Ross-Schiff Analysis of a Proposed Test of General Relativity: A Critique

IRWIN I. SHAPIRO
*Lincoln Laboratory,*Massachusetts Institute of Technology, Lexington, Massachusetts*
(Received 16 December 1965)

Using a model of circular coplanar orbits and an analysis accurate to first order in the sun's gravitational radius, Ross and Schiff discussed the recent proposal to test general relativity by measuring round-trip travel times of radar pulses transmitted from the earth towards an inner planet. Their main conclusion, that such measurements would be sensitive to a nonlinear term in Einstein's theory, we find to be invalid. Since first-order differences between Newtonian and Einsteinian orbits are well known to depend on a nonlinear term in the metric, one might expect the round-trip travel times also to depend in first order on such a term. Curiously, this expectation is not realized for circular orbits. When expressed as a function solely of clock readings, the first-order formula for travel time in the circular-orbit model is strictly independent of the nonlinear term. Even were the combined use of radar-pulse travel times and the results of "exact" optical measurements envisioned, their sensitivity to this nonlinear term would be masked almost completely by unavoidable uncertainties in the estimates of other unknown parameters such as the mass of the sun. For noncircular orbits, however, the travel-time measurements will be noticeably sensitive to this nonlinear term through its effect on the advance of the perihelion. In addition to re-examining the circular-orbit model, we describe the operational procedures that we have developed for testing general relativity with data obtained from actual planetary observations. These data cannot be expected in the near future to provide a significant test of more than the first-order influence of solar gravity on radar-pulse travel times and the non-Newtonian advance of Mercury's perihelion, as we previously pointed out.

INTRODUCTION

THE primary purpose of this paper is to discuss the reasons for our disagreement with the main conclusion recently reached by Ross and Schiff[1] who considered a simplified model of a radar experiment proposed to test Einstein's theory of general relativity.[2,3] This experiment involves transmitting radar pulses from the earth towards an inner planet and detecting the echoes. Ross and Schiff analyzed the first-order expression for these time delays derived under the assumption that the earth and target planet move in icrcular coplanar orbits; they concluded that measurements of such delays would be "sensitive to a nonlinear term in Einstein's theory." In the remainder of this section we first discuss briefly the general question of testing relativity with delay data and then examine the approach of Ross and Schiff, pointing out in detail the facts that lead to our opposite conclusion. We also describe the operational procedures that we have developed for making such tests with actual radar data. The last section is devoted to a detailed mathematical analysis of the circular-orbit model and provides the basis for many of the statements made below.

For our present discussion, the data accumulated from radar observations can be considered simply as a series of clock readings made by an observer on earth. That is, we may envision an earth-based atomic clock which records the time at the instant of transmission of each radar pulse as well as at the instant of reception of each echo. The question of which facets of general relativity such measurements will test is not amenable to a simple answer. The theory contains some initially unspecified parameters whose values must be obtained from the same basic data that will be used to test the predictions of the theory. The stringency of the test depends in a complex manner on the accuracy, type, and number of individual measurements included. In particular, a significant test requires redundant data, i.e., more than are necessary simply to determine specific values for the parameters of the theory. Currently accepted procedures call for the use of a technique such as the method of maximum likelihood to estimate these parameters which will then generally be theory-dependent as well as measurement-dependent numbers. Various formal devices or ordinary common sense may be used to decide whether the theory, augmented by the specific parameter values, is consistent with the data and hence "passes" the test.

