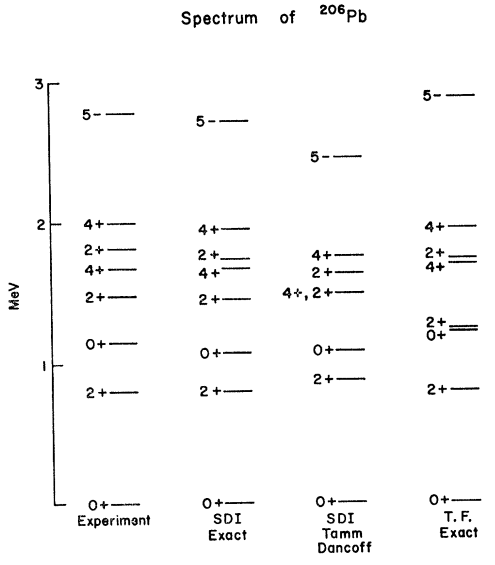

FIG. 4. A few levels of Pb^{206} can be compared here with experiment. The exact calculation with a pure SDI has been done (with a strength adjusted to fit the 2_1+ energy) and also the TD approximation. The last column on the right shows the results by True and Ford who have used a Gaussian interaction.

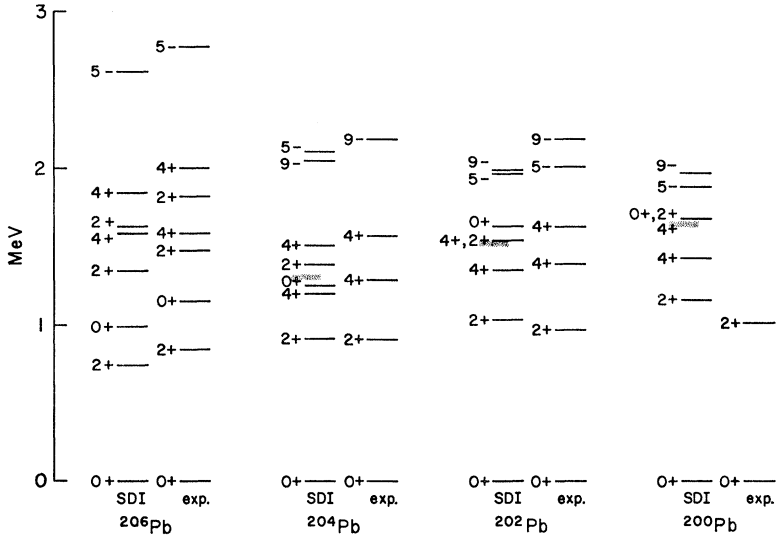

FIG. 5. Comparison with experiment of some levels of the lead isotopes. For $A<206$, the column noted SDI is the result of the Tamm-Dancoff method. Dotted strips represent calculated two-quasiparticle energies.

same order as the one obtained with the True and Ford interaction.[4,10(b)]

Next, the spectra of the odd lead isotopes were calculated on the basis of one-quasiparticle excitations (Fig. 6). Our results are not very different from those of Kisslinger and Sorensen, who had already obtained a very good fit for the states $p_{1/2}$, $p_{3/2}$, $f_{5/2}$, $i_{13/2}$ with a lower strength for the pairing interaction ($4\pi G=0.111$ instead of 0.140).

Our wave function of the ground state of Pb^{206} can be also compared to the wave function derived by Mukherjee and Cohen[19] from stripping experiments (Table II). These authors have stated that the actual configuration mixing in the ground state was much larger than expected from the calculation by True and Ford. It turns out that our wave function agrees much better with experiment than the True and Ford wave function, especially when the strength is adjusted to fit the energy levels of Pb^{206} ($4\pi G=0.154$). This results from the fact that the SDI has a much larger coherence in its nondiagonal elements.

It could be argued that the agreement we have for these isotopes, and especially for Pb^{206}, is not very significant since the states in these isotopes are not really collective ones. This argument has indeed some truth in it, since many states are only very slightly affected by the interaction. Therefore to find good agreement for these states is only a test of the goodness of the pure shell model. This is the case for the 2_2+, 2_3+, 4_1+,

TABLE II. Absolute value of the amplitude of the two-hole configurations in Pb^{206}. The column noted as experiment is taken from Ref. 19, the one noted as Gaussian gives theoretical amplitudes extracted from Ref. 4. The last two columns give these amplitudes in the present work with a pure SDI and two different strengths. The last line gives the overlap of the theoretical wave function with the experimental one.

