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Re-examination of Octet Dominance for Weak Interactions*

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Octet dominance for weak interactions is re-examined. The basis for discussion is that Cabibbo's octet-weak-current theory in correlating leptonic decays is to be regarded primarily as confirmatory evidence for an octet structure in strong interactions. For nonleptonic decays, a compact procedure for unifying the salient experimental features can be obtained with the assumption of the $\Delta T=\frac{1}{2}$ rule and T-$L(2)$ and time-reversal invariances, but without assuming an explicit hadron current-current structure. The status of R conjugation is briefly analyzed from the present viewpoint.

It is now generally accepted that $SU(3)$ symmetry is well established as an approximate symmetry of strong interactions. For most phenomena of strong interactions, with a judicious understanding of symmetry-breaking effects, the symmetry is correct to about 10–20%, which is about as much as we can generally hope for.[1]

An important question to ask at this stage is about the role of $SU(3)$ symmetry for the weak interactions, in particular the relevance of the current-current theory[2] for weak processes involving the hadron particles. It is the purpose of the following note to determine as crisply as possible, the set of propositions which can most adequately characterize the concept of octet dominance in weak interactions.

Cabibbo[3,4] proposed that the currents for the strongly interacting particles in the current-current theory have *octet transformation* properties, namely $J_\mu \sim O_j^i$ $(i, j=1, 2, 3)$—a traceless tensor. This is a very natural assumption since we are accustomed to associate the $\Delta S=0$ piece of the vector current J_μ^V as proportional to the isospin currents.[2] Extension of the components of J_μ to form an octet directly proportional to the octet of currents to which the isospin currents belong, is thus a compelling proposition. Application of the octet hypothesis to *leptonic* decays of hadrons in the current-current form,

$$J_\mu^\dagger L_\mu, \qquad (1)$$

where L_μ is the lepton current (transforming as a unitary singlet), is unambiguous and has in fact received very impressive experimental support.[5] It is important to emphasize however, that the success of the octet-current theory in correlating leptonic decays, is to be regarded primarily as confirmatory evidence for *an octet structure in strong interactions*, rather than as the germinating basis for an $SU(3)$ theory of weak interactions. In other words, this approach is very useful in the sense that it is a convenient probe of strong-interaction properties. Such a viewpoint need not be regarded as novel; it is in fact already implicit in some sense in Gell-Mann's proposal[6] to identify the weaker currents with the generators of $SU(3)$— thus defining a meaningful strong-interaction symmetry. As a corollary here, however, we do expect that the scale of symmetry breakdown for Cabibbo's proposals[3,4] should be the same as those for strong interactions.

Extension of the octet current structure to include a discussion of nonleptonic decay processes in the current-current picture, required that the interaction Lagrangian be proportional to

$$J_\mu^\dagger J_\mu \qquad (2)$$

The elements of $J_\mu^\dagger$ (Hermitian conjugate of J_μ) are members of the same octet, hence (2) must have transformation properties corresponding to the product of two identical octets **8**×**8**, namely **1**, **8**, and **27**. The **8** representation allows only $\Delta Y=\pm 1$ transitions, with $\Delta T=\frac{1}{2}$, and thus supplies a convenient basis for "explaining" the $\Delta T=\frac{1}{2}$ rule in the scheme of unitary symmetry. Cabibbo's additional hypothesis of *octet dominance*[4] requires then that the terms of $J_\mu^\dagger J_\mu$ in the octet representation are enhanced relative to those of the **27** representation. The concrete consequences of this assumption are:

(a) $K_1^0 \nrightarrow 2\pi$

(b) Coupled with CP invariance, the triangle relation Δ

$$3^{1/2}\langle\Sigma^+|p\pi^0\rangle - \langle\Lambda|p\pi\rangle = 2\langle\Xi^-|\Lambda\pi^-\rangle \qquad (3)$$

can be established for the parity-violating (p.v.) decay amplitudes only.[7]

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[1] A recent survey of the status of $SU(3)$ in strong interactions has been made by R. Socolow (private communication).

[2] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[3] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[4] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1963).

[5] R. H. Dalitz, in *Proceedings of the International School of Physics "Enrico Fermi" on Weak Interactions, Varenna, Italy, 1964* (Oxford University Press, New York, 1965), pp. 1–106.

[6] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964). Note also the analogy drawn by Sakurai [Phys. Rev. Letters **12**, 79 (1964)] that in a strong interaction theory in which the members of the vector-meson octet are coupled to the appropriate F-type currents, The sources of ρ^+ and M^+ [$=K^*(888)$] are precisely the currents that appear in Cabibbo's theory of weak interactions.

