# $f_{7/2}$ Model of $V^{46,47}$, $Cr^{49}$, and $Mn^{50}$†

J. N. GINOCCHIO
*Physics Department, Rutgers, The State University, New Brunswick, New Jersey*
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Using a pure $f_{7/2}$ configuration and an effective two-nucleon interaction, the energy spectra, magnetic moments, and beta decays of $V^{46,47}$, $Cr^{49}$, and $Mn^{50}$ are calculated. The empirical data available for comparison are sparse, but an attempt is made at comparison to point out both what would be of interest to look for experimentally and the deficiencies of the model. The spectacular features are predicted by the model, namely, the inversion of the lowest $T=0$ and $T=1$ levels of the self-mirror nucleus $V^{46}$, the isomeric $J=5$ level in $Mn^{50}$, and the "anomalous" $J=\frac{5}{2}$ ground-state spin of $V^{47}$ and $Cr^{49}$. It is more difficult to make judgments about the excited levels since the data are scarce; however, it is clear that the first excited ($J=5$) level for $V^{46}$ predicted by the model is pushed up above other excited levels.

## I. INTRODUCTION

IN a series of papers McCullen, Bayman, and Zamick[1] (hereafter referred to as MBZ) made a detailed study of the nuclear spectroscopy of the isotopes of calcium, scandium, titanium and other "$f_{7/2}$-shell" nuclei in this region of the periodic table, $56>A>40$. Empirical information is compared with theoretical calculations, thus helping to systematize the nuclear data and also to give some insight as to the physical processes involved. However, a few nuclei, namely $Ti^{46}$, $V^{47}$, $Cr^{49}$, and $Mn^{50}$, were not discussed. A similar study in which these nuclei were included was made independently[2]; in this paper the results for these particular nuclei are discussed.

The theoretical model used is the same as that of MBZ. Assuming a pure $f_{7/2}$ configuration and an inert $Ca^{40}$ core, a set of basis functions with good angular momentum, isospin, and seniority are constructed. An effective residual interaction is determined from the available information about the spectrum of the two-nucleon $(f_{7/2})^2$ nuclei, $Ca^{42}$ and $Sc^{42}$. Armed with these tools, an effective interaction matrix for a many-nucleon system is constructed. This matrix is diagonalized, thus obtaining the energy spectra and nuclear wave-functions from which spectroscopic factors, $\beta$-decay $ft$ values, magnetic moments, and M1 transitions are calculated.

Admittedly, the calculations are incomplete in that they neglect configuration admixing and core excitations. Also the energies of the $3^+$ and $5^+$ levels of $Sc^{42}$ are not yet determined. However it is hoped that the calculations will give an idea of the general trends to be expected and thereby aid the nuclear systematics and also point out "extra-$f_{7/2}$-shell" effects. The success of the MBZ study justifies this hope, and it is in this spirit that this paper is written.

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[1] B. Bayman, J. D. McCullen, and L. Zamick, Phys. Rev. Letters **11**, 215 (1963); J. D. McCullen, B. Bayman, and L. Zamick, Phys. Rev. **134**, B515 (1964).
[2] J. N. Ginocchio and J. B. French, Phys. Letters **7**, 137 (1963); J. N. Ginocchio, Ph.D. thesis, University of Rochester, 1964 (unpublished); J. N. Ginocchio, Nucl. Phys. **63**, 449 (1965).

