(3) Fitting the cross sections with the two states $DS3$ and $PS1$, the ratio $\sigma(+\text{op})/\sigma(++\text{n})$ for a third background amplitude must lie between $3 < R_{\text{B.G.}} < 6$. This rules out using $PP3$ or $DD5$ or possibly $FP5$ amplitudes as the background partial wave. Thus we establish that $PP1$ should be the background amplitude.

(4) It is possible to get an adequate fit to all the angular distributions at all energies using the three amplitudes $DS3$, $PS1$, and $PP1$. The partial cross sections and the phases relative to $DS3$ at 600 MeV are determined to be:

	$\sigma(+\text{op})$ (mb)	$\varphi_2-\varphi_1$	$\varphi_3-\varphi_1$
(1) $DS3$	3.0		
(2) $PS1$	0.09	180°	
(3) $PP1$	0.3		180°

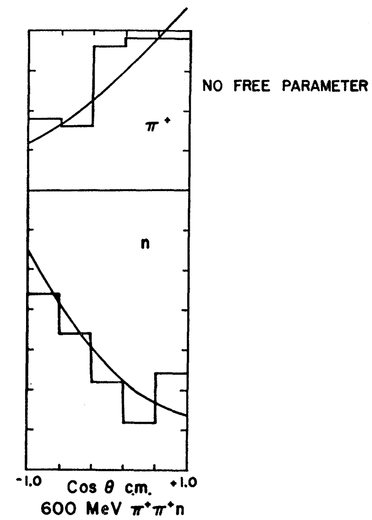

FIG. 8. Fit to the $\pi^+\pi^+n$ angular distributions at 600 MeV using a_3, c_3, and $PP1$ amplitudes. There are no adjustable parameters for this fit.

The amplitudes which give these cross sections also give results in good agreement with all the data from threshold to 800 MeV.

Any theory which is developed to describe single-pion production in π^+p collisions should have these partial waves dominating. Thus we have outlined a phenomenological procedure for analyzing single-pion production. It is a much harder task to carry out a similar analysis for π^-p systems as both $T=\frac{1}{2}$ and $T=\frac{3}{2}$ isotopic spins are present and one has many more parameters. We have not yet carried out a detailed least-squares fitting program to all the data to determine the significance of this fit, but observing the qualitative fit to many different distributions gives us confidence in the general validity of the analysis.

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Vector-Meson Couplings to the Baryons*

H. SUGAWARA AND FRANK VON HIPPEL
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

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We present some arguments by which the couplings of the vector mesons to the baryons are fixed. These arguments amount to an extension of the nonet scheme which has already been successfully applied by Okubo and by Glashow and Socolow to the 1^- and 2^+ mass formulas and decays. We find that, using the vector-meson couplings obtained and the *physical* vector masses, we can largely account for the following experimental observations: (1) the proportionality of the electric and magnetic nucleon form factors to each other and to the p-$(N_{33}^*)^+$ magnetic-dipole transition form factor; (2) the suppression of backward ϕ production in K^--p experiments; (3) the over-all suppression of ϕ production in π-p experiments; and (4) the isospin independence of the hard core in nucleon-nucleon scattering. We also estimate the relative leptonic decay rates of the ρ, ω, and ϕ and predict that there will be suppressions in $f'(1500)$ production corresponding to those observed in ϕ production.

THE VECTOR-MESON NONET

THERE are nine well-established vector mesons with masses of the order of 1 BeV: the ρ, ω, K^*, and ϕ. Their natural $SU(3)$ assignments are to an $SU(3)$ octet plus a singlet. Only one ambiguity remains: the correspondence of the physical ω and ϕ mesons with the $I=Y=0$ member of the octet (ω_8) and singlet (ω_1).

This ambiguity has been resolved by the ϕ-ω mixing theory[1] according to which the ω and ϕ are mixtures of

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[1] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); Phys. Rev. **132**, 434 (1962). The sign of θ is fixed by the nonet scheme (see Refs. 3 and 4).

the ω_1 and ω_8:

$$\omega = \cos\theta\omega_1 + \sin\theta\omega_8, \tag{1a}$$

$$\phi = -\sin\theta\omega_1 + \cos\theta\omega_8. \tag{1b}$$

The $(\text{mass})^2$ of the unmixed ω_8 is assumed to be given by the Gell-Mann–Okubo mass formula from the ρ and K^* $(\text{masses})^2$ and the mixing theory then gives θ from the physical ϕ and ω $(\text{masses})^2$. In our applications we will, for clarity, approximate θ ($\approx 40°$) by the angle θ_0 ($\approx 35.5°$) which has $\sin\theta_0 = \sqrt{\tfrac{1}{3}}$, $\cos\theta_0 = \sqrt{\tfrac{2}{3}}$. The error will usually be of order $\tan^2(\theta-\theta_0)^2 \approx 1\%$.