[7] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).

We are well aware that experimentally $K_1^0 \to 2\pi$ is very much the dominant decay mode.[8] To the extent that symmetry-breakdown effects in $SU(3)$ (with regard to decay properties) are generally expected to give corrections of not much more than 10–20% to conclusions reached on the basis of pure $SU(3)$, result (a) leaves much to be desired. Note that attempts[4,7] to take advantage of this forbiddenness to bring $K_1^0 \to 2\pi$ versus $K^+ \to \pi^+ + \pi^0$ in line with electromagnetic deviations only of $\Delta T = \frac{1}{2}$, make tacit use of the assumption that the scale of symmetry breakdown is of order of 10%. What is far from clear is the true significance of this result[9] since the primary $SU(3)$-symmetric decay is forbidden. Empirically the evidence for Δ [relation (3)] is rather good[10]; result (b) is, however, incomplete since only Δ_s (p.v.) is predicted—unless additional assumptions are introduced. Further arguments against this particular viewpoint of octet dominance, are given recently by Bailin.[11]

It is evident that we need to proceed from a less committed description for nonleptonic decays, than to postulate octet dominance *a priori*. Our present viewpoint of regarding the octet-current hypothesis for hadron particles as primarily associated with their strong-interaction properties, makes it less than compelling to "derive" the $\Delta T = \frac{1}{2}$ rule—since the latter is a selection rule for the weak interactions.

In order to obtain an integrated picture, the appropriate search is for those selection rules and symmetries of weak interactions [within the framework of $SU(3)$] which will predict the empirically satisfied Δ relation for both parity-conserving (p.c.) and parity-violating (p.v.) amplitudes. These proposed symmetries of weak interactions have of course to be consistent with electric-charge conservation. The solution is inherent in some recent work of Rosen.[12,13] We need, in addition to the $\Delta T = \frac{1}{2}$ rule, T-$L(2)$ invariance with the following transformation properties[14]:

$$(T, T_3, Y_T) \leftrightarrow (L, -L_3, Y_L) \qquad (4)$$
$$(K, K_3, Y_K) \leftrightarrow (K, -K_3, Y_K)$$

Explicitly:

$$T-L(2):\quad (B) \to (-p, \tfrac{1}{2}(\Sigma^0 - 3^{1/2}\Lambda^0), \Xi^-, -\tfrac{1}{2}(3^{\frac{1}{2}}\Sigma^0 + \Lambda^0); \Sigma^+, \Xi^0, n, -\Sigma^-)$$
$$(B) \equiv (\Sigma^+, \Sigma^0, \Sigma^-, \Lambda^0; p, n, \Xi^0, \Xi^-). \qquad (5)$$

The corresponding transformations for pseudoscalar mesons are obtained by substituting

$$(B) \to (\pi^+, \pi^0, \pi^-, \eta; K^+, K^0, -\bar{K}^0, K^-). \qquad (6)$$

For completeness and clarity, the conclusions will be stated in the form of a mathematical theorem.

Theorem 1. An effective Hamiltonian $H_{\rm NL}$ for nonleptonic weak decay (with derivative coupling) which satisfies the $\Delta T = \frac{1}{2}$ rule, $T-L(2)$ and time-reversal invariances, leads to the following consequences: (*i*) $H_{\rm NL}$ must transform according to the eight-dimensional representation of $SU(3)$. (ii) Δ is valid for both p.v. and p.c. amplitudes. (iii) $K_2^0 \nrightarrow 2\pi$, no restriction is imposed upon $K_1^0 \to 2\pi$. *Proof.* (i) and (ii) have already been given by Rosen.[12,13] For (iii) we recollect[13] that the most general form of an octet-type Hamiltonian is

