Absorption of Stopped π^- Mesons by He^3 and Nuclear Correlations

IL-TONG CHEON
Department of Physics, Kyoto University, Kyoto, Japan
(Received 28 December 1965)

The capture rates of stopped π^- mesons by He^3 leading to the final states $(p+2n)$ and $(d+n)$ are calculated by using the two-nucleon-capture model. The ratios of one-nucleon ejection to two-nucleon ejection and the branching ratio $W(pn\rightarrow nn)/W(pp\rightarrow np)$ are obtained. Furthermore, the effects of the short-range nuclear correlations on these values are investigated. It is shown that the correlation parameter can be determined from the relative momentum distributions of two ejected nucleons.

1. INTRODUCTION

RECENTLY, it has been noticed[1,2] that the capture of stopped π^- mesons by nuclei could be used as a powerful tool to get reliable information on nuclear properties such as the momentum distributions of nucleons, short-range pair correlations within the nucleus, and hole excitations in the shell model through the branching ratios and energy spectra.

Since it is commonly assumed that the absorption occurs preferentially on a proton in a strongly correlated nucleon pair and thus ejects this strongly correlated pair out of the nucleus leaving a two-hole excitation behind, one could possibly expect the absorption to be useful for the studies of two-hole excitations in the shell model. In fact, almost all experiments show a strong support for the two-nucleon ejection process, as result of the primary act of absorption. The lifetimes of highly excited hole states being likely to be short, the states have rather broad widths. These states should be investigated for the nuclei in which the energy levels can be resolved. From the reasons mentioned above, the study of the pion absorption is particularly interesting in investigations of the properties of light nuclei.

If the 139.2 MeV (pion rest mass) released as kinetic energy in the capture of a stopped pion is concentrated on a single nucleon, it moves with momentum over 500 MeV/c. The pion being captured at rest, this momentum must be the initial momentum of the nucleon by momentum conservation. For heavy nuclei it is hard to find such high-momentum components in the nuclear-momentum distribution, because the Fermi momentum is only of the order of 200 MeV/c. Even if the nucleons have higher Fermi momentum in light nuclei, it is unlikely that the individual nucleons move with momenta over 500 MeV/c. It is therefore impossible for a single nucleon in an average nuclear potential to capture the pion. Pion capture seems to be mainly a process involving, at least, a few particles, which exploits the high-relative-momentum components in nuclear interactions at short distances. Absorption by nucleon pairs, $\pi^-+n+p\rightarrow n+n$ or $\pi^-+p+p\rightarrow n+p$, is particularly important.[3] That the two nucleons share the available energy between themselves means a relative momentum of about 750 MeV/c. From this we can immediately conclude that the capture process involves relative distances of the order of 0.5 F. It is quite a short-range process, compared with the average internucleon distance in He^3 of about 2 F.

It is well known that the simple shell-model wave functions make no claim to describe the short-range N-N correlations correctly. In the two-nucleon-capture model under consideration, the effects of the p wave in the pion exchange between the nucleons and the effects of the hard core of the nuclear forces should be taken into the calculation of the pion-capture rate. These effects are important in the short-range part of the wave function describing the relative motion of two nucleons. The effects of the short-range N-N correlations were, at first, studied in nuclear matter.[4] Recently, the investigations of these effects on the cross sections in the (p,p') reactions,[5] the deuteron pick-up or stripping reactions,[6] and the photonuclear reactions[7] have been reported.

In addition to the above reasons, for the purpose of getting some information on the short-range correlations between the nucleons in the s shell, we take up He^3.

[1] T. Ericson, Phys. Letters **2**, 278 (1962).

[2] Il-T. Cheon, Nucl. Phys. (to be published); Il-T. Cheon, C. Nguyen-Trung, and Y. Sakamoto, Progr. Theoret. Phys. (Kyoto) **34**, 574 (1965); Il-T. Cheon, C. Nguyen-Trung, and Y. Sakamoto, Phys. Letters **19**, 232 (1965); Y. Sakamoto and Il-T. Cheon, Progr. Theoret. Phys. (Kyoto) **34**, 814 (1965); P. Ammiraju and S. N. Biswas, Nuovo Cimento **17**, 726 (1960).

[3] H. Byfield, J. Kessler, and L. M. Lederman, Phys. Rev. **86**, 17 (1952); S. Ozaki, R. Weinstein, G. Glass, E. Loh, L. Neimala, and A. Wattenberg, Phys. Rev. Letters **4**, 533 (1960); M. S. Kozadayev, M. M. Kulyukin, T. M. Sylyayev, A. A. Fillippov, and Yu. A. Shcherbakov, Zh. Eksperim. i Teor. Fiz. **38**, 409 (1960) [English transl: Soviet Phys.—JETP **11**, 300 (1960)]; V. S. Demidov, V. G. Kirillov-Ugryumov, A. K. Ponosov, V. P. Protasov and F. M. Sergeyev, Zh. Eksperim. i Teor. Fiz. **44**, 1144 (1963) [English transl: Soviet Phys.—JETP **17**, 773 (1963)].

[4] R. Jastrow, Phys. Rev. **98**, 1479 (1955); E. E. Salpeter, Ann. Phys. (N.Y.) **5**, 183 (1958); S. D. Drell and K. Huang, Phys. Rev. **91**, 1527 (1953); I. Iwamoto and M. Yamada, Progr. Theoret. Phys. (Kyoto) **17**, 543 (1957); A. de-Shalit and V. F. Weisskopf, Ann. Phys. (N.Y.) **5**, 282 (1958); J. B. Aviles, Jr., Ann. Phys. (N.Y.) **5**, 251 (1958).

[5] Il-T. Cheon, Progr. Theoret. Phys. (Kyoto) **32**, 74 (1964).

[6] Il-T. Cheon, Progr. Theoret. Phys. (Kyoto) **33**, 638 (1965).

[7] G. M. Shklyarevskii, Zh. Eksperim. i Teor. Fiz. **41**, 234 (1961) [English transl: Soviet Phys.—JETP **14**, 170, 324 (1962)].

