Elastic Neutron Scattering from B, C, K, and Ca at 14.0 MeV*

A. J. Frasca,† Roger W. Finlay, R. D. Koshel,‡ and R. L. Cassola
Department of Physics, Ohio University, Athens, Ohio
(Received 1 November 1965)

Differential cross sections of 14.0-MeV neutrons scattered elastically from B, C, K, and Ca have been measured with the Ohio University time-of-flight spectrometer. The associated-particle technique was used. A time resolution of 1.3 nsec was sufficient to guarantee discrimination against inelastic events at a flight path of 1.68 m. The data were corrected for finite geometry and double scattering. An automatic six-parameter search routine was used to calculate an optical-model fit to the data. Improved fits were obtained for the B, C, and Ca distributions. Results for the previously unmeasured K differential cross section are presented.

I. INTRODUCTION

CONSIDERABLE effort has recently gone into the measurement of neutron elastic-scattering cross sections. Comparison of the experimental results with the predictions of the optical model has been the primary motivation for these studies. The present series of experiments was initiated in order to further define the nature of the agreement between the model and the experimental results. The combination of associated-particle techniques and nanosecond time resolution provides certain advantages (for example: small scattering samples, resolved inelastic scattering, precise definition of neutron energy and scattering angle) which tend to clarify the application of the optical model. The results suggest that while the form of the optical potential used here can give excellent agreement for elastic-scattering differential cross sections, complete accord with experiment is not obtained.

II. EXPERIMENTAL ARRANGEMENT

The experimental arrangement used for all differential cross-section measurements is shown in Fig. 1. The 150-keV deuteron beam from the Ohio University Cockcroft-Walton accelerator was used to generate neutrons from the $T(d,n)He^4$ reaction. The tritium targets consisted of 1 to 4 Ci/in.^{2} of tritium adsorbed on titanium and backed by thin (0.010-in.) copper. Alpha particles emerging from the reaction at an angle of 79° were detected in a thin plastic scintillator for the dual purpose of providing a start signal for the time-of-flight spectrometer and an "electronic collimation" of the emerging neutrons. All neutrons associated with a detected alpha particle pass through the scattering sample. In this experiment, the samples subtended an angle of ±3.5° in the scattering plane. With this choice of angle for the emerging neutron, the neutron energy is nearly independent of the incident-deuteron energy; hence, the neutron energy can be taken as $E_n = 14.00 \pm 0.03$ MeV.

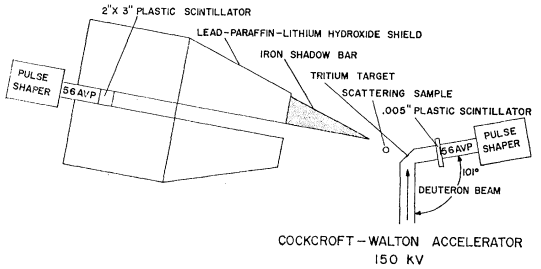

FIG. 1. Experimental arrangement.

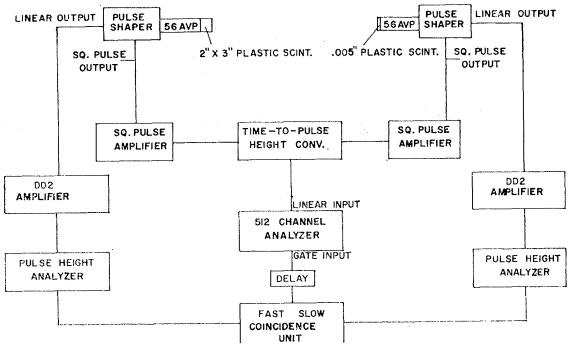

FIG. 2. Block diagram of time-of-flight electronics.

The samples used in this experiment were mechanically pressed into a cylindrical shape. All samples were approximately 3.18 cm in diameter and 7.62 cm long except for the calcium sample which was 3.8 cm long. The axis of the cylinder was oriented perpendicular to the scattering plane. All samples had the natural isotopic abundances. The calcium, boron, and potassium samples were wrapped with thin film to reduce surface oxidation.