Two-hole configuration	Expt.	Gaussian	SDI ($4\pi G=0.154$)	SDI ($4\pi G=0.140$)
$(p_{1/2})^{-2}$	0.730	0.865	0.741	0.794
$(f_{5/2})^{-2}$	0.444	0.308	0.474	0.437
$(p_{3/2})^{-2}$	0.345	0.376	0.382	0.256
$(i_{13/2})^{-2}$	0.345	0.122	0.333	0.295
$(f_{7/2})^{-2}$	0.173	···	0.185	0.161
Overlap		0.941	0.997	0.992

[19] P. Mukherjee and B. Cohen, Phys. Rev. **127**, 1284 (1962).

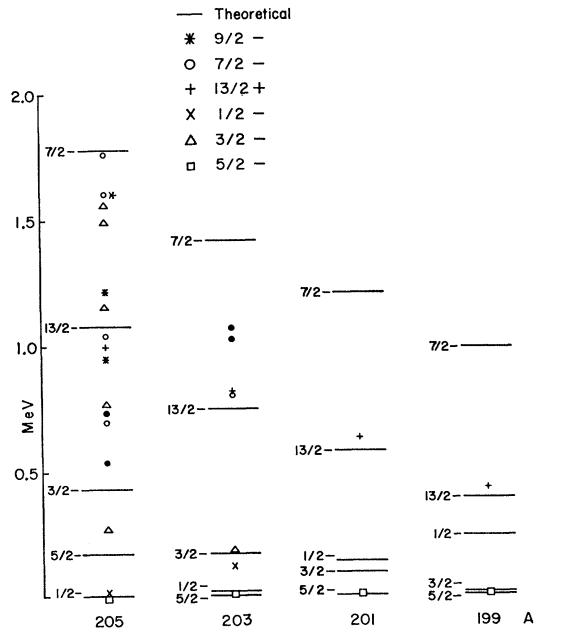

FIG. 6. Odd isotopes of Pb. One-quasiparticle levels calculated with a SDI. The results are in fact the same as those of a pairing force with same strength, but for which one neglects the self-energies.

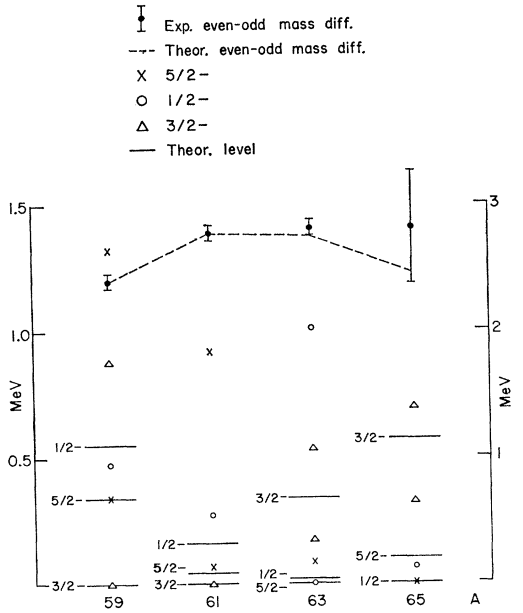

FIG. 7. Odd isotopes and odd-even mass differences in Ni. One-quasiparticle levels calculated with a SDI. The energies of the $3s_{1/2}$, $2d_{3/2}$, $1h_{11/2}$ are plotted with the left scale. The odd-even mass differences [Ref. 10(a)] are plotted above with error bars. The corresponding scale is on the right.

4_2+ states. The calculation attains its full significance when one considers the states which are most strongly affected by the residual interaction. We must, of course, not consider here the 2+ since its energy was artificially fitted. However the 0+ ground state and the 0+ first excited states are both strongly affected by the interaction. We can then consider it very encouraging that the excitation energy of the first-excited 0+ is in agreement with experiment and also that the excitation energy of weak collective states, e.g. 5−, with respect to a strongly perturbed ground state is also in good agreement.

(b) *Nickel Isotopes*

For the Ni isotopes we followed the usual procedure of fixing the strength by fitting odd-even mass differences and odd nuclei. We have been able to obtain good results for the spectra of the odd isotopes of Ni (Fig. 7). It is then extremely satisfactory to see that the energies of the first 2+ and the first 4+ compare also very well with experiment[20] whether calculated exactly or simply by the Tamm-Dancoff approximation (Fig. 3). (We have not, however, calculated the exact results for Ni^{64}.) It is not possible here to get an accurate description of the higher 0+, 2+, 4+ (or even 3+) states. The Tamm-Dancoff method gives results in poor agreement with the exact calculation and the distribution of levels, for example in Ni^{60}, does not agree with experiment.[11] Other calculations have been done on the Ni isotopes[5,21] with much more involved interactions. These calculations are in some ways more successful than ours or the one of Ref. 10(c). A very good accuracy has been obtained for some properties like the states of "seniority" 0 or 2 or the spectrum of Ni^{58}, but the errors can be very big also for the states of seniority four. This shows that the problem of the higher states of Ni isotopes is still open and probably that the core excitations should be included in some way.

