Dynamics of the $N^*(\frac{5}{2}^-)$†

P. R. AUVIL AND J. J. BREHM
Northwestern University, Evanston, Illinois
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A dynamical model of the D_{15} pion-nucleon resonance is considered. Since the branching ratio for decay into the πN channel is less than $\frac{1}{2}$, we first treat the case in which the resonance is produced in πN^* scattering. The coupling to the ρN^* inelastic channel is assumed to dominate the forces, and the N/D equations, solved in a pole approximation, are employed. We next consider the generalization of this model to $SU(3)$ symmetry and obtain a resonant octet. Finally we treat the three-channel problem with the πN channel included. The calculated mass and branching ratio of the resonance are in good agreement with experiment but the total width is too small.

I. INTRODUCTION

PHENOMENOLOGICAL analyses[1] of pion-nucleon scattering data have revealed the existence of a resonant D_{15} wave, obscured until recently by the well-known F_{15} resonance at 1688 MeV. The new resonant state occurs near 915 MeV (pion lab energy); its quantum numbers are $J^P=\frac{5}{2}^-$, $T=\frac{1}{2}$. Most significant for the subsequent discussion is the observation that it decays predominantly via inelastic modes. Thus, since the elastic branching fraction is less than $\frac{1}{2}$, the elastic phase is necessarily falling through 0 rather than rising through $\pi/2$ at resonance.

Dynamical models[2,3] exist for many of the higher πN resonances; it is the purpose of this paper to provide one for the $N^*(\frac{5}{2}^-)$. A mechanism, or force diagram, is proposed which favors the quantum numbers of this resonance and which, used in conjunction with dispersion relations and unitarity, determines the energy and width. Like other of the higher states, the attractive force producing the resonance involves a coupling to a specially chosen inelastic channel. The mechanism is, in fact, quite similar to the Cook-Lee model[2] of the $N^*(\frac{3}{2}^-)$.

Since the resonance is evidently coupled more strongly to the inelastic channels than to the πN channel, we first formulate the model in terms of elastic πN^* scattering, where N^* denotes the familiar (3,3) isobar. The dominant force is assumed to be provided by the coupling to the inelastic ρN^* channel given by the one-pion-exchange diagram of Fig. 1. As is well known, an off-diagonal force is always attractive in elastic scattering. Virtual absorption applies in this case since the ρN^* channel is closed at the energy of the desired resonance; unitarity causes it to manifest itself strongly below threshold. This particular choice of inelastic state is made because the parity and angular momenta of interest allow coupling to an s-wave ρN^* channel. This circumstance yields maximal absorption, favoring particularly the quantum numbers of the $N^*(\frac{5}{2}^-)$. Moreover, it is in the case of s-wave inelasticity that the simple-pole approximation of the Cook-Lee model is most justifiable.[4]

The formalism we adopt here is described in detail in AB1. Many of those results will be carried over bodily without change in notation. In Sec. II the model is implemented with the necessary calculations and is shown to yield the $N^*(\frac{5}{2}^-)$. In Sec. III the mechanism is generalized to $SU(3)$ symmetry, and it is shown that the resonant unitary multiplet is the octet. In Sec. IV the three-channel formalism of AB2 is used to see how the $N^*(\frac{5}{2}^-)$ appears in πN scattering. The extent to which the observed phase, branching ratio and total width are fit by this model is discussed there.

II. THE COUPLED πN^*, ρN^* PROBLEM

The coupled eigenamplitudes of angular momentum and parity are described by the matrix:

$$\mathfrak{F}=\begin{pmatrix} F_{22}{}^{sr} & F_{23}{}^{s\xi} \\ F_{32}{}^{\eta r} & F_{33}{}^{\eta\xi} \end{pmatrix}, \qquad (1)$$

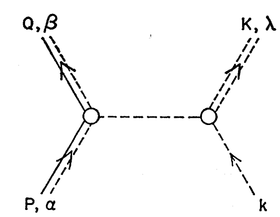
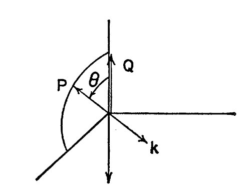


FIG. 1. The one-pion-exchange diagram, coupling πN^* to ρN^*.