To ascertain *a priori* whether or not a proposed set of delay measurements would constitute a test that could distinguish between two theories, one can proceed in several ways. The ambitious can generate a representative, artificial set of data assuming, for example, that general relativity is correct and including "errors" selected from an appropriate random distribution. By analyzing these "data" on the assumption that another theory is valid, one can then decide whether the post-fit residuals are large enough to enable a significant distinction to be made between the two theories. The less ambitious can choose to analyze simple models that involve the essence of the proposed experiment.[4] If used carefully, the latter approach will also lead to reliable results. Unfortunately the well-known principles of the operational method are too often either ignored or inconsistently adhered to in

* Operated with support from the U. S. Air Force.
[1] D. K. Ross and L. I. Schiff, Phys. Rev. **141**, 1215 (1966).
[2] I. I. Shapiro, Phys. Rev. Letters **13**, 789 (1964).
[3] I. I. Shapiro, Lincoln Laboratory Technical Report No. 368, 1964 (unpublished).
[4] I. I. Shapiro, Phys. Rev. **141**, 1219 (1966).

a priori analyses, leading authors to draw incorrect conclusions.

As mentioned above, Ross and Schiff chose to consider a model in which the earth and target planet are represented by test particles moving in coplanar circular orbits about the sun. The gravitational field of the latter is represented in their work by the generalized metric introduced by Eddington.[5] To first-order accuracy in the sun's gravitational radius, Ross and Schiff give the expression for the time delay in terms of the universal coordinates of this generalized metric. They point out correctly that these coordinates are not directly measurable quantities and then recast the formula, replacing the zero-order part by one depending on the individual planetary orbital periods and on the mass of the sun. In this form, the time delay appears to depend on a nonlinear term in the generalized metric, i.e., on a term in the metric that is proportional to the square of the sun's gravitational radius. To be assured of obtaining reliable results in *a priori* interpretations, one should of course express the theoretical predictions solely in terms of directly measurable quantities.[6] But neither the mass of the sun nor the individual orbital periods are measurable with radar. The synodic period is the only one that may be determined directly. It is the time interval between successive collinear alignments of the earth, the inner planet, and the sun, and is measurable since at the instants of such alignments the time delay is a minimum. When the first-order expression for time delays is cast purely in terms of clock readings for the model of circular orbits, the sensitivity to the nonlinear term vanishes. Since first-order differences between Einsteinian and Newtonian orbits depend on a nonlinear term in the metric, one would naturally have expected measurements of interplanetary-travel times of radar pulses to be sensitive in first order to this part of the metric. However, such an expectation is not realized for circular orbits; time-delay measurements accurate only to first order are insensitive to the nonlinear term.

Since Ross and Schiff mention explicitly only time-delay data and refer simply to a radar-reflection experiment, one might have thought that their analysis was indeed based solely on these radar measurements. In a private communication, however, Professor Schiff states that they "did not intend to exclude the use of optical data." Let us therefore examine in detail how the inclusion of such data affects the analysis. For the purpose of discussion, we may assume that the individual coordinate-time orbital periods have been determined exactly from optical observations. If we use these values in the Ross-Schiff expression for time delay, one important parameter will still remain unspecified—the mass of the sun. This mass is not known from any measurements with nearly the requisite accuracy in terms of the gravitational constant, the speed of light, and either the astronomical or the atomic unit of time. (Were one to choose units in which the product of the gravitational constant and the mass of the sun were unity, and in which, for example, the orbital period of the earth were unity, then the uncertainty would merely shift to the imprecisely known speed of light in these units.) One must therefore use the time-delay data not only to test for the presence of the nonlinear term, but to estimate the mass of the sun as well. A detailed examination shows, however, that the estimate of the coefficient of the contribution to the time delay of the nonlinear term in the metric will be highly correlated with the estimate of the mass of the sun. When one further considers that, e.g., the radius of the target planet must also be estimated from these data, the possibility of detecting reliably the effect of the nonlinear term in the metric all but vanishes. More precisely, whereas the maximum magnitude of the effect on time delay of the nonlinear term is 13 μsec in Einstein's theory, the deviation between the contribution to the time delay of this term and of a suitable linear combination of a change in solar mass and a change in, say, Mercury's radius will nowhere exceed about 0.4 μsec, as is shown below. The maximum deviation is even less when Venus is used or when the uncertainty in the target planet's mass is considered. For many reasons, the detection of such an effect is now well beyond experimental reach.

