Extended Electromagnetic Structure Effects on Low-Energy Proton-Proton Scattering

R. J. SLOBODRIAN
Lawrence Radiation Laboratory, University of California, Berkeley, California
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The corrections to low-energy $^1S_0$ p-p scattering parameters, arising from the extended electromagnetic structure of the proton, are calculated in the context of nuclear-interaction models without a static core, with a soft core, and also with a hard core of radius $r_c=0.40$ F. A comparison of the corrected $^1S_0$ p-p scattering length "neutralized" with an approximate formula $(a_p)^n$ is made with the $^1S_0$ n-n scattering length a_n, recently determined by reliable experimental methods with reasonable accuracy.

I. INTRODUCTION

THE accuracy of the measurements of proton-proton scattering cross sections in the low-energy region (between 0 and 5 MeV laboratory energy) has called in the past for refined calculations of departures from the simple Coulomb potential-energy term $V_C=e^2/r$, like the vacuum polarization contribution,[1,2] or the point magnetic-dipole-moment interaction effects,[3] the latter proven to be unimportant[4] on the assumption that the singlet-S nucleon-nucleon interaction is due to a static potential with a repulsive core.[5] Recently a calculation has been reported in order to determine the bearing of the corrections due to the electromagnetic structure of the nucleons[6] on charge independence.[7] Such calculation was carried out using a hard-core model for the nucleon-nucleon interaction. The shape-independent parameters for p-p scattering were kept constant and the corresponding parameters for the n-n and n-p scattering were calculated. However, it is accepted presently that there are no theoretical reasons to believe that real hard cores exist, although no clear statement can be made concerning the nucleon-nucleon interaction at very small distances.[8] The hard-core assumption has to be looked upon as a calculational simplification of the repulsive effects observed at about 300 MeV in the singlet S-wave phase shift.[9] Therefore it is advisable to explore the corrections to low-energy proton-proton scattering parameters in the light of soft-core potentials[10] and velocity-dependent potentials.[11,12] The latter have been preferred for mathematical reasons in calculations dealing with applications of nucleon-nucleon potentials to nuclear matter,[12,13] and also for calculations of the triton binding energy.[14] It has been suggested in a recent letter[12] that a major source of the reduced masses encountered in phenomenological nucleon-nuclear potentials arises from the explicit velocity dependence of the nucleon-nucleon potential itself. The velocity-dependent potential of Green[11] generates a central core dynamically, and at low energies the core is negligible. In order to illustrate this point Fig. 1 shows the radial dependence of the effective potential expressed by Eq. (2.4) of Ref. 11. The velocity-dependent potential of Green and Sharma[12] for $l=0$ is attractive at low energies

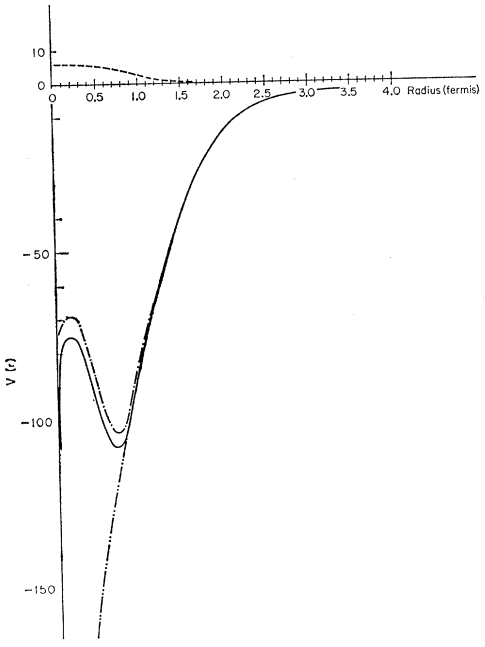

FIG. 1. Effective potential of Green (Ref. 11). The solid line is the static part $V'(r)$ for singlet even states. The dashed line is the small effect due to the dynamical core for $E_{lab}=6.9435$ MeV. The dash-dot line is the resulting $V'(r)$ adding the dynamical contribution at $E_{lab}=6.9435$ MeV. The dash-double-dot line is the radial dependence of the static central (tensor and spin-orbit) potentials $V(r)$. The units of $V(r)$ are 2.4=1 MeV.

