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Inelastic Scattering Based on a Microscopic Description of Nuclei*

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The inelastic scattering of nucleons and light nuclides from nuclei is formulated in terms of a general central two-body interaction between the scattered particle and the nucleons of the nucleus, whose motions are described by detailed shell-model wave functions. Form factors based on this microscopic description are obtained as closed expressions. The theory is applied to proton scattering from the even nickel isotopes. The constructive coherence in the transition to the collective 2^+ state leads to a form factor having the general shape of that used in the macroscopic description of collective motion. Unenhanced transitions, in contrast, are characterized by a variety not present in the macroscopic description. Nucleon scattering as here calculated is sensitive to the details constituting this variety, and therefore provides a means of subjecting microscopic descriptions of nuclei to detailed tests.

1. INTRODUCTION

THE importance of inelastic scattering as a means of investigating nuclear structure was recognized long ago, and the theory for single-nucleon transitions between shell-model states has been developed.[1,2] However, the important experimental discovery by Cohen and Rubin[3] that the same states that are strongly coupled to the ground state by the electromagnetic field are also strongly excited by inelastic scattering has focused the attention of both experimentalists and theorists on collective states. Until recently the only way of handling such states was through recourse to the Bohr-Mottelson macroscopic description of collective motion. In this picture, the incident particle interacts with the nucleus in its surface region, exciting the vibrations or rotations as the case may be through a one-body deformed optical potential.[4] The spherical part is fixed by elastic scattering. Each multipole of the deformed part is specified by one (deformation) parameter, β_λ, of which there is experimental evidence for quadrupole and octupole parts. These two parameters can be determined from the cross sections to the collective 2_1^+ and 3_1^- states. Cross sections to all other states based on these multipoles is now fixed. The information about nuclear levels that can be gained from such a treatment is meager. It includes the deformation parameter and in some odd nuclei, the parities and spins of those levels connected with the "one-phonon" states of the core. In even nuclei the collective 2^+ and 3^- states are often already known, but one can usually identify in addition the spins of several higher states by alpha scattering, especially if observations at several bombarding energies are made. The present state of this approach has been reviewed recently by Harvey.[5] To deal successfully with the so-called two-phonon states it has always been found necessary so far as we know, to introduce greater arbitrariness into the coupling strengths than the model allows. Buck,[6] in treating the 4_1^+ state of Ni^{58}, had to enhance the direct coupling by a factor 1.5. Dickens *et al.*[7] in treating the 11-MeV proton scattering on $Ni^{62,64}$ were obliged to take different quadrupole deformation parameters β for each of the 2_1^+, 0_2^+, 2_2^+ levels instead of a common value. Even with its shortcomings however, the macroscopic model has been fruitful, and since detailed structure calculations in the transition and deformed regions will not be forthcoming for some time, it will continue to be useful.

Fortunately there has been some progress during the last few years in describing even fairly complex nuclei in the vibrational regions, in terms of the underlying nucleon correlations, starting from a Hamiltonian for a system of fermions interacting in some average field.[8] Much of this activity centers around trying to reproduce the energy level systematics and electromagnetic transition rates in spherical nuclei, some of whose states exhibit collective properties. The energy-level systematics is of course the easiest part of such a program because the Hamiltonian is stationary at the eigenstates. Electromagnetic transitions, inelastic scattering and *several*-nucleon transfer probabilities each depend upon certain correlations among the nucleons. As a result transition rates can vary from strongly enhanced to strongly hindered when compared to the rate calculated for an

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1 C. A. Levinson and M. K. Banerjee, Ann. Phys. **2**, 471 (1957); **2**, 499 (1957); **3**, 67 (1958).

2 N. K. Glendenning, Phys. Rev. **114**, 1297 (1959).

3 B. L. Cohen and A. G. Rubin, Phys. Rev. **111**, 1568 (1958).

4 J. S. Blair, *Proceedings of the Conference of Padua* (Gordon and Breach, New York, 1962), p. 669; see also N. Austern in *Selected Topics in Nuclear Theory* (International Atomic Energy Agency, Vienna, 1962), p. 17; R. H. Bassel, G. R. Satchler, R. M. Drisco, and E. Rost, Phys. Rev. **128**, 2693 (1962); N. Austern and J. S. Blair, Ann. Phys. Academic (N. Y.) **33**, 15 (1965).

5 B. G. Harvey, in *Nuclear Structure and Nuclear Reactions*, Proc. 9th Summer Meeting at Hercegnovi, edited by N. Cindro (Federal Nuclear Committee, Yugoslavia, 1964).

6 B. Buck, Phys. Rev. **127**, 940 (1962).

7 J. K. Dickens, F. G. Perey, R. J. Silva and T. Tamura, Phys. Letters **6**, 53 (1963).

8 See, for example, the following reviews and references cited therein: M. Baranger in *Cargèse Lectures in Theoretical Physics*, edited by M. Lévy (W. A. Benjamin, Inc., New York, 1963); A. M. Lane, *Nuclear Theory* (W. A. Benjamin, Inc., New York, 1964); G. E. Brown, *Unified Theory of Nuclear Reactions* (North-Holland Publishing Company, Amsterdam, 1964).

uncorrelated state. These wide variations put a structure calculation to quite a severe test, much more so than the energy level systematics. Moreover, since the kinematics are at the control of the experimenter, in principle, reactions provide a more versatile tool than electromagnetic transitions in studying nuclear structure.