$$H_{\rm NL} = \alpha O_2{}^3 + \beta O_3{}^2, \qquad (7)$$

where, from time-reversal invariance, the coefficients α and β are real. Under the $T-L(2)$ transformation, $O_2{}^3 \leftrightarrow e^{i\phi} O_3{}^2$ with $\phi = \pi$, and $\alpha = -\beta$—i.e. antisymmetry under $2 \leftrightarrow 3$. In conjunction with the $\Delta T = \frac{1}{2}$ rule, we can relate $K_2^0 \to 2\pi$ to $K_2^0 \to K^0\bar{K}^0$. Briefly, we have first that $T-L(2)$ relates $\langle K_2^0 | \pi^+\pi^- \rangle$ to $\langle K_2^0 | K^+K^- \rangle$. If $|\pi^+\pi^-\rangle$ is a $CP = +1$ state, so is $|K^+K^-\rangle$. This means that it is symmetric in K^+ and K^-, and must be therefore $T = 1$. This then determines a unique branching ratio for $|K^+K^-\rangle$ to $|K^0\bar{K}^0\rangle$ final states. At this point we discover that $\langle K_2^0 | K^0\bar{K}^0 \rangle$ vanishes by virtue of $T-L(2)$ invariance. Notice that the same argument with $T-L(1)$ invariance[13] (corresponding to $\alpha = \beta$, $\phi = 0$) forces $\langle K_1^0 | \pi^+\pi^- \rangle$ to vanish.

Remarks. 1. The assumption of derivative coupling in Theorem 1 is no loss of generality. In terms for instance of nonleptonic hyperon decays, the most general, relativistically covariant Hamiltonian for $Y \to B + \pi$, includes both derivative and nonderivative coupling of the pion field. If Y and B are approximated as free particles on the mass shell, the derivative coupling can be reduced to a nonderivative form by means of the Dirac equation. It follows that the *effective* Hamiltonian need include only one type of coupling. Conclusions (i)–(iii) will follow from a nonderivative-type Hamiltonian with $T-L(2)$ invariance

[8] See, for instance, A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965).

[9] The branching ratio $\Gamma(K^+ \to \pi^+ + \pi^0)/\Gamma(K_1^0 \to 2\pi) \approx 1/500$ is suggestively close to (α/π) (where α is the fine-structure constant) as to warrant further consideration of its possible relevance to electromagnetic deviation of the $\Delta T = \frac{1}{2}$ rule. It is not ruled out that an explanation of this puzzling ratio may evolve along the lines of Bernstein *et al.* (Ref. 18) whereby electromagnetism violates CP, though the correct ingredient is not yet at hand. We see this because the *amplitude* for $K^+ \to \pi^+ + \pi^0$ must involve γ emission and re-absorption; hence we would expect $A(K^+ \to \pi^+\pi^0)/A(K_1^0 \to 2\pi) \sim \alpha/\pi$ from electromagnetic effects. But this gives a decay rate ratio $\Gamma(K^+ \to \pi^+\pi^0)/\Gamma(K_1^0 \to 2\pi) \sim (\alpha/\pi)^2$, which is a great deal smaller than the experimental 1/500. Thus despite the attractive numerical coincidence between α/π and 1/500, the detailed relationship is likely to be pretty complicated.

[10] D. D. Carmony *et al.*, Phys. Rev. Letters **12**, 482 (1964); D. D. Carmony *et al.* (unpublished); M. L. Stevenson *et al.*, Phys. Letters **9**, 349 (1964).

[11] D. Bailin, Nuovo Cimento **38**, 1342 (1965).

[12] S. P. Rosen, Phys. Rev. **135**, B1041 (1964).

[13] S. P. Rosen, Phys. Rev. **137**, B431 (1965).

[14] In the more familiar language of S. Meshkov, C. A. Levinson, and H. J. Lipkin [Phys. Rev. Letters **10**, 361 (1963)], $L \equiv$ (V spin) and $K \equiv$ (U spin).

replaced by $T-L(2)\times P$ invariance[13], where P is the parity operation.

2. $T-L(2)$ invariance violates the *octet-current rule*,[15] namely the hadron currents of the current-current picture shall be members of the *same* octet *with the same phase relations as those which occur in the strong interactions*. There is thus *no* easy connection between our discussion and those proposed earlier on the current-current picture[4,7]—unless the hadron currents are enlarged to include (experimentally absent) abnormal octets.[16] Note that a straightforward application of Cabibbo's hypothesis in form (2) gives rise to nonleptonic decays which are $T-L(1)$ invariant but not $T-L(2)$ *invariant*.[13] Dashen and Frautschi[17] have proposed that the known low-mass hadrons can impose a pattern of (at least approximate) bootstrap octet dominace on many phenomena including nonleptonic weak interactions. This is consistent with (i) of the above theorem, since the bootstrap approach is not dependent on a current-current formulation.