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and it becomes strongly repulsive at high energies, as illustrated in Fig. 2(a) of Ref. 12.

Several recent letters[15,16] and papers[17] have dealt with the analysis of the reputedly very accurate phase shifts obtained from low-energy p-p scattering experiments[18] in terms of scattering parameters, and also with comparisons of the latter with theoretical values obtained from different models for the p-p interaction.[15] It has been claimed that best agreement was found with the Coulomb-corrected partial-wave dispersion relation (PWDR).[15] The boundary-condition model (BC) was ruled out because it gave the wrong sign for the so-called shape parameter P.[15] The conclusion was based principally on the determination of the parameter P from the experimental data; such a determination has been the object of doubts by Breit[19] and of subsequent critical analysis.[16,17] It is no longer clear that the value of P is firmly established. The vacuum polarization correction (VPC) due to Foldy and Eriksen[1] dominates the curvature of the low-energy region, and as it is not yet possible to claim a stable value of the curvature[16] (and of P), the accuracy of this correction has not been established for S-wave p-p scattering. Finally, the comparison made in Ref. 15 did not include corrections of the scattering parameters due to the extended electromagnetic structure of the protons.

II. CALCULATION OF THE ELECTROMAGNETIC CORRECTIONS

If one believes in a quantum mechanical Hamiltonian formulation of the proton-proton interaction, it should be describable in terms of a potential

$$V_{pp}=V_N+V_{ES}+V_M+\sum_i V_i, \tag{1}$$

where V_N is attributed to nuclear effects, V_{ES} is the energy due to the electrostatic interaction, V_M is due to the magnetic-dipole interaction and $\sum_i V_i$ is the contribution, if any, of other interactions that may be present. It is usually assumed that $\sum_i V_i$ is very small compared with the remaining terms, and it is safe to ignore it. The S-wave scattering is described by the well-known amplitude

$$f(\theta)=f_c(\theta)+(1/2ik)e^{i\zeta_0}(e^{2i\delta_0}-1), \tag{2}$$

where $\zeta_0=\arg\Gamma(1+i\eta)$, η is the Coulomb parameter, $\eta=e^2/(\hbar^2 v_{\rm lab})$ ($\hbar$ is Planck's constant divided by 2π, e is the charge of the proton, and $v_{\rm lab}$ is the relative velocity), δ_0 is the S-wave "nuclear" phase shift, and f_c is the Coulomb-scattering amplitude. Of course (2) has to be adequately symmetrized. The accuracy of the experimental information has required the consideration of P-wave contributions and also relativistic corrections to η.[20]

Conventionally it has been assumed that $V_{ES}=V_C$ and the nuclear field was parametrized in order to obtain the phase shift δ_0 in agreement with the experimental value. This step, as is well known, is relevant for the comparison with n-p $^1S_0$ potentials,[21] and also in the future with n-n potentials, in view of the increasing accuracy and reliability of n-n scattering parameters,[22,23] in order to settle the old questions of charge independence and charge symmetry of the nucleon-nucleon interaction.

The high-energy electron scattering experiments[6] have proven beyond doubt that there is a positive extended charge in the proton with an rms radius of 0.8 F, and also a distributed magnetic moment of slightly larger radius. It is the purpose of this paper to present some calculations concerning the effects due to the electromagnetic structure of the protons on low energy S-wave p-p scattering parameters using the first-order perturbation technique employed earlier by Foldy and Eriksen[1] to correct for vacuum polarization effects. We will assume that the charge and magnetic-dipole distributions overlap at short distances without appreciable distortion.

