Analysis of the KN Scattering by the Exact N/D Method

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(Received 28 December 1965)

The KN-scattering phase shifts are calculated by considering the forces due to one-particle exchange, and using the exact N/D method. The particles exchanged are Λ, Σ, Y_1^* and Y_0^* in the u channel and ρ and ω in the t channel. The coupling constants are taken from $SU(3)$ symmetry and experiments. Two parameters are used for fitting the experimental data: the D-F mixing parameter f and the cutoff parameter W_c. Two sets of phase shifts exist in the case of the $I=0$ state for p and higher partial waves due to the Fermi-Yang ambiguity. It is found that the values of the parameters which give a good fit in the case of s-wave $I=0$ and $I=1$ phase-shift data (which are unique) lead us to a set of phase shifts which favors the Yang set.

I. INTRODUCTION

A NUMBER of workers[1–6] have studied the experimental data on KN-scattering phase shifts, but a clear understanding of the characteristic features has not been achieved so far. The study of this problem is quite interesting if one notes that the phase-shift analysis of the KN scattering presented by Stenger *et al.*[7] gives two different sets of phase shifts in the state $I=0$, in p and higher partial waves. The reason for such a discrepancy is the well-known Fermi-Yang ambiguity, which arises because of the lack of enough experimental information about the scattering process. One may then think of resolving the ambiguity with the help of the phase shifts calculated on the basis of a theory. Most of the studies made so far either do not take into account all the important one-particle-exchange forces,[1,2] or are confined to only one particular partial wave.[3,4]

Recently, Warnock and Frye[6] have made a phenomenological analysis of the problem and have tried to present quite a detailed analysis of the data. They have favored the Yang set of phase shifts in their analysis, but a very clear statement about it has not been possible since the number of parameters involved in doing this analysis is quite large.

It appears that the N/D method can serve as a very simple and advantageous technique to handle this problem. This method has the merit that it preserves the analyticity and the unitary properties of the amplitude. Several authors[4,5] have made use of this method in the present problem, but instead of solving the integral equations, they have had recourse to approximation schemes like that of Balázs. It is well known that the results of such an approximation as well as the one due to Zemach and Zachariasen depend strongly on the choice of the matching point and the subtraction point, respectively, and that they frequently bear little resemblance to the actual solution of the N/D equations.[8]

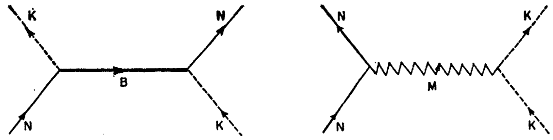
We have tried to analyze the problem by solving the integral equations exactly by numerical methods. Our analysis also involves a free cutoff parameter,[9] which arises because of the fact that the input forces considered by us involve the exchange of particles having spins greater than or equal to one. In such cases the kernels of the integral equations are no longer of the Fredholm type, and the range of integration must be terminated at a finite limit to ensure the existence of solutions.

We have done the analysis of the scattering for $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$, and $d_{5/2}$ partial waves by solving the equations exactly by the method of matrix inversion. We took into account only one-particle-exchange forces and used $SU(3)$ symmetry and the known decay widths to find out the various coupling constants involved. Among the one-particle exchanges we have omitted, for example, φ(1020 MeV), f_0(1250 MeV), B(1220 MeV), and Y_{05}^*(1815 MeV), one could expect a negligible contribution from f_0 and B since they are not known to decay into $K\bar{K}$. Because of the large mass, the contribution for Y_{05}^*(1815 MeV) might also be negligible. However, there are no arguments for neglecting some of the other exchanges. Inelastic effects must also be considered. However, this leads to a rather ambitious program plagued with very meager or no knowledge of some of the coupling constants and also with propagator ambiguities associated with large spins.[10]

We have also not taken into account the more-than-one-particle exchanges for reasons of simplicity.

The only exchanges considered here are Λ, Σ, Y_1^*(1385 MeV), and Y_0^* (1405 MeV), in the u channel and ρ(750 MeV) and ω(780 MeV) in the t channel. There are two

[1] F. Ferrari, G. Frye, and M. Pusterla, Phys. Rev. **123**, 315 (1961).
[2] M. M. Islam, Nuovo Cimento **20**, 546 (1961).
[3] A. D. Martin and T. D. Spearman, Phys. Rev. **136**, B1480 (1964).
[4] D. P. Roy, Phys. Rev. **136**, B804 (1964).
[5] S. Raichowdhary, Ph.D. thesis, University of Delhi, 1964 (unpublished).
[6] Robert L. Warnock and Graham Frye, Phys. Rev. **138**, B947 (1965).
[7] V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. **134**, B1111 (1964).
[8] See for example D. P. Roy (Ref. 4). He finds that the amplitude calculated by Balázs method has unwanted ghost poles. In the case of a multichannel problem, Zemach and Zachariasen technique gives rise to an amplitude which violates time-reversal invariance [F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962)].
[9] This parameter is present in the Zemach and Zachariasen approximation scheme also.
[10] G. Mohan and S. C. Agarwal, Nuovo Cimento **37**, 430 (1965).



(a) (b)

FIG. 1. Exchange graphs for KN scattering. (a) u-channel baryon exchange. (b) t-channel meson exchange.

parameters in our calculation: first, the D-F mixing parameter f of Martin and Wali,[11] and second, the cutoff parameter W_c which arises because of the exchange of particles of spin $\geqslant 1$ like ρ, ω, and Y_1^*. We varied f within a reasonable range ($0.25 \leqslant f \leqslant 0.55$) and found that our analysis favors the Yang set of phase shifts.