The reactions resulting from π^- capture by He^3 are

$$\pi^- + He^3 \to p+n+n\,, \tag{1.1}$$
$$\to d+n\,, \tag{1.2}$$
$$\to H^3+\pi^0\,, \tag{1.3}$$
$$\to H^3+\gamma\,, \tag{1.4}$$
$$\to d+n+\gamma\,, \tag{1.5}$$
$$\to p+n+n+\gamma\,. \tag{1.6}$$

The relative rates of these processes were calculated by Messiah[8] in the framework of the impulse approximation, the known data on meson capture in hydrogen and deuterium being used. The capture was also assumed to take place in the S state of the π-mesonic atom. The relative ratios of these processes calculated in Ref. 7 are 55.5, 27.8, 9.4, 4.8, 2.0, and 0.5%. Using the two-nucleon capture model, Divakaran[9] has obtained 12.3% for the ratio of radiative capture (1.4) to the absorption (1.1)+(1.2). The experimentally obtained rates of reactions (1.3) and (1.4) compared with the total capture rate are the following: $W_3=(13.5\pm0.9)\%$, $W_4=(6.2\pm0.7)\%$.[10]

In the present paper we calculate the absorption rates of reactions (1.1) and (1.2). We refer the processes (1.1) and (1.2) to the proton and deuteron modes, respectively. Divakaran used a simple Gaussian wave function which does not describe the short-range correlations for He^3. However, he emphasized that the effects due to the correlations are partly included in the coupling constant g determined phenomenologically. His theory has several defects:

(1) It is impossible to get information on the wave function of He^3.

(2) We can scarcely obtain information on the short-range correlations.

(3) The interaction used by him is not a simple sum of one-body operators, but a much more complicated one.

As for the wave function of He^3, we take the Gaussian type including the pair correlations due to the effects of the p wave in the pion exchange between the nucleons and to the hard core of the nuclear forces. We use a simple sum of one-body operators for the π-N interaction.

2. DERIVATION OF THE ABSORPTION RATES

The operator for the π-N interaction is taken to be the ordinary pseudoscalar interaction, which preserves Galilean invariance, in the nonrelativistic approximation[11]:

$$H=G\sum_{i=1}^{3}\left[M\boldsymbol{\tau}(i)\boldsymbol{\sigma}(i)\cdot\nabla^{(i)}\boldsymbol{\phi}(\mathbf{r}_i) -\mu\boldsymbol{\phi}(\mathbf{r}_i)\boldsymbol{\tau}(i)\boldsymbol{\sigma}(i)\cdot\nabla^{(i)}\right], \tag{2.1}$$

where G is the coupling constant, M and μ are masses of a nucleon and a pion, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are spin and isobaric-spin operators, respectively, and $\boldsymbol{\phi}(\mathbf{r}_i)$ denotes the pion wave functions which have the well-known forms

$$\begin{aligned}\phi_0(\mathbf{r})&=\left(\frac{Z}{a_0}\right)^{3/2}2\exp\left(-\frac{Z}{a_0}r\right)Y_{00}(\hat{r})\,,\\ \phi_1(\mathbf{r})&=\left(\frac{Z}{2a_0}\right)^{3/2}\frac{Z}{\sqrt{3}a_0}r\exp\left(-\frac{Z}{2a_0}r\right)Y_{1\nu}(\hat{r})\,,\end{aligned} \tag{2.2}$$

for $1S$ and $2P$ states, where Z is the charge of the initial nucleus and a_0 is the Bohr radius of the π-mesonic atom. The first term of (2.1) corresponds to capture of pions from P states and the second term to absorption from S states. Since the condition $ZR_0/a_0\ll1$ (with R_0 standing for the nuclear radius), is fulfilled for light nuclei under consideration, the meson wave function scarcely varies in the region of the nucleus, and we can confine ourselves only to the pion absorption from S states.[12] In liquid hydrogen the captures of both π^- and K^- mesons are known to occur from S orbits,[13] and helium nuclei capture the K^- meson predominantly from the S orbit.[14] Furthermore, we approximate the Hamiltonian as follows:

$$\begin{aligned}H&=-G\mu\sum_{i=1}^{3}\boldsymbol{\tau}(i)\boldsymbol{\sigma}(i)\cdot\nabla^{(i)}\boldsymbol{\phi}(\mathbf{r}_i)\\ &\approx-G\mu\left[\frac{\boldsymbol{\phi}(\mathbf{r}_1)+\boldsymbol{\phi}(\mathbf{r}_2)}{\sqrt{2}}\cdot\left\{\frac{\boldsymbol{\tau}(1)-\boldsymbol{\tau}(2)}{\sqrt{2}}\,\frac{\boldsymbol{\sigma}(1)+\boldsymbol{\sigma}(2)}{\sqrt{2}}\right.\right.\\ &\qquad\left.\left.+\frac{\boldsymbol{\tau}(1)+\boldsymbol{\tau}(2)}{\sqrt{2}}\,\frac{\boldsymbol{\sigma}(1)-\boldsymbol{\sigma}(2)}{\sqrt{2}}\right\}\cdot\frac{\nabla^{(1)}-\nabla^{(2)}}{\sqrt{2}}\right].\end{aligned}$$

As is well known, the totally antisymmetrized wave function of the $^{22}S_{1/2}$ state of He^3 takes the following

[8] A. M. L. Messiah, Phys. Rev. **78**, 639 (1952).

[9] P. P. Divakaran, Phys. Rev. **139**, B387 (1965).

[10] I. V. Falomkin, A. I. Filippov, M. M. Kulyukin, Yu. A. Shcherbakov, R. M. Sulya'ev, V. M. Tsupko-Sitnikov, and O. A. Zaimidoroga, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 14.

[11] The interaction Hamiltonian density is given by

$$\mathcal{H}=ig\psi^\dagger\gamma_5\psi\boldsymbol{\phi}\cdot\boldsymbol{\tau}. \tag{a}$$

As is well known, we may re-express this term in a nonrelativistic form,

$$\mathcal{H}\approx(ig/2MC)\psi^\dagger\boldsymbol{\sigma}\psi\cdot\mathbf{p}_\pi\boldsymbol{\phi}\boldsymbol{\tau}, \tag{b}$$

with $\boldsymbol{\sigma}$ given by the Pauli matrices and $\mathbf{p}_\pi$ the pion momentum operator. As (b) does not preserve Galilean invariance we must add a term. Thus we get

$$\mathcal{H}\approx(\hbar g/2MC)\psi^\dagger\boldsymbol{\sigma}\psi[\nabla_\pi-(\mu/M)\nabla_N]\boldsymbol{\phi}\boldsymbol{\tau}.$$

[12] T. Ericson, in Proceedings of the 1963 International Conference on High Energy Physics and Nuclear Structure, CERN Report 63-28 (unpublished), p. 47.