To conclude this part we must say that the interaction between the quasiparticles coupled to 2+ has in this case a very large effect on the energy of the first 2+. In Ni^{60}, for example, the lowest 2-q.p. energy is 2.6 MeV, first-order perturbation theory gives a first 2+ at 2.3 MeV, the diagonalization lowers this state down to 1.36 MeV (the experimental energy is 1.33). A conventional δ function acting throughout the nuclear volume presumably gives a much higher energy, as was already shown in Ref. 1 for some idealized configurations.

(c) *Tin Isotopes*

The strength here has been adjusted in the same manner as in the preceding case. The single-particle energies being taken from a previous calculation,[10(a)] we have fitted the odd-even mass differences and the one-quasiparticle states $d_{3/2}$, $h_{11/2}$, $s_{1/2}$ in the odd isotopes (Fig. 8). However, the 2_1+ turns out to be too high in the subsequent calculation of the even isotopes. Indeed the maximum error can be as big as 1 MeV [Fig. 9(a), (b), (c)]. It turns out that some other states like 4+, 5−, 7− lie also too high compared with experiment. The situation is thus very different from that in the nickel isotopes. Here the effect of the interaction between quasiparticles is much smaller. The following

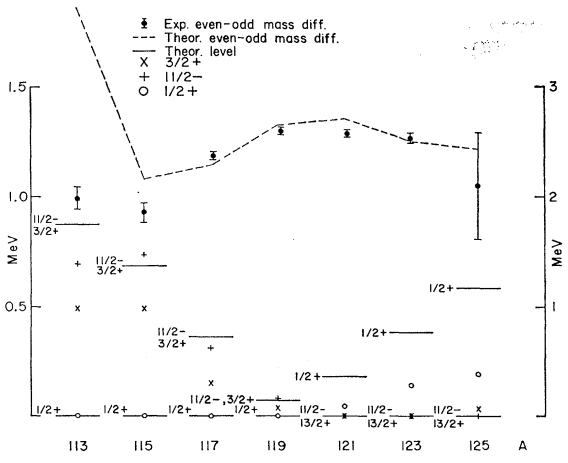

FIG. 8. Odd isotopes of Sn. See captions of Figs. 6 and 7 for the notations.

[20] R. K. Mohindra and D. M. Van Patter, Phys. Rev. **139**, 274 (1965).

[21] N. Auerbach, Nucl. Phys. **76**, 321 (1966).