The alpha particles were detected in a thin (0.005-in.) plastic scintillator which was mounted directly on the face of a 56 AVP photomultiplier tube. An aluminized Mylar film (0.80 mg/cm^2) served as a light-tight cover

* Work supported in part by the National Science Foundation and by the Ohio University Fund, Inc.
† This paper was presented by Albert J. Frasca to Ohio University in partial fulfillment of the requirements for the degree of Doctor of Philosophy. Present address: Industrial Nucleonics, Columbus, Ohio.
‡ Supported in part by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research, under AFOSR Grant Nr 854-65.

for the alpha-particle detector as well as an absorber of the elastically scattered deuterons. The neutron detector was another plastic scintillator 2 in. in diameter and 3 in. long coupled to a 56 AVP photomultiplier.

The neutron detector was housed in a large movable shield made of alternate castings of lead and LiOH dissolved in paraffin. Further shielding of the neutron detector from the direct beam was provided by a steel shadow bar 23 in. long. The entire shield assembly could be rotated through scattering angles from $-20°$ to $+160°$. The shield could also be moved radially to obtain flight paths between 1.6 and 4.0 m.

The time-of-flight spectrometer is based on the familiar associated particle technique. Timing signals are obtained from a two-stage tunnel-diode pulse-

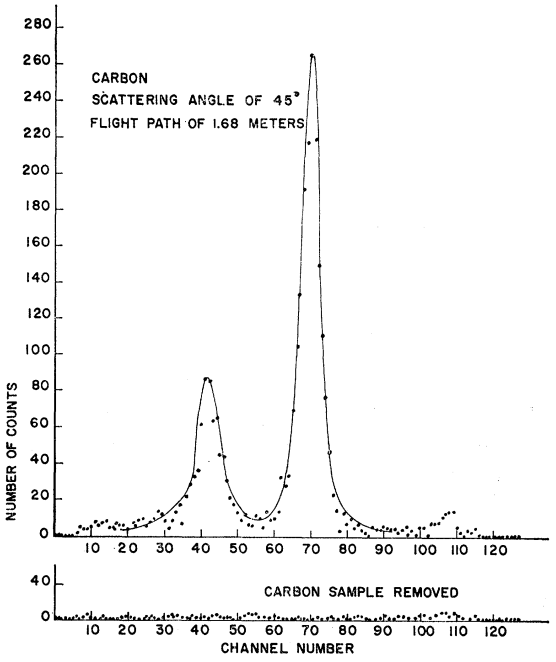

FIG. 3. Typical time spectrum of neutrons scattered from carbon and the associated background spectrum.

shaping circuit which is sensitive to the zero-crossing region of the differentiated anode signal of each detector. The timing information obtained in this way is remarkably insensitive to the amplitude of the anode pulse. Over-all time resolution better than 8×10^{-10} sec full width at half-maximum (FWHM) has been obtained for coincident gamma rays without the use of amplitude restriction.

When used as a practical neutron spectrometer, time resolution of about 1.3×10^{-9} sec was commonly experienced. Most of this additional linewidth is due to: (a) kinematic variations over the finite angular extent of the detectors, (b) dispersion in the alpha-particle velocities due to the thickness of the tritiated titanium

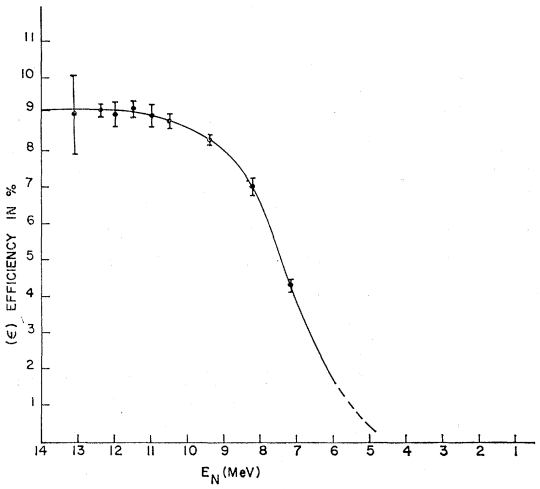

FIG. 4. Over-all efficiency of the neutron spectrometer as a function of energy.

target and, (c) dispersion of the photon transit times in the large neutron scintillator.