Section II contains the description of the input forces, choice of the amplitude, and the dispersion relations. Results and discussion are given in Sec. III. In Sec. IV we give a general summary of the results and conclusions.

II. DESCRIPTION

An arbitrary invariant amplitude for the general KN process is written as

$$T = A(s,t,u) + \tfrac{1}{2}B(s,t,u)\gamma\cdot Q\,, \tag{2.1}$$

where Q is the sum of the momenta of the two K mesons.

The partial-wave amplitude is defined as

$$f_{l_\pm} = (1/q)\exp(i\delta_{l_\pm})\sin\delta_{l_\pm}\,. \tag{2.2}$$

It is connected with $A(s,t,u)$ and $B(s,t,u)$ of Eq. (2.1) in the usual manner[12] and we shall not reproduce the expressions here. In the following we shall write down the expressions for $f_{l_\pm}$ directly.

A. Input Forces

We shall take the contributions from the Born diagrams in Figs. 1(a) and 1(b) and project them into the proper angular momentum and isotopic spin states in the s channel. We consider the forces arising from the following exchanges:

(1) Λ, Σ, Y_1^*, and Y_0^* in the u channel [Fig. 1(a)]
(2) ρ and ω in the t channel [Fig. 1(b)].

As indicated earlier, we shall not take into account the other possible one-particle exchanges like B, φ, and Y_{05}^*. We shall not consider the heavier and more-than-one-particle exchanges because much less is known about the treatment of such forces and their inclusion will make the problem tedious.

The contribution from Λ exchange to the partial-wave amplitude is given by

$$f_{l_\pm}{}^{\Lambda:(1,0)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{g_{KN\Lambda}{}^2}{4\pi}\frac{1}{4Wq^2} \times[(E+M_N)(W+M_\Lambda-2M_N)Q_l(\chi_s{}^\Lambda) + (E-M_N)(W-M_\Lambda+2M_N)Q_{l\pm1}(\chi_s{}^\Lambda)]\,, \tag{2.3}$$

where E is the energy of the nucleon and W and q are the energy and momentum of the KN system in the center-of-mass frame; further

$$\chi_s{}^\Lambda = 1 + [2(M_N{}^2+M_K{}^2) - M_\Lambda{}^2 - W^2]/2q^2 \tag{2.3a}$$

$g_{KN\Lambda}$ is the $KN\Lambda$ coupling constant, whose numerical value can be obtained with the help of the $SU(3)$ scheme. The factors 1 and -1 in the column are the isospin factors corresponding to the states of isospin 1 and 0 of the partial-wave amplitude.

The contribution of Σ exchange is easily obtained by replacing Λ by Σ everywhere and changing the isotopic factor for $I=0$ to 3 in the above expression. The Y_0^* contribution is given as

$$f_{l_\pm}{}^{Y_0^*:(1,0)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{g_{KNY_0^*}{}^2}{4\pi}\frac{1}{4Wq^2} \times[(E+M_N)(W-M_{Y_0^*}-2M_N)Q_l(\chi_s{}^{Y_0^*}) + (E-M_N)(W+M_{Y_0^*}+2M_N)Q_{l\pm1}(\chi_s{}^{Y_0^*})]\,, \tag{2.3b}$$

where all the symbols are the same as defined earlier with a change of suffix Λ to Y_0^* everywhere.

The Y_1^* contribution is

$$f_{l_\pm}{}^{Y_1^*:(1,0)} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \frac{g_{KNY_1^*}{}^2}{4\pi}\frac{1}{4Wq^2} \times[(E+M_N)\{A_l+(W-M_N)B_l\} + (E-M_N)\{-A_{l\pm1}+(W+M_N)B_{l\pm1}\}]\,, \tag{2.4}$$

where

$$A_l = -[(M_{Y_1^*}+M_N)\alpha(W) + (M_{Y_1^*}-M_N)\beta]Q_l(\chi_S{}^{Y_1^*})\,, \tag{2.4a}$$

$$B_l = [\alpha(W)-\beta]Q_l(\chi_S{}^{Y_1^*})\,,$$

and $\alpha(W)$ and β are defined as

$$\alpha(W) = q^{*2} + M_N{}^2 + M_K{}^2 - \tfrac{1}{2}(W^2 + M_{Y_1^*}{}^2)\,,$$
$$\beta = [(M_{Y_1^*}+M_N)^2 - M_K{}^2]^2/12M_{Y_1^*}{}^2\,. \tag{2.4b}$$

Here q^* is the center-of-mass momentum of the system at $W=M_{Y_1^*}$ and

$$\chi_S{}^{Y_1^*} = 1 + [2(M_N{}^2+M_K{}^2) - M_{Y_1^*}{}^2 - W^2]/2q^2\,. \tag{2.4c}$$

[11] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).
[12] See for example, S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).