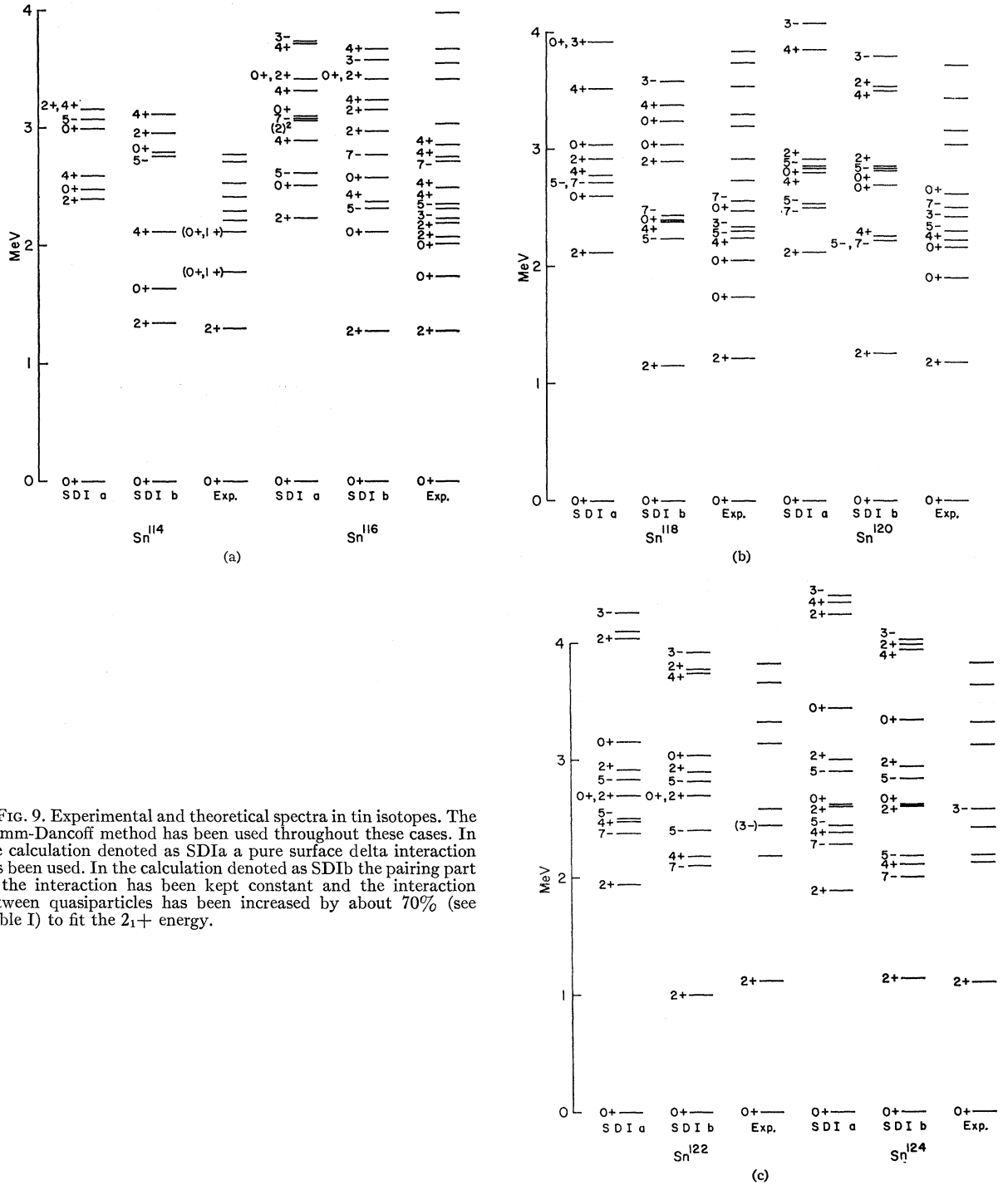

FIG. 9. Experimental and theoretical spectra in tin isotopes. The Tamm-Dancoff method has been used throughout these cases. In the calculation denoted as SDIa a pure surface delta interaction has been used. In the calculation denoted as SDIb the pairing part of the interaction has been kept constant and the interaction between quasiparticles has been increased by about 70% (see Table I) to fit the 2_1+ energy.

numbers for Sn^{118} are very characteristic: the lowest two-q.p. energy is around 2.86 MeV in this nucleus. First-order perturbation theory brings a 2+ at 2.70 MeV only, and the diagonalization of the residual interaction lowers the 2+ to 2.13 MeV. (Experimentally, it is 1.21.)

The calculation with the interaction SDI minus pairing allows, of course, much more reasonable results, but it is necessary to increase the strength of the SDI by 70% to lower sufficiently the 2+. This procedure lowers simultaneously the whole spectrum but the 2_1+ is by far the most sensitive of all. In Sn^{118} when the 2_1+ is

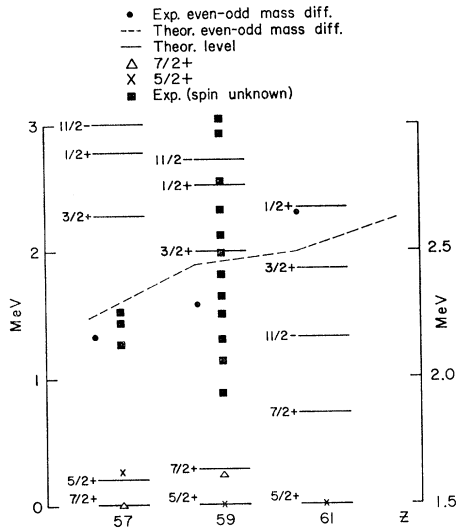

FIG. 10. Odd-even mass differences and odd nuclei for $N=82$ nuclei. See Figs. 6 and 7 for explanations of the calculation.

lowered by 960 keV, the 4_1+ is lowered by 400 keV, the first-excited 0+ by 160 keV, the 5_1- by 460 keV, the 7− by 260 keV (in other nuclei the last figures can be smaller). Then a more reasonable agreement with experiment is obtained for all these states [Fig. 9(b)].