Kisslinger[9] was the first to give a microscopic description of inelastic scattering from collective states in spherical nuclei. He emphasized, especially for the "two-phonon" states, that the particular shell-model orbits involved could influence very much the differential cross section, especially for nucleons.

In this paper we formulate the calculation of inelastic scattering based on a microscopic description of nuclear states in a form that is convenient for discussion and calculation.[10] The virtue of such an approach is that both the collective states about which the microscopic model is concerned, and the weaker noncollective and single-particle states are treated on the same footing in terms of their detailed structure. One can hope to say something through this approach about the success or failures of the existing microscopic calculations of nuclear structure, and perhaps indicate the directions in which improvements lie. Such a description certainly allows a richer variety of phenomenon than the rather restrictive phonon picture with its strict selection rules.

There are however ambiguities involved in this approach most important of which concerns the interaction between scattered particle and the nucleons of the nucleus. At high enough energies the impulse approximation may be valid and the free two-body t matrix can be used.[11] Then if the nuclear structure calculation includes the explicit participation of all the nucleons that are really involved in the collective motion, there should be no arbitrariness in the choice of the interaction. If however it has been found that an effective charge must be introduced to account for the observed electromagnetic transitions rates, then this approach will underestimate the cross section. Since in practice, aside from the very light elements, nuclear-structure calculations are performed in a truncated space involving the last one or two major shells, the ambiguity concerning the effective interaction will be present even at high energies. In this paper we are interested in lower energies commonly available on Van de Graaff and cyclotron accelerators, so that it certainly is present for us. We discuss it again in Sec. 3.2.

After the formulation of the problem found in Sec. 2 we apply the theory to proton scattering on the even nickel isotopes in Sec. 3. We have already reported some of our results for alpha scattering in the nickel region.[10]

2. FORM FACTORS FROM A MICROSCOPIC MODEL OF THE NUCLEUS

2.1 Background

To describe inelastic scattering from a nucleus, we need to know the matrix elements of the interaction of the scattered particle with the nucleus. For the form of the interaction between nucleons we adopt the potential

$$V(\mathbf{r}_j,\mathbf{r}_i)=(V_0+V_1\boldsymbol{\sigma}_j\cdot\boldsymbol{\sigma}_i)g(\mathbf{r}_j,\mathbf{r}_i). \qquad (1)$$

(Here V_0 and V_1 may depend in turn on $\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_i$.) The interaction of a scattered particle of mass number a and the nucleus A is therefore

$$V(\mathbf{a},\mathbf{A})=\sum_{j=1}^{a}\sum_{i=1}^{A}V(\mathbf{r}_j,\mathbf{r}_i). \qquad (2)$$

We shall neglect exchange effects which generally will be small as discussed later. (They are implicitly neglected in the macroscopic description). Therefore, the interaction, Eq. (2), is a sum of one-body operators on the nuclear coordinates, r_i. Hence, only components of the initial and final wave function that differ at most in the coordinates of one nucleon can be connected by the scattered particle. Consequently any excitation of the nucleus by the scattered particle consists at most of a superposition of elementary (single-particle) transitions. In the next section we recapitulate in a form suitable for our purposes, scattering by an odd nucleus in which the odd particle is excited. In terms of these results we will be able to express in a straight-forward way the inelastic scattering between any two nuclear states however complicated their structure, so long as their wave functions are known.

When dealing with a composite scattered particle we will suppress its structure and use a pseudo-interaction between its center of mass and the nucleons of the nucleus. Such a pseudopotential can, of course, be related to the interaction between nucleons, if we assume that the scattered particle exists only in its ground state. This is done in the appendix. Here we state the result. Using a Gaussian shape for the potential between nucleons

$$g(\mathbf{r}_j,\mathbf{r}_i)=\exp(-\beta|\mathbf{r}_j-\mathbf{r}_i|^2), \qquad (3)$$

and for the wave functions of the light nuclides, then the pseudopotential for the interaction between light nuclides and the nucleons of the nucleus is

$$V(\mathbf{a},\mathbf{A})=\sum_{i=1}^{A}(V_0'+V_1'\mathbf{S}\cdot\boldsymbol{\sigma}_i)\exp(-\beta'|\mathbf{R}-\mathbf{r}_i|^2), \quad (4)$$

[9] L. S. Kisslinger, Phys. Rev. **129**, 1316 (1963). See also E. A. Sanderson and N. S. Wall, Phys. Letters **2**, 173 (1962).

[10] A report of some of our work has already been made in N. K. Glendenning and M. Veneroni, Phys. Letters **14**, 228 (1965). Recent work by other authors includes: Rene Ceuleneev, thesis, University of Bruxelles, 1964 (unpublished); William P. Beres (to be published); V. A. Madsen and W. Tobocman, Phys. Rev. **139**, B864 (1965); and R. M. Haybron and H. McManus (to be published); M. B. Johnson, L. W. Owen, and G. R. Satchler (to be published); V. A. Madsen (to be published).