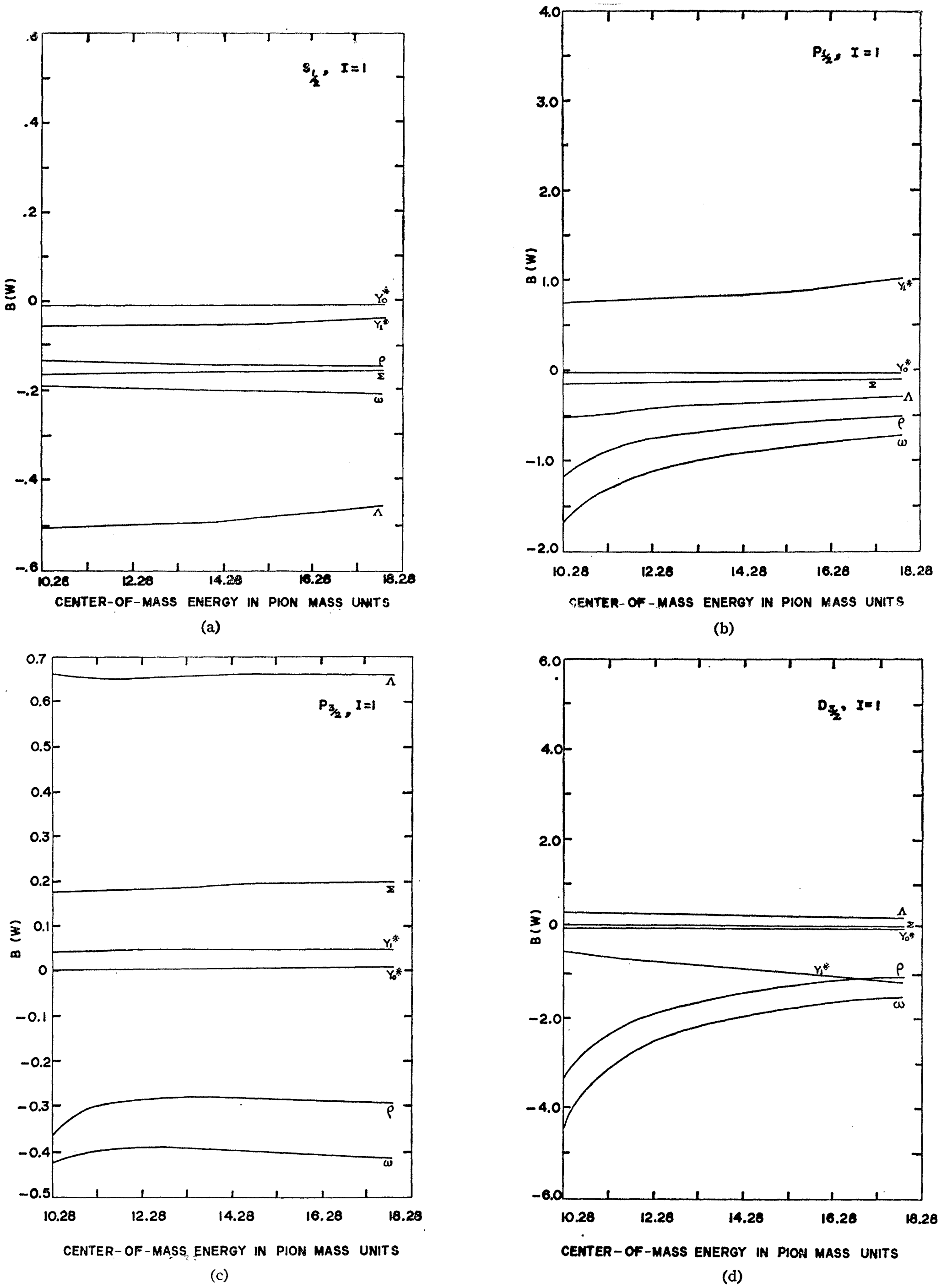


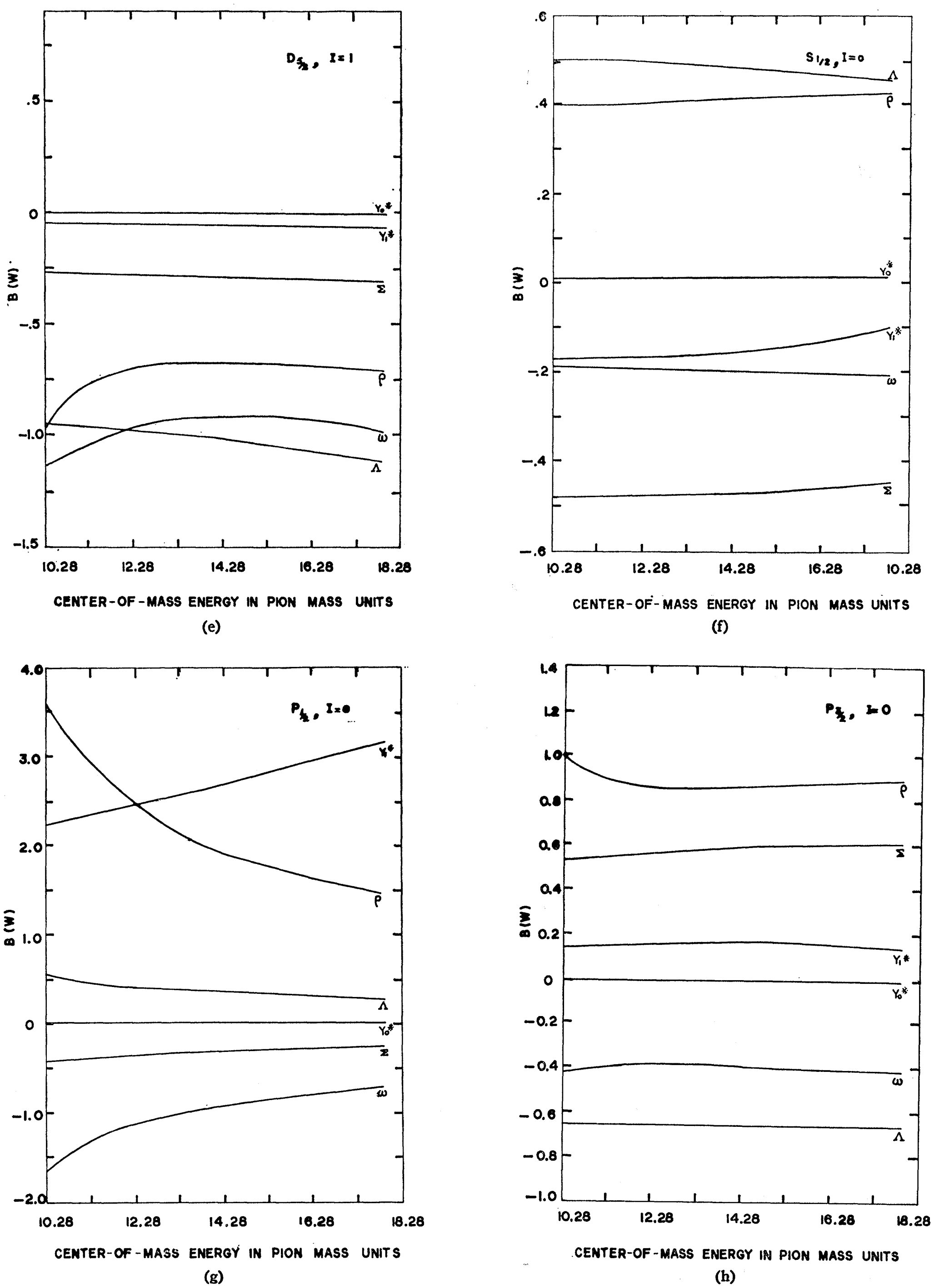
FIG. 2. (a)–(j) represent contributions from the various Born graphs projected in the different partial waves and isotopic-spin states plotted as a function of the center-of-mass energy after multiplying with the appropriate kinematic factor ρ.