We may conclude that measurements of interplanetary travel times of radar pulses are insensitive to a nonlinear term in the metric of Einstein's theory for the model of circular planetary orbits considered by Ross and Schiff. Augmenting the radar data with exact optical determinations of individual orbital periods would still not allow the effect of this nonlinear term to be distinguished experimentally.

What will in fact be tested by the radar time-delay observations? The actual physical situation is, of course, far more intricate than the model of test particles moving in coplanar circular orbits. Complications are introduced, for example, by the solar corona,[7]

[5] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, New York, 1957), p. 105.

[6] Ross and Schiff also stress the necessity for expressing predictions in terms of measurable quantities. However, they consider parameters such as orbital eccentricity to be measurable, whereas we find to the contrary that these are theory-dependent as well as measurement-dependent numbers (see, for example, Refs. 3 and 4).

[7] The group delay introduced by the solar corona may be determined by performing simultaneous observations at two frequencies (see Refs. 2 and 3); such measurements can even be made with one radar by alternating rapidly between transmissions at different frequencies. (Klystrons are tunable over a wide enough band to make such a procedure practical.) However, because of the time-varying characteristics of the corona, the spectrum of the radar echo will be broadened and consequently the determination of accurate time delays will be more difficult. Extrapolating from recent 430-Mc/sec measurements made at Cornell's Arecibo Ionospheric Observatory (R. B. Dyce and G. H. Pettengill, private communication), we find that at least at the present part of the solar cycle the problem of spectral broadening at the 8000-Mc/sec frequency of Lincoln Laboratory's Haystack facility will not be serious except extremely close to the solar limb. Studies of solar occulations of radio sources indicate that absorption by the corona will not be detectable at 8000 Mc/sec; the radar will be "blinded" by solar noise in the antenna sidelobes before the main lobe passes close enough to the limb to be noticeably affected by absorption. The neutral component of the corona appears to be far too small to be of significance.

by the rotation, shape and atmosphere of the earth, by the moon, and by the planets, all of which have nonnegligible masses and nonspherical shapes and move in noncircular noncoplanar orbits. Over the last several years, we have developed an array of double-precision computer programs to integrate the equations of motion of the moon, the earth, and the planets, to predict values for the radar observations, and to determine maximum likelihood estimates of the unknown parameters such as orbital initial conditions and planetary masses, shapes, and radii; we also evaluate and perform a "principal-axis" transformation on the correlation matrix of the parameter-estimate errors that are implied by the measurement errors.[8] As far as general relativity alone is concerned, preliminary studies based on actual data have convinced us that presently achievable radar-measurement and clock accuracies will allow in the near future a significant test only of the first-order effect of solar gravity on time delay and of the excess advance of the perihelion of Mercury,[9] as was previously mentioned.[2,3] Neither the corresponding secular advances of the perihelia of the three other inner planets, nor the slight nonsecular predicted differences between the Einsteinian and Newtonian planetary orbits and clock rates will be detectable reliably in the next several years. These conclusions are contingent on the use of optical data solely insofar as it is necessary to know the orbits of the outer planets, which are as yet inaccessible to radar, to calculate their perturbing effects on the inner planets. Errors in the conventional determinations of either the orbits or masses of the former will have only a higher order effect on the orbits of the latter.