Concentrating momentarily on the electrostatic effects, let us state that if the protons are pictured as uniformly charged spheres of radius R the potential energy is given by the well-known expressions

$$V_i(r)=\frac{e^2}{R^4}\left[(6/5)R^3-\tfrac{1}{2}Rr^2+\tfrac{3}{16}r^3-(1/160)(1/R^2)r^5\right], \quad 0\le r\le 2R, \tag{3}$$

$$V_0(r)=e^2/r, \quad r\ge 2R \tag{4}$$

where r is the distance between the centers of the spheres and e is the electric charge of each sphere. If the radius of the sphere R is taken equal to the rms radius of the actual charge distribution the results do not differ appreciably from a more realistic calculation, using a charge density $\rho=ke^{-\alpha r}$. The potential function of such a charge distribution is given by

$$U(r)=e[1/r-e^{-\alpha r}(1/r+\alpha/2)]. \tag{5}$$

The potential energy of two such exponential charge distributions can be obtained in closed form, but it is too lengthy to be presented here. The same applies to the term due to the magnetic interaction.[24] Figure 2

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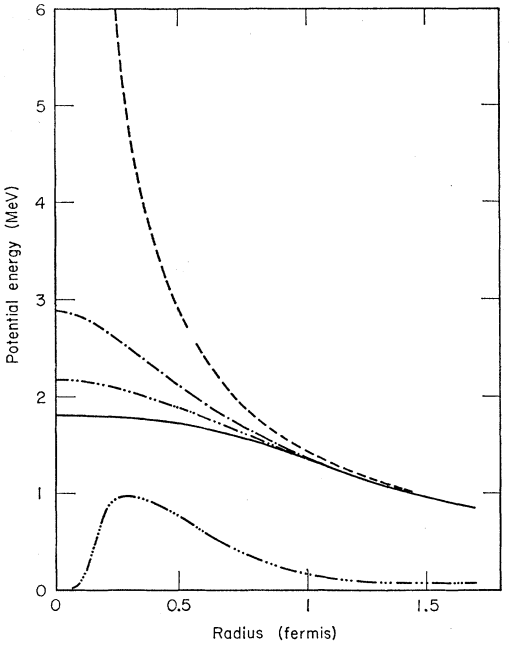

FIG. 2. Potential energy due to the electrostatic and magnetic-dipole interaction. The dashed line corresponds to the point-charge model; the dash-dot line corresponds to a point charge in the field of an exponential charge $\rho=ke^{-\alpha r}$; the dash-double-dot line corresponds to two uniformly charged spheres, the solid line corresponds to two exponential charge distributions consistent with the electron scattering experiments. The dash-triple-dot line corresponds to the extended magnetic-dipole interaction.

summarizes the results, it also includes the potential energy of a point charge in the field given by (5).

Presently it is customary to use the following representation for the low-energy p-p scattering[25]:

$$F=C^2k\cot\delta_0+(1/R)h(\eta)=-1/a_p+\tfrac{1}{2}r_ek^2-Pr_e^3k^4+Qr_e^5k^6+\cdots, \quad (6)$$

where

$$C^2=\frac{2\pi\eta}{e^{2\pi\eta}-1}, \quad k^2=\frac{M_pE}{2\hbar^2}, \quad R=\frac{\hbar^2}{M_pe^2},$$

$$h(\eta)=\mathrm{Re}\frac{\Gamma'(-i\eta)}{\Gamma(-i\eta)}-\ln\eta;$$

(E is the laboratory energy, M_p is the proton mass). η is again the Coulomb parameter, r_e is usually called effective range, and P is the shape parameter. The last term written explicitly in expansion (6) contains another shape-dependent parameter, Q. The very accurate data available today at five energies between 0 and 3.037 MeV, together with some higher energy data, do not favor a term in k^6 of comparable importance with the term in k^4 at 3 MeV.[26] The same conclusion is reached in the light of theoretical calculations for different well shapes.[27] Consequently we will ignore the k^6 term for the time being.

