Field-Theory Calculations in Broken Symmetries*

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A method based on field theory is developed for calculating corrections to broken-symmetry sum rules. The Gell-Mann–Okubo formulas for the bare masses of the baryons and mesons are found from equal-time commutators in the current algebra by using the canonical commutation relations for unrenormalized Heisenberg field operators. The corrections to the Gell-Mann–Okubo formula for the physical masses are calculated from the self-masses in perturbation theory. It is conjectured that the logarithmic infinity occurring in the second-order perturbation-theory result for the baryon self-mass cancels in the mass sum rule, and the corrections can be evaluated. For pure D coupling, the correction in second-order perturbation theory reduces the discrepancy in the baryon-mass sum rule from 12 to 7 MeV.

1. INTRODUCTION

ISOSPIN has played an important role in explaining the charge independence of strong interactions corresponding to complete unitary symmetry in $SU(2)$. When the electromagnetic field is "turned on" a preferred direction τ_3 is chosen in the space of the generator algebra of $SU(2)$ and the symmetry is broken. In view of the smallness of the ratio of the strengths of the two interactions the symmetry is only *weakly* broken.

With the advent of strangeness and the incorporating of this quantum number in $SU(3)$[1,2] it was realized that strongly broken symmetries play an important role in the world of elementary particles. The hadron mass spectrum is a result of a well-defined broken symmetry. When the breaking mechanism is turned on, a preferred direction $F_8=\frac{1}{2}\sqrt{3}Y$ is chosen in the space of the generator algebra of $SU(3)$. The breaking is stronger than in the case of $SU(2)$ as is reflected by the Gell-Mann–Okubo formula[3]

$$M_N+M_\Xi=\tfrac{3}{2}M_\Lambda+\tfrac{1}{2}M_\Sigma$$

which is satisfied to within 12 MeV.

The origin of the breaking of $SU(3)$ remains a mystery. Is there a field in nature that breaks the $SU(3)$ symmetry in the way the electromagnetic field breaks $SU(2)$? Perhaps it is something as simple as the splitting of the bare masses of the particles. Whatever the origin of the breaking the problem arises of finding ways to evaluate corrections to broken symmetry formulas like the Gell-Mann–Okubo formula. Recent attempts to solve this problem are exemplified by the work of Coleman, Glashow, and Socolow,[4] Fubini and Furlan,[5] and Fubini, Furlan, and Rossetti.[6] The latter authors have empolyed the equal-time commutators of the currents and have developed a dispersion-type formalism based on intermediate state expansions and completeness. Explicit calculation of the corrections to sum rules may be evaluated in terms of many-particle intermediate states including zero-mass charged particles. In the case of the derivation of the Adler-Weisberger[7] sum rule the experimental cross sections are used for scattering in the forward direction. It is assumed that there are no subtraction constants required in the calculations.

In the following, we shall pursue a different approach by developing a field-theory formalism in the framework of the broken symmetry. It is hoped that within this formalism strong interaction calculations may be viewed as *bona fide* perturbation calculations, since the exact unitary symmetry is treated as the zeroth approximation within the expansion scheme.

The mass sum rules for the baryons and mesons are studied in order to see how the method may be applied to specific problems. Sum rules for the bare masses are derived by evaluating exact, equal-time commutators using the canonical commutation relations for the unrenormalized field operators. We then replace the baryon bare masses M_{0B} by the differences of the physical masses M_B and the self masses δM_B. The corrections to the Gell-Mann–Okubo formula are due to the δM_B which are calculated from an $SU(3)$-invariant Yukawa coupling.

Because the expressions for the renormalization constants in field theory are obtained from the unrenormalized propagator the self-masses δM_B are not uniquely determined. It is conjectured that the logarithmic infinities that occur in δM_B, in second-order perturbation theory, cancel in the mass sum rule. A calculation of the corrections to the baryon mass sum rule can then be performed.

2. EQUAL-TIME COMMUTATION RELATIONS

We shall consider the eight-dimensional algebra of $SU(3)$ and the set of infinitesimal transformations

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[1] M. Gell-Mann, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1964).

[2] Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[3] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[4] See the review article, S. Coleman, in Proceedings of the Eastern United States Theoretical Physics Conference, 1964 (unpublished).

[5] S. Fubini and G. Furlan, Physics 4, 229 (1965).

[6] S. Fubini, G. Furlan, and C. Rossetti, CERN Report No. 65/998/5-Th. 578, 1965 (unpublished).