† Supported in part by the National Science Foundation.
[1] P. Bareyre, C. Brickman, A. V. Stirling, and G. Villet, Phys. Letters **18**, 342 (1965); J. P. Merlo and G. Valladas, Proc. Roy. Soc. (London) (to be published).
[2] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 283 (1962); **127**, 297 (1962).
[3] P. R. Auvil and J. J. Brehm, Phys. Rev. **138**, B458 (1965); **140**, B135 (1965); and Ann. Phys. (N. Y.) **34**, 505 (1965). The second and third of these papers will be referred to hereafter as AB1 and AB2, respectively.
[4] F. T. Meiere, Phys. Rev. **136**, B1196 (1964). Meiere treats the same problem as Cook and Lee but without recourse to the pole approximation; where s-wave inelasticity is allowed he reproduces their results.

in which the subscripts 2, 3 refer to the πN^* and ρN^* channels, respectively. The superscripts s, r (η,ξ) refer to the multiplicity of the πN^* (ρN^*) channel for a given J^P. The F's incorporate the correct threshold behavior so that, e.g.,

$$F_{22}{}^{sr}=\frac{2M}{P_0+M}\left(\frac{P_0+M}{P}\right)^{2L}M_{22}{}^{sr} \tag{2}$$

and

$$F_{32}{}^{\eta r}=\left(\frac{2M}{P_0+M}\right)^{1/2}\left(\frac{2M}{Q_0+M}\right)^{1/2}\left(\frac{P_0+M}{P}\right)^{L}M_{32}{}^{\eta r}, \tag{3}$$

where (P,P_0) and (Q,Q_0) denote the (momentum, energy) of the initial and final N^*.[5] The M's are the transition amplitudes in the helicity representation, projected onto states of definite J^P. In Table I we identify the off-diagonal block M_{32}, for negative parity, in terms of the helicity amplitudes. These in turn are given by

$$\langle\beta\lambda|M_{32}|\alpha\rangle=2\pi\int_{-1}^{1}d\cos\theta\, d_{\beta-\lambda,\alpha}{}^{J}(\theta)M_{32}. \tag{4}$$

In this notation the S matrix is

$$S=1+2\pi i\rho^{1/2}\mathfrak{F}\rho^{1/2}, \tag{5}$$

where ρ is the diagonal phase space matrix.

$$\rho=\begin{pmatrix}\rho_2\delta_{sr} & 0\\ 0 & \rho_3\delta_{\eta\xi}\end{pmatrix}, \tag{6}$$

with

$$\rho_2=\frac{2}{(4\pi)^3}\frac{P^{2L+1}}{W(P_0+M)^{2L-1}}, \tag{7}$$

and

$$\rho_3=\frac{2}{(4\pi)^3}\frac{Q(Q_0+M)}{W}. \tag{8}$$

Note that in formulas (3) and (8) we have built in s-wave inelastic threshold behavior.

TABLE I. J^P eigenamplitudes in terms of helicity amplitudes.

r	η	Combinations for negative parity J^-
$\frac{3}{2}$	1	$\langle\frac{3}{2}1\|M_{32}\|\frac{3}{2}\rangle-(-)^{J-3/2}\langle-\frac{3}{2}-1\|M_{32}\|\frac{3}{2}\rangle$
	2	$\langle\frac{1}{2}1\|M_{32}\|\frac{3}{2}\rangle-(-)^{J-3/2}\langle-\frac{1}{2}-1\|M_{32}\|\frac{3}{2}\rangle$
	3	$\langle-\frac{1}{2}1\|M_{32}\|\frac{3}{2}\rangle-(-)^{J-3/2}\langle\frac{1}{2}-1\|M_{32}\|\frac{3}{2}\rangle$
	4	$\langle-\frac{3}{2}1\|M_{32}\|\frac{3}{2}\rangle-(-)^{J-3/2}\langle\frac{3}{2}-1\|M_{32}\|\frac{3}{2}\rangle$
	5	$\langle\frac{3}{2}0\|M_{32}\|\frac{3}{2}\rangle-(-)^{J-3/2}\langle-\frac{3}{2}0\|M_{32}\|\frac{3}{2}\rangle$
	6	$\langle\frac{1}{2}0\|M_{32}\|\frac{3}{2}\rangle-(-)^{J-3/2}\langle-\frac{1}{2}0\|M_{32}\|\frac{3}{2}\rangle$
$\frac{1}{2}$	1	$\langle\frac{3}{2}1\|M_{32}\|\frac{1}{2}\rangle-(-)^{J-3/2}\langle-\frac{3}{2}-1\|M_{32}\|\frac{1}{2}\rangle$
	⋮	⋮
	6	$\langle\frac{1}{2}0\|M_{32}\|\frac{1}{2}\rangle-(-)^{J-3/2}\langle-\frac{1}{2}0\|M_{32}\|\frac{1}{2}\rangle$