It is convenient for the following calculation to use the function

$$K=RF=A+BE+CE^2 \quad (7)$$

employed also by Foldy and Eriksen[1,2] for the VPC. E is the laboratory energy of the protons. It is clear that if the electrostatic energy, within some range, is not given by e^2/r, but rather by functions like those of Fig. 2, the electrostatic energy has been overestimated. Consequently what is conventionally attributed to "nuclear" interaction contains in part effects of the excess of electrostatic energy

$$V_d=V_C-V_{ES}. \quad (8)$$

V_d can be classified as a short-range perturbation. To correct this we can calculate the change induced in the function K by a potential-energy change $\Delta V_{ES}=-V_d$. Analogously we can handle the effect due to the extended magnetic dipole potential V_M; here we would have $\Delta V_M=V_M$. As long as the quadratic term of (7) is small we can make use of first-order perturbation theory and calculate ΔK ($\Delta K\cong B\Delta E$) as

$$\Delta K=\frac{M_pC^2R}{\hbar^2}\int_0^{R_f}\Delta V(r)u^2(r)dr, \quad (9)$$

where

$$u(r)=[F(r)\cot\delta_0+G(r)][1-e^{-\beta(r-r_c)}], \quad r\geq r_c$$
$$u(r)=0, \quad r\leq r_c. \quad (10)$$

$F(r)$ and $G(r)$ are the regular and irregular Coulomb wave functions adequately normalized. The parameter β is chosen so as to give the correct effective range r_e for the nuclear potential if the Coulomb interaction is neglected within the range of the nuclear forces. The parameter r_c is the core radius, which can be taken to be zero for an effective potential like the one proposed by Refs. 11 or 12. For a soft-core potential of the Gartenhaus type,[10] $u(r)$ was calculated numerically. The cutoff radius R_f was taken to be 1.6 F for the electrostatic correction calculation, and 4.0 F for the magnetic-dipole contribution, in order to achieve a relative accuracy of about 0.01% in the calculation of both corrections. Table I contains the results. The energy dependence of the different ΔK corrections is weak, and therefore a linear interpolation is adequate to obtain the values at energies other than those listed in the table. It is apparent that the electrostatic correction due to extended charge effects is largely compensated by the magnetic-dipole contribution. Table II exhibits the total $\Delta K_T=\Delta K_{ES}+\Delta K_M$ (sum of the

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[26] R. J. Slobodrian (to be published) and UCRL-16690 (unpublished).

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TABLE I. Corrections of the values of the K function[a]: Columns I to V contain the corrections due to extended charge effects: Column I corresponds to a point charge in the field of an exponential distribution, column II corresponds to the overlap of two uniformly charged spheres of radius $R=0.8$ F, column III corresponds to the overlap of two exponential charge distributions, columns IV and V contain the corrections due to the overlap of exponential charge distributions with a hard core of radius $r_c=0.4$ F and with a soft core, respectively. Columns VI, VII, and VIII contain the corrections due to magnetic-dipole effects; column VII contains the corrections with a hard core of radius $r_c=0.4$ F, and column VIII corresponds to a soft core.

E_{lab} (MeV)	ΔK I	II	III	IV	V	VI	VII	VIII
0.1000	−0.092715	−0.106502	−0.121335	−0.015260	−0.032300	0.097406	0.033855	0.056744
0.1562	−0.092793	−0.106571	−0.121375	−0.015282	−0.032340	0.097483	0.033873	0.056770
0.2777	−0.092929	−0.106741	−0.121666	−0.015309	−0.032386	0.097741	0.033953	0.056901
0.3999	−0.093058	−0.106879	−0.121875	−0.015345	−0.032469	0.097881	0.034053	0.057071
0.6249	−0.093289	−0.107144	−0.122167	−0.015400	−0.032591	0.098199	0.034180	0.057286
0.9764	−0.093708	−0.107619	−0.122733	−0.015509	−0.032782	0.098835	0.034486	0.057664
1.7359	−0.094558	−0.108574	−0.123855	−0.015730	−0.033206	0.100092	0.034954	0.058555
2.4997	−0.095437	−0.109554	−0.124989	−0.015946	−0.033614	0.101370	0.035466	0.059407
3.9057	−0.097049	−0.111369	−0.127077	−0.016336	−0.034380	0.103718	0.036339	0.060971
6.9435	−0.100477	−0.115221	−0.131603	−0.017204	−0.036087	0.108665	0.038373	0.064266