[13] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters **3**, 61 (1959).

[14] T. B. Day and G. A. Snow, Phys. Rev. Letters **5**, 112 (1960).

form[15]:

$$\Psi_I=\psi^s\xi^a-\psi'\xi''+\psi''\xi',$$

where ψ represents the space part of the wave function, and the ξ's contain the spin and isobaric-spin wave functions of the proper symmetries. The above form represents the most important part of the ground-state wave function of He^3. We approximate the ground-state wave function of He^3 by a product of a totally symmetric space part ψ^s and a completely antisymmetric spin and isobaric-spin part. By the above approximation, the wave function of He^3 takes the convenient form

$$\Psi_I=-\tfrac{1}{2}(|{}^{13}S_0{}^{pp}\rangle-|{}^{31}S_1{}^{pn}\rangle+|{}^{13}S_0{}^{pn}\rangle), \qquad (2.3)$$

where

$$\begin{aligned}|{}^{13}S_0{}^{pp}\rangle&=\psi_I(123)(00\tfrac{1}{2}S|\tfrac{1}{2}S)(11\tfrac{1}{2}-\tfrac{1}{2}|\tfrac{1}{2}\tfrac{1}{2})\\&\quad\times\chi_{12}(00)\zeta_{12}(11)\chi_3(S)\zeta_3(-),\\|{}^{31}S_1{}^{pn}\rangle&=\psi_I(123)\sum_{\sigma\sigma'}(1\sigma\tfrac{1}{2}\sigma'|\tfrac{1}{2}S)(00\tfrac{1}{2}\tfrac{1}{2}|\tfrac{1}{2}\tfrac{1}{2})\\&\quad\times\chi_{12}(1\sigma)\zeta_{12}(00)\chi_3(S)\zeta_3(+),\end{aligned} \qquad (2.4)$$

and

$$\begin{aligned}|{}^{13}S_0{}^{pn}\rangle&=\psi_I(123)(00\tfrac{1}{2}S|\tfrac{1}{2}S)(10\tfrac{1}{2}\tfrac{1}{2}|\tfrac{1}{2}\tfrac{1}{2})\\&\quad\times\chi_{12}(00)\zeta_{12}(10)\chi_3(S)\zeta_3(+).\end{aligned}$$

$\chi_{12}(1\sigma)$ and $\zeta_{12}(00)$ represent, respectively, the triplet spin and singlet isobaric-spin states of two nucleons, $\zeta_3(-)$ means the isobaric-spin state with $-\frac{1}{2}$ z component, and the symbols $(\,|\,)$ are Clebsh-Gordan coefficients.

A Gaussian type with rms radius $R=1.60$ F is chosen for the space part of the He^3 wave function. As is seen in the previous section, the π-meson absorption is a very short-range process. These short-range correlations due to the effects of the p wave of pions exchanged between the nucleons and to the hard core of the nuclear forces will strongly affect the pion-capture rates. The sensitivity of the absorption rate to the short-range behavior of the wave function can already be seen in the case of π^- capture by deuterons[16]; the nonradiative capture depends sensitively on the probability of finding a high relative momentum of the nucleons in the deuteron. Therefore, we have to introduce the short-range correlations correctly. Introducing the correlation function of the following form:

$$f(r_{ij})=1-\exp(-\beta_0 r_{ij}^2), \qquad (2.5)$$

we have

$$\begin{aligned}\psi_I(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)&=N(\alpha,\beta_0)\prod_{i<j}\exp(-\alpha r_{ij}^2)\\&\quad\times\{1-\exp(-\beta_0 r_{ij}^2)\}\\&=N(\alpha,\beta)\prod_{i<j}\{\exp(-\alpha r_{ij}^2)\\&\quad-\exp(-\beta r_{ij}^2)\}, \qquad (2.6)\end{aligned}$$

[15] M. Verde, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 144.

[16] K. Brueckner, R. Serber, and K. Watson, Phys. Rev. **81**, 575 (1951).

TABLE I. Possible transitions of the nucleon pair for S-state absorption.

Nucleon pair	Transition
$(pp)\rightarrow(pn)$	${}^{13}S_0\rightarrow{}^{33}P_0$
$(pn)\rightarrow(nn)$	${}^{31}S_1\rightarrow{}^{33}P_1$
	${}^{13}S_0\rightarrow{}^{33}P_0$

for the ground state of He^3, where $\beta=\alpha+\beta_0$ and $N(\alpha,\beta)$ is the normalization constant which has a rather complicated form:

$$\begin{aligned}\frac{1}{N^2(\alpha,\beta)}&=\left(\frac{\pi}{2}\right)^3[(3\alpha^2)^{-3/2}+3\{\alpha(\alpha+2\beta)\}^{-3/2}\\&\quad+3\{\beta(\beta+2\alpha)\}^{-3/2}+(3\beta^2)^{-3/2}\\&\quad-6\{\alpha(2\alpha+\beta)\}^{-3/2}-24\sqrt{2}(\alpha^2+4\alpha\beta+\beta^2)^{-3/2}\\&\quad-6\{\beta(2\beta+\alpha)\}^{-3/2}+96(\alpha+\beta)^{-3/2}\\&\quad\times\{(\alpha+5\beta)^{-3/2}+(5\alpha+\beta)^{-3/2}\}\\&\quad-(64/3\sqrt{3})(\alpha+\beta)^{-3}]. \qquad (2.7)\end{aligned}$$

In the two-nucleon capture model under consideration, the possible transitions of the nucleon pair are given in Table I (see Ref. 17).