The many-channel N/D method[6] is used to satisfy the unitarity relations. We set

$$\mathfrak{F}=\mathfrak{N}\mathfrak{D}^{-1},$$

where

$$\text{disc}\,\mathfrak{D}=-2\pi i\rho\mathfrak{N} \tag{9}$$

on the right-hand cuts. The model is driven by the force assumed on the left-hand cuts. We express this in a pole approximation by writing

$$\begin{aligned}\text{disc}_L F_{22}{}^{sr}&=0=\text{disc}_L F_{33}{}^{\eta\xi},\\ \text{disc}_L F_{32}{}^{\eta r}&=2\pi i c_\eta b_r\delta(W-W_2),\\ \text{disc}_L F_{23}{}^{s\xi}&=2\pi i b_s c_\xi\delta(W-W_2).\end{aligned} \tag{10}$$

The determinant of the $\mathfrak{D}$ matrix is then

$$\mathfrak{D}_0=\det\mathfrak{D}=1-b^2c^2S_2(W)T_2(W) \tag{11}$$

in which

$$b^2=\textstyle\sum_r b_r{}^2,\qquad c^2=\sum_\xi c_\xi{}^2, \tag{12}$$

and where

$$S_2(W)=\int_{M+\mu}^{\infty}\frac{dx}{x-W}\frac{\rho_2(x)}{(x-W_2)^2},$$

and

$$T_2(W)=\int_{M+\rho}^{\infty}\frac{dx}{x-W}\frac{\rho_3(x)}{(x-W_2)^2}. \tag{13}$$

A resonance occurs at an energy W^* below the inelastic threshold when

$$\text{Re}\,\mathfrak{D}_0(W^*)=0. \tag{14}$$

Explicit evaluation of the diagram of Fig. 1 yields the determination of the parameters of the model. Since F_{32} is singular at $M+\mu$ we choose the pole position to be $W_2=M+\mu$. We then span the region of interest by evaluating b^2 and c^2 at the inelastic threshold $W_3=M+\rho$ so that

$$c_\eta b_r=(W_2-W_3)[F_{32}{}^{\eta r}(W_3)]_{\text{diagram}}. \tag{15}$$

The $N^*N^*\pi$ vertex was evaluated in AB1. If we neglect the f-wave coupling, it is given by

$$\begin{aligned}V_{\beta\alpha}&=g_{33}\bar{\Psi}_\mu{}^{(\beta)}(Q)\gamma_5\Psi_\mu{}^{(\alpha)}(P)\\ &=(-)^{\alpha+1/2}\epsilon_\alpha g_{33}\left(\frac{P_0+M}{2M}\right)^{1/2}\frac{P}{P_0+M}d_{\beta,\alpha}{}^{3/2}(\theta)\end{aligned} \tag{16}$$

for $Q=0$. In (16), the factor ϵ_α is

$$\begin{aligned}\epsilon_{\pm3/2}&=1,\\ \epsilon_{\pm1/2}&=\tfrac{1}{3}(1-2P_0/M).\end{aligned} \tag{17}$$

The $\rho\pi\pi$ vertex is readily evaluated to be

$$\begin{aligned}U_\lambda&=fe_\mu{}^{(\lambda)}(K)k_\mu\\ &=(-)^\lambda fPd_{\lambda,0}{}^{1}(\theta),\end{aligned} \tag{18}$$

[5] We use m, M, μ, and ρ for the masses of the nucleon, (3,3) isobar, pion, and ρ meson, respectively. The pion mass is taken as the unit of energy.