The ρ contribution is given by

$$f_{l\pm}{}^{\rho;(1,0)}=-\begin{pmatrix}1\\-3\end{pmatrix}\frac{g_{KK\rho}g_{NN\rho}}{4\pi}\frac{1}{2Wq^2}[(E+M_N)(W-M_N)Q_l(\chi_s{}^\rho)+(E-M_N)(W+M_N)Q_{l\pm1}(\chi_s{}^\rho)], \tag{2.5}$$

where

$$\chi_S{}^\rho=1+M_\rho{}^2/2q^2; \tag{2.5a}$$



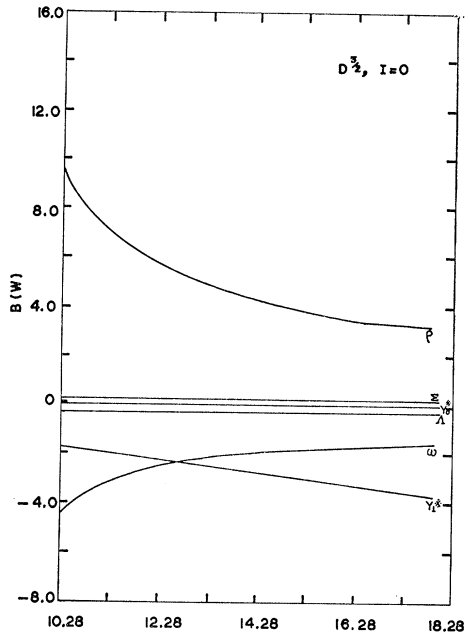
(*Fig. 2 continued on next page.*)

$g_{KK\rho}g_{NN\rho}$ here refers to rationalized coupling strength for charge coupling. We do not take into consideration the magnetic coupling.

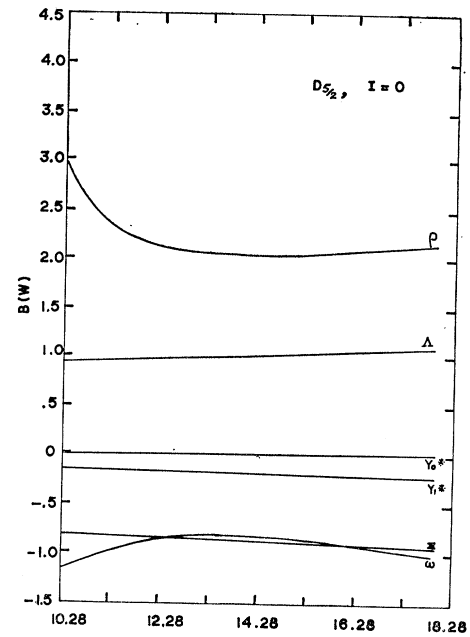
The ω contribution is trivially obtained by replacing ρ by ω everywhere in the above expression and replacing both isospin factors by unity.

B. Choice of the Amplitude and the Dispersion Relations

Although the driving force is limited to single-particle exchange, and therefore contains the correct threshold zeros, the amplitude obtained from the N/D integral-equation formalism may not possess the re-



(i)



(j)

(*Fig. 2 continued from previous page.*)

quired threshold behavior unless the amplitudes are redefined, so that they are nonvanishing at the threshold. For this we write $h=\rho f$, where ρ is a kinematic factor which takes care of the threshold behavior and the behavior at the infinity of the amplitude, so that h is now a well-behaved smooth function of energy and contains the dynamical singularities only.