A. Proton Mode

We first derive the transition rate of ${}^{13}S_0\rightarrow{}^{33}P_0$ in the proton mode, $\pi^-+p+n\rightarrow n+n$. By separating the center-of-mass and relative motions of the outgoing nucleons, the final state is given by the form

$$\begin{aligned}\Psi_F(123)&=V^{-3/2}\exp[i(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathbf{R}_0]\\&\quad\times\prod_{ijk=1}^{3}\phi_{lJ}{}^{ST}(\mathbf{P}_k\mathbf{r}_{ij})X(123), \qquad (2.8)\end{aligned}$$

where $\mathbf{P}_k=\frac{1}{3}(\mathbf{k}_i-\mathbf{k}_j)$, $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$, $\mathbf{R}_0=\frac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)$, and the $X(123)$'s denote the spin and isobaric-spin functions. Taking the final-state interactions into account, we take the asymptotic form[18] for the wave functions describing the relative motions of outgoing nucleons:

$$\begin{aligned}\phi_{lJ}{}^{ST}(\mathbf{Pr})&=4\pi\sum i^l\exp(i\delta_{lJ}{}^{ST})[A_{lJ}{}^{ST}(P)j_l(Pr)\\&\quad-B_{lJ}{}^{ST}(P)n_l(Pr)]Y_{lm}{}^*(\hat{r})Y_{lm}(\hat{P}),\end{aligned}$$

composed of the spherical Bessel and Neumann functions, where

$$\begin{aligned}&A_{ll}{}^{ST}(P)=\cos\delta_{ll}{}^{ST}, &&B_{ll}{}^{ST}(P)=\sin\delta_{ll}{}^{ST},\\&A_{10}{}^{11}(P)=\cos\delta_{10}{}^{11}, &&B_{10}{}^{11}(P)=\sin\delta_{10}{}^{11},\end{aligned}$$

$$A_{01}{}^{ST}(P)=\cos\epsilon_1\cos\delta_{01}{}^{ST}-\sin\epsilon_1\cos\delta_{21}{}^{ST},$$

$$B_{01}{}^{ST}(P)=\cos\epsilon_1\sin\delta_{01}{}^{ST}-\sin\epsilon_1\sin\delta_{21}{}^{ST}.$$

[17] P. Huguenin, Nucl. Phys. **41**, 534 (1963); T. Kohmura, Progr. Theoret. Phys. (Kyoto) **34**, 234 (1965).

[18] J. J. deSwart and R. E. Marshak, Phys. Rev. **111**, 272 (1958); A. Reitan, Nucl. Phys. **36**, 56 (1962).

Here, $\delta_{lJ}{}^{ST}$ and ϵ denote, respectively, the phase shifts and phase parameters, which are determined from the nucleon-nucleon scattering analysis.

In the spin and isobaric-spin spaces, the matrix element becomes

$$\langle {}^{33}P_0{}^{nn}|H|{}^{13}S_0{}^{pn}\rangle_p = \tfrac{1}{6}G\mu\phi_0(0)\langle\psi_F(123)-\psi_F(213)|\nabla_0{}^{(2)}-\nabla_0{}^{(1)}|\psi(123)\rangle\,. \tag{2.9}$$

Since the meson wave function scarcely varies in the region of the nucleus, it has been approximated by its value at the origin. And in the derivation of the matrix element (2.9), we have made use of the following relations:

$$\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}=\textstyle\sum_\mu \sigma_{-\mu}\nabla_\mu\,,$$

$$\sigma_{\pm1}=\pm\tfrac{1}{2}\sqrt{2}(\sigma_x\pm i\sigma_y)\,,\qquad \boldsymbol{\nabla}_{\pm1}=\mp\tfrac{1}{2}\sqrt{2}(\boldsymbol{\nabla}_x\pm i\boldsymbol{\nabla}_y)\,,$$

$$\sigma_0=\sigma_z\,,\qquad \boldsymbol{\nabla}_0=\boldsymbol{\nabla}_z\,.$$

It is straightforward to evaluate the matrix element with respect to the space parts of the initial and final states, using the gradient formula

$$\begin{aligned}\nabla_q\phi_l(r)Y_{lm}(\hat{r})=&(-)^{q+1}\left(\frac{l+1}{2l+1}\right)^{1/2}(l+1\;m+q\;1\;-q|l\;m)\\&\times\left(\frac{d}{dr}-\frac{l}{r}\right)\phi_l(r)Y_{l+1m+q}(\hat{r})\\&-(-)^{q+1}\left(\frac{l}{2l+1}\right)^{1/2}(l-1\;m+q\;1\;-q|l\;m)\\&\times\left(\frac{d}{dr}+\frac{l+1}{r}\right)\phi_l(r)Y_{l-1m+q}(\hat{r})\,.\end{aligned}$$

Thus, the matrix element describing the transition ${}^{13}S_0\longrightarrow{}^{33}P_0$ becomes

$$\begin{aligned}\langle {}^{33}P_0{}^{nn}|H|{}^{13}S_0{}^{pn}\rangle_p&=-i(32/9\sqrt{2})G\mu\phi_0(0)(2\pi)^5V^{-3/2}\\&\times N(\alpha,\beta)\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)Y_{10}(\hat{P}_3)P_3\\&\times\prod_i\exp\{-i\delta_{l_iJ_i}{}^{S_iT_i}(P_i)\}\\&\times\left\{A_{l_iJ_i}{}^{S_iT_i}(P_i)\frac{\sqrt{\pi}}{4}\left[\frac{1}{\alpha\sqrt{\alpha}}\exp\left(-\frac{P_i^2}{4\alpha}\right)\right.\right.\\&\left.-\frac{1}{\beta\sqrt{\beta}}\exp\left(-\frac{P_i^2}{4\beta}\right)\right]+B_{l_iJ_i}{}^{S_iT_i}(P_i)\\&\left.\times\frac{1}{2}\left[\frac{1}{P_i}(1/\alpha-1/\beta)-F(\alpha,\beta;P_i)\right]\right\}\,,\end{aligned}\tag{2.10}$$

where

$$(S_1,T_1,l_1,J_1)=(S_1,T_1,0,S_1)\,,$$
$$(S_2,T_2,l_2,J_2)=(S_2,T_2,0,S_2)\,,$$
$$(S_3,T_3,l_3,J_3)=(1,1,1,0)\,,$$

and

$$\begin{aligned}F(\alpha,\beta;P)=&\frac{1}{\alpha\sqrt{\alpha}}\exp\left(-\frac{P^2}{4\alpha}\right)\int_0^{P/2\sqrt{\alpha}}\exp(t^2)dt\\&-\frac{1}{\beta\sqrt{\beta}}\exp\left(-\frac{P^2}{4\beta}\right)\int_0^{P/2\sqrt{\beta}}\exp(t^2)dt\,.\end{aligned}$$