For our convenience we write out below the $SU(3)$ wave functions of the vector mesons in (3×3) matrix form. The matrix indices may be thought of as representing quark and antiquark indices. The members of the vector-meson octet may then be represented by linear combinations of Gell-Mann's eight traceless 3×3 $SU(3)$ generators[2] and the $SU(3)$ singlet may be represented as proportional to the 3×3 unit matrix. The nonet wave functions may then be written out as[3,4]

$$(V_9)_j{}^i = \begin{bmatrix} (\omega+\rho^0)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (\omega-\rho^0)/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\phi \end{bmatrix}, \tag{2}$$

where the simplicity of the diagonal entries is due to the fact that we have already used Eq. (1) with $\theta=\theta_0$.

The baryon octet and decuplet wave functions also have a standard form in $SU(3)$. They are assumed to transform under $SU(3)$ as three quark states $B^{ij,k}$, D^{ijk} with Young diagrams

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \\ \cline{1-1} \end{array} \quad \text{and} \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline \end{array},$$

respectively, for the permutation symmetries of the three indices.[5]

THE NONET SCHEME

Our extension of the nonet assumption made for the vector-meson mass matrix and decays[3,4] is simply the assumption that the matrix $(V_9)_j{}^i$ couples directly to the baryon wave-function indices with no extra terms proportional to the trace of V_9. This assumption fixes the relative $SU(3)$ couplings of the vector octet and singlet to the baryon octet ($B^{ij,k}$), decouplet (D^{ijk}), and octet-decuplet transition vertices through the expressions,

$$(f+d)\bar{B}_{ij,k}(V_9)_l{}^k B^{ij,l} + 2(f-d)\bar{B}_{ij,k}(V_9)_l{}^i B^{lj,k}, \tag{3}$$

$$\bar{D}_{ijk}(V_9)_l{}^k D^{ijl}, \tag{4}$$

and

$$\bar{D}_{ijk}(V_9)_l{}^k B^{li,j} + \bar{B}_{ij',k}(V_9)_l{}^i D^{jkl}.$$

We now make a key observation: *The couplings of the nonstrange baryons and baryon resonances to the ϕ are zero.* This is obvious from the fact that the nonstrange baryons have no components in their wave functions with an index 3 (see Ref. 5) while the ϕ wave function (in the approximation that $\theta=\theta_0$) is just $(V_9)_3{}^3$. It should be emphasized that this result for the $\bar{B}B\phi$ vertex is independent of the F/D ratio. It therefore applies to both the vector and tensor couplings of the ϕ or to any of their linear combinations, such as electric or magnetic coupling.

Note that different results for the coupling of the vector mesons to the baryon octet would have been obtained if the baryon octet had been represented by a 3×3 matrix $(B_8)_l{}^k$ in analogy to the vector-meson octet rather than by the Gell-Mann–Zweig three-quark wave function $B^{ij,k}$ which we have chosen,

$$B^{ij,l} = \epsilon^{ijk}(B_8)_k{}^l. \tag{5}$$

The 3×3 representation of the baryon octet would result in the suppression of the ϕ-Σ coupling rather than of the ϕ-N coupling.[6] The experimental results which we discuss below therefore combine with the results of $SU(6)$ in favoring the three-quark representation of the baryons.

Results for most vertices will depend upon the D/F ratios assumed, but we may already explain several observations by this decoupling of the ϕ.

UNIVERSALITY OF THE NONSTRANGE BARYON FORM FACTORS

The nucleon magnetic and electric form factors and the p-$(N_{33}{}^*)^+$ magnetic-dipole transition form factors have all been measured and are consistent with a universal scaling law

$$\frac{G_M{}^p(q^2)}{\mu_p} = \frac{G_M{}^n(q^2)}{\mu_n} = \frac{3}{2\sqrt{2}}\frac{G_M{}^T(q^2)}{\mu_p} = G_E{}^p(q^2), \qquad G_E{}^n(q^2) = 0 \tag{6}$$

typically to within 10% up to momentum transfers in the BeV/c region.[7,8] The vector mesons might be expected to contribute strongly to these form factors; the ρ and ω masses differ by only 20% from the mass required by the form-factor slope at $q^2=0$.