As has been done by Ball and Wong,[13,14] we write

$$h_J(W)=[W^{2J}/(E+M_N)q^{2J-1}]f_{l=J-1/2}(W). \quad (2.6)$$

We assume that the function $h_J{}^L(W)$, which contains the correct left-hand cut discontinuities, is obtained by summing the six Born amplitudes given in the previous section, multiplied by the appropriate kinematical factor [as given in Eq. (2.6)]. Now $h_J{}^L$ being known, the amplitude h_J can be written as

$$h_J(W)=N(W)/D(W), \quad (2.7)$$

where

$$D(W)=1-\frac{W}{\pi}\int_U dW'\left(\frac{E'+M_N}{W'^{2J+1}}\right)\frac{|q'|^{2J}N(W')}{W'-W} \quad (2.8)$$

and

$$N(W)=h_J{}^L(W)+\frac{1}{\pi}\int_U dW'\left\{\frac{W'h_J{}^L(W')-Wh_J{}^L(W)}{W'-W}\right\}\times\left(\frac{E'+M_N}{W'^{2J+1}}\right)|q'|^{2J}N(W'), \quad (2.9)$$

where U stands for the integration over the physical cut.

These dispersion integrals are, however, divergent and in order to get some physically acceptable results one has to introduce a cutoff parameter by terminating the upper limit of integration at a finite value W_c. This feature is common to all calculations of the present type where particles with spins one or higher are exchanged.

The Eq. (2.9) is a familiar Fredholm-type integral equation and can be solved numerically. The resulting solution can be fed into Eq. (2.8) so that $D(W)$ can also be computed. The phase shifts are given by

$$\cot\delta=(\rho/q)\,\mathrm{Re}D(W)/N(W). \quad (2.10)$$

The various coupling constants that occur have been found by using $SU(3)$ and experimental decay widths.[15] The Y_0^*(1405 MeV) coupling constant was determined by assuming the Dalitz-Tuan model, as has been done by Warnock and Frye.[6] The values used are

$$\begin{aligned} g_{KNY_0^*}{}^2/4\pi&=0.32,\\ g_{KNY_1^*}{}^2/4\pi&=0.05,\\ g_{KK\rho}g_{NN\rho}/4\pi&=0.56,\\ g_{KK\omega}g_{NN\omega}/4\pi&=1.68. \end{aligned} \quad (2.11)$$

The coupling constants involved in the exchanges of Λ and Σ particles are related to the well-known $g_{\pi NN}{}^2$ coupling constant through the mixing parameter f in

[13] J. S. Ball and D. Y. Wong, Phys. Rev. 133, B179 (1964).

[14] It may be mentioned that the amplitude h defined in this way gives rise to poles in f at $W=0$. However, this difficulty is common to all the amplitudes which have been used so far by different authors in solving such problems.

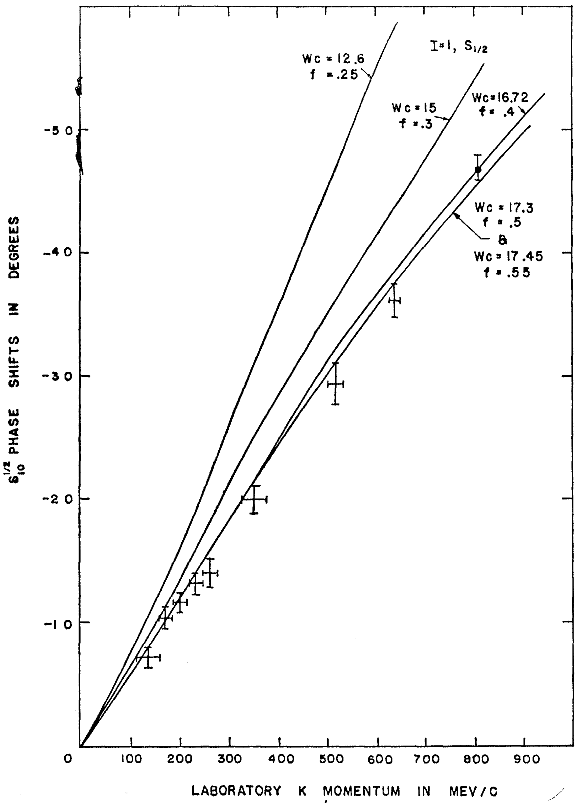
[15] A. W. Martin and K. C. Wali, Nuovo Cimento 31, 1324 (1964).

the following way:

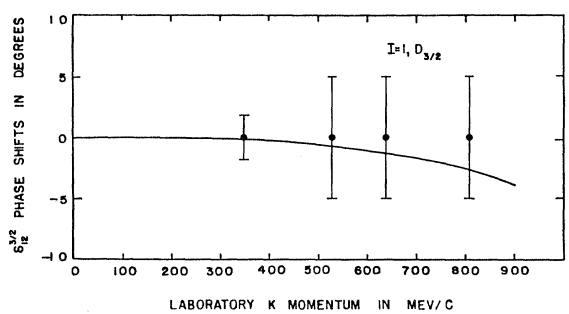
$$g_{KN\Lambda}^2=\tfrac{1}{3}(1+2f)^2g_{\pi NN}^2, \tag{2.12}$$

$$g_{KN\Sigma}^2=(1-2f)^2g_{\pi NN}^2. \tag{2.13}$$

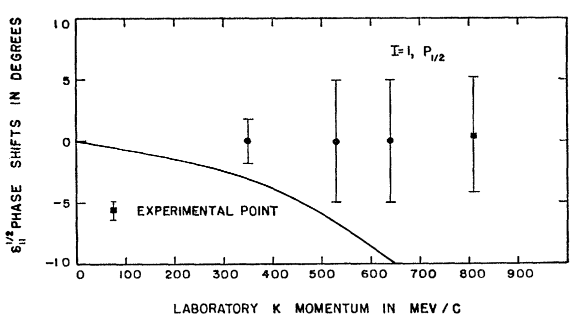
The contributions from the various exchanges, projected in the different partial waves and isotopic spin states, have been shown in the graphs of Fig. 2[(a)–(j)]. The function $B(W)$ which has been plotted as a function of the center-of-mass energy W is the contribution from a particular Born diagram multiplied by the appropriate kinematic factor ρ. In the case of the con-