In a similar way we can evaluate the matrix elements,

$$\langle {}^{33}P_1{}^{nn}|H|{}^{31}S_1{}^{pn}\rangle_p\quad\text{and}\quad\langle {}^{33}P_0{}^{nn}|H|{}^{13}S_0{}^{pp}\rangle_p\,,$$

corresponding to the transitions ${}^{33}S_1\longrightarrow{}^{33}P_1$ and ${}^{13}S_0\longrightarrow{}^{33}P_0$. Fortunately, no interferences among these three matrix elements exist. Therefore the calculation of the transition rates may be performed independently, by using the Golden rule

$$W=(2\pi/\hbar)\sum|\langle f|H|i\rangle|^2\times\delta((3\hbar^2/2M)(\mathbf{P}_1{}^2+\mathbf{P}_2{}^2+\mathbf{P}_3{}^2)+E_s-\mu c^2)\rho_f\,,$$

where E_s is the separation energy to the final state $p+n+n$. In this framework the capture rates of π^- mesons by the nucleon pairs in He^3 are obtained in the following forms:

$$\begin{aligned}dW({}^{33}P_0{}^{nn}|{}_p{}^{pn})/dP_3&=(128M/3\hbar^3)(2\pi)^3(G\mu)^2|\phi_0(0)|^2\\&\times N^2(\alpha,\beta)D_{10}{}^{11}(\alpha,\beta;P_3)\\&\times\int_{\min}^{\max}\left[\frac{1}{P_1P_3}\left\{\frac{M}{3\hbar^2}(\mu c^2-E_s)-P_1{}^2-P_3{}^2\right\}\right]^2\\&\times M^2(S_1,T_1,S_2,T_2;\alpha,\beta;P_1,P_3)P_1dP_1\,,\end{aligned}\tag{2.11}$$

$$\begin{aligned}dW({}^{33}P_1{}^{nn}|{}_p{}^{pn})/dP_3&=(256M/3\hbar^3)(2\pi)^3(G\mu)^2|\phi_0(0)|^2\\&\times N^2(\alpha,\beta)D_{11}{}^{11}(\alpha,\beta;P_3)\\&\times\int_{\min}^{\max}\left[1-\frac{1}{P_1{}^2P_3{}^2}\left\{\frac{M}{3\hbar^2}(\mu c^2-E_s)-P_1{}^2-P_3{}^2\right\}^2\right]\\&\times M^2(S_1,T_1,S_2,T_2;\alpha,\beta;P_1,P_3)P_1dP_1\,,\end{aligned}\tag{2.12}$$

and

$$\begin{aligned}dW({}^{33}P_0{}^{pn}|{}_p{}^{pp})/dP_3&=(256M/3\hbar^3)(2\pi)^3(G\mu)^2|\phi_0(0)|^2\\&\times N^2(\alpha,\beta)D_{10}{}^{11}(\alpha,\beta;P_3)\\&\times\int_{\min}^{\max}M^2(S_1,T_1,S_2,T_2;\alpha,\beta;P_1,P_3)P_1dP_1\,,\end{aligned}\tag{2.13}$$

where $D_{lJ}{}^{ST}(\alpha,\beta;P_3)$ and $M(S_1,T_1,S_2,T_2;\alpha,\beta;P_1,P_3)$

have rather complex forms. They are defined as follows:

$$D_{lJ}^{ST}(\alpha,\beta;\,P_3)=P_3^3\biggl\{A_{lJ}^{ST}(P_3)\frac{\sqrt{\pi}}{4}\times\biggl[\frac{1}{\alpha\sqrt{\alpha}}\exp\biggl(-\frac{P_3^2}{4\alpha}\biggr)-\frac{1}{\beta\sqrt{\beta}}\times\exp\biggl(-\frac{P_3^2}{4\beta}\biggr)\biggr]+B_{lJ}^{ST}(P_3)\times\frac{1}{2}\biggl[\frac{1}{P_3}\biggl(\frac{\beta-\alpha}{\alpha\beta}\biggr)-F(\alpha,\beta;\,P_3)\biggr]\biggr\}^2,$$

and

$$M(S_1,T_1,S_2,T_2;\,\alpha,\beta;\,P_1,P_3)=P_3^{-3}(2ME-P_1^2-P_3^2)^{-3/2}D_{0S_1}^{S_1T_1}(\alpha,\beta;\,P_1)\times D_{0S_2}^{S_2T_2}(\alpha,\beta;\,(2ME-P_1^2-P_3^2)^{1/2}),$$

with $E=(1/3\hbar^2)(\mu c^2-E_s)$. Of course, the capture rates obtained above must be summed up over the allowed values of quantum numbers, S_1, S_2, T_1, and T_2 after being multiplied by their respective weights.

B. Deuteron Mode

In this subsection we derive the transition rate leading to the final state of a deuteron and a neutron. The deuteron mode means the one-nucleon ejection produced by π^--meson absorption, while the proton mode evaluated in Subsec. 2.A corresponds to the two-nucleon ejection.

From the qualitative arguments of the π^--absorption rate by He^4 on the basis of the two-nucleon capture model, Ammiraju and Biswas[2] concluded that the deuteron and triton modes should be the dominant processes. On the other hand, Eckstein[19] obtained results in complete disagreement with their conclusion. In the two-nucleon capture model it is likely that two fast nucleons should be ejected from the nucleus, and the probability that one of these nucleons would then stick to the residual nucleus would be very small indeed. It is, however, important to note that the two-nucleon capture model means not only that the available momentum is shared by two nucleons, but that the capture of a pion proceeds primarily in the presence of two correlated nucleons. As is seen in Sec. 1, the stopped π^- meson is absorbed by the nucleons moving with momenta over 500 MeV/c in the nucleus. It is unlikely that the individual nucleons move with such a momentum in the nucleus. The strongly correlated nucleon pair, however, can have relatively high momentum components. It is possible that one nucleon of this pair remains in the nucleus with a momentum below the Fermi momentum and the other is ejected out of the nucleus with high relative momentum after the π^--meson absorption. Thus, there is no reason to believe *a priori* that the two-nucleon capture model necessitates two fast nucleons in the final state.

For the reasons mentioned above, it may be very interesting to evaluate the ratio of the capture rate for the two-nucleon ejection to that for the one-nucleon ejection.