## II. THE NUCLEI ($V^{46}$,$Mn^{50}$)

Because of the presence of large attractive Wigner and Majorana forces, one of the large effects of nuclear forces is to favor the spatially symmetric states. The more symmetric the spatial wave function, the lower the isospin. It happens that, for nuclei with $|M_T|>0$, the wave function with the lowest possible space symmetry has $T=|M_T|$. However for nuclei with $M_T=0$ and atomic mass of the form $A=4k+2$, where k is a (non-negative) integer, there are two possible isospins for the most spatially symmetric wave function,[3] namely $T=0$ or $T=1$. Thus in odd-odd self-conjugate nuclei, both the $T=0$ and $T=1$ levels are expected to lie low in the energy spectra. The simplest example, of course, is the deuteron. Since the two-body triplet force dominates the singlet force, the (bound) $T=0$ state lies about 2.3 MeV lower than the (unbound) $T=1$ state. Examining the measured energy spectra of the other self-conjugate nuclei, this splitting is gradually decreased (see Table I) so that for $Cl^{34}$ ($k=8$) there is an inversion and the $T=1$ ($J=0^+$) level becomes the ground state. This inversion occurs again for $Sc^{42}$.

TABLE I. Relative energies of lowest $T=0$, 1 levels in $M_T=0$ nuclei.[a]

| Nucleus | $J^\pi$ | $E(J=0, T=1) - E(J^\pi, T=0)$ (MeV) |
|---|---|---|
| ${}_1H_1{}^2$ | $1^+$ | 2.3 |
| ${}_3Li_3{}^6$ | $1^+$ | 3.56 |
| ${}_5B_5{}^{10}$ | $3^+$ | 1.74 |
| ${}_7N_7{}^{14}$ | $1^+$ | 2.311 |
| ${}_9F_9{}^{18}$ | $1^+$ | 1.08 |
| ${}_{11}Na_{11}{}^{22}$ | $3^+$ | (0.666) |
| ${}_{13}Al_{13}{}^{26}$ | $5^+$ | 0.229 |
| ${}_{15}P_{15}{}^{30}$ | $1^+$ | 0.686 |
| ${}_{17}Cl_{17}{}^{34}$ | $3^+$ | −0.145 |
| ${}_{19}K_{19}{}^{38}$ | $(3^+)$ | 0.123 |
| ${}_{21}Sc_{21}{}^{42}$ | $1^{+b}$, $7^+$ | −0.60 |

[a] Data from *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.), 61-5, 6.
[b] R. W. Zurmuhle *et al.*, Bull. Am. Phys. Soc. **9**, 456 (1964).

[3] See, for example, M. Hamermesh, *Group Theory* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1964), Chap. 11.

TABLE II. Log$ft$ values for $Mn^{50}$ $\beta^+$ $Cr^{50}$: theoretical and empirical.

| Theoretical | | | | | Empirical[a] | | |
|---|---|---|---|---|---|---|---|
| | $Cr^{50}$ | | log$ft$ | | | $Cr^{50}$ | log$ft$ |
| $E$ (MeV) | $J_f^{\pi}$ | $Ji=5$ | $Ji=6$ | $Ji=7$ | $E$ (MeV) | $J_f^{\pi}$ | $Ji=(5,6,7)$ |
| 0.0 | $0^+$ | | | | 0.0 | $0^+$ | |
| 1.104 | $2^+$ | | | | 0.79 | $2^+$ | |
| 2.176 | $4^+$ | 4.9 | | | 1.90 | $4^+$ | |
| 2.745 | $2^+$ | | | | 2.94 | | |
| 3.210 | $4^+$ | 4.2 | | | 3.18[b] | | |
| 3.264 | $6^+$ | 4.7 | 6.8 | 4.2 | 3.35 | $(6^+)$ | 5.1 |
| 3.675 | $2^+$ | | | | 3.64[b] | (1,2) (2,3,4) | |
| 3.721 | $3^+$ | | | | 3.72 | | |
| | | | | | 3.81 | (2) | |
| 3.876 | $4^+$ | 5.2 | | | 3.87 | | 4.9 |
| 3.962 | $1^+$ | | | | 3.92 | | |
| | | | | | 3.96 | $(2^+)$ | |
| 4.013 | $5^+$ | 5.1 | 4.1 | | 4.00 | | |
| | | | | | 4.08 | | |
| 4.170 | $6^+$ | 4.1 | 4.6 | 6.1 | 4.16 | | |
| | | | | | Many levels No levels measured above 4.67 MeV | | |
| 4.881 | $6^+$ | 6.2 | 5.7 | 5.1 | | | |
| 4.991 | $8^+$ | | | 4.9 | | | |