It was also necessary to use a certain amount of amplitude restriction in the neutron experiments. A block diagram of the electronics is shown in Fig. 2. The lower level of the neutron discriminator was set so that the lower energy neutrons from the $D(d,n)He^3$ reaction were not counted. The alpha-particle spectrum was contaminated slightly on the high-pulse-height side by neutron pulses and on the low-pulse-height side by noise. Hence, a broad ($\approx40\%$) pulse-height window around the alpha-particle peak was used throughout the experiment. A coincidence requirement of about 20 nsec was imposed on the discriminator outputs. This coincidence pulse then gated the multichannel analyzer for an input pulse from the time converter. A typical time spectrum taken under these conditions is shown in Fig. 3. A background spectrum is also shown.

The absolute efficiency of the spectrometer was determined by measuring the differential elastic scattering of 14-MeV neutrons from hydrogen. A polyethylene target of dimensions identical to the graphite target was prepared. Time spectra were taken with both targets at several angles between 15 and 45 deg. Experimental conditions (such as counting rates, flight path, etc.) were held fixed. Then the known response of the system to inelastic and elastic scattering from carbon was subtracted from the polyethylene spectra. These results were then compared to a recent phase-shift analysis (YLAN4M, June 1965) by Breit *et al.*[1] By this method it was possible to determine the absolute efficiency of the spectrometer as a function of energy to better than 5%. Figure 4 shows the spectrometer efficiency as a function of neutron energy.

[1] G. Breit (private communication).

III. EXPERIMENTAL PROCEDURE

The number of angles for which time spectra were taken varied from 14 and 16 for boron and carbon to 22 for potassium and calcium where the angular distributions show more rapid flucuations. Measurements at many angles were repeated several times. "Sample-out" background spectra were taken in a large number of cases, but as can be seen from Fig. 3, the background had no particular structure and was small compared to the signal except for the very backward angles.

The neutron flight path was chosen to be either 1.60 (the shortest available) or 1.68 m. Careful preliminary measurements were made to determine the certainty with which the first inelastic contribution could be subtracted from the elastic scattering peak. Stelson *et al.*[2] have discussed the importance of such a precaution. At these flight paths, boron was the most difficult to resolve for two reasons. First, the sample was made from natural boron which is 80.22% B^{11} and 19.78% B^{10}. The first excited state of B^{11} is at 2.14 MeV and is resolved under these conditions. However, the first excited state of B^{10} is at 0.717 MeV and could not be resolved. Schrank *et al.*[3] have shown that the cross section for inelastic proton scattering to this state is always less than 5 mb/sr, and can therefore contribute no more than 0.2% to the "elastic" peak in the natural-boron time spectrum.

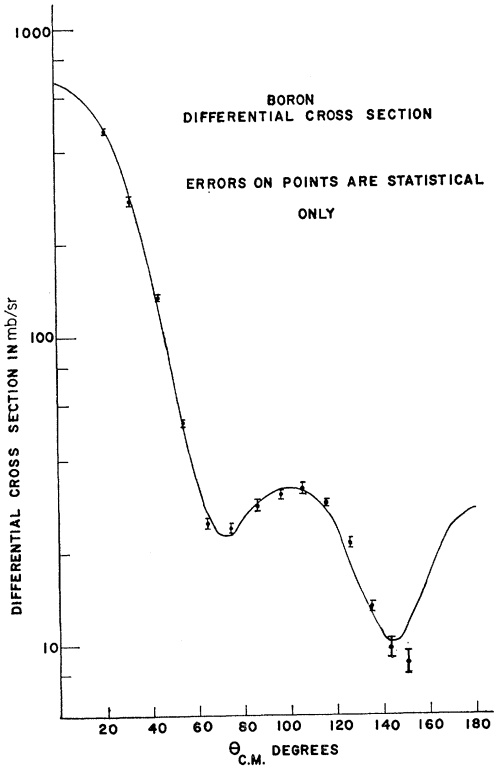

FIG. 5. Differential cross section for elastic scattering from boron. The solid curve is the optical-model prediction as described in the text.