[7] S. A. Adler, Phys. Rev. Letters 14, 1051 (1965); W. I. Weisberger, *ibid.* 14, 1047 (1965).

generated by the eight independent Hermitian operators F_i, which may depend on the time.[1] The eight components of the total unitary F-spin current of the meson-baryon system are defined by[8]

$$\mathcal{J}_{i\mu}=i\bar{B}\gamma_\mu F_i B-f_{ijk}\Pi_j\partial_\mu\Pi_k\,,\quad (i,j,k=1\cdots 8)\,,\quad (1)$$

where B_i and Π_i denote the octets of baryon and meson fields, respectively. The Lagrangian density $\mathcal{L}$ is a function of a number of field operators and their gradients. If the Lagrangian is invariant under the infinitesimal unitary transformations, then all eight components of the unitary spin are conserved:

$$\partial_\mu\mathcal{J}_{i\mu}=0\,. \quad (2)$$

If the Lagrangian has a part $\mathcal{L}_b$ which is not invariant under the unitary transformations, then not all the components $\mathcal{J}_{i\mu}$ are conserved. Gell-Mann[9] has emphasized that even though the symmetry may be badly violated by $\mathcal{L}_b$ this does not prevent us from defining $\mathcal{J}_{i\mu}$ as in (1), nor does it affect the commutation rules of the unitary spin density. The total unitary spin F_i is defined by

$$F_i=-i\int\mathcal{J}_{i4}d^3x \quad (3)$$

at any time. For equal times the commutation rules for the F_i are

$$[F_i,F_j]=if_{ijk}F_k\,. \quad (4)$$

It is observed that the components F_4, F_5, F_6, and F_7 of the unitary spin change strangeness by one unit and isotopic spin by a half unit. These components are not conserved when there is a term $\mathcal{L}_b$ in the Lagrangian.

The unitary spin densities satisfy the equal time commutation relation[9]

$$[\mathcal{J}_{i4}(\mathbf{x},t),\mathcal{J}_{j4}(\mathbf{x}',t)]=-f_{ijk}\mathcal{J}_{k4}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{x}')\,. \quad (5)$$

By hypothesis, the noninvariant term $\mathcal{L}_b$ contains no gradients and is therefore the negative of the noninvariant term $\mathcal{H}_b$ in the Hamiltonian density $\mathcal{H}$ which we define to be

$$\mathcal{H}=\mathcal{H}_s+\mathcal{H}_b\,, \quad (6)$$

where $\mathcal{H}_s$ is the symmetry-preserving part of $\mathcal{H}$. We have

$$\partial_\mu\mathcal{J}_{i\mu}=-i[F_i(t),\mathcal{H}(\mathbf{x},t)]\,. \quad (7)$$

By integrating (7) over space, we obtain the familiar equations of motion

$$\dot{F}_i=\int\partial_\mu\mathcal{J}_{i\mu}d^3x=-i\left[F_i,\int\mathcal{H}d^3x\right]. \quad (8)$$

[8] J. W. Moffat, in *Proceedings of the Fifth International Winter School in Nuclear Physics, 1966*, edited by P. Urban (University of Graz Publishers, Austria, 1966).
[9] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

As we shall be concerned with $SU(3)$ let us employ the de Swart convention for the unitary spin components F_i.[10] The spin components corresponding to the nonzero roots of $SU(3)$ are defined by

$$F_A{}^{\pm}=-i\int\mathcal{J}_4{}^{A\pm}d^3x\sim A^{\pm}\,, \quad (9)$$

where $A=I$, K, L and $\sim$ denotes that $F_A{}^{\pm}$ in (9) has the same $SU(3)$ transformation properties as $A^{\pm}$. The translation operators in the isospin subspace connect states with $\Delta I=1$. The L-like operators are translation operators in the U-spin subspace and the K-like operators are translation operators in the V-spin subspace.

The assumption of the broken eightfold way is that the energy density is the sum of two terms, one of which is invariant under the F spin, and the other transforming like the $I=0$, $Y=0$ component of an octet. Thus,

$$\mathcal{H}_b\sim F_8\,. \quad (10)$$

In terms of group theory it can be proved that if

$$\partial_\mu\mathcal{J}_\mu{}^{K+}(\mathbf{x},t)=-i[F_K{}^+(t),\mathcal{H}_b(\mathbf{x},t)]\,, \quad (11)$$

and $\mathcal{H}_b$ satisfies (10), then

$$\partial_\mu\mathcal{J}_\mu{}^{K+}\sim K^+\,. \quad (12)$$

It follows that

$$[\partial_\mu\mathcal{J}_\mu{}^{K+}(\mathbf{x},t),F_K{}^+(t)]=0\,. \quad (13)$$

The result (13) holds for a more general $\mathcal{H}_b$ than the one given by (10).