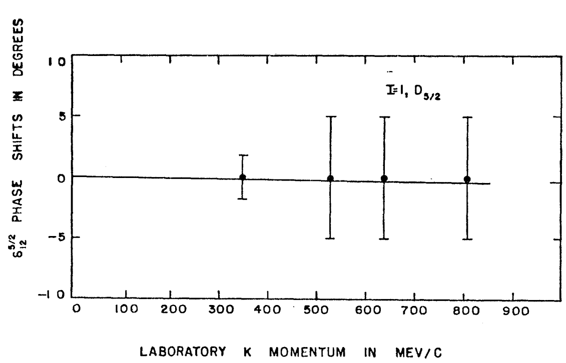
(a)



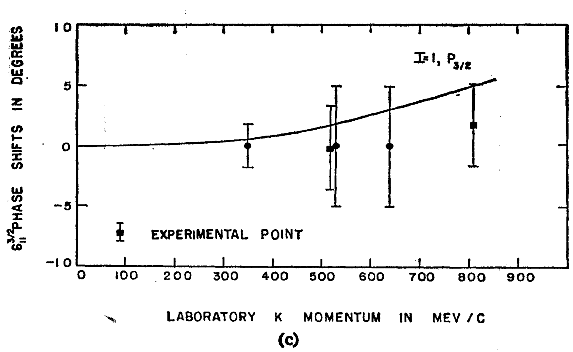
(d)



(b)



(e)



(c)

FIG. 3. (a)–(e) represent the $I=1$ phase-shift data and the theoretical curves.

tributions from the Λ and Σ exchanges, the value of the mixing parameter f has been taken to be 0.25 in plotting these graphs.

III. RESULTS AND DISCUSSION

The phase shifts for the $I=1$ s-wave KN scattering have been taken from the analysis of K^+p scattering by

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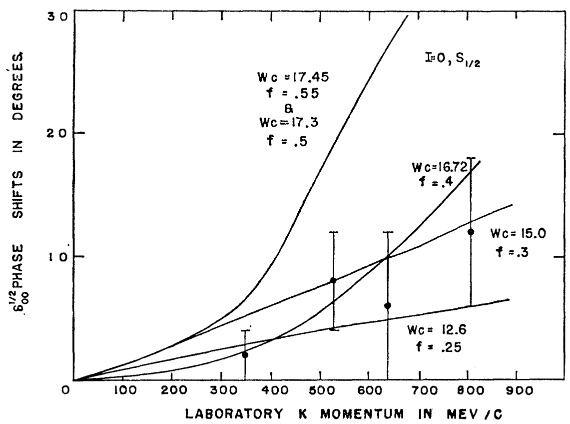