[6] R. Blankenbecler, Phys. Rev. **122**, 983 (1961).

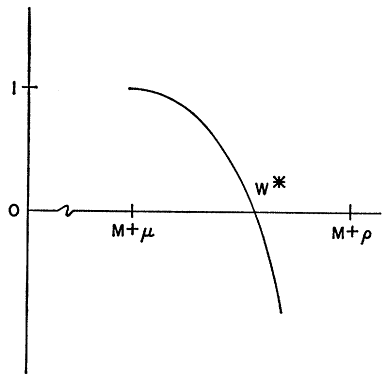


FIG. 2. $\text{Re}\mathfrak{D}_0$ versus W for $J^P=\frac{5}{2}^-$, $T=\frac{1}{2}$.

where f is related to the width of the ρ by

$$\Gamma_\rho=(f^2/4\pi)k^3/6\rho^2 \tag{19}$$

which yields $f^2/4\pi=7.93$. A straightforward calculation then gives

$$b_r=\epsilon_r(J,r,\tfrac{3}{2},-r|J,\tfrac{3}{2},1,0) \tag{20}$$

for $r=\frac{3}{2},\frac{1}{2}$, and

$$c_\eta=(-)^{\beta-1/2}\frac{8\pi}{3}h_T f g_{33}\frac{P^2}{P_0+M}\frac{\rho-\mu}{2M(P_0-M)+\mu^2}\times\left(\frac{P_0+M}{P}\right)^L(J,\beta-\lambda,\tfrac{3}{2},-\beta|J,\tfrac{3}{2},1,-\lambda) \tag{21}$$

for $\eta=1$ to 6. Table I gives the appropriate β and λ for a given η. The factor h_T is an isospin coefficient given by

$$h_T=((5/3)^{1/2},(4/15)^{1/2},-(3/5)^{1/2}),\quad \text{for}\quad T=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2}. \tag{22}$$

A crucial feature of the model is the dominance of the $T=\frac{1}{2}$ coefficient. The value of L is the minimum orbital angular momentum and depends on the value of J, given negative parity. Since we are also given s-wave inelasticity we need only consider $J^P=\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$; for these cases $L=2$, 0, and 2, respectively. The factor c^2 is maximal for $L=2$, and the factor b^2 favors $J=\frac{5}{2}$ over $J=\frac{1}{2}$. Thus the model singles out $J^P=\frac{5}{2}^-$, $T=\frac{1}{2}$ as the most attractive configuration.

An estimate of the coupling constant $g_{33}^2/4\pi$ may be obtained, for example, by resorting to static $SU(6)$ symmetry.[7] This was done in AB1 with the result[8]:

$$g_{33}^2/4\pi=(27/5)g^2/4\pi=81, \tag{23}$$

in which g denotes the $NN\pi$ coupling constant.

We can now calculate $\text{Re}\mathfrak{D}_0(W)$ for $J^P=\frac{5}{2}^-$, $T=\frac{1}{2}$, and look for $W=W^*$ satisfying Eq. (14). In Fig. 2 we have plotted the results. The desired resonance occurs for $W^*=12.34\mu=1730$ MeV. The phenomenological analysis of the experimental data[1] yields a resonance near 915-MeV pion lab energy, corresponding to a mass of 1700 MeV. The width of the resonance can be read off from the results by computing $\text{Im}\mathfrak{D}_0$ and the slope of $\text{Re}\mathfrak{D}_0$. This quantity is more meaningful in the context of the three-channel problem, where the πN state is included; therefore, mention of it will be deferred to Sec. IV.

We view the results as very encouraging; the model produces a reasonance which is less than $\frac{1}{4}$ of a pion mass off the mark. The important points are that the mechanism prefers the right quantum numbers and provides what can be assumed to be the bulk of the attraction. The calculated W^* is a little too large; the coupling of other inelastic channels would lower it toward the desired position.