In order to calculate the absorption rate for the deuteron mode, we take the final state in the form

$$\Psi_F(\mathbf{r}_1,\mathbf{r}_2;\,\mathbf{r}_3)=V^{-1}\exp[i\{\mathbf{k}\cdot\mathbf{r}_3+\mathbf{K}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2\}]\times\varphi_d(\mathbf{r}_{12})X(123)\,,\quad(2.14)$$

where $\varphi_d(\mathbf{r}_{12})$ is the wave function of a deuteron. The effects of final-state interactions are expected to be very important in the proton mode. However, in the deuteron mode we do not take final-state interactions into account because in this case the relative momentum of the final-state particles would seem to be sufficiently high so that the effects of final-state interactions would be small. This is reasonable because the deuteron-neutron elastic scattering near 100 MeV can be interpreted by the impulse approximation.[20]

Thus, the matrix element for the deuteron mode yields

$$\langle f|H|i\rangle=-\tfrac{1}{2}(\langle f|H|{}^{13}S_0{}^{pp}\rangle-\langle f|H|{}^{13}S_0{}^{pn}\rangle+\langle f|H|{}^{31}S_1{}^{pn}\rangle)\,.\quad(2.15)$$

The absorption rate is, then,

$$W_d=\frac{2\pi}{\hbar}\int|\langle f|H|i\rangle|^2\delta\biggl(\frac{\hbar^2K^2}{2M_d}+\frac{\hbar^2k^2}{2M}+E_B-\mu c^2\biggr)\times\frac{V}{(2\pi)^6}d\mathbf{k}d\mathbf{K}$$
$$=\frac{0.5185}{4184\hbar}(G\mu)^2|\phi_0(0)|^2N^2(\alpha,\beta)(2\pi)^6\frac{2M}{3\hbar^2}\times\biggl[\frac{4M}{3\hbar^2}(\mu c^2-E_B)\biggr]^{3/2}\times(\alpha^{-3/2}-\beta^{-3/2})\mathfrak{F}^2(\alpha,\beta;\,[(4M/3\hbar^2)(\mu c^2-E_B)]^{1/2})\times\biggl\{\alpha^{-3/2}\exp\biggl[-\frac{M(\mu c^2-E_B)}{3\alpha\hbar^2}\biggr]-\beta^{-3/2}\exp\biggl[-\frac{M(\mu c^2-E_B)}{3\beta\hbar^2}\biggr]\biggr\}^2,\quad(2.16)$$

where E_B is the separation energy of He^3 into $d+n$ and

$$\mathfrak{F}(\alpha,\beta;\,K)=\int_0^\infty\varphi_d(r)j_0(\tfrac{1}{2}Kr)\times[\exp(-\alpha r^2)-\exp(-\beta r^2)]r^2dr\,.$$

[19] S. G. Eckstein, Phys. Rev. **129**, 413 (1963).

[20] Y. Sakamoto and T. Sasakawa, Progr. Theoret. Phys. (Kyoto) **27**, 879 (1959).

Taking the limiting value of $\beta=\infty$, we can obtain the rate for π^--meson capture containing no correlations.

3. NUMERICAL RESULTS AND DISCUSSION

In the present calculation, we used $\alpha=0.26\ F^{-2}$, which was determined from the electron-scattering data about the nucleon distribution for He^3 and the coupling constant $G^2/4\pi=(\mu c^5/\hbar)f^2$ ($f^2=0.08$).[21] Using the Hulthén-type wave function[22]

$$\varphi_d(\mathbf{r}_{12})=1.85(1/r)[\exp(-0.232r)-\exp(-1.202r)]Y_{00}(\hat{r})$$

for the outgoing deuteron, we obtain the result that the absorption rate for the deuteron mode is

$$W_d=1.03\times10^{17}\ \mathrm{sec}^{-1} \tag{3.1}$$

without the nuclear correlations, i.e., with $\beta_0=\infty$. Figure 1 shows the ratio of the capture rate $W_d(\beta_0)$ with the correlation effects to $W_d(\beta_0=\infty)$ without the correlation. When we use the zero-range approximation for the relative motion of the two nucleons which absorb the pion,

$$W_d{}^0=0.58\times10^{14}\ \mathrm{sec}^{-1}. \tag{3.2}$$

This value is about 24 times smaller than that obtained by Divakaran.[9]

If we think of the capture as taking place on a pair of nucleons and then one of those nucleons picking up the third nucleon to come out as a deuteron, we would expect that the high-momentum components or short-range correlations in the deuteron wave function should also play an important role. In order to estimate these effects on the capture rate, we adopt the following form as the deuteron wave function with the hard core.[23]

$$\varphi_d(r)=N\frac{1}{r}\{\exp[-a(r-r_c)]-\exp[-b(r-r_c)]\}\,,\quad r\geq r_c$$
$$=0,\quad r<r_c$$
$$N^2=(ab/2\pi)(a+b)/(a-b)^2.$$

r_c is the core radius and it is taken to be 0.4×10^{-13} cm.

$$a^{-1}=4.31\times10^{-13}\ \mathrm{cm},$$
$$b=\frac{3}{\rho'}[1+4/9a\rho']^{-1}=3.050\times10^{13}\ \mathrm{cm}^{-1},$$
$$\rho'=\rho-2r_c=0.90\times10^{-13}\ \mathrm{cm}.$$

ρ is the triplet effective range, 1.7×10^{-13} cm. The result is

$$\bar{W}_d(\beta_0=\infty)=3.59\times10^{15}\ \mathrm{sec}^{-1}. \tag{3.3}$$

[21] R. M. Spector, Phys. Rev. **134**, B101 (1964).
[22] M. J. Moravcsik, Nucl. Phys. **7**, 113 (1958).
[23] H. Feshbach, Phys. Rev. **107**, 1626 (1957).

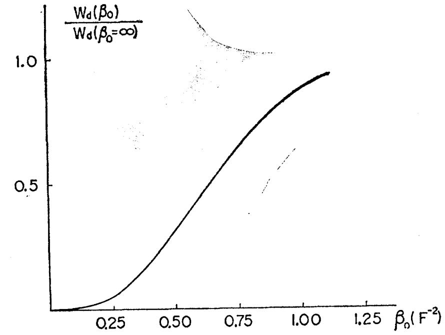

FIG. 1. The ratio of the capture rate for the deuteron mode $W_d(\beta_0)$ with the correlation effects to $W_d(\beta_0=\infty)$ without the correlations.

Capture rates for the proton mode in the case without the final-state interactions and their ratios to that for the deuteron mode (3.1) are listed in Table II. These values seem to be consistent with those obtained by Spector[21] for O^{16} nuclei and by Jibuti and Kopaleishvili[24] for C^{12} nuclei.