[11] R. M. Haybron and H. McManus, Phys. Rev. **136**, B1730 (1964).

where **S** is the spin operator of the nuclide a, and R is its center-of-mass coordinate. The constants V_0', V_1', and β' are related to the corresponding quantities in the nucleon-nucleon potential, the range being longer for the pseudopotential.

It is convenient to expand the potential Eq. (1) or (4) in multipoles (Slater expansion). To treat both spin-dependent and independent terms on the same footing we define the one-body operators: $\mathbf{\Sigma}_0(k)$ is unity and $\mathbf{\Sigma}_1(k)$ is the spin vector operator of the kth particle appearing in the potential. Define then the tensors

$$\mathfrak{Y}_{LSJ}=[\mathbf{Y}_L\mathbf{\Sigma}_S]_J\,, \tag{5}$$

where the square bracket denotes vector coupling.[12] Also let the Legendre transform of the space part of the potential be:

$$v_L(r,r')=\frac{2L+1}{2}\int g(\mathbf{r},\mathbf{r}')P_L(\cos\omega)d\cos\omega\,. \tag{6}$$

We have then the expansion

$$V(\mathbf{r},\mathbf{A})=\sum_{LSJ}(-)^{L+S+J}V_S\mathfrak{J}_{LSJ}(r,\mathbf{A})\cdot\mathfrak{Y}_{LSJ}(\hat{r})\,, \tag{7}$$

where

$$\mathfrak{J}_{LSJ}(r,\mathbf{A})=\frac{4\pi}{2L+1}\sum_{i=1}^{A}v_L(r,r_i)\mathfrak{Y}_{LSJ}(\hat{r}_i)\,. \tag{8}$$

To describe the scattering, one needs to know the matrix elements between the nuclear states of the interaction V and hence of $\mathfrak{J}$. The reduced matrix elements of $\mathfrak{J}$ are often called form factors in this context. They are, so to speak, the way in which the nucleus appears to the scattering particle. They appear in the coupling terms between the various open inelastic channels when the Schroedinger equation is written as a set of coupled equations. All of the nuclear information enters the description of the scattering through these quanitities, and we shall discuss them in great detail.

However, before doing so, we write down expressions for the differential cross-section. We shall calculate cross sections in the distorted-wave approximation which is the solution of the scattering problem to first order in V. It will be valid as long as the state we are interested in is not strongly coupled to other excited states when compared to its coupling to the ground state. This will almost always be true of the first 2^+ level in the vibrational regions. For the transition from the nuclear state α_1J_1 to α_2J_2 (where α denotes all quantum numbers additional to the total spin and its Z projection) we find for the cross section for particles of spin s_1

$$\frac{d\sigma}{d\Omega}(\alpha_1J_1\rightarrow\alpha_2J_2)=\frac{1}{(2s_1+1)(2J_1+1)}\sum_{LSJ}\frac{1}{2S+1}\times V_S{}^2\langle s_1\|\mathbf{\Sigma}_S\|s_1\rangle^2\sigma_{LSJ}\,, \tag{9}$$

where

$$\sigma_{LSJ}=\frac{k_2}{k_1}\left(\frac{m^*}{2\pi\hbar^2}\right)^2\sum_M|B_{LSJ}{}^M|^2 \tag{10}$$

$$B_{LSJ}{}^M=\frac{i^{-L}}{(2L+1)^{1/2}}\int\psi^{(-)*}(\mathbf{k}_2,\mathbf{r})\times\mathfrak{F}_{LSJ}{}^{\alpha_2\alpha_1}(r)Y_L{}^{M*}(\hat{r})\psi^{(+)}(\mathbf{k}_1,\mathbf{r})d\mathbf{r}\,. \tag{11}$$

Here $\psi^{(\pm)}$ are distorted waves describing the motion of the scattered particle under the influence of the optical potential which is assumed not to include a spin-orbit term. This neglect is not serious, except possibly at large angles, if we confine ourselves to calculating only cross sections, but not polarizations. The sum on L and S is incoherent as long as this term is absent. Otherwise there are cross terms. Also

$$\begin{aligned}\langle s_1\|\mathbf{\Sigma}_S\|s_1\rangle&=\delta_{S0} && \text{if } s_1=0\\ &=(2s_1+1)^{1/2}(\delta_{S0}+(2S+1)^{1/2}\delta_{S1}) && \text{if } s_1=\tfrac{1}{2}\\ &=(2s_1+1)^{1/2}(\delta_{S0}+\sqrt{2}\delta_{S1}) && \text{if } s_1=1\,,\end{aligned} \tag{12}$$

which would hold for alpha particles, nucleons (or t and He^3) and deuterons, respectively.

In the above, $\mathfrak{F}(r)$ denotes the form factor[13] or integral over the nuclear coordinates of $\mathfrak{J}$, Eq. (8):

$$\mathfrak{F}_{LSJ}{}^{\alpha_2\alpha_1}(r)\equiv\langle\Psi_{\alpha_2J_2}(\mathbf{A})\|\mathfrak{J}_{LSJ}(r,\mathbf{A})\|\Psi_{\alpha_1J_1}(\mathbf{A})\rangle\,. \tag{13}$$

For convenience, we shall refer to the form factor with $S=0$ as scalar since it is the matrix element of a scalar in spin space, and to the three form factors with $S=1$ (hence $L=J$ or $J\pm1$) as vector form factors. The latter arise of course from the spin-dependent part of the potential, Eq. (4).