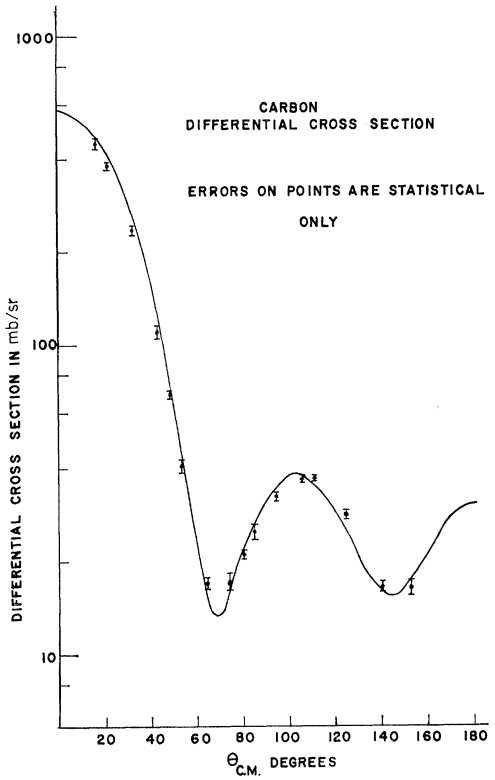

FIG. 6. Differential cross section for elastic scattering from carbon. The solid curve is the optical-model prediction as described in the text.

IV. RESULTS

The angular distributions of neutrons scattered from boron, carbon, potassium, and calcium are shown in Figs. 5 to 8. The errors shown represent the statistical errors only.

Each of the angular distributions was corrected for the finite angular resolution of the spectrometer. The method described by Eckart[4] was used for this correction.

The angular distributions for boron and carbon were further corrected for double scattering using the method of Blok and Jonker.[5] No double-scattering corrections were made for the calcium and potassium angular distributions. In these cases the scattering samples were thin enough that they elastically scattered only about 5% of the incident neutrons. Cross and Jarvis[6] have examined the importance of double-scattering corrections in this case. It would seem that the double-

[2] P. H. Stelson *et al.*, Nucl. Phys. **68**, 97 (1965).
[3] G. Schrank, E. K. Warburton, and W. W. Daehnick, Phys. Rev. **127**, 2159 (1962).
[4] C. Eckart, Phys. Rev. **51**, 735 (1937).
[5] J. Blok and C. C. Jonker, Physica **18**, 809 (1952).
[6] W. G. Cross and R. G. Jarvis, Nucl. Phys. **15**, 155 (1960).

scattering correction will affect the total elastic cross section by a negligible amount and will only affect the angular distributions at the sharp minima. Therefore, the minima are probably somewhat deeper than shown in Figs. 7 and 8.

Elastic scattering of 14-MeV neutrons from boron has been reported by Tesch.[7] He reported a value of 900±50 mb for the integrated elastic-scattering cross section which is in very good agreement with the value of 938±20 given here.

Previous measurements of neutron scattering from carbon at this energy have been reported by several authors.[7–9] The integrated elastic cross section obtained here (785±30 mb) compares very well with the numerical average (790 mb) of five separate experiments in the references cited. Since carbon measurements were included primarily for standardization of the spectrometer and the correction programs, this close agreement was entirely satisfactory.

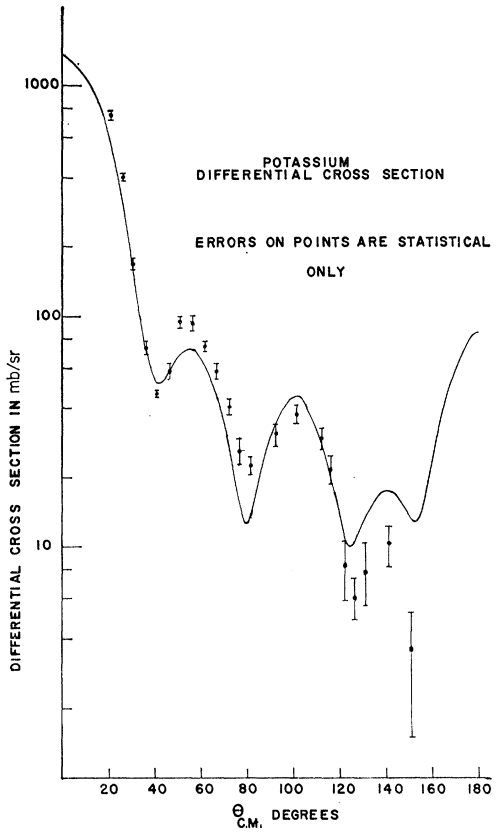

FIG. 7. Differential cross section for elastic scattering from potassium. The solid curve is the optical-model prediction as described in the text.