[a] Empirical $\beta$-decay analysis by D. C. Sutton *et al.*, (see Ref. 4), and empirical $Cr^{50}$ levels from P. J. Twin and J. C. Willmott, Nucl. Phys. 53, 484 (1964).
[b] There is believed to be a doublet at these energies.

Speaking crudely, the lowering of this state is due to the pairing property of nuclear forces which tends to energetically lower states which have their angular momentum coupled to zero.

The next self-mirror nuclei are $V^{46}$ and $Mn^{50}$. In a pure $f_{7/2}$-picture $V^{46}$ has three $f_{7/2}$ neutrons and three $f_{7/2}$ protons while $Mn^{50}$ has three $f_{7/2}$-neutron-holes and three $f_{7/2}$-proton holes. Thus a pure $f_{7/2}$ picture would predict identical spectra for these two nuclei. The theoretical $f_{7/2}$ spectrum is given in Fig. 1. We see that an inversion is predicted, i.e., a T=1 (J=$0^+$) ground state lies 320 keV below the first T=0 (J=$5^+$) level. Altogether a total of four T=0 levels are predicted below the next T=1 (J=$2^+$) level.

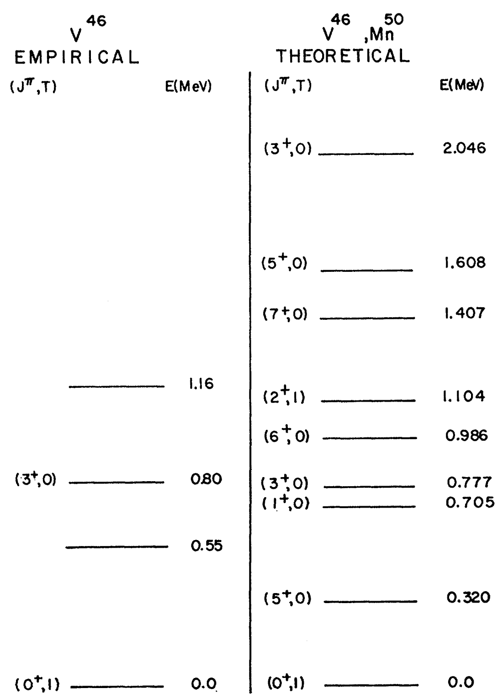


FIG. 1. Empirical spectrum of $V^{46}$ and theoretical spectrum of $V^{46}$ and $Mn^{50}$.

The experimental evidence is sparse. However for $Mn^{50}$ there is known to be a $(J^{\pi},T)=(0^+,1)$ level and a 1.7 minute isomeric T=0 level within 200 keV of each other but the order is not certain.[4] This T=0 level is thought to have a high spin because a triple gamma-cascade follows the $\beta$-decay to $Cr^{50}$. Our calculation suggests that this isomeric T=0 level has spin $5^+$; the $6^+$ and $7^+$ are predicted to lie more than 700 keV higher than the $5^+$. In Table II the calculated $\beta$-decay scheme for $Mn^{50} \rightarrow Cr^{50}$, assuming these three spins for $Mn^{50m}$, is compared with the empirical $\beta$-decay scheme. The J=$6^+$ assumption gives too high an $ft$ value to the 3.3-MeV level and no decay to the 3.8-MeV level. The J=$7^+$ assumption gives too low an $ft$ value to the 3.3 level and no decay to the 3.8-MeV level. The J=$5^+$ assumption gives reasonable $ft$ values to both of these levels, but it also suggests that the level at 3.2 MeV should have been populated. However, the experimental error does not rule out this possibility. The overestimation of the extent of population relative to the 3.3-MeV level may be accountable to the high sensitivity of $ft$-values to configuration admixtures.[5]

