Since s_0 is not the pole of $\Gamma_\alpha^{II}(s)$, we have

$$\gamma_1/g_{01}=\gamma_\alpha/g_{0\alpha} \tag{4.6}$$

from Eq. (4.5). On the other hand, s_0 is the pole of $\Gamma_\alpha(s)/K_\alpha(s)$,[14] and hence we have

$$\left(\frac{1}{\mu^2-s_0}\frac{\gamma_1}{g_{01}}\right)^2=\left(\frac{1}{\mu^2-s_0}\frac{\gamma_\alpha}{g_{0\alpha}}\right)^2=\frac{1}{\mu^2-s_0}\frac{\gamma_\alpha}{K_\alpha(s_0)},$$

or

$$-g_{0\alpha}g_{0\beta}+\gamma_\alpha\frac{1}{\mu^2-s_0}K_\beta(s_0)=0.$$

This relation is just the condition (3.3). Therefore, it is true also in the multichannel case that if the proper vertex pole comes out from the second Riemann sheet, then the pole is not the pole of the scattering amplitudes.

Note that this pole thus coming out can exist only in the interval $\mu^2\leqq s\leqq s_1$, but not in the region $s<\mu^2$. Therefore, for the pole appearing below μ^2 the above interpretation is not valid. We have simply assumed that the pole below μ^2 also is not the pole of the scattering amplitudes.[15]

Note added in proof. Now we can discuss the problem without any approximations. The details will be reported elsewhere.

ACKNOWLEDGMENTS

The author would like to thank Professor Geoffrey F. Chew, Dr. G. Konisi, and Dr. W. Rarita for their kind interest and valuable comments. Thanks are also due to Dr. David L. Judd for his hospitality at the Lawrence Radiation Laboratory.

[14] We have already shown in Eq. (4.3) that $\Gamma_\alpha(s)/K_\alpha(s)$ should have the pole s_0, and hence $|D_0(s_0)|\neq 0$. Nevertheless $T_0(s)$ may have the pole s_0 due to the pole of $N_0(s)$. The condition (3.3) assures that the residue of the pole of $N_0(s)$ at $s=s_0$ vanishes.

[15] Let us suppose that $\Gamma(s)$ has two poles, s_0 and s_0'. If we put $\mu^2=s_0$ and $g=g_0$, then the other pole s_0' disappears by virtue of the condition (3.3).

PHYSICAL REVIEW VOLUME 145, NUMBER 4 27 MAY 1966

Generalized Wigner-Bargmann Equations*

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(Received 12 November 1965)

Generalized Wigner-Bargmann equations are constructed, for the **143** meson multiplet in $U(6,6)$, that have the effect of replacing the mean $U(6,6)$ mass in the free-field expansion by the appropriate particle masses [in first- or second-order $SU(3)$ breaking]. Used in conjunction with existing generalized equations for the **364** baryon multiplet, modified $U(6,6)$ coupling constants for the three-meson and baryon-meson vertices are derived.

SEVERAL authors[1–3] have constructed generalized Wigner-Bargmann equations to be satisfied by the finite-dimensional irreducible representations of $U(6,6)$,[4] which break the $U(6,6)$ mass degeneracy in a required way. The free-field solutions of the equations are then used to construct form factors.

There is considerable ambiguity in this construction for the following reason. The choice of the mass matrix in the generalized equations determines the scale factors between the fields and their subsidiary fields in the $U(6,6)$ multiplets. These scale factors are *not* physical observables, the physical observables being the mass-squared matrices, which are products of scale factors. However, since the free-field expansion of the $U(6,6)$ multiplets is a linear combination of fields and their subsidiary fields, the form factors and propagators depend crucially on the choice of scale factors that is made.

For the case when $SU(3)$ symmetry is taken to be exact, many authors[3,5] have suggested on phenomenological grounds that wherever masses occur in the free-field expansion of multiplets as denominators of $SU(3)$ multiplets, these should be replaced by the mean masses of the respective multiplets. That is, we take the scale factors between fields and their subsidiary fields to be equal. This corresponds to a mass-matrix of the form

$$M=\textstyle\sum_i m_iP(i)\,, \tag{1}$$

where $P(i)$ is the unitary singlet projection operator

* The research reported in this document has been sponsored in part by the Air Force Office of Scientific Research OAR through the European Office of Aerospace Research, U. S. Air Force.