By means of the zero-range and plane-wave approximations, Divakaran obtained the result that the ratio of the capture rate for the proton mode to that for the deuteron mode is 4.6. The result obtained by Messiah is 2. In the case without short-range nuclear correlations in the nucleus, our results are 1.49 for the Hulthén-type deuteron wave function and 42.9 for the deuteron wave function with the hard core. However, these ratios become smaller the strength of nuclear correlations increases. This fact may be interpreted as follows: For the deuteron mode, the momentum of outgoing particles takes the only value determined by the momentum-conservation law; while, for the proton mode, since the relative momentum-distributions of the two outgoing nucleons are very sensitive to nuclear correlations, the capture rate obtained by integrating the momentum distribution depends sensitively on the correlations.

In the case with final-state interactions, the branching ratios $W(pn\rightarrow nn)/W(pp\rightarrow pn)$ are 1.64, 0.91, 1.17, and 2.81 for $\beta_0=0.25\ F^{-2}$, $0.55\ F^{-2}$, $0.95\ F^{-2}$, and ∞, respectively, and the capture rates for the proton mode

TABLE II. Results for the case without final-state interactions.

β_0 (F^{-2})	Capture rates for the proton mode (sec^{-1})	Their ratios to the capture rates for the deuteron mode W_p/W_d	$W_p/\bar{W}_d$	Branching ratios $\frac{W(pn\rightarrow nn)}{W(pp\rightarrow pn)}$
0.25	1.16×10^{15}	0.25	7.25	0.85
0.55	9.35×10^{15}	0.24	6.93	1.27
0.95	3.63×10^{16}	0.42	12.1	1.31
∞	1.54×10^{17}	1.49	42.9	2.50

[24] R. I. Jibuti and T. I. Kopaleishvili, Nucl. Phys. **55**, 337 (1964).

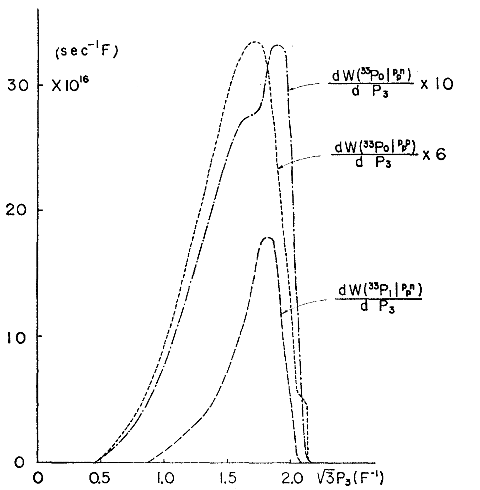

FIG. 2. Relative momentum distributions of two emitted nucleons without the final-state interactions and the nuclear correlations. The factor 10^{16} is multiplied.

become 1.51×10^{15}, 6.62×10^{15}, 9.85×10^{15}, and 1.38×10^{17} sec^{-1} for each value of the correlation parameter β_0. In order to consider the final-state interactions we have used the values of the phase shifts and phase parameters determined from the proton-proton scattering and the proton-neutron scattering analyses.[25] Figures 2 to 10 show the relative-momentum distributions of the two ejected nucleons in the stopped π^--meson absorption. The dotted curves correspond to the transition ${}^{13}S_0\rightarrow{}^{33}P_0$ produced by (pp) pairs. The dashed and dot-dashed curves correspond to the ${}^{31}S_1\rightarrow{}^{33}P_1$ and ${}^{13}S_0\rightarrow{}^{33}P_0$ transitions produced by (np) pairs, respectively. The solid curves represent the total

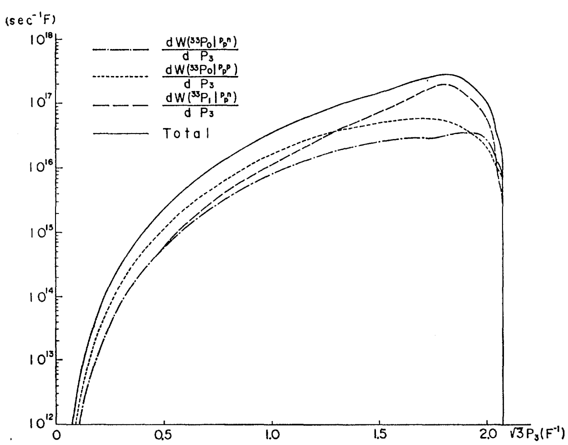

FIG. 3. Relative momentum distributions of two emitted nucleons without the final-state interactions and the nuclear correlations. The logarithmic scale is adopted.

[25] G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960); M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, *ibid.* **122**, 1606 (1961).

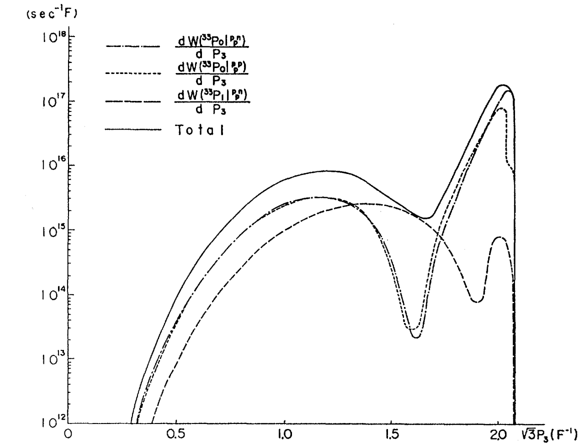

FIG. 4. Relative momentum distributions of two emitted nucleons in the case of $\beta_0=0.95$ F^{-2} without the final-state interactions.

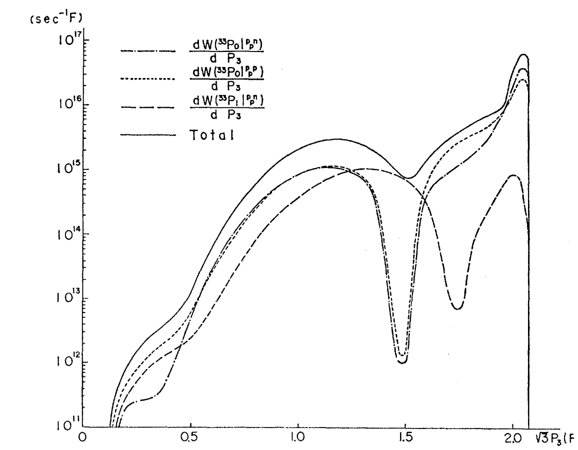

FIG. 5. Relative momentum distributions of two emitted nucleons in the case of $\beta_0=0.55$ F^{-2} without the final-state interactions.