3. HEISENBERG FIELDS AND CANONICAL COMMUTATION RELATIONS

We shall use unrenormalized Heisenberg operators to describe the interacting fields. The kinetic-energy part of the Lagrangian is

$$\mathcal{L}_0=-[\bar{B}(\gamma_\mu\partial_\mu+M_{0B})B+(\partial_\mu\Pi_i\partial_\mu\Pi_i+m_{0b}{}^2\Pi_i\Pi_i)]\,, \quad (14)$$

where M_{0B} and m_{0b} describe the bare masses of the baryons and mesons, respectively. The Lagrangian for the Yukawa coupling is

$$\mathcal{L}_I=iG\bar{B}\gamma_5[\alpha D_i+(1-\alpha)F_i]B\Pi_i\,. \quad (15)$$

This interaction is assumed to be invariant under F-spin transformations. As the bare masses M_{0B} and m_{0b} are different in (14) the kinetic-energy part $\mathcal{L}_0$ of the Lagrangian contains a mass-splitting term $\mathcal{L}_b$ and the unitary symmetry is reduced.

The assumption that $\mathcal{L}_I$ is invariant under the F-spin transformations is a strong one as evidence from photoproduction of K particles and the binding of hyperfragments seems to indicate that the couplings ΣKN

[10] J. J. De Swart, Rev. Mod. Phys. **35**, 916 (1963).

and ΛKN are both much smaller than the $N\pi N$ coupling and, therefore, the mechanism responsible for the mass differences breaks the coupling constants as well.[1] However, it is hoped that the splitting of the coupling constants occurs in a higher order in the symmetry breaking.

The equal-time canonical commutation relations for the unrenormalized Heisenberg operators are

$$[\Pi_b(\mathbf{x},t),\partial_t\Pi_{b'}(\mathbf{x}',t)]=i\delta_{bb'}\delta(\mathbf{x}-\mathbf{x}')\,,$$
$$\{B_a(\mathbf{x},t),\bar{B}_{a'}(\mathbf{x}',t)\}=\gamma_4\delta_{aa'}\delta(\mathbf{x}-\mathbf{x}')\,, \qquad (16)$$

and

$$[\Pi_b(\mathbf{x},t),\Pi_{b'}(\mathbf{x}',t)]=0\,, \qquad (17)$$
$$\{B_a(\mathbf{x},t),B_{a'}(\mathbf{x}',t)\}=0\,,$$
$$\{\bar{B}_a(\mathbf{x},t),\bar{B}_{a'}(\mathbf{x}',t)\}=0\,. \qquad (18)$$

The F-spin current density $\mathcal{J}_{\mu B}{}^{K^+}$ of the baryons with the transformation property K^+ in V-spin space is given by

$$\mathcal{J}_{\mu B}{}^{K^+}=-\sqrt{3}\bar{p}\gamma_\mu\Lambda+\sqrt{3}\bar{\Lambda}\gamma_\mu\Xi^- -\bar{p}\gamma_\mu\Sigma^0 -\sqrt{2}\bar{n}\gamma_\mu\Sigma^- +\bar{\Sigma}^0\gamma_\mu\Xi^- +\sqrt{2}\bar{\Sigma}^+\gamma_\mu\Xi^0\,. \qquad (19)$$

The corresponding F-spin current density of the mesons is

$$\mathcal{J}_{\mu M}{}^{K^+}=-\sqrt{3}K^-\partial_\mu\eta+\sqrt{3}\eta\partial_\mu K^- -K^-\partial_\mu\pi^0+\pi^0\partial_\mu K^- -\sqrt{2}\bar{K}^0\partial_\mu\pi^- +\sqrt{2}\pi^-\partial_\mu\bar{K}^0\,. \qquad (20)$$