As mentioned before, increase of the SDI by 70% seems to account for all the effects which have not been included (e.g., core excitations). This figure could certainly be reduced by using a finite-range interaction as was done by several authors. However, since they have not succeeded in getting the energy of the 2+ with good accuracy, and since they have been obliged to use anomalous values of the triplet odd interaction to improve the fit, it can be said that this assumption hides also a large part of core excitation.

(d) *Isotones of $N=82$*

In this case the SDI gives very similar results to those of the Sn isotopes. This was also the case for the calculations with a Gaussian interaction.[11] Once the strength of the interaction has been determined by fitting the odd-even mass differences (Fig. 10) the 2+ is found 200 to 300 keV higher than experiment, which may appear as a more acceptable result [Fig. 11(a), (b)]. The percentage of increase of the strength of the SDI is then only 38%. Here also it was found by Rho that a larger value of the triplet odd-even singlet ratio had to be introduced (larger than 1) to account for the 2_1+ energy.

(e) *Spectrum of Pb^{210} and Po^{210}*

Only a few experimental levels are known in these two nuclei: a 2+, 4+, 6+ in both of them and a 5− in Po^{210}. There are essentially two main points to be noted (from Fig. 12). First the impossibility of fitting the excitation energy of both Po^{210} and Pb^{210} with the same strength. Pb^{206} and Po^{210} require a very similar strength for fitting the 2_1+ energy (with, however, different

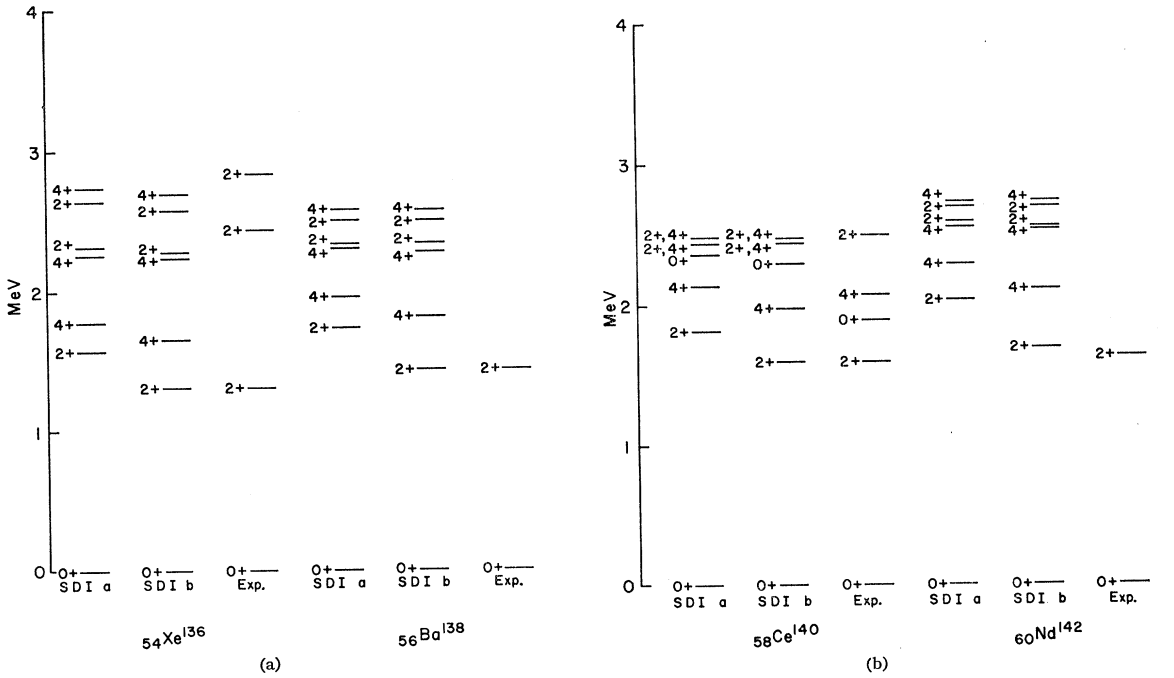

FIG. 11. Experimental and theoretical spectra in even isotones of $N=82$. The notations are the same as in Fig. 9. Here it has been necessary to increase the strength by about 35%. (See Table I.)

numbers of single-particle states), but Pb^{210} needs a much smaller strength. This is illustrated in Table III. This illustrates the fact that the interaction is really an effective interaction or at least that a more subtle dependence of the interaction with respect to the single-particle states is needed. The second fact which supports this conclusion is the only fair fit of the 4+ and the 6+ in both of these nuclei, compared to the almost perfect fit obtained in Pb^{206}. Indeed in Pb^{210} the 4+ and the 6+ are, respectively, 200 keV and 250 keV too low. In Pb^{206} the largest deviations with respect to the experiment are only a few tenths of keV. These deviations are, however, very reasonable. Note also that the 5− in Po^{210} is accurately described.