In $V^{46}$ the inversion has definitely been established empirically. The ground-state spin has been measured

[4] D. C. Sutton *et al.*, Bull. Am. Phys. Soc. 4, 278 (1959).
[5] Incidentally we notice how well the $f_{7/2}$ model predicts the pattern of the $Cr^{50}$ spectrum up to 4.17 MeV. This correspondence should help in determining the angular momenta of these levels.

to be $0^+$, and after a thorough search no $\beta$-decaying isomeric level has been found.[6] Hence the $J=5^+$ must be pushed up relative to the ground state as compared to $Mn^{50}$.

Jänecke[7] has tried to determine the energies of the excited states by looking at thresholds of the induced $\beta$-activity in the reaction $Ti^{46}(p,n)(V^{46}(\beta^+)Ti^{46})$ as a function of proton energy. Thresholds of activity are seen at 0.55, 0.81, and 1.16 MeV. He assumes that these thresholds correspond to energy levels in $V^{46}$ and goes on to speculate as to the possible spins and isospins of these levels. Justifiably he suggests that the 0.55-MeV level has $J=1^+$, $T=0$ since a higher spin would be isomeric with a possible $\beta$-decay. However he goes on further to infer that the 0.81-MeV level is the $J=2^+$, $T=1$ analog of the 0.88-MeV level in $Ti^{46}$ and that the 1.16-MeV level has $(J,T)=(5^+,0)$ from the fact that Leipunskii *et al.*[8] have seen a $(1.4\pm0.3)$ msec gamma ray of about 1 MeV. However, more recent experiments[9] measure a gamma ray with energy of $(0.801\pm0.001)$ MeV and with half-life of about $(1.04\pm0.10)$ msec. Thus the isomeric transition is not from the 1.16-MeV level but from the 0.8-MeV level which indicates that this level has higher spin than two, very probably three. In comparing the empirical spectrum in Fig. 1 with the $f_{7/2}$-model spectrum and assuming that the 0.55 level has $J=1^+$, we see that the $J=1^+$, $3^+$ are reproduced fairly well, but that apparently the $J=5$ has been pushed up quite high, at least above 0.8 MeV.

TABLE III. Theoretical spectrum of $V^{46}$ and $Mn^{50}$ with $E\leqslant 5.140$ MeV. Magnetic moments (nm): $J=5$, 2.700; $J=1$, 0.540.

| $J^a$ | $T$ | $E$(MeV) | $J^a$ | $T$ | $E$(MeV) |
|---|---|---|---|---|---|
| 0 | 1 | 0.0 | 2 | 1 | 3.675 |
| 5 | 0 | 0.321 | 3 | 1 | 3.721 |
| 1 | 0 | 0.705 | 7 | 0 | 3.824 |
| 3 | 0 | 0.777 | 4 | 1 | 3.876 |
| 6 | 0 | 0.986 | 1 | 1 | 3.962 |
| 2 | 1 | 1.104 | 5 | 1 | 4.013 |
| 7 | 0 | 1.407 | 3 | 0 | 4.066 |
| 5 | 0 | 1.608 | 1 | 0 | 4.142 |
| 3 | 0 | 2.046 | 6 | 1 | 4.170 |
| 1 | 0 | 2.165 | 4 | 0 | 4.416 |
| 4 | 1 | 2.176 | 5 | 0 | 4.418 |
| 4 | 0 | 2.328 | 7 | 0 | 4.496 |
| 2 | 1 | 2.745 | 6 | 1 | 4.881 |
| 9 | 0 | 3.079 | 3 | 0 | 4.905 |
| 4 | 1 | 3.210 | 8 | 0 | 4.948 |
| 6 | 1 | 3.264 | 8 | 1 | 4.991 |
| 3 | 0 | 3.285 | 4 | 0 | 5.032 |
| 2 | 0 | 3.352 | 11 | 0 | 5.046 |
| 5 | 0 | 3.352 | 6 | 0 | 5.068 |
| 8 | 0 | 3.370 | 0 | 1 | 5.140 |
| 2 | 0 | 3.621 | | | |

ᵃ All levels have positive parity.