III. THE COUPLED P_8B_{10}, V_8B_{10} PROBLEM

In Sec. II we invoked $SU(2)$ symmetry. The calculation describes the coupled channel scattering of particles in the relevant isospin multiplets. We can generalize this readily to $SU(3)$ symmetry. We let all the particles in the baryon decuplet (B_{10}), pseudoscalar-meson octet (P_8), and vector-meson octet (V_8) be degenerate with masses M, μ, and ρ, respectively. We then require that the $B_{10}B_{10}P_8$ and $V_8P_8P_8$ couplings be $SU(3)$ symmetric. The model may then be carried over bodily to describe the coupled-channel scattering in the P_8B_{10} and V_8B_{10} representations. The force, as shown in Fig. 3, is given by P_8 exchange.

Only two modifications of Sec. II are necessary: The primary change is to be made in Eq. (21); the isospin coefficient h_T is replaced by the unitary spin coefficient h_F where F denotes each of the $SU(3)$ multiplets occurring in the product $8\otimes10$:

$$8\otimes10=8\oplus10\oplus27\oplus35. \tag{24}$$

The other modification is the trivial one of selecting representative values for the masses M, μ, and ρ. Once these alterations are made we need only remark that, since the determinant of the $\mathfrak{D}$ matrix is invariant under the orthogonal transformation from the particle basis to the unitary spin basis, then Eq. (11) may be simply reinterpreted with h_T replaced by h_F.

The determination of the h_F's is facilitated by select-

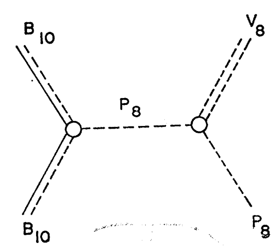


FIG. 3. P_8-exchange diagram, coupling P_8B_{10} to V_8B_{10}.

[7] R. H. Capps, Phys. Rev. Letters **14**, 31 (1965).

[8] An error of a factor of 3 was made in obtaining this number in AB1. The result may also be derived from the work of B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965).

ing the following wave functions[9]:

$$|\eta\Omega\rangle=[|35,-2,0\rangle+|10,-2,0\rangle]/\sqrt{2}, \quad (25)$$

$$|\pi N^*, T=\tfrac{1}{2}\rangle=[|27,1,\tfrac{1}{2}\rangle-2|8,1,\tfrac{1}{2}\rangle]/\sqrt{5}, \quad (26)$$

$$|\pi N^*, T=\tfrac{3}{2}\rangle=[-|35,1,\tfrac{3}{2}\rangle-\sqrt{5}|27,1,\tfrac{3}{2}\rangle +\sqrt{10}|10,1,\tfrac{3}{2}\rangle]/4, \quad (27)$$

$$|\pi N^*, T=\tfrac{5}{2}\rangle=|35,1,\tfrac{5}{2}\rangle. \quad (28)$$

The notation on the right hand side is $|F,Y,T\rangle$ in which Y denotes hypercharge. Given these expressions, the calculation of the h_F's reduces to one of computing isospin coefficients. If we start with Eq. (25) we find that all the h_F's are determinable from the h_T's, Eq. (22) The results are:

$$h_F=(\sqrt{(12/5)},\sqrt{(3/5)},\sqrt{(1/15)},-\sqrt{(3/5)}) \quad \text{for} \quad F=8,10,27,35. \quad (29)$$

Thus the octet is the most attractive multiplet.

The mass M for the degenerate B_{10} multiplet is determined by averaging the decuplet masses, weighted by the isospin multiplicity. The masses μ and ρ, for P_8 and V_8, are computed similarly, but in terms of the squares of the octet masses. The numbers are:

$$\mu=2.92 \text{ pion masses}, \quad \rho=2.06\mu, \quad M=3.38\mu. \quad (30)$$

To determine whether the most attractive multiplet resonates, we calculate $\mathrm{Re}\mathfrak{D}_0(W)$ for the octet occurring in (24), with $J^P=\frac{5}{2}^-$. As shown in Fig. 4, there is a resonance occurring at $W^*=5.18\mu=2120$ MeV. The significant result is that a resonant octet exists by virtue of this model; it is not clear what should be made of the numerical result.