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[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[3] S. Okubo, Phys. Letters **5**, 165 (1963).

[4] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[5] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN report No. 8419/Th 412, 1964 (unpublished).

[6] By using a nonet representation for the baryons and assuming $F/D=1$, J. Schwinger [Phys. Rev. **135**, B816 (1964)] was able to decouple the ϕ from the nucleons. This ratio is in contradiction with the D/F ratios of 0 and $\frac{3}{2}$ indicated by the nucleon form-factor results for the electric and magnetic couplings, however.

[7] L. H. Chan, K. W. Chen, J. R. Dunning, Jr., N. F. Ramsey, J. K. Walker, and Richard Wilson, Phys. Rev. **141**, 1298 (1966).

[8] B. V. Geshkenbein, Phys. Letters **16**, 323 (1965). *Note added in proof.* R. H. Dalitz [Clarendon Laboratory, Oxford, 1966 (unpublished)] has re-analyzed the $N_{33}{}^*$ photoproduction and electroproduction data and obtains a coefficient of $G_M{}^T(q^2)$ from experiment (1.28±0.02) times that in Eq. (16). His re-analysis does not affect Geshkenbein's conclusion that the $N_{33}{}^*$ electroproduction and nucleon form factors are proportional.

Our first observation is that the nonet scheme gives directly a proportionality between the vector-meson pole contributions to the isoscalar and isovector nucleon form factors. This follows simply from the fact that, since the ϕ does not couple to the nucleons, it does not contribute to their form factors[9]; the only vector mesons which can then contribute are the ρ and ω which are almost degenerate in mass. The situation would presumably be much changed if we were measuring the form factor of a strange baryon or if θ were not approximately equal to θ_0. [Then, taking as an extreme example a case in which the ϕ dominanted the isoscalar form factor and a ρ dominated the isovector form factor, the slope of the isoscalar form factor at $q^2=0$ would differ by a factor of $(m_\phi/m_\rho)^2\approx 2$ from the slope of the isovector form factor.]

Our second observation concerns the collinear $SU(6)$ predictions for the form factors.[10] Collinear $SU(6)$ gives all the results of Eq. (6) in the symmetry limit, with the exception of the proportionality between proton electric and magnetic form factors. This means that, applied to the vector-meson couplings it constrains the electric coupling to be pure F, the magnetic couplings to have $F/D=\frac{2}{3}$ and the magnetic octet-decuplet couplings to be related by the appropriate constant to the magnetic octet-octet couplings.[10] The collinear $SU(6)$ form-factor predictions have been confirmed while virtually all the subsequent applications of collinear $SU(6)$ to production processes[11] which have been tested, have been found to disagree totally with experiment.[12] Several arguments have been advanced to the effect that, even if collinear $SU(6)$ predictions hold for vertices and form factors, they will not hold for collinear production processes.[12] Here we may make the further observation that it requires an accident such as the decoupling of the ϕ from the nucleons and the degeneracy of the ρ and ω, for the collinear $SU(6)$ predictions for vertices to work as spectacularly as they have been found to work for the nucleon and electroproduction form factors.

We note, incidentally, that if the usual recipe is used for breaking the symmetry of the vector-meson pole residues in the form factors (6), i.e., the symmetric residues are multiplied by $(m_i/\bar{m})^2$, where m_i is the physical meson mass and $\bar{m}$ is the symmetric mass,[13] the vector-meson poles still do not break the collinear $SU(6)$ predictions. This is because the near degeneracy of the ρ and ω poles preserves the symmetric relation between their residues and also because the form of the symmetry breaking does not make the ϕ coupling nonzero.

Thus we come to the remarkable conclusion that, in spite of symmetry breaking in both the form-factor pole positions and their residues of order $(m_\phi/m_\rho)^2\approx 2$, the symmetric predictions (6) are preserved for the measurable form factors.