Clearly more empirical evidence is needed concerning these two very interesting nuclei. The $f_{7/2}$-model reproduces the most spectacular features, namely, the inversion of the $T=0$ and $T=1$ levels in $V^{46}$ and the existence of a low-lying isomeric level in $Mn^{50}$, but nothing substantial can be said about the details of the spectra. In Table III the theoretical magnetic moments of the lowest $J=5$ and $J=1$ levels and the energy spectrum up to 5 MeV are tabulated.

### III. THE NUCLEI ($V^{47}$,$Cr^{49}$)

One of the significant features of the particle-hole conjugate nuclei $V^{47}$ and $Cr^{49}$ is that the measured spin of their ground states is $\frac{5}{2}^-$, whereas a spin of $\frac{7}{2}^-$ would be expected in the simple Mayer-Jensen single-particle picture. The $f_{7/2}$-model does in fact predict a $\frac{5}{2}^-$ ground state with a $\frac{7}{2}^-$ level lying very near at 0.170 MeV as shown in Fig. 2. Comparing the theoretical and empirical spectrum of $V^{47}$ in more detail, it appears very likely that the 0.155-MeV level is the $\frac{7}{2}^-$ level which is predicted to be at 0.170 MeV.[10] Also it is quite possible that either the 0.693- or 0.665-MeV level corresponds to the $\frac{3}{2}^-$ level predicted to be at 1.44 MeV; MBZ have found depressions of the $\frac{3}{2}^-$ level

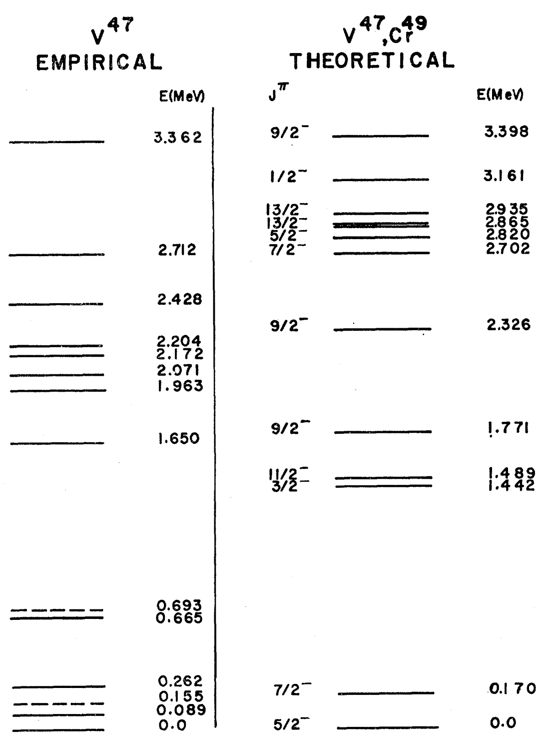


FIG. 2. Empirical spectrum of $V^{47}$ and theoretical spectrum of $V^{47}$ and $Cr^{49}$. Dashed levels are not seen in $Ti^{46}(p,\gamma)V^{47}$ (see Ref. 11).

[6] J. H. Miller and D. C. Sutton, Bull. Am. Phys. Soc. **3**, 206 (1958).
[7] J. Jänecke, Phys. Letters **6**, 69 (1963).
[8] O. I. Leipunskii *et al.*, Dokl. Akad. Nauk SSSR **109**, 935 (1956) [English transl.: Soviet Phys.—Doklady **1**, 505 (1956)].
[9] T. Conlon (private communication).