The quantities L, S, and J are, respectively, the orbital, intrinsic, and total angular momentum transferred between the nucleus and scattered particle. Their possible values are limited by several obvious selection rules. First J must connect the spins of the two nuclear states

$$|J_1-J_2|\leq J\leq J_1+J_2,\quad \mathbf{J}=\mathbf{L}+\mathbf{S}\,. \tag{14}$$

Second, since the nuclear force conserves parity then

$$(-)^L=\pi_1\pi_2\,, \tag{15}$$

where π is the parity of a nuclear state. We have used a central interaction[14] so that S can have only the values 0 and 1.

For scattering from an even nucleus we write separately the cross-section for natural parity states, J, $\pi=(-)^J$ and for unnatural parity states. The latter

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[12] Our notation means $[\mathbf{Y}_L\mathbf{\Sigma}_S]_J{}^M\equiv\sum_\mu C_{M-\mu,\mu,M}{}^{LSJ}Y_L{}^{M-\mu}\Sigma_S{}^\mu$.

[13] We use Racah's original definition of reduced matrix elements, cf. A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, 1957), p. 75. There are several currently used definitions.

[14] If a tensor part is included in the interaction then we could also have $L=J\pm2$.

can be reached directly only through the spin-dependent part of the interaction. For spinless particles

$$\frac{d\sigma}{d\Omega}(0\rightarrow J(-)^J)=V_0^2\sigma_{J0J}. \quad (16)$$

For spin $\frac{1}{2}$ particles

$$\frac{d\sigma}{d\Omega}(0\rightarrow J(-)^J)=V_0^2\sigma_{J0J}+V_1^2\sigma_{J1J}, \quad (17)$$

$$\frac{d\sigma}{d\Omega}(0\rightarrow J(-)^{J+1})=V_1^2(\sigma_{J-1,1,J}+\sigma_{J+1,1,J}), \quad (18)$$

while for spin 1 particles

$$\frac{d\sigma}{d\Omega}(0\rightarrow J(-)^J)=V_0^2\sigma_{J0J}+\tfrac{2}{3}V_1^2\sigma_{J1J}, \quad (19)$$

$$\frac{d\sigma}{d\Omega}(0\rightarrow J(-)^{J+1})=\tfrac{2}{3}V_1^2(\sigma_{J-1,1,J}+\sigma_{J+1,1,J}). \quad (20)$$

In these equations, V_0 and V_1 are to be interpreted as the pseudo-potential depths discussed in connection with Eq. (4) whenever the scattered particle is composite.

2.2. Single-Particle Transitions

Here we consider inelastic scattering from an odd nucleus with closed shells plus one valence nucleon. The valence nucleon is excited by the interaction. Let its state be described by ψ_a with radial part $u_a(r)$, where a stands for all quantum numbers n_a, l_a, j_a except the projection m_a. The form factor for a single-particle transition is analogous to the general form factor, Eq. (13)

$$F_{LSJ}{}^{ab}(r)\equiv\langle\psi_a(\mathbf{r}')\|\mathfrak{I}_{LSJ}(r,\mathbf{r}')\|\psi_b(r')\rangle =4\pi R_{ab}{}^L(r)\langle j_a\|\mathfrak{Y}_{LSJ}\|j_b\rangle. \quad (21)$$

Here $R_{ab}{}^L$ is a radial integral

$$R_{ab}{}^L(r)=\frac{1}{2L+1}\int u_a(r')v_L(r,r')u_b(r')r'^2dr' \quad (22)$$

and

$$\langle j_a\|\mathfrak{Y}_{J0J}\|j_b\rangle=(-)^{j_a+1/2}\left[\frac{\hat{\jmath}_a\hat{\jmath}_b\hat{L}}{4\pi}\right]^{1/2}\begin{pmatrix} j_a & L & j_b\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix},$$

$$\langle j_a\|\mathfrak{Y}_{LSJ}\|j_b\rangle=(-)^{l_a}\left[\frac{\hat{\jmath}_a\hat{l}_a\hat{\jmath}_b\hat{l}_b\hat{L}\hat{S}\hat{J}}{2\pi}\right]^{1/2}\begin{pmatrix} l_a & L & l_b\\ 0 & 0 & 0\end{pmatrix}\times\begin{Bmatrix} l_a & l_b & L\\ \frac{1}{2} & \frac{1}{2} & S\\ j_a & j_b & J\end{Bmatrix},$$

$$=(-)^{j_a-j_b+L+S+J}\langle j_b\|\mathfrak{Y}_{LSJ}\|j_a\rangle, \quad (23)$$

where $\hat{\jmath}\equiv 2j+1$ and the order of coupling s and l is, $\mathbf{l}+\mathbf{s}=\mathbf{j}$. There are obvious selection rules which restrict the multipolarity when the single-particle transition is known, or in the case of an even nucleus for which the rules of Eq. (14), (15) are more stringent, eliminate contributions from those elementary transitions which are not compatible with Eq. (14) and (15). They are

$$l_a+L+l_b=\text{even}, \quad |l_a-l_b|\leq L\leq l_a+l_b \quad |j_a-j_a|\leq J\leq j_a+j_b. \quad (24)$$

There are other special selection rules which can be derived from those stated and the properties of the 3-j and 9-j symbols in Eq. (23). For example, the transition involving only the recoupling of a group of equivalent particles, $(j^n)_0\rightarrow(j^n)_J$, receives no contribution from the spin-dependent part of the force (i.e., $F_{J1J}\equiv 0$).