There are only a few candidates for membership in this $\frac{5}{2}^-$ octet. Two of the states would, of course, be the $N^*(\frac{5}{2}^-)$ isospin doublet. Another recent experimental discovery seems to fit into this scheme very well. This is the $Y^*(1760)$ of Armenteros *et al.*[10]; it has been determined to be an isospin triplet and it has been found that the assignment $J^P=\frac{5}{2}^-$ provides the best fit to the data. Unfortunately, two known masses are insufficient to give a prediction for the missing members.

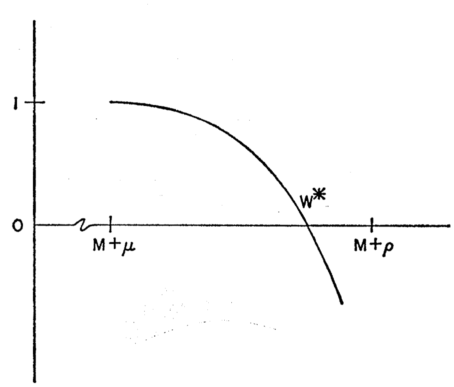


FIG. 4. $\mathrm{Re}\mathfrak{D}_0$ versus W for $J^P=\frac{5}{2}^-$ in the octet representation.

IV. COUPLING TO THE πN CHANNEL AND CONCLUSION

In order to compare the result of Sec. II with the experimentally determined branching ratio and width of the $N^*(\frac{5}{2}^-)$, we must couple our solution to the πN state. Any of a variety of diagrams can be chosen to couple the πN, πN^*, and ρN^* channels. At this juncture we do not undertake an exhaustive survey to find the optimum choice; instead, we simply select the coupling shown in Fig. 5 as representative and base our three-channel calculation on the addition of it alone. In fact, in the context of a three-channel problem, it is quite unlikely that *any* such optimum choice will give a good total width. Many two-body channels are open and probably significant at this energy, but we make no effort to estimate their importance. The point of view we take is that the resonance mechanism has been found. This fixes the quantum numbers for any subsequent discussion pertaining to any other coupled channels. Accordingly, we consider only $J^P=\frac{5}{2}^-$, $T=\frac{1}{2}$ in what follows.

Once the selection of a coupling to πN states has been made, as in Fig. 5, the extension to three channels is straightforward. The techniques employed are described in detail in AB2. Diagrams coupling πN to πN^* would involve, e.g., nucleon exchange; these have been left out, as they were in AB2, since they do not allow s-wave absorption. The $N^*N\pi$ vertex is taken to be

$$V_{\beta s}=g_3\bar{\Psi}_\nu^{(\beta)}(Q)u^{(s)}(p)p_\nu, \quad (31)$$

where Q and p are the momenta of the N^* and N, and, using the known width of the N^* ($\Gamma=0.89\mu$), $g_3^2/4\pi=0.39\mu^{-2}$.

Since the essential elements of this calculation may be found in AB2, we limit ourselves here to a discussion of the results. If we use the above value for g_3 and the

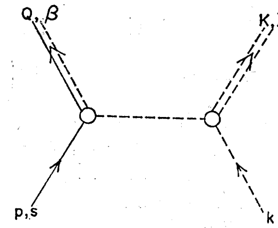


FIG. 5. The one-pion-exchange diagram, coupling πN to ρN^*.

[9] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963). The formulas are for P_8B_{10} states; the V_8B_{10} wave functions are in complete parallel with these.

[10] R. Armenteros, M. Ferro-Luzzi, D. W. G. Leith, R. Levi-Setti, A. Minten, R. D. Tripp, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, J. Meyer, and J. P. Porte, Phys. Letters 19, 338 (1965).

value of g_{33} from Eq. (23), the resonance occurs at approximately the correct position in the πN channel but with a branching ratio greater than $\frac{1}{2}$ (i.e., with phase $=\pi/2$). If we vary our couplings slightly so as to fit exactly the two parameters, $W^*=12.1\mu$ and $\Gamma_e/\Gamma=0.35$,[1] then we need the couplings: $g_3'=0.53g_3$ and $g_{33}'=1.01g_{33}$. However, for these new couplings the total width is only $\Gamma=40$ MeV as compared to the experimental value of $\Gamma=325$ MeV.