[a] Tabulated values of the Coulomb wave functions were used for the calculation: M. Abramovitz, *Tables of Coulomb Wave Functions* Vol. I (National Bureau of Standards, Washington, D. C., 1952); A. V. Luk'yanov, I. V. Teplov, and M. K. Akimova, *Whittaker Functions* (The Macmillan Company, New York, 1965), and references therein.

electrostatic and magnetic corrections), the function K contains already the VPC. It is worth while noting that the correction ΔK_T calculated using a model without a static core is of opposite sign to the one given by models with a soft or a hard core. Reducing the core radius would therefore produce a cancellation of the magnetic and electrostatic corrections at a finite value of r_c. Table III shows the parameters obtained through shape-dependent (SD) and shape-independent (SI) fits to the values of K' given in Table II. The SD fits are also given excluding the point at 0.3825 MeV, in order to indicate the rather strong dependence of the parameter P on it,[16] and also because the method of determination of the $^1S_0$ phase shift for such a point differs from the more conventional one employed for the remaining four points. The exclusion of this point from the SI fits produces small changes in the SI scattering parameters, quite close to the changes induced by such exclusion in the SI parameters of the SD fits, and therefore they are not reproduced in Table III.

An inspection of Table III reveals that the uncertainty of the scattering length due to the extended electromagnetic structure of the proton is much greater than the errors quoted in Ref. 15, because of the lack of a precise knowledge of the wave function at small distances. It is about 1.3%, whereas the errors are 0.1% for the SD fit and 0.06% for the SI fit.

III. CHARGE SYMMETRY

In order to establish the degree of validity of the principle of charge symmetry it is necessary to compare the "neutralized" proton-proton $^1S_0$ scattering length $(a_p)^n$ with the neutron-neutron scattering length, for which significantly more accurate values have been produced recently.[22,23] To that effect it is advisable to perform the corrections ΔK_{ES} due only to the extended charge of proton on the function K, because the n-n electromagnetic corrections and the p-p magnetic-dipole effects are nearly equivalent,[7] and thus the resulting $(a_p)^n$ can be compared directly with the recent values for a_n. Table IV contains the values K'' of the corrected function, and Table V shows the resulting scattering parameters. Presently the comparison can be based on the SI parameters and therefore on the SI fits to the experimental data. The recent reliable values of a_n are as follows: $a_n=16.4\pm1.3$ F,[22] and $a_n=16.1\pm1.0$ F.[23] It seems permissible to average both values, and consequently we can adopt a value $a_n=16.25\pm0.8$ F. If

TABLE II. Extended electromagnetic structure correction to the values of the K function ΔK_T. Column I corresponds to a "no-core" model (or to a model with a dynamic core like the one due to Green[a]); column II corresponds to a hard-core calculation with $r_c=0.4$ F; column III corresponds to a soft-core calculation. The table is given for the energies at which very accurate experimental cross sections have been measured.[b]

E_{lab} (MeV)	K	I ΔK_T	K'	II ΔK_T	K'	III ΔK_T	K'
0.3825	3.86501	−0.023984	3.88899	0.018699	3.84631	0.024590	3.84042
1.397	4.35428	−0.023820	4.37810	0.019114	4.33517	0.025140	4.32914
1.855	4.57406	−0.023741	4.59780	0.019267	4.55479	0.025418	4.54864
2.425	4.84212	−0.023633	4.86575	0.019492	4.82263	0.025751	4.81637
3.037	5.13318	−0.023520	5.15670	0.019705	5.11347	0.026098	5.10708

[a] See Refs. 11 and 12. [b] See Ref. 18.