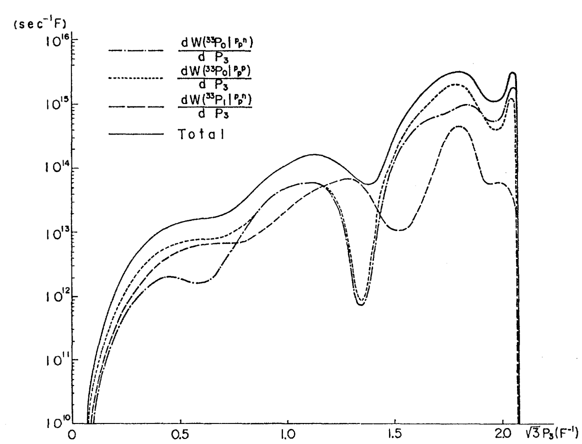

FIG. 6. Relative momentum distributions of two emitted nucleons in the case of $\beta_0=0.25$ F^{-2} without the final-state interactions.

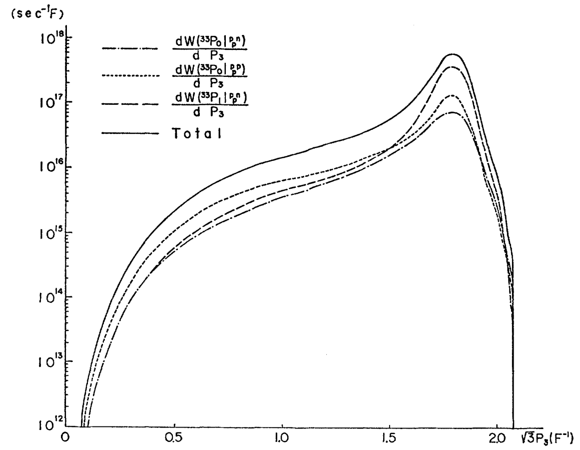

FIG. 7. Relative momentum distributions of two emitted nucleons without the nuclear correlations but with the final-state interactions.

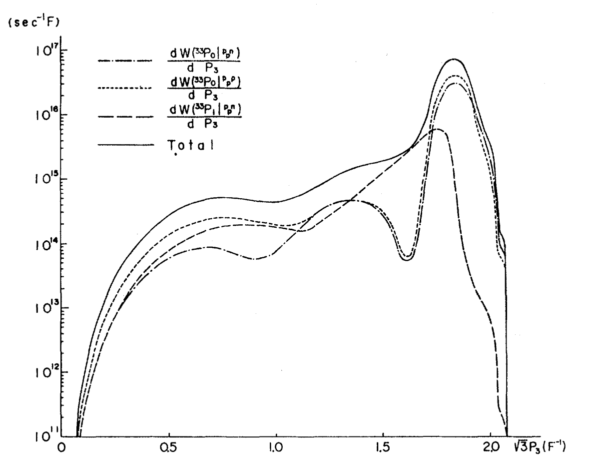

FIG. 8. Relative momentum distributions of two emitted nucleons in the case of $\beta_0=0.95$ F^{-2} with the final-state interactions.

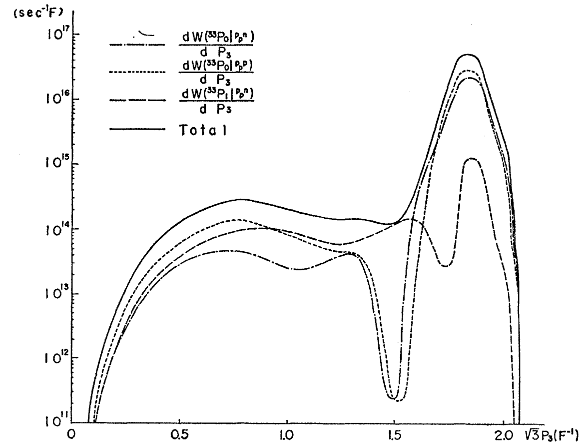

FIG. 9. Relative momentum distributions of two emitted nucleons in the case of $\beta_0=0.55$ F^{-2} with the final-state interactions.

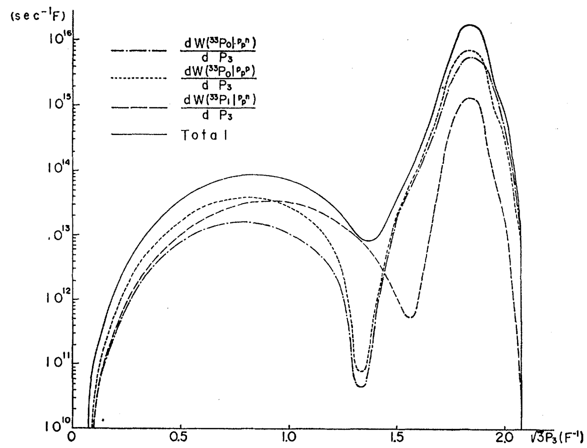

FIG. 10. Relative momentum distributions of two emitted nucleons in the case of $\beta_0=0.25$ F^{-2} with the final-state interactions.

distributions of the relative momentum of the two ejected nucleons.

Figures 2 to 6 show relative-momentum distributions for each value corresponding to parameter β_0 without the final-state interactions, while Figs. 7 to 10 correspond to those with the final-state interactions. If the effects of correlations are not considered, the relative-momentum distributions are shown by simple curves. However, a large dip appears in the distributions because of the effects of nuclear correlations. This fact is very interesting. As the strength of the nuclear correlations increases the depth of the dip becomes larger and its position shifts toward the low-momentum region. This means that the high-momentum parts of the rates increase with the effects of the nuclear correlations. It is reasonable that the two nucleons with high relative momenta should be ejected because they have high-momentum components within the nucleus owing to the existence of the short-range nuclear correlations. It is concluded from such a situation that the determination of the correlation parameter β_0 may be possible by comparison of the position and depth of the dip with the experimental data. Thus we expect to obtain some information about the ground-state wave function of He^3.

ACKNOWLEDGMENTS

The author wishes to express his sincere thanks to Professor M. Kobayasi for constant encouragement. Thanks are due to Dr. Y. Sakamoto for many valuable discussions and to Dr. M. Ozaki for his help in the numerical computation. The author is indebted to C. Nguyen-Trung for preparation of the present paper.