3. Time reversal or equivalently (via the CPT theorem) CP invariance is assumed for the weak interactions. This is in accord with recent propositions[18] that the observed violation of CP invariance from the small decay amplitude[19] of $K_2^0\to\pi^++\pi^-$ are due to C violation in electromagnetism (or perhaps semistrong interaction), rather than to the weak interactions themselves. Consequence (iii) thus fits well with this picture.

4. We are well aware that the problem of "deriving" the Δ relation is far from unique. In addition to derivations based on the current-current picture with abnormal octets mentioned above[16], several alternative formulations in terms of either bad symmetries (e.g. hypercharge reflection R) or specific dynamical pole models (which have no gentle connection with symmetry) have been examined.[20] A particularly aggrevating "bête noire" of particle physics, is the concept of R-conjugation invariance ($R: O_\nu^\mu\to O_\nu^\mu$, $O\equiv\bar{B}, B, \pi$). It is one of the two weak symmetries available in $SU(3)$; the other is the $T-L(2)$ (or $T-L(1)$) transformation. Aside from the known violations of R—symmetry in strong and electromagnetic interactions, the hypothesis of R (or RP)—invariance for the weak hadron currents appears to be invalidated by our knowledge of the *weak* baryon leptonic decay processes as well.[5] However, it must be emphasized that Theorem 1 makes no pretense at a connection with the Cabibbo type of current-current description for nonleptonic weak processes. It is therefore not ruled out that R conjugation may prove useful as an heuristic invariance for our less committed phenomenological viewpoint in terms of an effective Hamiltonian. Imposition of the *additional* constraint of RP (or R) invariance, has in fact led to further and successful predictions between asymmetry parameters for nonleptonic decay modes.[13] It is well to remind ourselves that a "bad" symmetry of strong interaction can sometimes nevertheless yield measured success, when used judiciously, in weak interactions. A case in mind is the defunct global symmetry.[21] Here, three weak symmetries are available, and when combined with the $\Delta T=\frac{1}{2}$ rule, they predict all the properties of non-leptonic hyperon decays.[22]

In summary, Theorem 1 represents a tight and economical set of proposals (and consequences) for unifying the salient features of non-leptonic weak interactions. It is at least a compact answer to the "meaning of the meaning of Δ" raised sometimes ago by Pais and Treiman.[15]

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Note added in proof. The following features deserve emphasis. (a) The conclusions of Theorem 1, which assume T-invariance for the normal weak interactions ($\Delta S=\pm1$ transitions allowed, $\Delta S=\pm2$ transitions forbidden) are also consistent with the alternative proposition of Wolfenstein [Phys. Rev. Letters **13**, 562 (1964), see also Lee and Wolfenstein of Ref. 18] that the observed violation of CP (or Time Reversal via CPT Theorem) invariance of Ref. 19 is due to a super weak interaction ($\Delta S=\pm2$ transitions allowed). (b) We are aware that the $\Delta T=\frac{1}{2}$ rule may be violated in the realm of normal weak interactions itself. The existing data suggest however approximate $\Delta T=\frac{1}{2}$ rule in non-leptonic processes, hence Theorem 1, in this case can be regarded as a leading approximation. (c) In a recent bootstrap study, Dashen, Dothan, Frautschi, and Sharp ["Self-Consistent Determination of Coupling Shifts in Broken SU(3)," Phys. Rev. (to be published)] conclude

[15] A. Pais and S. B. Treiman, in *Proceedings of the 12th Annual Conference on High Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[16] S. Coleman, S. L. Glashow, and B. W. Lee, Ann. Phys. (N. Y.) **30**, 348 (1964). References to past literatures are given here. Note also the *second-class currents* [S. Weinberg, Phys. Rev. **112**, 1375 (1958)] would have the same effect as abnormal octets.

[17] R. F. Dashen and S. C. Frautschi, Phys. Rev. **140**, B698 (1965).

[18] T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965); J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965). See also J. Prentki and M. Veltman, Phys. Letters **15**, 88 (1965).

[19] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).

[20] A summary is given by S. P. Rosen, Phys. Rev. **140**, B326 (1965). See also B. W. Lee, *ibid.* **140**, B152 (1965).

[21] Another example, in the context of R-invariance itself, is the Bronzan-Low-A parity [Phys. Rev. Letters **12**, 522 (1964)] with $R=CA$, where C is the charge conjugation. The case here is even more remarkable since *strong* mesonic decays are involved in the use of this selection rule.