4. BARE-MASS SUM RULES

Let us consider first the derivation of the sum rule for the baryons. From (19) and the Heisenberg equations of motion obtained from (14), we get

$$\partial_\mu\mathcal{J}_{\mu B}{}^{K^+}=-\sqrt{3}(M_{0p}-M_{0\Lambda})\bar{p}\Lambda+\sqrt{3}(M_{0\Lambda}-M_{0\Xi})\bar{\Lambda}\Xi^- -(M_{0p}-M_{0\Sigma})\bar{p}\Sigma^0-\sqrt{2}(M_{0n}-M_{0\Sigma})\bar{n}\Sigma^- +(M_{0\Sigma}-M_{0\Xi})\bar{\Sigma}^0\Xi^-+\sqrt{2}(M_{0\Sigma}-M_{0\Xi})\bar{\Sigma}^+\Xi^0\,. \qquad (21)$$

We have used the circumstance that the F-spin invariant interaction terms in $\mathcal{H}_s$ do not contribute to $\partial_\mu\mathcal{J}_\mu{}^{K^+}$ as $\mathcal{H}_s$ commutes with $\mathcal{J}_4{}^{K^+}$. Moreover, in the following we shall take advantage of the fact that the bare mass sum rules for the baryons and mesons, obtained from the commutation relations for the currents, are independently satisfied.

In order to derive mass sum rules we shall calculate equal-time commutators of current densities. By virtue of the commutation relations (16)–(18) it follows that

$$\int d^3x d^3x'[\partial_\mu\mathcal{J}_{\mu B}{}^{K^+}(\mathbf{x},t),\mathcal{J}_{4B}{}^{K^+}(\mathbf{x}',t)]=\int d^3x\,\bar{p}(x)\Xi^-(x) \times(6M_{0\Lambda}-4M_{0p}+2M_{0\Sigma}-4M_{0\Xi})=0\,. \qquad (22)$$

The product of field operators $\bar{p}(x)\Xi^-(x)$ is not zero for all x, and (22) yields the Gell-Mann–Okubo sum rule for the bare masses

$$M_{0p}+M_{0\Xi}=\tfrac{3}{2}M_{0\Lambda}+\tfrac{1}{2}M_{0\Sigma}\,. \qquad (23)$$

This result has been derived to all orders in the $SU(3)$ invariant interaction assuming that the breaking Hamiltonian $\mathcal{H}_b$ is consistent with (12) and (13).

Let us now consider the pseudoscalar mesons. By using the Heisenberg equations of motion for the mesons and (20), we get

$$\partial_\mu\mathcal{J}_{\mu M}{}^{K^+}=\sqrt{3}(M_{0K}{}^2-M_{0\eta}{}^2)K^-\eta+(M_{0K}{}^2-M_{0\pi}{}^2)K^-\pi^0 +\sqrt{2}(M_{0K}{}^2-M_{0\pi}{}^2)K^-\bar{K}^0\,. \qquad (24)$$

Moreover,

$$\mathcal{J}_{4M}{}^{K^+}/i=\sqrt{3}K^-\partial_t\eta-\sqrt{3}\eta\partial_tK^-+K^-\partial_t\pi^0 -\pi^0\partial_tK^-+\sqrt{2}\bar{K}^0\partial_t\pi^- -\sqrt{2}\pi^-\partial_t\bar{K}^0\,. \qquad (25)$$

The canonical commutation relations (16) and (17) yield

$$\int d^3xd^3x'[\partial_\mu\mathcal{J}_{\mu M}{}^{K^+}(\mathbf{x},t),\mathcal{J}_{4M}{}^{K^+}(\mathbf{x}',t)]=\int d^3x\,K^-(x) \times K^-(x)(4M_{0K}{}^2-3M_{0\eta}{}^2-M_{0\pi}{}^2)=0\,. \qquad (26)$$

We obtain the Gell-Mann–Okubo sum rule for the pseudoscalar mesons

$$4M_{0K}{}^2-3M_{0\eta}{}^2-M_{0\pi}{}^2=0 \qquad (27)$$

to all orders in the F-spin invariant interactions.

5. FIELD-THEORETICAL CALCULATIONS OF CORRECTIONS TO MASS SUM RULES

We shall now describe a method of calculating corrections to the Gell-Mann–Okubo sum rule for the physical masses.

Mass renormalization in field theory is based on the assumption that the observed mass M_B can be written

$$M_B=M_{0B}+\delta M_B\,. \qquad (28)$$

The free-field part of the Lagrangian $\mathcal{L}_0$ contains a term $M_0\!:\!\bar{\psi}\psi\!:$, where M_0 is the bare mass. This term is rewritten as $M\!:\!\bar{\psi}\psi\!:-\delta M\!:\!\bar{\psi}\psi\!:$. The quantity δM is determined for a free fermion such that the self-energy diagram is cancelled by the counter-term diagram, and the resulting mass M is the observed mass of the fermion. The bare mass M_0 and the self-mass δM are not uniquely defined, because they pertain to the unrenormalized propagator and they are infinite in every order of perturbation theory.