One might object to such a narrow viewpoint and suggest that optical data be incorporated in a more fundamental way. Indeed we intend to attempt to utilize the optical observations fully and, in fact, our computer programs were written to process these simultaneously with the radar data and with Doppler data from Mariner-type space probes which should prove very useful, especially in estimating planetary masses. However, because of the extreme sensitivity of the optical results to nonuniformities in earth rotation, to stellar location, to planetary phase, to atmospheric turbulence, etc., the proper interpretation of these observations is a nontrivial task. Care must be taken not to weight these data too heavily; their number well outstrips the number of radar measurements, but the individual measurement accuracies are far inferior to those obtained by radar. Therefore danger exists in too strong a reliance on the "square root of N," since unsuspected systematic errors in the optical data may then dominate. The probable errors astronomers quote with their processed results are, because of this reliance, often far below the actual errors as determined by independent means.[10] A now classic example is provided by the astromical unit, the very accurate radar value differing from the result based on optical observations by ten times the quoted error of the latter determination.[11] More recently, radar observations of Mercury made at the Arecibo Ionospheric Observatory[12] indicate that some of the "Newtonian" orbital elements require corrections in the sixth significant figure[13] although the conventional determinations were stated to be accurate to eight. The astronomical determinations of planetary orbital periods are supposedly far more accurate. Yet radar observations indicate, for example, that relative to earth, Mercury's orbital longitude is ahead of predictions by about $1''$ of heliocentric arc. Although it might be argued that this displacement should be interpreted in other ways, the logical possibility nevertheless remains that it could signify an error in the orbital period. A difference in Mercury's orbital period of several parts in 10^9 would cause a difference in orbital position of $1''$ of arc after 100 years. Mercury's proximity to the sun makes optical observations notoriously difficult and perhaps one should not be too surprised to uncover such a relatively large error.

There is at least one more important problem involved in using the optical data either directly or after processing: The astronomer's time scale must be reconciled with the atomic-time scale used for the radar measurements. A comparison made between three years of lunar observations and corresponding atomic-clock readings achieved such a reconciliation[14] to a stated accuracy of two parts in 10^9. Of course, extended series of radar observations will themselves allow accurate inferences to be made of the values of atomic-time orbital periods consistent, say, with general relativity.

To summarize, we intend to test several predictions of general relativity utilizing in essence only round-trip time-delay data recorded in atomic time.[15] An attempt to incorporate the optical data will be made as well but, because of both intrinsic and practical difficulties, we

[8] For general relativity and for Newtonian theory, suitably augmented by either of the two obvious assumptions concerning the interaction of light with matter, the mathematical analysis was carried out by M. Ash, I. Shapiro, and M. Tausner (unpublished) and the computer programs were written by M. Ash, A. Rasinski, and W. Smith.

[9] At present, we have neither the right type nor amount of data necessary for a significant test.

[10] For this reason especially, we feel that an independent determination of the advance in the perihelion of Mercury's orbit is highly desirable.

[11] See, for example, I. I. Shapiro, Bull. Astron. **25**, 177 (1965).

[12] R. B. Dyce and G. H. Pettengill (private communication).

[13] M. Ash, A. Rasinski, I. Shapiro, and W. Smith (to be published).

[14] W. Markowitz, R. G. Hall, L. Essen, and J. V. L. Parry, Phys. Rev. Letters **1**, 105 (1958).

[15] Doppler shits are also measured and there is the faint possibility that a third order term in v/c may be detectable (Ref. 3). Spectral broadening introduced by the corona and by the target planet's apparent rotation may make necessary the use of millimeter wave or optical transponders on natural or artificial planets to detect this third order contribution. Since in theory it is proportional to the time derivative of the "extra" time delay introduced by general relativity, this contribution would be maximized near superior conjunction by placing an artificial planet in a retrograde orbit that lies in the ecliptic.

are less than sanguine about the improvements to be obtained.