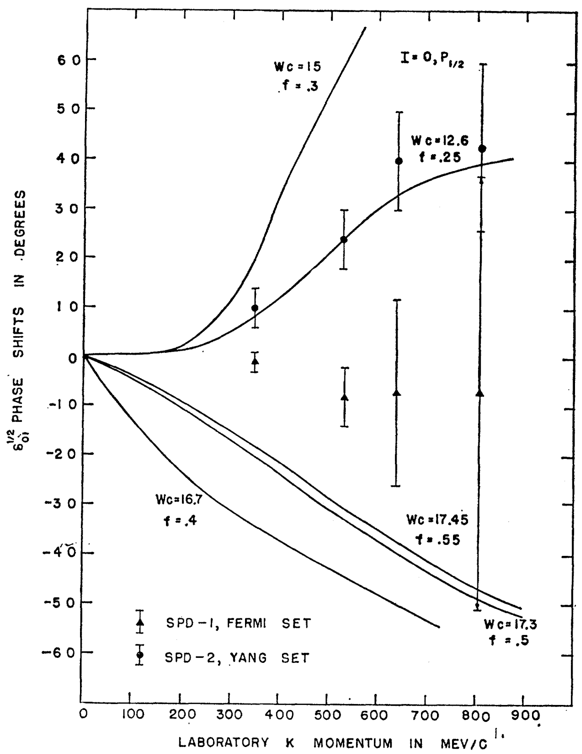

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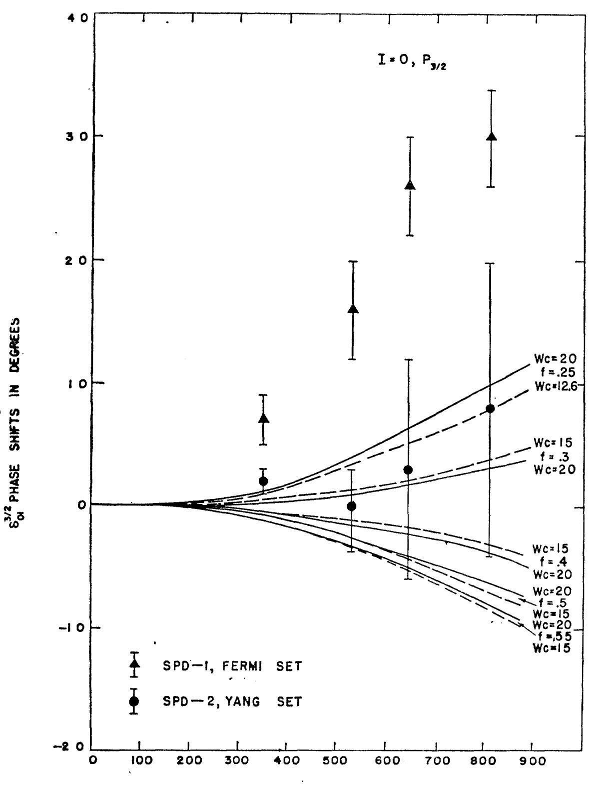
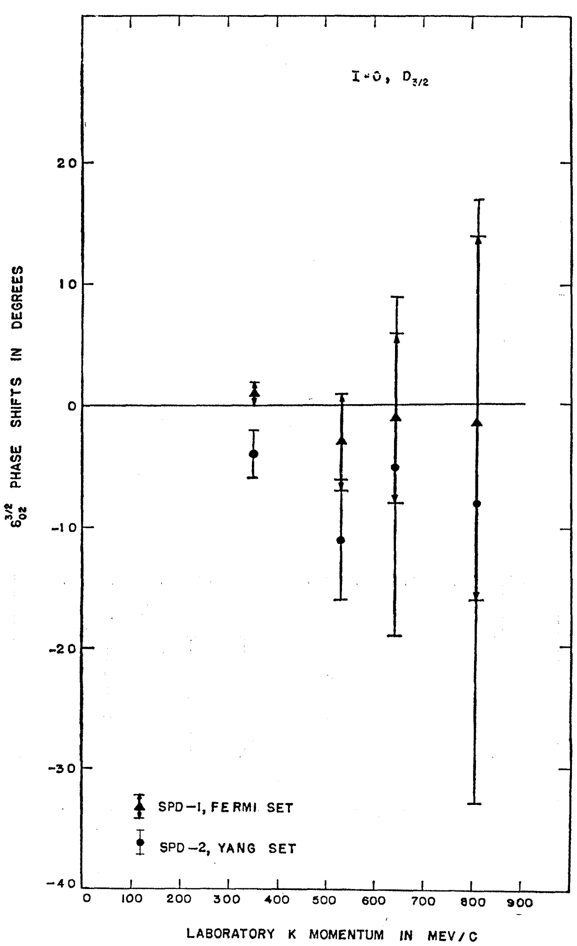
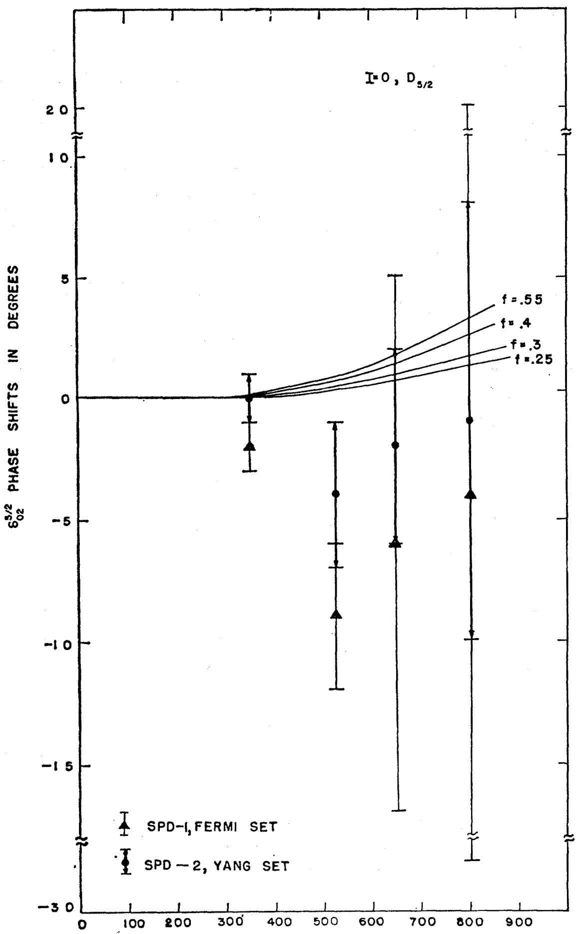




(d)

FIG. 4. (a)–(e) represent the $I=0$ phase-shift data and the theoretical curves. The Fermi-type phase shifts are represented by triangles whereas the Yang-type are denoted by circles.



(e)

Goldhaber *et al.*[16] The p-wave phase shifts at 810 MeV/c are from Stubbs *et al.*[17] and the $p_{3/2}$ phase shift at 520 MeV/c is from Kycia *et al.*[18] The other phase shifts have been taken from Warnock and Frye.[19]

The $I=0$ s-wave phase shifts have been obtained quite uniquely by Stenger *et al.*[7] However, in the case of p and higher partial waves, two sets of phase shifts have been obtained. One, in which the d and higher partial waves are neglected (named SP), and the other in which d waves are also considered but the higher ones are not taken into account (named SPD). We have compared our results with the SPD set, because that looks more reasonable to us. Again the Fermi-Yang ambiguity gives rise to two sets (named 1—the Fermi set, and 2—the Yang set) in each case. We shall try to decide between SPD-1 and SPD-2 phase shifts.