Let us consider the baryon sum rule (23) for the bare masses. Substituting (28) into (23), we get

$$M_p+M_\Xi-\tfrac{3}{2}M_\Lambda-\tfrac{1}{2}M_\Sigma-\delta A=0\,, \qquad (29)$$

where δA is given by

$$\delta A=\delta M_p+\delta M_\Xi-\tfrac{3}{2}\delta M_\Lambda-\tfrac{1}{2}\delta M_\Sigma\,. \qquad (30)$$

For a given baryon a calculation of δM_B in second-order perturbation theory gives

$$\delta M_B=\frac{G^2}{32\pi^2\hbar c}\sum_i C_i^2\Big\{M_B\ln\Big(\frac{\lambda^2}{M_B^2}\Big)+M_B\Big[-\tfrac{1}{2}+\Big(\frac{m_i}{M_B}\Big)^2+2\Big(\frac{m_i}{M_B}\Big)^2\Big(1-\frac{m_i^2}{2M_B^2}\Big)\times\ln\Big(\frac{m_i}{M_B}\Big)-2\Big(\frac{m_i}{M_B}\Big)^3\Big(1-\frac{m_i^2}{2M_B^2}\Big)^{1/2}\cos^{-1}\Big(\frac{m_i}{2M_B}\Big)\Big]\Big\}, \quad (31)$$

where λ is the invariant Feynman cutoff, the C_i are certain Clebsh-Gordan coefficients and the m_i $(i=1\cdots 8)$ denote the meson masses of the octet of pseudoscalar mesons. We have assumed in (31) that λ is the same for all the baryons in the limit as $\lambda\to\infty$, and we have kept only the leading term in an expansion of δM_B in the mass difference of the external and internal baryon masses in the self-energy graph.

We shall now make the conjecture that we can replace M_B in the logarithmically divergent term $M_B\ln(\lambda^2/M_B^2)$ by the mean mass of the baryons $M=1.141$ BeV. The logarithmic infinity now cancels in the limit as $\lambda\to\infty$, if we substitute δM_B into the expression δA in (30).

The term δA can now be evaluated for the case of pure D coupling with $G^2/\hbar c\simeq 15$ and the result is

$$\delta A\simeq -5\text{ MeV}. \quad (32)$$

Up to second order in G, for F-spin–invariant interactions, the sum rule for the baryon masses is now obeyed to within 7 MeV. For $F/D\sim\frac{1}{5}$ a calculation including both F and D coupling gives $\delta A\simeq -2$ MeV and the discrepancy in the Gell-Mann–Okubo sum rule is 10 MeV.

It is interesting to note that in our field-theory formalism the mass sum rules occur unambiguously with linear masses for the baryons and with squares of masses for the mesons. It is also important to recognize that our mass sum rules for the bare masses and the result for the corrected Gell-Mann–Okubo formula are not dependent on the energy and momentum. The Lagrangian and the Heisenberg equations of motion are automatically covariant, and we rely on calculations of equal time commutators without the use of intermediate state expansions and completeness.

6. CONCLUDING REMARKS

We have obtained bare mass sum rules, to all orders in the unitary-symmetric Yukawa interactions, from the equal-time commutators in the algebra of currents and the canonical commutation relations for the Heisenberg field operators. The current algebra must be supplemented by a dynamical formalism either in the language of dispersion relations, as was proposed by Fubini, Furlan, and Rossetti,[6] or in the language of field theory. The methods of dispersion relations are troubled by subtraction constants and analytical continuation to zero-mass charged particles, while the field-theory methods are not always well-defined, because of cutoffs and the nonuniqueness of renormalization constants.

We have conjectured that the logarithmic infinity in the baryon self-mass cancels in the mass sum rule, and find that the finite correction to the sum rule is of the right order of magnitude and in the right direction.

It is evident that a new approach to the masses of elementary particles, or a more well-defined solution of the mass problem in field theory will resolve many of the problems in broken symmetry theories.

The field-theoretical methods proposed in this paper to calculate sum rules, and their corrections within broken symmetry schemes, can be applied to other problems such as the electromagnetic mass differences, and the calculation of magnetic moments in $SU(6)$. Applications of this type are presently being studied.

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