When the potential has a Gaussian shape, and harmonic-oscillator wave functions are used for the bound states, as is usual in nuclear-structure calculations, the radial integral of Eq. (22) can be obtained as a polynomial in r^2 times an exponential factor as was shown in earlier work.[2] The result takes the form

$$R_{ab}{}^L(r)=e^{-\gamma r^2}\sum_{m=0}^{\bar{m}}G_m{}^L(a,b)\left(\frac{\beta r}{\sqrt{\nu}}\right)^{2m+L}, \quad (25)$$

where $\nu=m\omega/\hbar$ is the oscillator parameter, $\beta^{-1/2}$ is the force range of Eq. (3), and $\gamma=\nu\beta/(\nu+\beta)$. Also the range on the sum is quite small; $\bar{m}=\frac{1}{2}(N_a+N_b-L)$ where $N=2(n-1)+l$ is the oscillator quantum number. The coefficients G are rather complicated and are given in the Appendix.

The shape of a single-particle form factor is given by $R_{ab}{}^L(r)$ and does not depend on S and J as shown by Eq. (21) (although its magnitude does). Qualitatively, the shape can be surmised easily since in the limit of a zero-range force it is just proportional to the product of the radial functions of the bound particle in its initial and final state

$$R_{ab}{}^L(r)\rightarrow u_a(r)u_b(r)/4\pi. \quad (26)$$

Form factors for several single-particle transitions of multipolarity $L=2$ or rather their shape as given by Eq. (22), are shown in Fig. 1 for three force ranges, including 1.85 F which is the range used in the nuclear structure calculations. A finite-range potential without core is seen to wash out considerably the finer details.

It is perhaps worth commenting on an objection often raised in connection with the use of harmonic oscillator functions. It concerns their asymptotic behavior; they vanish more rapidly at large distance than is expected. We grant this fact, but its effect on our results is rather small, especially for collective states because their form factors are so large at their peak near the surface that it dominates the distant tail that is small in any case.

2.3. Transitions between States in Even Spherical Nuclei

Consider now transitions between any nuclear states $\alpha_1 J_1$ and $\alpha_2 J_2$ (where α denotes all quantum numbers additional to JM needed to specify the state). We want the form factor

$$\mathfrak{F}_{LSJ}{}^{\alpha_2\alpha_1}(r)=\langle\psi_{\alpha_2J_2}(\mathbf{A})\|\mathfrak{J}_{LSJ}(\mathbf{r},\mathbf{A})\|\psi_{\alpha_1J_1}(\mathbf{A})\rangle. \quad (27)$$

By virtue of the fact that the interaction between scattered particle and nucleus is a one-body operator on the nuclear coordinates, this form factor can be written as some linear combination of the elementary form factors considered in the last section. The particular linear combination depends of course on the detailed structure of the two nuclear states. Nuclei for which detailed structure calculations have been done, lie in the single-closed-shell regions. The following development is designed to make use of these wave functions. Such nuclei have been treated by the BCS theory with the addition of an interaction between quasiparticles. The wave functions have the form

$$|\alpha JM\rangle=\tfrac{1}{2}\sum_{a,b}\eta_{ab}{}^{\alpha J}A_{JM}{}^{\dagger}(ab)|0\rangle, \quad (28)$$

where the η's are configuration amplitudes, $|0\rangle$ denotes the ground state, which is here the vacuum for quasiparticles of which a pair creation operator is

$$A_{LM}{}^{\dagger}(a,b)=(-)^{j_a-j_b+J}A_{JM}{}^{\dagger}(b,a)=-[\alpha_a{}^{\dagger}\alpha_b{}^{\dagger}]_J{}^M, \quad (29)$$

with $\alpha^{\dagger}$ a quasiparticle creation operator. Define also the scattering operator

$$N_{JM}{}^{\dagger}(a,b)=(-)^{j_a-j_b+M}N_{J-M}(b,a)$$
$$=-[\alpha_a{}^{\dagger}\tilde{\alpha}_b]_J{}^M, \quad (30)$$

where

$$\tilde{\alpha}_{am_a}=(-)^{j_a-m_a}\alpha_{a-m_a}. \quad (31)$$

The transformation connecting particles $\beta^{\dagger}$ and quasiparticles, $\alpha^{\dagger}$ is

$$\beta_{jm}{}^{\dagger}=U_j\alpha_{jm}{}^{\dagger}+V_j\tilde{\alpha}_{jm}. \quad (32)$$