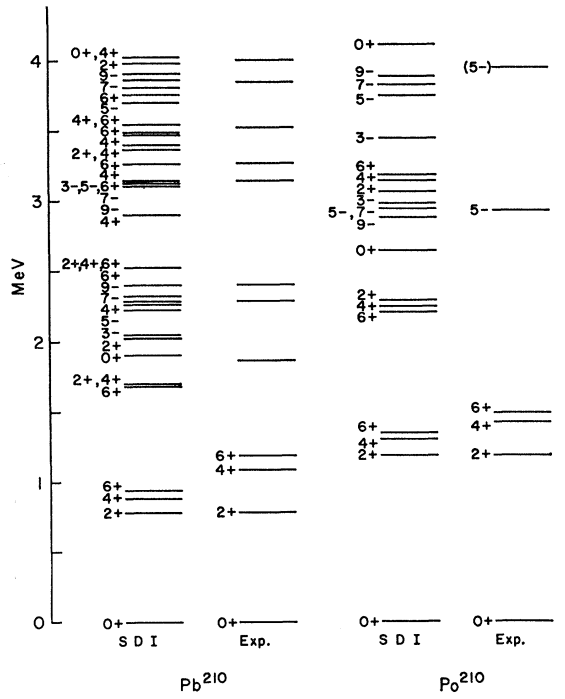

FIG. 12. Spectrum of two particles outside Pb^{208} with a SDI, and comparison with experiment. An exact shell-model calculation has been performed here.

4. Comparison with Previous Calculations

In Fig. 13 are plotted the level spectra of Pb^{204}, Sn^{120}, Ce^{140}, and Ni^{62}. The results of a SDI are compared with some of those of previous calculations [respectively, Refs. 10(b), 10(a), 11, 10(c)] in which a Gaussian interaction was used. For Sn^{120} and Ce^{140} the results are those for SDI minus pairing. One can note the big similarity between the two calculations. The most notable difference is that the first-excited 0+ lies consistently much lower when a Gaussian is used. The fact is particularly striking in Sn^{120} and in Ni^{62} but not so important in Pb^{204}. (The calculation of the 0+ for the $N=82$ isotones, e.g., Ce^{140}, was not done in Ref. 11.) This seems to be a typical finite-range effect. The monopole component is much more important with respect to the other components as far as the range of the interaction increases.

In particular, the diagonal elements of the interaction are much larger. In Sn^{120}, for example, the lowest two-q.p. energy is 2.80 for a SDI, 2.60 for a Gaussian. The first-order perturbation theory lowers the state down to 1.46 MeV for a SDI, to 1.2 MeV for a Gaussian. (The diagonalization of the matrix lowers the state further, 0.52 MeV in the first case, but to −3 MeV in the second case.) It turns out that these states are mainly spurious. (The spurious state is an exact eigenstate of the two-q.p. matrix at zero energy in

TABLE III. Excitation energy of the first 2+ for several strengths of the interaction in an exact two-particles (or two-holes) shell-model calculation. Each strength fits the energy of the 2_1+ in one of these nuclei. Note that Po^{210} and Pb^{206} require a similar strength but that Pb^{210} needs a much smaller one. All the energies are in MeV.

$4\pi G$ \ 2_1+	Pb^{206}	Po^{210}	Pb^{210}
+0.154	0.803	1.13	1.75
+0.159	0.834	1.18	1.80
+0.092	0.580	0.53	0.794
Experimental energy	0.803	1.18	0.794

TABLE IV. Lower excitation energies of Pb^{210} and Sn^{120}. The case of Pb^{210} is an exact shell-model calculation with the SDI. The first column includes 7 single-particle (s.p.) states; in the second one, three of the s.p. states have been neglected. The case of Sn^{120} is a Tamm-Dancoff calculation with 8 or 5 single-particle states. In each case the strength of the interaction has been modified to get, in the first case the same 2_1+, in the second one the same quasi-particle energies (these have not been listed here).