Finally, in this connection, we point out that collinear $SU(6)$[10] and the nonet scheme do not conflict when their constraints overlap. For example, the zero magnetic coupling of the ϕ to the baryons is predicted by both the nonet scheme and collinear $SU(6)$. Where the collinear $SU(6)$ and nonet productions do not overlap [collinear $SU(6)$ makes no predictions about the electric coupling of the ϕ, for instance], they complement each other.[14]

SUPPRESSION OF BACKWARD ϕ PRODUCTION IN K^--p REACTIONS

Backward ϕ production in the reaction $K^-+p\to\Lambda+\phi$ has been observed to be only a few percent of backward ω production in the reaction $K^-+p\to\Lambda+\omega$ at 2.24 and 3 BeV/c.[15,16] At 3 BeV/c the backward ω production is peaked around 180°,[16] suggestive of a strong nucleon pole contribution in the u channel. (N_{33}^* exchange is forbidden by isospin conservation.)

This pole is suppressed in ϕ production by the smallness of the ϕ-nucleon coupling.[17] In its absence, backward ϕ production can certainly be expected to be lower than backward ω production, the exact amount depending upon just how dominant the nucleon exchange is for backward ω production.

SUPPRESSION OF ϕ PRODUCTION IN π-p INTERACTIONS

The production of ϕ's in the $\pi-p$ reaction $\pi+p\to\pi+p+\phi$ is observed to be only about a percent of ω

[9] In contradiction to this conclusion, the A quantum number [J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964)] predicts that the ϕ and not the ω contributes to the nucleon isoscalar form factor. We would like to point out, however, that the experimental evidence for the A quantum number is now rather weak. It was introduced originally to explain the suppression of the $\phi\to\rho+\pi$ and $\pi^0\to 2\gamma$ decay rates. The first decay rate has now been explained by the nonet formalism (Refs. 3 and 4) while estimates of the second decay rate by the M. Gell-Mann, D. Sharp, W. G. Wagner mechanism [Phys. Rev. Letters **8**, 261 (1962)] now agree in order of magnitude with experiment [K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters **14**, 264 (1965)].

[10] K. J. Barnes, P. A. Carruthers, and F. von Hippel, Phys. Rev. Letters **14**, 82 (1965).

[11] See, e.g., J. C. Carter, J. J. Coyne, S. Meshkov, D. Horn, M. Kugler, and H. J. Lipkin, Phys. Rev. Letters **15**, 373 (1965).

[12] J. D. Jackson, Phys. Rev. Letters **15**, 990 (1965).

[13] This prescription is attractive because the symmetric contribution of the vector mesons to the static charge and magnetic moment are then unaffected by symmetry breaking. See, e.g., J. J. Sakurai, Nuovo Cimento **34**, 1582 (1964); R. Dashen and D. Sharp, Phys. Rev. **133**, B1585 (1964); S. Coleman and H. Schnitzer, *ibid.* **134**, B863 (1964).

[14] $M(12)$ also predicts that the ϕ does not couple to the nucleons. [R. L. Warnock, in *Second Topical Conference on Resonant Particles* (Ohio University, Athens, Ohio, 1965), p. 397.]

[15] G. W. London, R. R. Rau, N. P. Samios, S. S. Yamamoto, M. Goldberg, S. Lichtman, M. Primer, and J. Leitner, Phys. Rev. **143**, 1034 (1966).

[16] J. Badier *et al.*, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[17] H. M. Fried and J. G. Taylor, Phys. Rev. Letters **15**, 709 (1965), have suggested this explanation but they assumed that only suppression of the Dirac couplings of the ϕ need occur.

production in the reaction $\pi+p\to\pi+p+\omega$ at 3.43, 3.54, and 3.7 BeV/c.[18]

If we consider the simple one-particle-exchange diagrams describing ϕ production in π-p interactions, we are restricted to nucleon exchange and ρ exchange. Since, according to the nonet scheme, ϕ coupling to nucleons is suppressed, the nucleon pole should not contribute importantly. This point has already been made above in our discussion of backward ϕ production in K^--p interactions. The nonet scheme also suppresses $\phi\rho\pi$ coupling,[3,4] which means that ρ exchange should not contribute strongly to ϕ production either.

If these two poles are important in ω production, their suppression in ϕ production could produce the observed low ϕ production cross section. Evidence for their importance in ω production is, in fact, found in the production angular distribution of the reaction $\pi^++p\to\omega+(N_{33}^*)^{++}$ at 4 GeV/c. This angular distribution is dominated by strong forward and backward peaks.[19]

ISOSPIN INDEPENDENCE OF THE NUCLEON-NUCLEON HARD CORE

The nucleon-nucleon phase shifts at high energy ($\approx$200 MeV c.m. kinetic energy) exhibit a behavior most easily explicable by a strong short-range repulsion with a radius of about 0.4 F. The effects is quite similar in both $I=0$, 1 states.