TABLE III. Calculated scattering parameters from least-squares fits to the corrected values K' listed in Table II. The lines numbered 1–4, 2–5, 3–6 correspond respectively to the K' values of columns I, II, and III of Table II. The lines numbered 4, 5 and 6 are obtained excluding the experimental point at 0.3825 MeV. For comparison the parameters obtained from the point-charge assumption are transcribed, together with predictions of the BC and PWDR models.[a]

Parameter		A	B (MeV^{-1})	C (MeV^{-2})	$-a_p$ (F)	r_e (F)	P
Model: Extended charge							
SD	1	3.70339	0.48674	−0.002771	7.7827	2.8052	0.03004
	2	3.66085	0.48640	−0.002734	7.8731	2.8033	0.02970
	3	3.65499	0.48632	−0.002764	7.8857	2.8028	0.03005
	4	3.71093	0.47966	−0.001227	7.7668	2.7645	0.01391
	5	3.66881	0.47895	−0.001109	7.8560	2.7604	0.01262
	6	3.66277	0.47900	−0.001164	7.8690	2.7606	0.01324
SI	1	3.70904	0.47734	0	7.7708	2.7511	0
	2	3.66630	0.47713	0	7.8611	2.7499	0
	3	3.66063	0.47669	0	7.8736	2.7488	0
Point charge							
SD[b]		3.67934	0.48690	−0.002767	7.8332	2.8062	0.0299
SD[a]					7.8284±0.0080	2.794±0.026	0.026±0.014
SI[a]					7.8163±0.0048	2.745±0.014	0
BC[a]					7.8009	2.687	−0.036
PWDR[a]					7.8259	2.786	0.024

[a] See Ref. 15.
[b] Values calculated by the present author. The slight discrepancy with the values of Ref. 15 is well within experimental errors and is of no consequence to the arguments presented in this paper.

TABLE IV. Extended charge correction ΔK_{ES} to the function K. Column I corresponds to a "no-core" model (or to a model with a dynamic core like the one due to Green[a]); column II corresponds to a hard-core calculation with $r_c=0.4$ F; column III corresponds to a soft-core calculation. The function K is already corrected for vacuum polarization effects.

E_{lab} (MeV)	K	I ΔK_{ES}	I K''	II ΔK_{ES}	II K''	III ΔK_{ES}	III K''
0.3825	3.86501	−0.121845	3.98685	−0.015340	3.88035	−0.032457	3.89747
1.397	4.35428	−0.123354	4.47763	−0.015631	4.36991	−0.033017	4.38730
1.855	4.57406	−0.124032	4.69809	−0.015763	4.58982	−0.033270	4.60733
2.425	4.84212	−0.124879	4.96700	−0.015925	4.85804	−0.033574	4.87569
3.037	5.13318	−0.125787	5.25897	−0.016095	5.14928	−0.033907	5.16709

[a] See Ref. 11.

TABLE V. Calculated scattering parameters from least-squares fits to the corrected values K'' listed in Table IV. The lines numbered 1, 2, 3 correspond, respectively, to the K'' values of columns I, II, and III of Table IV, line 4 contains the values of the fit to the uncorrected function K. The column $(-a_p)^n$ contains the "neutralized" $^1S_0$ p-p scattering length, to be compared with the average value of two recent reliable experiments $a_n=-16.25\pm0.8$.

Parameter		A	B (MeV^{-1})	C (MeV^{-2})	$-a_p$ (F)	r_e (F)	P	$(-a_p)^n$ (F)
SD	1	3.80060	0.48841	−0.002772	7.5835	2.8148	0.02975	
	2	3.69462	0.48712	−0.002748	7.8011	2.8075	0.02974	
	3	3.71149	0.48762	−0.002820	7.7657	2.8103	0.03041	
	4	3.67934	0.48690	−0.002767	7.8332	2.8062	0.02990	
SI	1	3.80626	0.47900	0	7.5723	2.7607	0	16.10
	2	3.70022	0.47779	0	7.7893	2.7537	0	17.14
	3	3.71728	0.47806	0	7.7536	2.7552	0	16.96
	4	3.68771	0.47657	0	7.8157	2.7466	0	17.29

we now use the relation[27]

$$1/(a_p)^n=1/a_p+(1/R)[\ln(R/r_e)-0.330] \quad (11)$$

(all the symbols are as defined earlier), we can calculate the "neutralized" values of the proton-proton scattering length, and compare with the value of the neutron-neutron scattering length a_n. The last column of Table V shows the values of $(a_p)^n$. The best agreement corresponds to the value corrected for extended charge effects in a model without static core. The total spread of values of $(a_p)^n$ is about 7.2%, thus corresponding to changes in the potential parameters of a few tenths of a percent.