In conclusion we have seen that a simple model of inelastic coupling in a pole approximation gives an excellent prediction of the position of the $N^*(\frac{5}{2}-)$ resonance. Thus both the $N^*(\frac{3}{2}-)$[2,11] and $N^*(\frac{5}{2}-)$ resonances and their octet character are understandable from a dynamical model. The width of the $N^*(\frac{5}{2}-)$, as calculated in the three-channel case, is far too small as was that of the $N^*(\frac{7}{2}+)$, discussed in AB2. In that paper, it is argued that this is probably the result of having only one inelastic channel open for the decay. Other states are going to play a role and we would expect the resonance width to be increased by this Ball-Frazer[12] effect, but we know of no simple way to estimate it. A test of this model would be to observe that the dominant decay product of the $N^*(\frac{5}{2}-)$ is $\pi+N^*(\frac{3}{2}+)$.

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One of the authors (P.R.A.) would like to thank Professor M. Ross for an interesting discussion of this problem.

[11] J. J. Brehm, Phys. Rev. **136**, B216 (1964).

[12] J. S. Ball and W. R. Frazer, Phys. Rev. Letters **7**, 204 (1961).

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Electromagnetic Interaction in Static Strong-Coupling Theory

S. K. Bose†

Centre for Advanced Study in Theoretical Physics and Astrophysics, University of Delhi, Delhi, India

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Electromagnetic properties of isobars are studied using the group-theoretic formulation of the strong-coupling theory due to Cook, Goebel, and Sakita. Expressions for magnetic moment and electromagnetic mass shifts for isobars are obtained in the scalar and the pseudoscalar theories. The significance of these results is briefly discussed.

1. INTRODUCTION

RECENTLY, Cook, Goebel, and Sakita[1] have given a group-theoretic formulation of the static strong-coupling theory. As shown by these authors the strong-coupling approximation is closely related to the mathematical concept of group contraction and the "dynamical group" which emerges in the strong-coupling limit has the structure of a semidirect product of the "primitive group of invariance" with a suitable Abelian invariant subgroup. In this formulation the infinite number of isobar states that occur in the static scalar strong-coupling theory are put in a single unitary irreducible representation of the group $SU(2)\times T_3$. Similarly, the isobar states of pseudoscalar strong-coupling theory are put in a single unitary irreducible representation of the group $SU(2)\otimes SU(2)\times T_9$. The purpose of this paper is to study the electromagnetic properties of isobars in the static scalar and pseudoscalar strong-coupling theories using this group-theoretic formulation. In Sec. 2 we construct the explicit matrix representation of the infinitesimal generators of $SU(2)\times T_3$. These results are then utilized in Sec. 3 to obtain a magnetic-moment formula and an electromagnetic mass formula in static scalar theory. The pseudoscalar theory is studied in Sec. 4. The relevant representation of the group $SU(2)\otimes SU(2)\times T_9$ is first constructed and then these results are applied to obtain expressions for magnetic-moment and electromagnetic mass shifts of isobars. The significance of these results is then briefly discussed.

2. REPRESENTATIONS OF $SU(2)\times T_3$

Let us denote the six infinitesimal generators of this group by M_i and A_i. The commutation relations for these generators are

$$[M_i,M_j]=i\epsilon^{ijk}M_k, \tag{1}$$

$$[M_i,A_j]=i\epsilon^{ijk}A_k, \tag{2}$$

$$[A_i,A_j]=0. \tag{3}$$

We now define appropriate "ladder" generators:

$$M_\pm=M_1\pm iM_2;\quad A_\pm=A_1\pm iA_2. \tag{4}$$

The commutation relations of these follow from Eqs.

† Present address: International Center for Theoretical Physics, Trieste, Italy.

[1] T. Cook, C. Goebel, and B. Sakita, Phys. Rev. Letters **15**, 35 (1965).