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[5] R. Delbourgo. Phys. Letters **15**, 347 (1965); F. Hussain and P. Rotelli, Nucl. Phys. **74**, 669 (1965); M. Suzuki (unpublished); R. J. Rivers, Nuovo Cimento (to be published).

of the $SU(3)$ multiplet of mean mass m_i in the $U(6,6)$ representation considered. For example, for the **143** meson multiplet, we take the generalized mass matrix to be of the form

$$M = m_P P(0)\otimes 1\otimes 1' + m_V P(1)\otimes 1\otimes 1', \qquad (2)$$

where $P(0)$ and $P(1)$ are the $U(2,2)$ projection operators for the pseudoscalar and vector mesons of masses m_P and m_V, respectively. This gives the free-field expansion

$$\Phi = (1+\not{p}/m_P)\gamma_5\otimes P + (1+\not{p}/m_V)\gamma_\mu\otimes V_\mu. \qquad (3)$$

The notable exception to this choice of mass-matrix [after switching off $SU(3)$ breaking] is that of Ref. 1.

On the same phenomenological grounds, we suggest that the most plausible choice of mass matrix that incorporates $SU(3)$ breaking for an arbitrary $U(6,6)$ multiplet should still leave the scale factors between fields and their subsidiaries equal. Having decided *a priori* the mass formulas to be satisfied by the fields, this makes the free-field expansion unique. Reverting to the example of the **143** multiplet, we take the mass matrix as

$$M = P(0)\otimes(M_P + M_X) + P(1)\otimes M_V, \qquad (4)$$

where M_P and M_V are $SU(3)$ matrices containing terms transforming as $T_3^3\otimes 1 + 1\otimes T_3^3$, and M_X is a matrix that displaces the pseudoscalar singlet X. The pseudoscalar and vector fields thus satisfy Klein-Gordon equations of the form

$$(p^2 - m_1^2)P = 0 \text{ (for the octet)}, \qquad (4a)$$
$$(p^2 - m_2^2)V_\mu = 0,$$

where m_1^2 and m_2^2 are $SU(3)$ matrices.

We now encounter a difficulty peculiar to the fact that we are working with meson fields. This is that the usual first-order $SU(6)$ and $SU(3)$ mass formulas are taken to be quadratic in the masses, whereas the generalized Wigner-Bargmann equations are linear. Since the $SU(3)$ multiplets' mass-squared matrices will thus contain terms transforming as $T_3^3\otimes T_3^3$, we obtain second-order $SU(3)$ mass formulas. These are

$$3m_\eta^2 = 8m_K(m_K - m_\pi) + 3m_K^2 \qquad (5)$$

for the pseudoscalar octet, which, on substituting the physical η, K masses, gives

$$m_\pi \approx m_{\pi\text{G.M.O.}}, \qquad (6)$$

(where $m_{\pi\text{G.M.O.}}$ is the pion mass calculated from the Gell-Mann–Okubo mass formula), and

$$m_\rho^2 = m_\omega^2,\ (2m_{K^*})^2 = (m_\varphi + m_\rho)^2, \qquad (7)$$

and the usual $\omega - \varphi$ mixing angle.[6]

It is very easy to choose M so that the additional relations

$$m_{K^*}^2 - m_\rho^2 = m_K^2 - m_\pi^2 \qquad (8)$$

are satisfied.

This gives a free-field expansion [no summation over $SU(3)$ indices] of Φ as

$$\Phi_{\beta,b}{}^{\alpha,a} = \left[\left(1+\frac{p}{m_{Pb}{}^a}\right)\gamma_5\right]_\beta^\alpha P_b{}^a + \left[\left(1+\frac{p}{m_X}\right)\gamma_5\right]_\beta^\alpha \frac{X}{\sqrt{3}}\delta_b{}^a + \left[\left(1+\frac{p}{m_{Vb}{}^a}\right)\gamma_\mu\right]_\beta^\alpha V_{\mu b}{}^a, \qquad (9)$$

where $P_b{}^a$ is the pseudoscalar octet, $V_{\mu b}{}^a$ the vector nonet with the usual ω-φ mixing, and

$$m_{Pb}{}^a = \begin{bmatrix} m_\pi & m_\pi & m_K \\ m_\pi & m_\pi & m_K \\ m_K & m_K & m_\eta' \end{bmatrix}_b^a \qquad (10)$$
$$m_{Vb}{}^a = \begin{bmatrix} m_\rho & m_\rho & m_{K^*} \\ m_\rho & m_\rho & m_{K^*} \\ m_{K^*} & m_{K^*} & m_\varphi \end{bmatrix}_b^a$$

where the masses satisfy (5), (7), (8) and $m_\eta' = 2m_K - m_\pi$.