[10] It may not be significant but it is curious that the two levels at 0.693 and 0.155 MeV are *not* seen in the reaction $Ti^{46}(p,\gamma)V^{47}$ but are seen in the reaction $Ti^{47}(p,n)V^{47}$ as reported by H. Albinsson and J. Dubois, Phys. Letters **15**, 260 (1965). Apparently none of the levels in the $\gamma$-decay sequence decays to these levels.

of as much as 1 MeV in their survey of other odd nuclei. Possibly the other level at 600 keV is the $11/2^-$ predicted to be at 1.49 MeV. Similar arguments can be made about the higher levels; nothing more definitive can be said about these levels until more information is available. However, the presence of the very low-lying levels at 0.272 and 0.089 MeV presents a mystery and is in no way explainable by the $f_{7/2}$ model.

In Fig. 3 the $f_{7/2}$-model spectrum of $Cr^{49}$ is compared with the spectrum measured from the neutron pickup.[11] The $f_{7/2}$ model predicts that the only level that should be seen in the neutron pickup at this energy is the $J^\pi=\frac{7}{2}^-$ level at 0.170 MeV (see Table IV). Empirically the reaction was dominated by a strong $l=3$ transition to the 0.270-MeV level which agrees well with the model; all other excitations are weak. The weak excitation to the ground state is consistent with a $J^\pi=\frac{5}{2}^-$ for this level. The 1.10-MeV level could very well be the $J^\pi=\frac{3}{2}^-$ level at 1.44 MeV, a depression of 0.300 MeV being consistent with $J^\pi=\frac{3}{2}^-$ levels found in other odd nuclei. It is possible that the levels seen at 1.58 and 1.76 MeV are $d_{3/2}$ and $s_{1/2}$-hole states with $J^\pi=\frac{3}{2}^+$ and $\frac{1}{2}^+$, respectively. The systematics of these hole states predicts[12] them to be at about 1.4 and 1.5 MeV, respectively. The nature of the level at 2.0 MeV is not clear. Only very high spins are predicted in this vicinity, and it is very unlikely that $Cr^{50}$ contains $h_{9/2}$ neutrons. Finally the level seen at 2.63 MeV agrees well with the 2.702-MeV level predicted by the model.

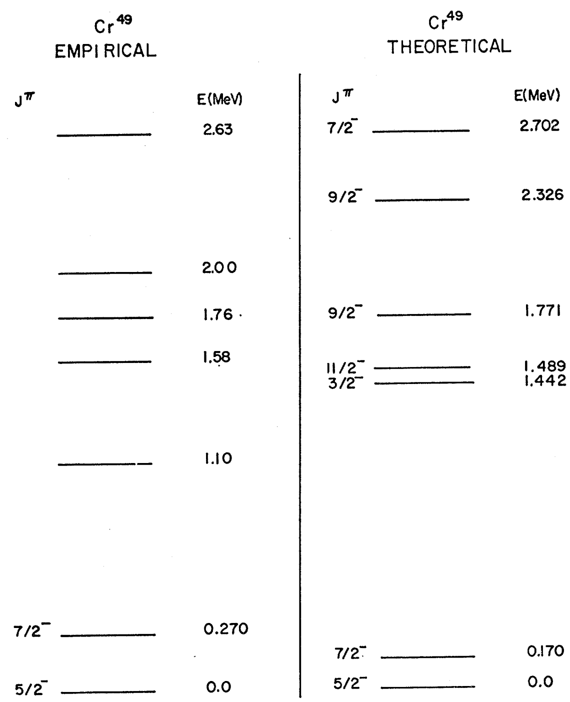


FIG. 3. The empirical spectrum of $Cr^{49}$ from the neutron pickup reaction $Cr^{50}$(p,d)$Cr^{49}$ compared to the theoretical spectrum. The only strong excitation is the $l=3$ transition to the 0.270-MeV level.