7 s.p. states $4\pi G=0.086$		4 s.p. states $4\pi G=0.092$		8 s.p. states $4\pi G=0.180$		5 s.p. states $4\pi G=0.230$	
Pb^{210}				Sn^{120}			
$J\pi$	E_J (MeV)	$J\pi$	E_J (MeV)	$J\pi$	E_J (MeV)	$J\pi$	E_J (MeV)
0+	1.89	0+	1.91	0+	2.84	0+	2.84
0+	3.26	0+	3.30	0+	2.84	0+	2.86
2+	0.79	2+	0.79	2+	2.11	2+	2.02
2+	1.70	2+	1.71	2+	2.84	2+	2.84
2+	2.01	2+	2.11	2+	2.87	2+	2.88
2+	2.51	2+	2.52	2+	3.82	2+	3.81
3−	2.04	3−	2.00	3−	4.03	3−	4.08
3−	3.14	3−	3.14	3−	4.29	3−	4.33
4+	0.88	4+	0.89	4+	2.64	4+	2.59
4+	1.69	4+	1.71	4+	3.87	4+	3.85
4+	2.25	4+	2.34	4+	4.26	4+	4.29
4+	2.52	4+	2.52	4+	4.45	4+	4.49
5−	2.22	5−	2.20	5−	2.55	5−	2.53
5−	3.13	5−	3.12	5−	2.85	5−	2.85
6+	0.93	6+	0.93	6+	2.74	6+	2.72
6+	1.68	6+	1.69	6+	5.47	6+	5.49
6+	2.39	6+	2.42	6+	5.71	6+	5.76
6+	2.52	6+	2.52				
7−	2.28	7−	2.28	7−	2.57	7−	2.51
7−	3.11	7−	3.11	7−	4.18	7−	4.20
9−	2.32	9−	2.31				
9−	3.10	9−	3.10				

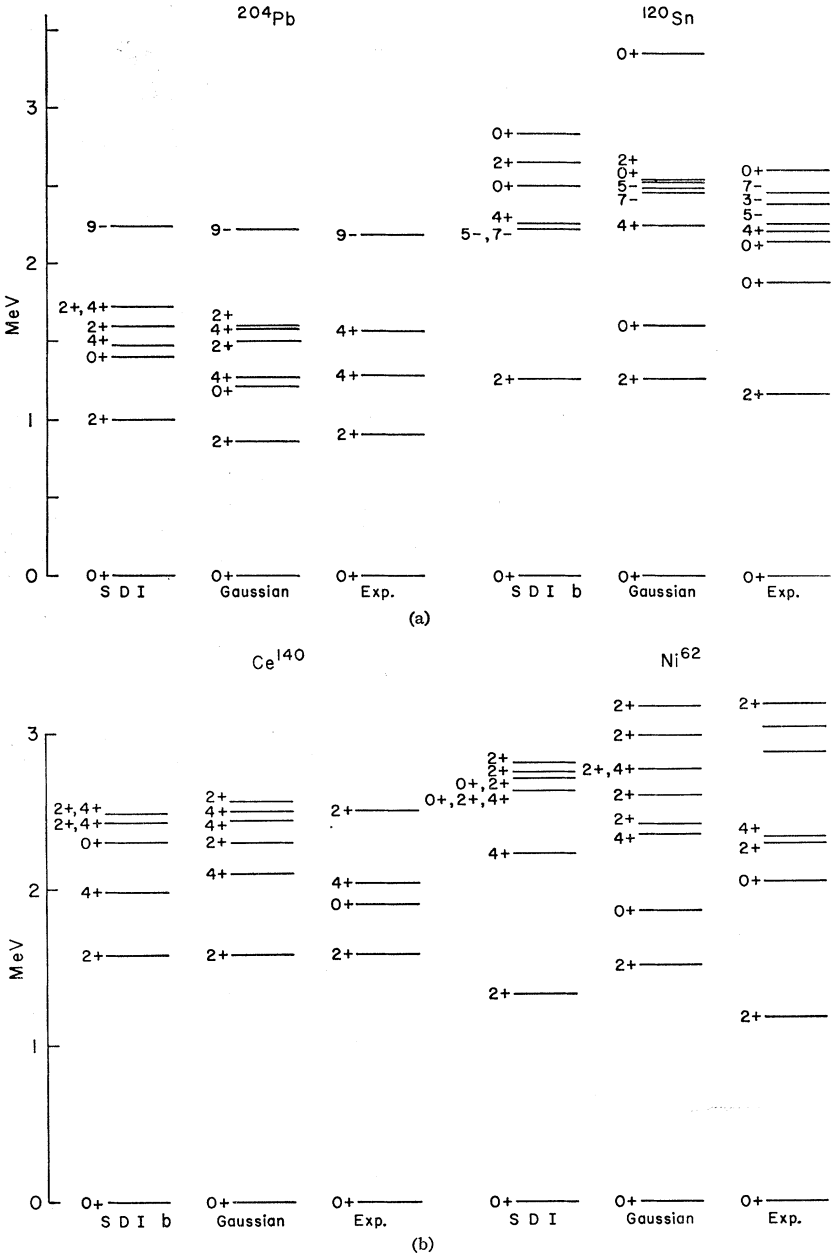

FIG. 13. Comparison of the SDI with a Gaussian interaction (see also Fig. 4) and with experiment for each region studied. The calculations noted as Gaussian are extracted from Ref. 10(a) for Sn^{120}, 10(b) for Pb^{204}, 10(c) for Ni^{62}, from Ref. 11 for Ce^{140}.