[22] S. P. Rosen, Phys. Rev. Letters **9**, 186 (1962).

that the $\Delta T=\frac{1}{2}$ rule cannot be predicted on the basis of a single dominant octet enhancement, for the parity-conserving decays. This is intuitively suggestive that breakdown of $\Delta T=\frac{1}{2}$ rule is likely to be more prominent in P-wave amplitudes. Study of sum rules for deviations from $\Delta T=\frac{1}{2}$ of the parity-conserving amplitudes, using the effective Hamiltonian approach, will thus be of great interest.

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Auxiliary-Field Method for Vector Field Theories

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The theory of a neutral vector field is studied by introducing an auxiliary scalar field interacting with it. The vector field propagator of the theory is the same as that obtained by Yang and Lee using the ξ-limiting process. The parameter ξ now enters as an interaction parameter. We show, further, that it is a gauge parameter and the physical content of the theory is independent of its value. Using a canonical quantization scheme, it is found that the contribution of the scalar field to the energy of the system is negative, and one is forced to introduce an indefinite metric. To have a positive norm of the states for arbitrary interactions involving production of these hypothetical scalar particles, the values of ξ have to be restricted so that the energy of the system is less than $m^2\xi^{-1}$, m being the mass of the vector meson. Most of the formalism can be easily extended to the theory of charged vector mesons. The main purpose of the theory is to investigate the possibility of renormalizing the interactions of the vector field with currents whose divergences do not vanish. It is found that an extra term in the interaction Lagrangian makes the auxiliary-field equation independent of the current and hence of the additional interaction. It is shown that the S matrix cannot produce these particles and that there need be no restrictions on the parameter ξ for this case so long as one deals with initial states having no scalar mesons.

1. INTRODUCTION

The nonrenormalizability of a class of massive-vector-meson theories has been established by a number of recent investigators.[1] There have been some successful attempts by Salam and Delbourgo[2] to devise ways to extract finite results from spin-1 electrodynamics. The main difficulty in renormalizing such theories is the behavior of the propagation function at large momenta. Explicitly it is

$$\Delta_{\mu\nu}{}^F=(g_{\mu\nu}+k_\mu k_\nu/m^2)/(k^2-m^2-i\epsilon)\,, \qquad (1)$$

and tends to a constant for large values of k_μ. To get around this difficulty, Yang and Lee[3] formulated a ξ-limiting process for charged vector mesons which leads to a propagation function falling off as $1/k^2$ for large k_μ. In their method one introduces a ξ-dependent kinetic energy term in the Lagrangian. On quantizing the system by the well-known canonical procedure[4] one finds that the system consists of scalar particles possessing negative energy. This difficulty is removed by introduction of an indefinite metric and the S matrix is found to be unitary only if the system has energy less than $m^2\xi^{-1}$. The theory is renormalizable for $\xi>0$ whereas for dealing with systems with arbitrary energy one has to take the limit $\xi\rightarrow 0$.

We shall report here a method of suitably parametrizing the vector meson gauge so that the modified propagator is identical to that obtained by Yang and Lee, and we shall discuss some definite advantages of this parametrization. We shall deal with a neutral vector field, although most of our formalism can be carried over to the theory of charged vector fields. Besides the electromagnetic interactions of charged vector mesons, there exists another class of nonrenormalizable interactions with neutral vector mesons in which the interacting current has a nonvanishing divergence. We shall use our theory to see whether they can be renormalized.

After necessary notations and preliminaries, we shall give the Lagrangian density of our theory in Sec. 2 where we shall also obtain the field equations. This will make obvious the nature of the propagation function that one would expect from the theory. In Sec. 3 we shall show that the interaction parameter is actually a gauge parameter by following a method due to Johnson.[5] In Sec. 4, canonical quantization of the system is carried out. It is seen that the auxiliary scalar field contribution to the energy of the system is negative exactly like the contribution from the fourth component of the electromagnetic field potential. To deal with the system covariantly one uses the method of the

[1] A. Kumar and A. Salam, Nucl. Phys. **21**, 624 (1960); S. Kamefuchi and H. Umezawa, *ibid.* **23**, 399 (1961); A. Salam, Phys. Rev. **127**, 331 (1962); **130**, 1287 (1963).
[2] A. Salam and R. Delbourgo, Phys. Rev. **135**, B1398 (1964).
[3] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).
[4] See, for instance, G. Wentzel, *Quantum Theory of Fields* (Interscience Publishers, Inc., New York, 1949).
[5] K. Johnson, Brandeis Summer School Lectures, 1964 (unpublished); Lectures delivered at Bangalore Summer School, 1965 (unpublished) and private communications.