MODEL ANALYSIS

Following Eddington[5] and Robertson,[16] Ross and Schiff[1] employ the general static spherically symmetric metric in the isotropic form

$$ds^2=[1-(2\alpha m/r)+(2\beta m^2/r^2)+\cdots]dt^2 -[1+(2\gamma m/r)+\cdots](dx^2+dy^2+dz^2), \quad (1)$$

where $r^2=x^2+y^2+z^2$, where m represents the mass of the sun (or, equivalently, its gravitational radius), and where the gravitational constant and the speed of light have been set equal to unity. In Einstein's theory of gravitation, the coefficients α, β, and γ are all unity. For the model of test particles in coplanar circular planetary orbits, the first-order coordinate-time round-trip interplanetary time delay T can be expressed in coordinate time t as

$$T=2R+2(1+\gamma)m \times\ln\left[\frac{r_e}{r_p}\left(\frac{R+r_e-r_p\cos(\varphi_p-\varphi_e)}{R+r_e\cos(\varphi_p-\varphi_e)-r_p}\right)\right], \quad (2)$$

where

$$R^2=r_e^2+r_p^2-2r_er_p\cos(\varphi_p-\varphi_e), \quad (3)$$

$$\varphi_{p,e}=\dot{\varphi}_{p,e}t\approx\frac{m^{1/2}}{r_{p,e}^{3/2}}\left[1-\left(\frac{\gamma+2\beta}{2}\right)\frac{m}{r_{p,e}}\right]t, \quad (4)$$

and where r_p and r_e are the orbital radii and φ_p and φ_e the angular positions of the inner planet and the earth, respectively. For convenience, and since the conclusions will be unaffected by the approximations, we ignored planetary motions during the travel time of the radar pulse and followed Ross and Schiff by setting $\alpha=1$ and by not converting Eq. (2) to proper time. The formula for $T(t)$ differs from the corresponding Ross-Schiff equation only in that polar coordinates have been used here.[17] By choosing the directly measurable synodic period as the unit of time and inferior conjunction as the origin of time, we may replace $\varphi_p-\varphi_e$ by $2\pi t$ since, by definition, the angle between the two orbital radii increases from 0 to 2π during a synodic period. Equation (2) is now manifestly independent of β, the coefficient of the nonlinear term in m appearing in the expression for the generalized metric. But the equation does depend on two quantities which have not been operationally defined, namely r_p and r_e. These, of course, may be estimated from the data at the same time that the theoretical predictions are being tested. For the purposes of this model study we may determine r_p and r_e from two measurements of time delay, say T_0 and T_1, made at $t=0$ and at $t=\frac{1}{4}$.[18] This determination will yield theory-dependent as well as measurement-dependent numbers. We may write

$$\begin{aligned} r_p&=r_p^N+\Delta r_p,\\ r_e&=r_e^N+\Delta r_e, \end{aligned} \quad (5)$$

where r_p^N and r_e^N are the Newtonian values determined from $T^N(t)=2R^N$, and where Δr_e and Δr_p are of order m. Solving, we find

$$\begin{aligned} r_p^N&=\tfrac{1}{4}[(2T_1^2-T_0^2)^{1/2}-T_0],\\ r_e^N&=\tfrac{1}{4}[(2T_1^2-T_0^2)^{1/2}+T_0], \end{aligned} \quad (6)$$

and

$$\begin{aligned} \Delta r_p&=(1+\gamma)m\left\{\frac{r_e}{r_e+r_p}\ln\left(\frac{r_e}{r_p}\right) -\frac{(r_e^2+r_p^2)^{1/2}}{r_e+r_p}\ln\left[\frac{r_e}{r_p}\left(\frac{(r_e^2+r_p^2)^{1/2}+r_e}{(r_e^2+r_p^2)^{1/2}-r_p}\right)\right]\right\},\\ \Delta r_e&=-(1+\gamma)m\left\{\frac{r_p}{r_e+r_p}\ln\left(\frac{r_e}{r_p}\right) +\frac{(r_e^2+r_p^2)^{1/2}}{r_e+r_p}\ln\left[\frac{r_e}{r_p}\left(\frac{(r_e^2+r_p^2)^{1/2}+r_e}{(r_e^2+r_p^2)^{1/2}-r_p}\right)\right]\right\}. \end{aligned} \quad (7)$$