If we require that the first-order $SU(6)$ mass formulas[7] in the squared masses be obtained exactly, we are compelled to introduce additional terms in M_P and M_V in (4) transforming as $T_3^3\otimes T_3^3$ with suitable coefficients so that the second-order terms in the mass-squared matrices cancel. In this case, the free-field expansion is again given in Eqs. (9) and (10), except that now $m_\eta' = (2m_K^2 - m_\pi^2)^{1/2}$ and the masses satisfy

$$3m_\eta^2 = 4m_K^2 - m_\pi^2, \qquad (11)$$
$$m_\rho^2 = m_\omega^2, \quad 2m_{K^*}^2 = m_\rho^2 + m_\varphi^2,$$

and (8).

For baryons belonging to the **364** representation, a purely $SU(3)$ mass matrix can be chosen[1] that enables the fields and their subsidiaries to be chosen with equal scale factors.

THREE-MESON VERTEX

Taking the effective three-meson vertex as[8]

$$\mathcal{L}(\Phi\Phi\Phi) = \tfrac{1}{4} i m_\rho g \operatorname{Tr}[\Phi\Phi\Phi], \qquad (12)$$

the coupling constants g_{ABC} [where A, B, C denote $SU(2)$ multiplets] are given by

$$g_{ABC} = g_{ABC}{}^0 X_{ABC}, \qquad (13)$$

where

$$g_{\rho\pi\pi}{}^0 = (9/\sqrt{2})g, \qquad (14)$$
$$g_{\rho\pi\omega}{}^0 = (9\sqrt{2}/m_\rho)g,$$

[6] If we had not artificially split off the X^0, the pseudoscalar nonet would have also satisfied a linear relation of the form (7).

[7] A. Pais, Phys. Rev. Letters 13, 175 (1965); M. A. Bég and V. Singh, *ibid.* 13, 418 (1965); T. K. Kuo and T. Yao, *ibid.* 13, 415 (1965).

[8] There is difficulty in deciding how to extract dimensionless coupling constants—see M. Suzuki (Ref. 5). In this paper we follow the formalism of Ref. 1 to facilitate comparison.

and all other $g_{ABC}{}^0$ (for $1^-0^-0^-$ and $1^-1^-0^-$ couplings) are obtained from these by exact $SU(3)$ symmetry and ω-ϕ mixing.

For $(1^-0^-0^-)$ couplings,

$$X_{VP_1P_2}=m_\rho(m_V+m_{P_1}+m_{P_2})/3m_{P_1}m_{P_2}, \quad (15)$$

provided that neither P_1 nor P_2 is η. For this case

$$X_{K^*K\eta}=\frac{m_\rho}{9}\left[\frac{m_{K^*}+m_K+m_\pi}{m_\pi m_K}+\frac{2(m_{K^*}+2m_K)}{m_K m_\eta{}'}\right], \quad (16)$$

where $m_\eta{}'$ is $2m_K-m_\pi$ or $(2m_K{}^2-m_\pi{}^2)^{1/2}$, depending on which mass formulas we wish to have satisfied.

For the $(1^-1^-0^-)$ couplings we have

$$X_{V_1V_2P}=m_\rho{}^2\frac{(m_{V_1}+m_{V_2}+m_P)}{3m_{V_1}m_{V_2}m_P}. \quad (17)$$

We thus have

$$\frac{g_{\rho\pi\omega}}{g_{\rho\pi\pi}}=\frac{2m_\pi(m_\pi+2m_\rho)}{m_\rho{}^2(m_\rho+2m_\pi)}\approx(750\text{ MeV})^{-1}, \quad (18)$$

if the π mass is evaluated from the mass formulas. This agrees very closely with the value

$$g_{\rho\pi\omega}/g_{\rho\pi\pi}\approx(780\text{ MeV})^{-1}, \quad (19)$$

obtained without $SU(3)$ splitting and a mean pseudoscalar mass of 400 MeV.[9] This gives a low $\omega\to3\pi$ width of 1.4 MeV using the Gell-Mann, Sharp, and Wagner model[10] for ω decay and the known $\rho\to2\pi$ decay rate. This suggests a fairly large contribution from many-particle intermediate states. Since such states violate the truncated inhomogeneous $U(6,6)$ theory it is not certain how meaningful it is to approximate these by an effective interaction of the form $\mathrm{Tr}[\Phi\Phi\Phi]$.