[7] K. Tesch, Nucl. Phys. **37**, 412 (1962).
[8] Brookhaven National Laboratory Report No. BNL 325 (U. S. Government Printing and Publishing Office, Washington, D. C., 1964), Suppl. No. 2.
[9] R. Brouchez, Nucl. Phys. **43**, 628 (1963).

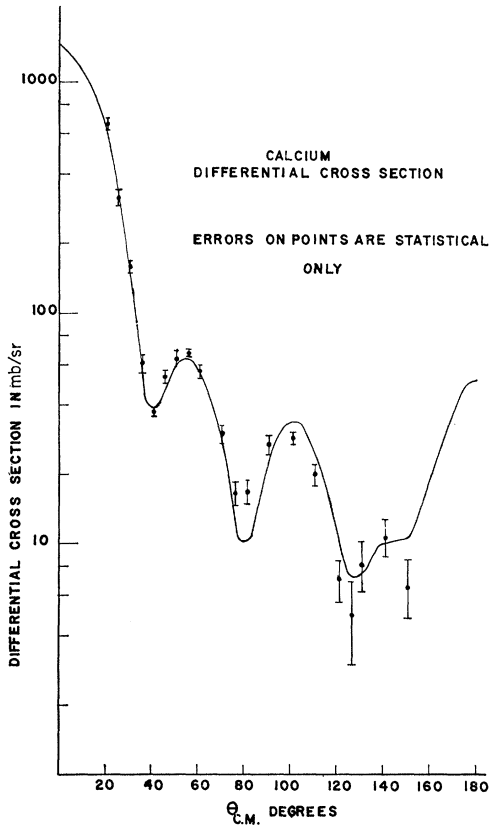

FIG. 8. Differential cross section for elastic scattering from calcium. The solid curve is the optical-model prediction as described in the text.

Calcium elastic scattering has been previously measured by Cross and Jarvis[6] and McDonald and Robson.[10] The integrated elastic cross section obtained here (918±52 mb) is in better agreement with the latter (930±100 mb) than with the former (820±70 mb).

V. DISCUSSION

The experimentally determined angular distributions for carbon, boron, potassium and calcium obtained in the present experiments have been compared with optical-mode calculations. A computer program has been developed for this purpose. The complex potential used has the form

$$V(r) = -Vf(r) - iW_v f(r) - iW_s g(r) - V_{so}, \quad (1)$$

where

$$f(r) = [1+\exp((r-R)/a)]^{-1}, \quad (2)$$

$$g(r) = \exp[-((r-R)/b)^2], \quad (3)$$

$$V_{so} = V_s(\hbar/m_\pi c)^2(1/r)[df(r)/dr]\boldsymbol{\sigma}\cdot\mathbf{l}, \quad (4)$$

[10] W. J. McDonald and J. M. Robson, Nucl. Phys. **59**, 321 (1965).

TABLE I. Optical-model parameters, predicted cross sections plus experimental elastic cross sections. The quoted errors represent counting statistics only.

Element	V (MeV)	W_s (MeV)	V_s (MeV)	r_0 (F)	a (F)	b (F)	σ_{el}^{th} (mb)	σ_r^{th} (mb)	σ_t^{th} (mb)	χ^2	σ_{el}^{expt} (mb)
B	48.2	6.87	−4.72	1.28	0.52	0.70	913	434	1347	34	938±20
C	52.1	7.66	−5.58	1.25	0.31	0.42	836	316	1152	65	785±30
K	46.1	6.70	−2.09	1.30	0.58	0.88	897	908	1805	307	1038±57
Ca	45.4	8.54	−2.57	1.29	0.60	0.90	835	1033	1868	74	918±52

and m_π is the mass of the pion. The nuclear radius R is

$$R=r_0A^{1/3}, \tag{5}$$

where A is the mass number of the target nucleus.