Here U, V, are the coefficients of the Bogoliubov-Valatin transformation found by solving the BCS equations for the nucleus in question. The Condon-Shortley phases are used with a consequence that

$$V_aU_a(-)^{l_a}\geq 0. \quad (33)$$

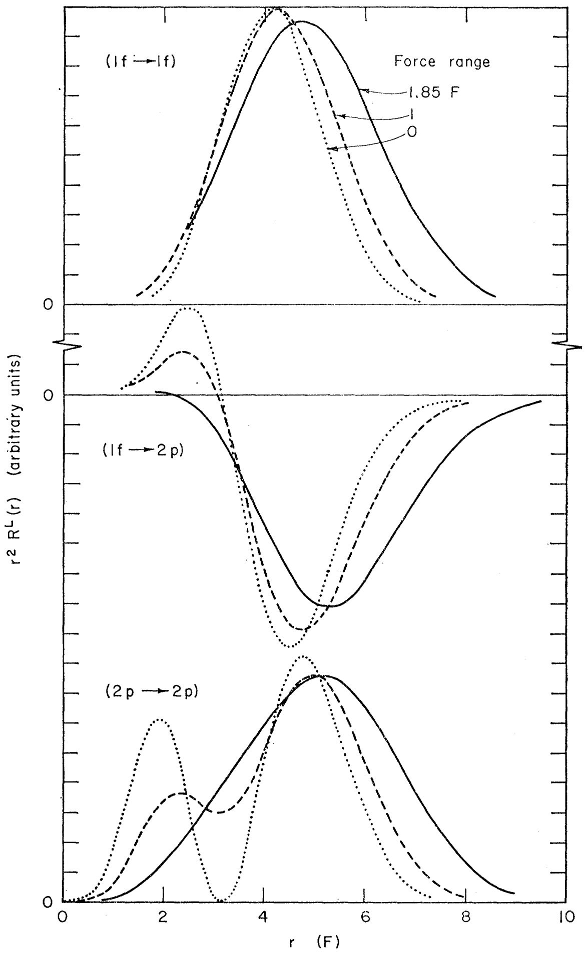

FIG. 1. Shapes of quadrupole single-particle form factors for several transition are shown for three values of the force range. The oscillator parameter $\nu=m\omega/\hbar$ has the value 0.25 F^{-2} for the nickel isotopes.

In addition they satisfy the normalization condition

$$U^2+V^2=1. \quad (34)$$

Any one-body tensor operator T_J on the nuclear particles can be written in terms of the quasiparticles as

$$T_J{}^M=\sum_{ab}\frac{1}{(2J+1)^{1/2}}\langle a\|\mathbf{T}_J\|b\rangle\{V_a{}^2(2j_a+1)^{1/2}\delta_{ab}\delta_{J0}$$
$$+\tfrac{1}{2}[U_aU_b-(-)^{J+\sigma}V_bV_a][N_{JM}{}^{\dagger}(a,b)+(-)^{M+\sigma}N_{J-M}(a,b)]$$
$$-\tfrac{1}{2}[U_aV_b+(-)^{J+\sigma}U_bV_a][A_{JM}{}^{\dagger}(a,b)+(-)^{M+\sigma}A_{J-M}(a,b)]\}, \quad (35)$$

where σ is defined by[15]

$$\langle a\|T_J\|b\rangle=(-)^{j_a-j_b+\sigma}\langle b\|T_J\|a\rangle.$$

The form factor for transitions from the ground state to an excited state are now easily obtained as

$$\mathfrak{F}_{LSJ}(r)=\langle\alpha J\|\mathfrak{J}_{LSJ}(r,\mathbf{A})\|0\rangle=-\tfrac{1}{2}\sum_{ab}\eta_{ab}{}^{\alpha J}[U_aV_b+(-)^{L+S}U_bV_a]F_{LSJ}{}^{ab}(r), \quad (36)$$

[15] See Eq. (23) for the phase in our case.

where $F_{LSJ}{}^{ab}$ is the form factor for the single-particle, transition $b \rightarrow a$ which was discussed in the preceding section. Using Eqs. (21), (25) we obtain finally

$$\mathfrak{F}_{LSJ}(r) = e^{-\gamma r^2} \sum_{m=0}^{\bar{m}} d_m \left(\frac{\beta r}{\sqrt{\nu}}\right)^{2m+J}, \tag{37}$$

where the coefficients d_m are given by

$$d_m \equiv d_m(\alpha LSJ) = \sum_{a,b} A_{ab} G_m{}^L(a;b), \tag{38}$$

$$A_{ab} \equiv A_{ab}(\alpha LSL) = -2\pi[U_a V_b + (-)^{L+S} U_b V_a] \times \eta_{ab}{}^{\alpha J} \langle j_a \| \mathfrak{Y}_{LSJ} \| j_b \rangle. \tag{39}$$

The range on m is quite small since

$$\bar{m} = \tfrac{1}{2} \max(N_a + N_b - L),$$

where N is the oscillator quantum number.

The form factors connecting two excited states also has the simple structure of Eq. (37) but with coefficients d defined in a different way. The explicit closed form for the form factors is exceedingly useful from a numerical point of view when cross sections are calculated in distorted wave approximation or in the solution of the coupled equations. It means that each form factor can be summarized by a half dozen constants or so as compared with a large table as a function of r.

The scalar form factor ($S=0$), which comes from the spin-independent part of the interaction, alone is present in the scattering of spinless projectiles. The vector form factors ($S=1$) arising from the spin-dependent part of the interaction are associated with spin flip transitions of scattered particles having spin. In the second case the UV factor of Eq. (36) is different for the scalar form factor than for the vector so that one or the other may dominate for some nuclear levels. This can tell us which part of the force played the most important role in defining the properties of the nuclear state in question. In fact the structure calculation[16,17] for the nickel isotopes indicates that the spin-independent part seems to be most important for the lowest level of each spin while the spin-dependent part often plays the most important role in one of the higher levels.