The quantities r_p and r_e appearing in Eq. (7) should each bear a superscript N. The sought for first-order accuracy in m, will, however, not be affected by deleting the superscript. Inserting these results in Eq. (2) yields

$$T(t)=2R^N+2(1+\gamma)m\left\{\ln\left[\frac{r_e}{r_p}\left(\frac{R+r_e-r_p\cos2\pi t}{R+r_e\cos2\pi t-r_p}\right)\right]-\left(\frac{r_e-r_p}{R}\right)\cos2\pi t\ln\left(\frac{r_e}{r_p}\right) -\frac{(r_e^2+r_p^2)^{1/2}(1-\cos2\pi t)}{R}\ln\left[\frac{r_e}{r_p}\left(\frac{(r_e^2+r_p^2)^{1/2}+r_e}{(r_e^2+r_p^2)^{1/2}-r_p}\right)\right]\right\}, \quad (8)$$

with

$$R^N=\{(r_e^N)^2+(r_p^N)^2-2r_e^Nr_p^N\cos2\pi t\}^{1/2}, \quad (9)$$

and with r_p^N and r_e^N defined in terms of measurements by Eq. (6).

For this model we see that to first order in m, the

[16] H. P. Robertson, in *Space Age Astronomy*, edited by A. J. Deutsch and W. E. Klemperer (Academic Press Inc., New York, 1962), p. 228.

[17] Ross and Schiff also present a formula for the time delay in isotropic coordinates which in this context are completely equivalent to harmonic coordinates. This expression appears to differ from the corresponding one involving standard Schwarzschild coordinates. Both results had been obtained and explained previously (see Ref. 3); in brief, the difference has no operational significance. It is simply a question of different coordinate systems leading to different coordinate values for the positions of the planets. The formulas, of course, become identical when expressed solely in terms of measurable quantities.

[18] This analysis follows closely the approach used in Ref. 4.

radar measurements of time delay are in principle insensitive to the nonlinear term in the generalized metric. Given noncircular orbits, the conclusion will of course be different; the nonlinear term contributes, for example, to the secular advance of the perihelion which can be detected from the radar data.[2,3]

To decide whether a test of the influence of solar gravity on time delay will be significant, we must determine the sensitivity of $T(t)$ to errors in either T_0 or T_1. A simple calculation shows that such errors would be magnified by less than a factor of two at most in their effects on $T(t)$. Since for Einstein's theory the first-order part of the expression for $T(t)$ varies from zero to about 200 μsec at superior conjunction, the test will clearly be significant if the errors in individual time-delay measurements are only of the order of 10 μsec.[19] Such accuracy is presently achievable at Cornell's Arecibo Ionospheric Observatory for observations of Mercury near inferior conjunction[12]; we expect that it will soon be achievable near superior conjunction at Arecibo, at Lincoln Laboratory's Haystack and at the Jet Propulsion Laboratory's new Goldstone facility as well.

How would the inclusion of optical data affect this analysis? With certain important qualifications, we could then consider the individual planetary orbital periods P_p and P_e to be directly measurable quantities. Casting the zero-order part of Eq. (2) in terms of these, we find

$$T(t)=\{2m^{1/3}/(2\pi)^{2/3}\}\{P_e^{4/3}+P_p^{4/3}-2(P_eP_p)^{2/3}\cos 2\pi t\}^{1/2}$$
$$+2(1+\gamma)m\ln\left[\frac{r_e}{r_p}\left(\frac{R+r_e-r_p\cos 2\pi t}{R+r_e\cos 2\pi t-r_p}\right)\right]$$
$$-\tfrac{2}{3}(\gamma+2\beta)m((r_e+r_p)/R)(1-\cos 2\pi t)\,, \quad (10)$$

with P_p and P_e given in units of the synodic period. The orbital radii r_p and r_e now appear only in the first-order term and may be given their Newtonian values without disturbing the first-order accuracy of Eq. (10).