A hard core of the correct range is produced by vector-meson exchange. Since the ϕ does not couple to the nucleon, it would be due to ρ and ω exchange. The ω exchange potential is repulsive in both $I=0$, 1 states while the ρ exchange potential changes sign. In fact the potential is proportional to $(g_{\omega NN}{}^E)^2+(g_{\rho NN}{}^E)^2$ in the $I=1$ state and $(g_{\omega NN}{}^E)^2-3(g_{\rho NN}{}^E)^2$ in the $I=0$ state. (The superscript E indicates the electric coupling of the vector meson.)

Therefore, only if $(g_{\omega NN}{}^E)^2>3(g_{\rho NN}{}^E)^2$, will the vector mesons produce a repulsive core at all in the $I=0$ state and only if $(g_{\omega NN}{}^E)^2\gg(g_{\rho NN}{}^E)^2$, will it have a comparable effect to that in the $I=1$ state. Indeed we find from the nonet coupling scheme and the assumption that the electric coupling of the vector octet is dominantly F-type [as is indicated by the electric nucleon form-factor relation in (6)] that

$$(g_{\omega NN}{}^E)^2=9(g_{\rho NN}{}^E)^2. \tag{7}$$

Recent estimates of this ratio[20] based on simple meson-exchange models for the nucleon-nucleon potential have yielded values of this ratio of 8 and 17 in agreement with our qualitative physical arguments based upon the isospin independence of the hard core and also in rough agreement with our prediction (7). A more complete discussion of the vector meson nonet's contributions to baryon-baryon and baryon-antibaryon interactions will be presented elsewhere.

LEPTONIC DECAY RATES OF THE VECTOR MESONS

We note here that, assuming the photon transforms as the $SU(3)$ operator Q, and the octet-singlet mixing picture of the vector mesons reviewed above, we obtain directly the matrix element ratios

$$\frac{\langle\omega|\gamma\rangle}{\langle\rho|\gamma\rangle}=\frac{\sin\theta}{\sqrt{3}},\quad \frac{\langle\phi|\gamma\rangle}{\langle\rho|\gamma\rangle}=\frac{-\cos\theta}{\sqrt{3}}. \tag{8}$$

Upon insertion into the expression for the decay,[21] $V\to e^++e^-$ and using $\theta=40°$, these ratios give

$$\frac{(\omega\to e^++e^-)}{(\rho\to e^++e^-)}=0.14,\quad \frac{(\phi\to e^++e^-)}{(\rho\to e^++e^-)}=0.26. \tag{9}$$

[To within a percent $\Gamma(V\to\mu^++\mu^-)=\Gamma(V\to e^++e^-)$.] We have used the physical masses in estimating (9) but have made no attempt to estimate symmetry breaking of the $\langle V|\gamma\rangle$, which may be large. These estimates are independent of any pole-dominance mode of the nucleon form factors such as has previously been used.[21] No experimental numbers are available for the ratios but several experiments are in progress.

THE 2^+ MESON NONET

The $f'(1500)$ and $f(1250)$ in the 2^+ nonet appear to play identical roles to the ϕ and ω in the 1^- nonet.[4] We therefore expect that backward f' production will be suppressed relative to backward f production in the reaction $K^-+p\to(f\text{ or }f')+\Lambda$ and that there will be an over-all suppression of f' production relative to f production in the reactions $\pi+p\to(f\text{ or }f')+N$ and $\pi+p\to(f\text{ or }f')+\pi+N$.

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[18] M. Abolins, R. Lander, W. Mehlhop, N. Xuong, and P. Yager, Phys. Rev. Letters **11**, 381 (1963); Y. Y. Lee, W. Moebs, B. Rose, D. Sinclair, and J. VanderVelde, *ibid.* **11**, 508 (1963).

[19] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I. C.) München collaboration, Phys. Rev. **138**, B897 (1965).

[20] We have converted the Dirac and Pauli coupling constants obtained by R. A. Bryan and B. L. Scott, Phys. Rev. **135**, B434 (1964). Other references may be found in this article.

[21] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **8**, 79 (1962).