the degenerate case, as pointed out in the first part.) A similar explanation can be used for the differences between the physical first 0+ states.

However, despite the fact that a finite-range interaction accounts better for the first 0+ excited states in Sn isotopes, for example, this systematic effect does not necessarily support the assumption of such a finite-range interaction. Our previous study shows that these 0+ states are poorly given by the Tamm-Dancoff approximation.

From these results it can be deduced that none of the interaction used can be excluded, if one allows the subtraction of a certain amount of pairing to the SDI. More accurate calculations and a more systematic use of the wave function, as in Ref. 22, have to be made.

5. Renormalization of the Strength

To complete this discussion we will present the results of a calculation which shows that in two cases, Sn^{120} and Pb^{210}, an increase in the strength of the interaction

[22] N. K. Glendenning and M. Veneroni, Phys. Letters **14**, 228 (1964); R. Arvieu and E. Salusti, Nucl. Phys. **66**, 305 (1965).

can simulate very accurately for most of the levels an increase in the number of single-particle states one could take into account (Table IV). These calculations were not only done for the two nuclei quoted above, but for all the Sn isotopes with the same results (in all cases with a pure SDI). For Sn the following states were added to the one plotted in Table I, with the following single-particle energies:

$$\epsilon(1g_{9/2})=-3.97, \quad \epsilon(1h_{9/2})=+8.23, \quad \epsilon(2f_{7/2})=+7.74,$$

deduced from the Nilsson spectrum. In the case of Pb^{210} the states $4s_{1/2}$, $2g_{7/2}$, and $3d_{3/2}$ were subtracted from those plotted in Table I. The results from Table IV show then that the spectrum of lowest states, once conveniently normalized, depends only on the few single-particle states nearest to the Fermi surface for a SDI in the two-quasiparticle formalism. Would these results still hold in a more exact calculation including core excitations and the neutron-proton interaction? This is a very interesting question, the answer to which would give much information concerning the effective interaction.

IV. CONCLUSIONS

The SDI has been used to calculate the spectra of nuclei which can be treated in terms of identical particle configurations like the single-closed-shell nuclei. The spectra have been calculated by the BCS+Tamm-Dancoff method, but in some cases an exact shell-model diagonalization has also been performed. The comparison between the two sets shows that the approximation method works quite well for the two-particle case. However, for more than two particles outside closed shells, the approximation works well for the 2_1+ or the 4_1+ but not so well for the other states as already pointed out.[5] Therefore for the following the emphasis should be put more on the 2_1+ and the 4_1+ and therefore on the ability of the interaction to fit both the even-odd mass differences and the energy of these states. This has been seen possible for Ni and Pb isotopes, but not for Sn or for the isotopes of $N=82$. It shows that in the nuclei which are well described by few configurations of the shell model the SDI is able to give a similar (or even better) agreement with experiment than other interactions. In Sn and in the $N=82$ nuclei it confirms that some more studies have to be made concerning core excitation. In Ni isotopes the agreement for the 2_1+ energy is quite surprising. Indeed it has been seen that, contrary to the case of the Pb isotopes (for which, however, an exact calculation has still to be done), the highest states are poorly fitted even by a very complicated interaction.[21] Some core excitation is needed therefore and it might also have been expected that this core excitation be needed to explain also the first 2+. It seems that the SDI acts in a particular coherent way for this set of $\frac{1}{2}-$, $\frac{3}{2}-$, $\frac{5}{2}-$ orbitals, much more coherently than for the set $\frac{1}{2}+$, $\frac{3}{2}+$, $11/2-$ in the Sn isotopes. In the last cases it seems that finite-range interactions give slightly better agreement. It is hoped that the SDI can be used in the future in shell-model calculations. It can serve at least as an excellent tool to understand simply the geometrical and most coherent effects of the interaction. Since in some cases an exact solution is easily found[2] it can also serve to test new approximation methods of the shell model which would replace an extended diagonalization.

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