Aside from the use of polar coordinates and the synodic period unit of time, our expression (10) for $T(t)$ differs from the corresponding one in the Ross-Schiff paper only in that the zero-order term is given explicitly here, whereas there it was simply described verbally. As is evident, the zero-order term now depends on the sun's mass which is not a directly measurable quantity. One might at first think that m may be set equal to unity and that this problem would then disappear. Second thoughts show that our freedom of choice in selection of units has already been fully exploited by setting the gravitational constant, the speed of light, and the synodic period all equal to unity. Even were we instead to set only the synodic period and the product of the gravitational constant and the mass of the sun equal to unity, we would still be faced with the necessity of determining the speed of light accurately in terms of the units of length and time so defined. We may, of course, estimate m from the data. Following the procedure used above leads to

$$T(t)=T_0\frac{\{P_e^{4/3}+P_p^{4/3}-2(P_eP_p)^{2/3}\cos 2\pi t\}^{1/2}}{\{P_e^{2/3}-P_p^{2/3}\}}$$
$$-2(1+\gamma)m(R/(r_e-r_p))\ln(r_e/r_p)$$
$$+2(1+\gamma)m\ln\left[\frac{r_e}{r_p}\left(\frac{R+r_e-r_p\cos 2\pi t}{R+r_e\cos 2\pi t-r_p}\right)\right]$$
$$-\tfrac{2}{3}(\gamma+2\beta)m((r_e+r_p)/R)(1-\cos 2\pi t)\,, \quad (11)$$

where $m^{1/3}$ in the zero-order part has been evaluated in terms of T_0, the time delay at inferior conjunction.

Equation (11) depends explicitly on β. However, we must still inquire whether or not radar measurements of time delay could in practice detect the presence of the β term. First we note that its contribution vanishes at inferior conjunction and increases monotonically to a maximum value at superior conjunction; for $\beta=1$ its magnitude there is 13 μsec. Second we must consider the possibility that the β term may be indistinguishable because its effects on $T(t)$ are nearly identical to those of small changes in other parameters which must also be estimated from the data. More precisely, if the estimate of β is too highly correlated with the estimates of other parameters, say p_i, then the β term cannot be detected; one could not distinguish, for example, between the two possibilities $\beta=1$, $p_i=p_{0i}$ and $\beta=0$, $p_i=p_{i0}+\Delta p_i$, where the Δp_i are suitably chosen to minimize the effect on $T(t)$ of having set β equal to zero instead of unity. As two examples of such parameters, consider the mass of the sun and the radius of the target planet, neither of which can yet be determined with sufficient accuracy for our purposes by an independent means; they must be estimated from the radar data. We see from a comparison of Eqs. (10) and (11) that any slight change in the estimate of the sun's mass (or, equivalently, of T_0) would introduce a first-order term proportional to R in the expression for $T(t)$. Similarly, any change in the estimate of the target planet's radius would introduce a constant term. A simple analysis shows, in fact, that estimates of these

[19] R. H. Dicke and P. J. Peebles [Space Sci. Rev. 4, 419 (1965)] have stated that this test of the effect of solar gravity on time delay is "completely equivalent" to the test involving the deflection of light passing by the sun. They used an index-of-refraction analogy to argue that the deflection would necessarily be accompanied by an increase in time delay. Such a conclusion has some validity if a theory in accord with the generalized metric is assumed to be valid *a priori*. But one should not try to prejudge nature. There is certainly no difficulty in imagining a theory that would not be consistent even qualitatively with predictions based on the generalized metric. For example, were we to assume that in a gravitational field photons behaved like mass particles, then the "extra" delay would be approximately one half the Einstein prediction, but with the opposite sign. The photons would speed up rather than slow down. The deflection, although again twice as large in the Einstein prediction, is towards the sun in both theories.