We note that if we forsake equal scale factors between the fields and their subsidiaries and require only that the mass matrix reduce to (2) on switching off the $SU(3)$ breaking (and a reasonable mass formula), it is impossible to increase the ratio (18) appreciably. If we want to increase the ratio (18) at all significantly we are compelled to discard the phenomenological argument and require that we do *not* recover (2) on switching off the $SU(3)$ breaking.

BARYON-MESON VERTEX

Taking the baryon-meson vertex as

$$\mathcal{L}(\bar{\Psi}\Psi\Phi)=ih\,\mathrm{Tr}[\bar{\Psi}\Psi\Phi], \quad (20)$$

where we use the free-field expansion for the **364** multiplet Ψ_{ABC} given in Ref. 1, the coupling constants are given by

$$g_{ABC}=g_{ABC}{}^0X_{ABC}, \quad (21)$$

where

$$g_{N^*N\pi}{}^0=(\sqrt{2}/5)g_{NN\pi}{}^0=h, \quad (22)$$

and the other $g_{ABC}{}^0$ are given by $SU(3)$ and a D/F ratio of $\frac{3}{2}$. The functions X_{ABC} are given by

$$X_{DBP}=\frac{2}{3}\frac{1}{m_B}\left(1+\frac{m_B+m_D}{m_{DBP}}\right), \quad (23)$$

where

$$m_{DB\pi}=m_\pi,$$
$$m_{DBK}=m_K, \quad (24)$$
$$m_{\Xi^*\Xi\eta}=m_{Y^*\Sigma\eta}=3m_\pi m_\eta{}'/(2m_\pi+m_\eta{}'),$$

and

$$X_{B_1B_2P}=\frac{[(m_{B_1}+m_{B_2})^2-m_P{}^2]}{18m_{B_1}m_{B_2}}\left(1+\frac{m_{B_1}+m_{B_2}}{m_{B_1B_2P}}\right), \quad (25)$$

where

$$m_{B_1B_2\pi}=m_{\Sigma\Sigma\eta}=m_\pi,$$
$$m_{B_1B_2K}=m_K,$$
$$m_{NN\eta}=3m_\pi m_\eta{}'/(5m_\eta{}'-2m_\pi), \quad (26)$$
$$m_{\Xi\Xi\eta}=9m_\pi m_\eta{}'/(10m_\pi-m_\eta{}'),$$
$$m_{\Lambda\Lambda\eta}=3m_\pi m_\eta{}'/(4m_\pi-m_\eta{}').$$

The baryon masses satisfy the G. M. O. relations and

$$m_{\Xi^*}-m_{Y^*}=m_\Xi-m_\Sigma. \quad (27)$$

The values of $g_{DB\pi}$ and $g_{B_1B_2\pi}$ are not appreciably altered if, as before, we only require that the mass matrix reduce to (1) on switching off the $SU(3)$ breaking. Using the experimental value of $g_{NN\pi}$, the calculated N^* width is very low—about 18 MeV. This is comparable with the value obtained if we switch off the $SU(3)$ breaking and take mean multiplet masses of $m_P=400$ MeV, $m_B=1100$ MeV.

Thus generalizing the mass matrix from (2) to (4) for the **143** mesons and taking a similar generalization for the baryons does not appreciably alter predictions, although superficially the results look worse, since we cannot mask them by varying multiplet masses from situation to situation. As seen, the results do not seem to be in good agreement with experiment. To the extent that they all contain pion masses in denominators they are very susceptible to the smallness of even the G. M. O. pion mass in comparison to the other meson masses. It is to be hoped that better predictions can be made for those decays involving only K's and η of the pseudoscalar mesons, or only vector mesons.

ACKNOWLEDGMENTS

The author would like to thank Professor P. T. Matthews for helpful discussions concerning this work.

[9] R. Delbourgo. Phys. Letters **15**, 347 (1965).
[10] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962).