2.4. Transitions in Odd Nuclei

We considered earlier the case of a pure single-nucleon transition. This is an idealization hardly realized in nature because of the internucleon interactions. It would be best realized when the core is doubly closed. In most cases the Fermi surface is diffuse and this can have important effects on those properties deriving from the odd nucleon. Such effects can be most conveniently treated in the framework of the BCS theory and its extensions. According to this theory the odd nuclei are described by wave functions containing an odd number of quasiparticles. The lower levels could even be single quasiparticle states. For transitions between such states we easily obtain from Eq. (35)

$$\mathfrak{F}_{LSJ}{}^{a,b}(r) \equiv \langle 0 \| \alpha_a \mathfrak{F}_{LSJ} \alpha_b{}^\dagger \| 0 \rangle = [U_a U_b - (-)^{L+S} V_a V_b] F_{LSJ}{}^{a,b}(r). \tag{40}$$

That is, the form factor for a quasiparticle transition is, to within the factor shown, given by the corresponding particle form factor. The factor does not exceed unity in absolute value and so, as could be anticipated on intuitive grounds, the properties deriving from the odd nucleon tend to be suppressed by the residual interaction (exclusion principle). Note also that except in the limit in which the quasiparticle is a particle ($U^2 \equiv 1$) the multiplicative factor in Eq. (40) is larger for spin-flip transitions ($S=1$) than for ordinary transitions. [Note that our form factors are defined for unit potential depth, and that ultimately the relative strengths of V_0 and V_1 have to be considered in Eqs. (17)–(20].

3. PROTON SCATTERING ON NICKEL ISOTOPES

3.1. Structure of the Nickel Isotopes

We recall briefly the method used to calculate the nuclear wave functions[16] which for these nuclei was performed by Arvieu, Salusti, and Veneroni.[17] The doubly closed shells at 28 nucleons were regarded as inert, contributing only to the central field in which the outer neutrons move. A finite-range interaction was considered to act between the outer neutrons

$$V = -26e^{-(r/1.85)^2}[P_{\mathrm{SE}} + \tfrac{1}{2} P_{\mathrm{TO}}] \quad (\mathrm{MeV}) \tag{41}$$

(where P is a projection operator for the singlet-even or triplet-odd state). Its effects were taken into account in two steps. First the Bogoliubov-Valatin transformation was calculated to extract the pairing effects of this interaction. In the second step, the residual interaction between the resultant quasiparticles was taken into account by diagonalizing the interaction in the truncated space of two-quasiparticle configurations corresponding to the major unfilled neutron shell. In fact at the second step the more complicated equations of the random-phase approximation (RPA) were also solved, but their solutions did not differ significantly from the two-quasiparticle diagonalization. In other words the BCS vacuum is a very good representation of the ground state. However by solving the RPA, including the so-called exchange terms, one is able to isolate the spurious 0^+ state introduced by the non-conservation of particle number. This separation turns out to be crucial for the 0^+ states, since the spurious state is completely coherent for scattering.

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[16] R. Arvieu and M. Veneroni, Compt. Rend. **250**, 992 (1960); **250**, 2155 (1960); **252**, 670 (1960); R. Arvieu, Ann. Phys. (Paris) **8**, 407 (1963).

[17] R. Arvieu, E. Salusti, and M. Veneroni, Phys. Letters **8**, 334 (1964).

It is worth noting the difference between this treatment and that of several other authors including Kisslinger.[18] The latter authors use the quasiboson approximation to obtain a vibrational spectrum. The quasiboson operator corresponds to the collective 2_1 state. Application of the operator twice to the vacuum leads to the two-*phonon* triplet. These states are therefore a linear combination of 4 quasiparticle configurations. The description of the 2_1 collective state is similar in both approaches. The differences are in the other states. In the work of Arvieu *et al.*, which we use for the nickel isotopes, all excited states are combinations of two-quasiparticle configurations. Therefore our 0_2, 2_2, 4_1 states have nothing to do with two-*phonon* states, as far as their microscopic description is concerned. On the contrary the 0_2 and 4_1 are each more analogous to a one-phonon state of multipolarity equal to its spin.

The two-quasiparticle description has been quite successful as far as the tin and lead isotopes are concerned.[19] For the nickel isotopes it is not so good evidently because the nucleons within the closed shells participate to a non-negligible extent. Unlike the tin isotopes, Ni^{60} and Ni^{62} have a vibration-like spectrum which is an indication of the participation of the core particles. However, the energies of the 0_2, 2_2, and 4_1 states are approximately correct which suggests at any rate that these levels have large two-quasiparticle admixtures. If the ground state quasiparticle correlations are really small, as suggested,[17] the four-quasiparticle admixtures would not contribute in the ground-state–excited-state transition. Their presence would only suppress the cross section because of the normalization.

In spite of the possible deficiencies of the nuclear structure calculation in nickel isotopes we shall illustrate the theory of Sec. 2 by application to these isotopes because of the experimental activity in this region.