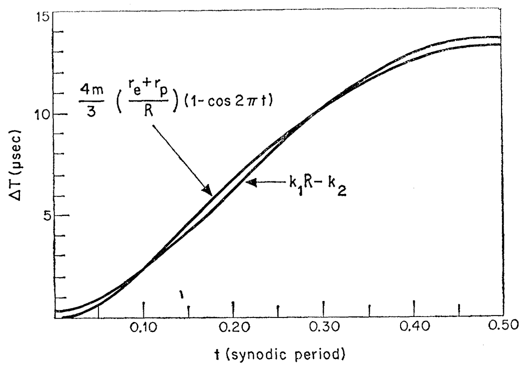

FIG. 1. Illustration of the high correlation between estimates of the β coefficient in the generalized metric and estimates of the sun's mass and the target planet's radius (see text). The upper label on the graph denotes the negative of the coefficient of β in Eq. (10). The lower label has two parts; the first refers to the effect on $T(t)$ of a small change in the estimate of the sun's mass and the second to the corresponding effect of a small change in the estimate of the target planet's radius. The constants k_1 and k_2 were chosen to most closely match the curve defined by the coefficient of β. Including other relevant parameters would lead to a far better match.

two parameters will be very highly correlated with the estimate of β.[20] For circular orbits an hypothesis such as $\beta=1$, $m=m_0$, $\rho_p=\rho_{p0}$, where ρ_p denotes the target planet's radius, would lead to predictions for earth-Mercury time delays that nowhere differ by more than about 0.4 μsec from predictions based on the alternative hypothesis $\beta=0$, $m\approx m_0(1-5\times10^{-8})$, $\rho_p\approx\rho_{p0}(1-6\times10^{-4})$, as is illustrated in Fig. 1. (The corresponding differences are even smaller for earth-Venus time delays.) Not only is this maximum difference more than an order of magnitude less than the expected standard deviation of the time-delay measurement errors, but at the microsecond level and below many other effects also contribute seriously to the masking problem. To be definite, we mention the variations in planetary topography,[3] and the previously discussed uncertainties in the determinations of the target planet's mass and of the atomic-time planetary orbital periods.

For the model of circular orbits, we may conclude that measurements of interplanetary radar-pulse time delays are sensitive to the γ term in the generalized metric, but are insensitive to the β term. Even were we to consider an experiment based on "exact" optical determinations of individual planetary orbital periods in addition to the radar measurements of time delay, distinguishing reliably a β-dependent term for circular orbits would not be feasible. For actual noncircular orbits, the β term in the metric will in fact be detectable from several years of radar data since it affects the advance of planetary perihelia. The results of optical observations will also be required in the interpretation, but only to determine the effects of the outer planets on the orbits of the inner ones. The other nonsecular first-order differences between Einsteinian and Newtonian planetary orbits will probably not be distinguishable with present-day radar techniques. Future use of radio or optical devices on other natural planets and of artificial planets with which radio or laser contacts are maintained should enable significant improvements to be made in the accuracy of these tests of general relativity and may also allow some of the smaller orbital differences to be detected.[21]

[20] The mass of the target planet affects its orbit and must also be estimated from the observations. The estimate of this parameter, too, is very highly correlated with the estimate of β. One hopes that a sufficiently accurate value for the inner planet's mass may be obtained in the future by discerning the perturbations introduced into the orbit of either a neighboring planet or a space probe that passes reasonably close to the inner planet. At present, for example, the mass of Mercury has been determined with little more than one significant figure of accuracy.

[21] With an artificial planet, conventional optical data for determining orbital periods will, of course, not be available. In addition an artificial planet will most likely have an area-to-mass ratio about 10^6 times higher than that of a natural one and will therefore be subject to important sunlight-pressure-induced orbital perturbations. To calculate these accurately, the orientation history and reflection properties of the artificial planet must be known. Perhaps it might someday be feasible to protect such a body from these perturbations by providing an orbiting solar screen that is servo-controlled.