IV. CONCLUSIONS

There are two main consequences of the effects due to the extended electromagnetic structure of the protons. The first is that the corrections to be performed to account for such a structure bear an uncertainty due to the lack of knowledge of the interaction at very small distances, much greater than the present accuracy permitted by the low-energy experimental data. Due to the approximate nature of Eq. (11), the second consequence can be stated tentatively as follows: A model without a static core is more consistent with the principle of charge symmetry than models with a static core, or than a model neglecting the electromagnetic structure of the protons. Conversely, if the principle of charge symmetry is taken for granted, the evidence seems to favor models without a static core.

Finally, one should also remark that there are some effects on the remaining scattering parameters, but they are presently within the experimental errors. However it would be valuable to attempt a measurement of the *n-n* effective range with high accuracy, in order to throw additional light on the validity of the principle of charge symmetry.[28] It would make possible a more categorical statement concerning the choice of model for the nucleon-nucleon interaction in general, and also about the interaction at very small distances.

[28] Such a measurement seems feasible using the comparison technique of Ref. 23.

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Giant Dipole Resonances in the *s-d* Shell and Their Electromagnetic Properties*

W. H. Bassichis† and F. Scheck‡
The Weizmann Institute of Science, Rehovoth, Israel
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The Hartree-Fock self-consistent-field calculations of the single-particle oribtals in Ne^{20}, Mg^{24}, and Si^{28} including, besides the $(2s,1d)$ shell, the $(1p)$ and the $(2p,1f)$ shells, is performed. Using these single-particle energies and wave functions we then calculate the giant dipole resonances within the framework of the particle-hole model and their electromagnetic properties such as oscillator strengths, γ widths, and cross sections for the inelastic scattering of high-energy electrons. For the interaction (for both the Hartree-Fock treatment and the particle-hole calculation) a standard Rosenfeld force was chosen, which fits low-energy data in the *s-d* shell. In Mg^{24} and Si^{28} the resulting giant-resonance states form essentially two groups: The first, with lower energies, is built mainly out of particle-hole transitions from the $(2s,1d)$ to the $(2p,1f)$ shell; the second one is built mainly out of $(1p)^{-1}$ $(2s,1d)$ states. This result is in agreement with experimental studies of (p,γ), (γ,p), and (p,n) reactions in these nuclei. For the fine structure within each group, however, the agreement with the experimental data is poorer. This however is not surprising in view of the lack of knowledge about the *radial* shape of the wave functions, the proper form of the interaction, etc. It turns out, moreover, that the lower group contains mainly $K=0$ states whereas in the higher group the $K=1$ states are dominant. This splitting between $K=0$ and $K=1$ states is expected from the collective model and from the experimental situation in the strongly deformed nuclei of the rare-earth and the transuranium group.

I. INTRODUCTION

THE giant dipole resonances in the deformed nuclei of the *s-d* shell are of particular interest in view of the relation between the shell-model and the collective-model description of these states. On the one hand there is clear experimental evidence[1] that the observed splitting of the giant resonance in *heavy* deformed nuclei corresponds to a splitting between the lower ($K=0$) and the higher ($K=1$) states (K being the projection of the angular momentum on the intrinsic z axis) as predicted by the collective model.[2] On the other hand, for the deformed nuclei of the $s-d$ shell, it is known from several experiments[3–7] that the lower of the two observed groups of states is built out of particle-hole configurations where the hole is in the $(2s,1d)$ shell and the particle in the $(2p,1f)$ shell whereas the higher group contains mainly $(1p)^{-1}(2s,1d)$ configurations.

The only theoretical approach to the giant resonances in the deformed $s-d$ shell so far has been the work of

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† Present address: Massachusstts Institute of Technology, Cambridge, Massachusetts.
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