As mentioned earlier, we have two parameters f and W_c at our disposal. The reasonable range for f has been found to be $0.25 \leqslant f \leqslant 0.55$ by Martin and Wali. If we choose $g_{\pi NN}^2/4\pi$ to be 15, this implies that

$$\begin{aligned} 11.25 \leqslant g_{KN\Lambda}^2/4\pi \leqslant 22.05\,, \\ 0 \leqslant g_{KN\Sigma}^2/4\pi \leqslant 3.75\,, \end{aligned} \tag{3.1}$$

which is so large a variation, that with the help of this and the cutoff parameter, one may fit the Fermi as well as the Yang type of phase shifts in almost all the cases. However, one thing that helps us in arriving at a definitely favored solution is the uniqueness of the s-wave phase-shift data in the isotopic spin states $I=0$ and $I=1$. The signs of the phase shifts in the two isotopic spin states are opposite and help us to fix up the two parameters quite definitely. However, while quite a good fit is obtained for s-wave at $f=0.4$, and $W_c=16.72$ pion masses [Figs. 3(a) and 4(a)], the phase shifts in the $p_{1/2}$ and $p_{3/2}$ partial waves for $I=0$ which are obtained by using these values of the parameters do not fit well the experimental data [Fig. 4(b) and (c)]. Moreover the $p_{3/2}$ phase shifts favor the Yang set of solutions whereas the $p_{1/2}$ show a trend towards the Fermi set. This is undesirable because the correct angular distribution requires that both phase shifts together belong either to the Yang set or to the Fermi set. We also noticed that the phase shifts in the $p_{3/2}$ partial wave are more or less independent of the cutoff parameter (within a reasonable range) and change only with various values of the mixing parameter f.[20] The variation of this $p_{3/2}$ phase shift for various values of f is shown in Fig. 4(c). It is clear that the phase shifts have a trend towards the Yang set (which is small and positive) rather than the Fermi set (which is large and positive). Therefore in order that the results may carry any meaning, one should try to adjust the parameters in such a way that one gets the Yang set in all cases. The $d_{3/2}$ and $d_{5/2}$ partial-wave phase shifts depend little on the two parameters and are small [Fig. 4(d) and (e)]. The errors reported in these cases are very large and hence the phase shifts determined theoretically may favor the Fermi as well as the Yang set. In the case of the $p_{1/2}$ partial wave, on the other hand, the two solutions are characteristically different. The Fermi set is small and negative while the Yang set is large and positive. We therefore varied our parameters in such a way that while keeping the signs in the s wave correct, we could favor the Yang set in $p_{1/2}$. It was found that for the value of $f \geqslant 0.4$ it was not possible to get a value of the cutoff parameter which would give correct signs in s-wave $I=0$ and $I=1$ states, and the Yang set in

[16] S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, and T. F. Stubbs, Phys. Rev. Letters **9**, 135 (1962).

[17] T. F. Stubbs, H. Bradner, W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **7**, 188 (1961).

[18] T. F. Kycia, L. T. Kerth, and R. G. Baender, Phys. Rev. **118**, 553 (1960).

[19] Private communication with R. L. Warnock. See also Ref. 6.

[20] The reason for limiting the value of the cutoff parameter between 15 and 20 pion masses in most of the cases is that the s-wave fit becomes extremely bad for the values of W_c outside this region. Especially in the case of s-wave $I=0$ state, the sign of the phase shifts and consequently of the scattering length becomes wrong.

$p_{1/2}$, $I=0$ and $p_{3/2}$, $I=0$ partial waves. With the value of f at 0.3 and $W_c=15$ pion masses we found that the phase shifts obtained are all of Yang type and give correct signs in s wave. However, the fits in case of s-wave $I=1$ and $p_{1/2}$, $I=0$ are not good although a qualitative agreement is there. If the value of the mixing parameter is further reduced, a good fit is obtained for all, except s-wave $I=1$, at $f=0.25$ and $W_c=12.6$ pion masses. As is clear from Fig. 3(a), this is quite a bad fit, but the qualitative agreement still exists. The phase shifts in the case of $I=1$, p and higher partial waves are small and do not vary much with the values of the two parameters in the given range. The fits have been shown in Fig. 3 and are reasonably good.

We have thus been able to fit the Yang type of phase shifts (at least qualitatively) with the help of only two parameters.

IV. CONCLUDING REMARKS

In the present calculation we have shown that if the same values of the two parameters f and W_c are used in all the partial waves, we can get a qualitative agreement in the case of the Yang set only. No values of the parameters allow us to favor the Fermi set consistently and hence this set is ruled out in this calculation. By this simple approach it seems that the most probable set of correct phase shifts is of the Yang type. As is evident from the graphs, the agreement in some cases is not as good as one would expect; the fits may be improved by considering the exchanges that have not been taken into account or by considering the inelastic channels like $KN \rightarrow K^*N$. Use of the values of the coupling constant determined with the help of higher symmetry schemes may also prove to be helpful. An attempt in this direction has already been made by Warnock[21] in which the broken $M(12)$ symmetry has been put to the test. The analysis resembles the one in Ref. 6. Both these analyses do not unitarize the amplitude (though it has been said that the unitarity corrections are small) and are therefore quite different from the present one. It may be quite accidental but is worth noticing that both these calculations as well as the one by Raichowdhary[5] (who has used the Balázs approximation scheme) have a tendency to favor the Yang set, which is an essential conclusion of the present model also.

ACKNOWLEDGMENTS

The author is indebted to Dr. Gyan Mohan for his kind interest and continued guidance and to Dr. Lalit Kumar Pande for suggesting the problem. It is a pleasure to thank Shri Aditya Kumar and Dr. Virendra Singh for some helpful discussions. Thanks are also due to the Structural Engineering Research Centre, Roorkee, and the Tata Institute of Fundamental Research, Bombay, for making available for their electronic computers IBM 1620 and CDC 3600, respectively, for solving the problem.

[21] Robert L. Warnock, in *Proceedings of the Second Topical Conference on Resonant Particles* (Ohio State University Press, Columbus, Ohio, 1965), p. 397.