In order to compare the optical model calculations with the experimental results the automatic search program STEPIT[11] was used to minimize χ^2 for each angular distribution. χ^2 is given by

$$\chi^2=\sum_{i=1}^{N}\left(\frac{\sigma^{th}(\theta_i)-\sigma^{expt}(\theta_i)}{\Delta\sigma^{expt}(\theta_i)}\right)^2, \tag{6}$$

where N is the number of experimental points, $\sigma^{th}(\theta_i)$ is the optical model prediction at θ_i, $\sigma^{expt}(\theta_i)$ is the corresponding experimental value, and $\Delta\sigma^{expt}(\theta_i)$ is the experimental error at θ_i. In all the calculations best fits were obtained only with regard to the angular distributions. No attempt was made to obtain a best value for the reaction or total cross section.

In all the calculations W_v was set to zero so that only surface absorption was included. In order to use as few parameters as possible, the first calculations were made for carbon, potassium, and calcium with the radial parameter r_0 fixed at 1.25 F. This choice appeared to be justified in view of the analysis by Nodvik, Duke, and Melkanoff[12] of proton scattering from carbon and the work of Bjorklund and Fernbach[13] on 14-MeV neutron-scattering data. In the case of boron this parameter was allowed to vary as there appeared to be no justification for fixing the value. Except for carbon no satisfactory fits were found when the radial parameter was fixed. Subsequent calculations for potassium and calcium allowed all the parameters to vary. For boron, potassium, and calcium there were a total of six parameters varied and for carbon five.

The results of the calculations for the four elements are shown in Figs. 5 to 8. The solid line in each case is the minimum χ^2 fit to the experimental data. As can be seen the agreements between the experimental and theoretical cross sections are very good. The parameters found in the calculations are presented in Table I along with the theoretical elastic and reaction cross sections.

A theoretical analysis of boron using the same optical model has been previously carried out by Lutz, Mason, and Karvelis.[14] Their fit is not quite as good as the one obtained here since they fixed the value of V_s at −2.5 MeV. On comparing the experimental value of the total cross section $\sigma_t^{exp}=1.35$ mb [8] with the theoretical value in Table I, it is seen that there is excellent agreement.

For carbon, potassium, and calcium the comparison of the theoretical reaction cross sections with the experimental reaction cross sections, or, if experimental reaction cross sections were not available, the comparison of theoretical total cross sections with experimental ones was not quite as good. In all cases the theoretical values were lower by several hundred millibarns. In previous calculations on carbon, Lutz *et al.*[14] and Clarke and Cross[15] found better values for the reaction and total cross sections. However, the calculations were biased in such a way that only good values for the reaction and total cross sections were allowed. The resulting angular distributions obtained by them are not as good as the ones shown here.

Nodvik *et al.*[12] in their analysis of proton scattering from carbon also found theoretical values for the reaction cross section which were smaller than the experimental values. In their analysis, they used volume plus Gaussian surface absorption. However, in most cases they found that best fits were obtained when the volume term was absent. This is the model used here.

There is good agreement between the parameters obtained here on calcium and those obtained by McDonald and Robson[10] in their analysis of 14.1-MeV elastic neutron scattering from calcium. They also obtained low values for the calculated total and reaction cross sections.

The poorest value for χ^2 was found for potassium. This was a surprising result in so far as potassium is adjacent to calcium in the periodic table and a small value of χ^2 was found for calcium. However, there are no significant differences in the parameters obtained for calcium and potassium. This is necessary if the

[11] J. P. Chandler (private communication).
[12] J. S. Nodvik, C. B. Duke, and M. A. Melkanoff, Phys. Rev. **125**, 975 (1962).
[13] F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).
[14] H. F. Lutz, J. B. Mason, and M. D. Karvelis, Nucl. Phys. **47**, 521 (1963).
[15] R. L. Clarke and W. G. Cross, Nucl. Phys. **53**, 177 (1964).

optical model is to be a useful description of elastic scattering.