TABLE IV. Theoretical spectrum of $V^{47}$ and $Cr^{49}$ with $J\leqslant 13/2$ and $E\leqslant 4.437$ MeV. Magnetic moments (nm): For $V^{47}$, $J=\frac{5}{2}^-$, 3.103; $J=\frac{7}{2}^-$, 3.826. For $Cr^{49}$, $J=\frac{5}{2}^-$, −0.407; $J=\frac{7}{2}^-$, −0.050.

| $J^a$ | $T$ | $E$ (MeV) | Reaction strengths $Ti^{46}\to V^{47}$ $Cr^{50}\to Cr^{49}$ |
|---|---|---|---|
| $\frac{5}{2}$ | $\frac{1}{2}$ | 0.0 | |
| $\frac{7}{2}$ | $\frac{1}{2}$ | 0.170 | 4.158 |
| $\frac{3}{2}$ | $\frac{1}{2}$ | 1.442 | |
| $\frac{11}{2}$ | $\frac{1}{2}$ | 1.489 | |
| $\frac{9}{2}$ | $\frac{1}{2}$ | 1.771 | |
| $\frac{9}{2}$ | $\frac{1}{2}$ | 2.326 | |
| $\frac{7}{2}$ | $\frac{1}{2}$ | 2.702 | 0.0 |
| $\frac{5}{2}$ | $\frac{1}{2}$ | 2.820 | |
| $\frac{3}{2}$ | $\frac{1}{2}$ | 2.865 | |
| $\frac{13}{2}$ | $\frac{1}{2}$ | 2.868 | |
| $\frac{13}{2}$ | $\frac{1}{2}$ | 2.935 | |
| $\frac{1}{2}$ | $\frac{1}{2}$ | 3.161 | |
| $\frac{9}{2}$ | $\frac{1}{2}$ | 3.398 | |
| $\frac{11}{2}$ | $\frac{1}{2}$ | 3.614 | |
| $\frac{7}{2}$ | $\frac{1}{2}$ | 3.684 | 0.352 |
| $\frac{11}{2}$ | $\frac{1}{2}$ | 3.692 | |
| $\frac{3}{2}$ | $\frac{1}{2}$ | 4.013 | |
| $\frac{7}{2}$ | $\frac{3}{2}$ | 4.065 | 1.299 |
| $\frac{5}{2}$ | $\frac{3}{2}$ | 4.067 | |
| $\frac{7}{2}$ | $\frac{1}{2}$ | 4.116 | 0.0 |
| $\frac{5}{2}$ | $\frac{1}{2}$ | 4.191 | |
| $\frac{9}{2}$ | $\frac{1}{2}$ | 4.292 | |
| $\frac{11}{2}$ | $\frac{1}{2}$ | 4.431 | |
| $\frac{13}{2}$ | $\frac{1}{2}$ | 4.437 | |

a All levels have negative parity.

The theoretical spectra for $V^{47}$ and $Cr^{49}$ are summarized in Table IV along with the magnetic moments of the lowest $J=\frac{5}{2}$ and $J=\frac{7}{2}$ levels. Reaction strengths[13] for the stripping of a proton to $Ti^{46}$ and the pickup of a neutron from $Cr^{50}$ are given. The total strength must be six since in the first case six $f_{7/2}$-proton holes are open to receive another proton, and, in the second, six $f_{7/2}$ neutrons are present to be picked up.


## ACKNOWLEDGMENTS

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[11] L. C. McIntyre and C. A. Whitten, Bull. Am. Phys. Soc. **10**, 480 (1965).

[12] R. K. Bansal and J. B. French, Phys. Letters **11**, 145 (1964).

[13] The pickup (stripping) strength to a level is the number of active particles (holes) in the target which contribute to making that particular level in the residual nucleus. For a precise definition see J. B. French and M. H. Macfarlane, Nucl. Phys. **26**, 168 (1961).