VI. CONCLUSION

The optical model with surface absorption very well describes the angular distributions of 14-MeV neutrons elastically scattered from boron, carbon, potassium, and calcium. In addition the model gives excellent values for the total cross section for boron. This would seem to indicate that the optical model can successfully describe the elastic scattering from boron.

The model gave consistently lower values for the reaction and total cross sections for carbon, potassium, and calcium. This indicates that the use of surface absorption alone may not be the complete answer. A good model should not only give good fits to the angular distributions but should also give good values for the reaction and total cross sections. This matter requires further investigation.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to J. P. Chandler of Indiana University for making available his search routine STEPIT and to C. J. Slavik for his useful discussions regarding the optical-model program. We would also like to thank P. L. Beach and M. W. Gilpatrick for their assistance throughout all phases of the experiment. The authors are indebted to M. J. Hayes for performing the double-scattering calculation for boron and carbon.

Finally, we are happy to thank the computer staff at the Battelle Memorial Institute for their continued interest in this work and for the use of their excellent facilities.

PHYSICAL REVIEW VOLUME 144, NUMBER 3 22 APRIL 1966

Elastic Electron Scattering from the Magnetic Multipole Distributions of Li^6, Li^7, Be^9, B^{10}, B^{11}, and N^{14} †

R. E. RAND, R. FROSCH, AND M. R. YEARIAN
High-Energy Physics Laboratory and Department of Physics, Stanford University, Stanford, California

(Received 1 December 1965; revised manuscript received 10 January 1966)

The magnetic form factors of the $1p$-shell nuclei Li^6, Li^7, Be^9 B^{10}, B^{11}, N^{14}, have been measured by high-energy electron scattering at an angle of 180°. Incident electron energies up to 230 MeV ($q^2 \simeq 5.2$ F^{-2}) were used. The incident- and scattered-electron paths were separated by use of a uniform magnetic field. Background due to charge scattering was subtracted by varying the scattering angle around 180°. At the higher momentum transfers the scattering from those nuclei having spin $\geq \frac{3}{2}$, was dominated by the octupole moment distributions. Results are interpreted in terms of harmonic-oscillator and finite-square-well shell models. Parameters dependent only on the coupling of the angular momenta in the $1p$ shell may be obtained from the data. They show a general trend from L-S coupling to j-j coupling as the mass number increases, in agreement with calculations of magnetic dipole moments and energy-level schemes. The outstanding exception is Li^7, whose form factor is accurately described by the extreme independent-particle model. (L-S coupling is quite incompatible with the data.) The Li^6 results may also be interpreted in terms of an "alpha plus deuteron" model. Numerical results for the harmonic-well strengths are obtained and compare favorably with electron-charge-scattering values. It is also possible to estimate magnetic octupole moments, where applicable, with fair accuracy, although these estimations are to a certain extent model-dependent.

I. INTRODUCTION

AN investigation of the magnetic properties of most of the stable $1p$-shell nuclei has been made by measuring their magnetic form factors at high momentum transfers. Previously reported preliminary data[1] on Be^9 and B^{11} have now been completed and are presented together with results from Li^6, Li^7, B^{10}, and N^{14}. The magnetic form factors are measured by high-energy elastic electron scattering at an angle of 180° (where "charge" scattering is very small), using the electron beam from the Stanford Mark III Linear Accelerator. The results are complemented by various low-energy measurements (using a similar technique on the Stanford Mark II Linac) by Peterson,[2] Goldemberg and Torizuka,[3] and Vanpraet and Kossanyi-Demay.[4] Comparison is also made with the results of Goldemberg *et al.*,[5] at Orsay.

† Work supported in part by the U. S. Office of Naval Research, Contract No. Nonr 225(67).

[1] R. E. Rand, R. Frosch, and M. R. Yearian, Phys. Rev. Letters **14**, 234 (1965).

[2] G. A. Peterson, Phys. Letters **2**, 162 (1962).

[3] J. Goldemberg and Y. Torizuka, Phys. Rev. **129**, 312 (1963).

[4] G. J. Vanpraet and P. Kossanyi-Demay, Nuovo Cimento **39**, 388 (1965).

[5] J. Goldemberg, D. B. Isabelle, T. Stovall, D. Vinciguerra, and A. Bottino, Phys. Letters **16**, 141 (1965).