3.2. The Direct Interaction

We need to know the interaction between the scattered proton and the extra-core neutrons of the nickel isotopes. Unfortunately it is not clear what this interaction is. If one had a complete theory of the nucleus and the reaction mechanism, and if in addition one knew that the meson cloud surrounding each nucleon was not distorted by the proximity of others, then the vacuum interaction would be used (if it were known). But this is not the case in practice. Nuclear structure calculations are performed in a highly truncated pseudo-Hartree-Fock space. It is believed that many of the important correlations caused by the mutual interactions of the nucleons are nonetheless reproduced. But it is recognized that the residual interaction appropriate in such a model may be different from the vacuum force. It can in fact be more complicated than the vacuum interaction, depending for example on the local density of nucleons. Moreover, one must anticipate that the residual interaction will be different in different parts of the periodic table, just because the truncation involves different shells.

Perhaps we can guess one modification introduced by the truncation. It is known that electromagnetic transition rates of some states in the nickel isotopes are enhanced over single-particle rates. In the nuclear model calculation outlined in Sec. 3.1, only *neutrons* participate in the excitations. The core therefore does play an important role in the correlations and motions of the extra-core neutrons. This participation of the core, for electromagnetic transitions, can be accounted for approximately by endowing all nucleons with an additional effective charge. We anticipate therefore that the direct interaction in our calculations should be stronger than the vacuum interaction to simulate the participation of the core nucleons in the excitation.

As a first orientation however, we shall use a force suggested by the two-body problem as a guide in our calculation, and then see by how much it must be augmented to reproduce the experimentally observed cross sections. It is known for example that the singlet to triplet strength in the even states is about 0.6 and that the force is weak and possibly repulsive in odd states. Let us assume therefore that there is no interaction in odd states and in even states that

$$V(r)=-52e^{-(r/1.85)^2}[P_{\mathrm{TE}}+0.6P_{\mathrm{SE}}]\ (\mathrm{MeV}). \quad (42)$$

This potential approximately reproduces the low-energy neutron-proton data.[20] For the reaction calculation it is more convenient to use a different parametrization, namely

$$V(r)=\{V_{00}+V_{01}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+(V_{10}+V_{11}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\}g(r). \quad (43)$$

For the present case of protons scattered from bound neutrons, the two constants V_0 and V_1 of Eq. (1) are given by

$$\begin{aligned} V_0&=V_{00}-V_{01}\equiv\tfrac{1}{8}(3TE+3TO+SE+SO),\\ V_1&=V_{10}-V_{11}\equiv\tfrac{1}{8}(TE+TO-SE-SO), \end{aligned} \quad (44)$$

(where TE stands for the triplet-even strength, etc.). Corresponding, therefore, to the potential, Eq. (42), we have

$$V_0=-23\ \mathrm{MeV},\quad V_1=-2.6\ \mathrm{MeV}\quad (\text{for } p\text{-}n). \quad (45)$$

For completeness we add that for nucleon scattering from like nucleons, the constants are

$$\begin{aligned} V_0&=V_{00}+V_{01}\equiv\tfrac{1}{4}(SE+3TO),\\ V_1&=V_{10}+V_{11}\equiv\tfrac{1}{4}(TO-SE), \end{aligned} \quad (46)$$

[18] See Ref. 9. Also M. Baranger, Phys. Rev. **120**, 957 (1960); S. Yoshida, Nucl. Phys. **38**, 380 (1962).

[19] R. Arvieu, E. Baranger, M. Baranger, V. Gillet and M. Veneroni, Phys. Letters **4**, 119 (1963); R. Arvieu and M. Veneroni, Phys. Letters **8**, 407 (1963).

[20] W. W. True and K. W. Ford [Phys. Rev. **109**, 1675 (1958)] used the singlet part.

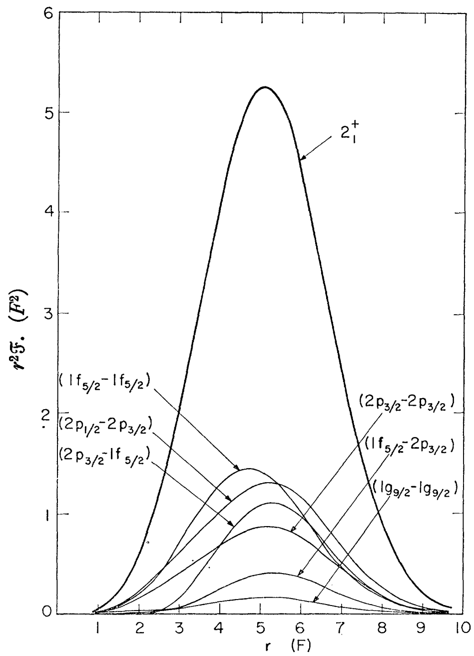

FIG. 2. The quadrupole single-particle form factors that contribute to the 2^+ states of Ni^{60} are shown with the magnitudes and signs appropriate to the collective state showing how they all contribute constructively to the collective form factor shown by the heavy line labeled 2_1^+.

which, corresponding to Eq. (42), have the values

$$V_0=-7.8 \text{ MeV}, \quad V_1=7.8 \text{ MeV} \quad (\text{for } n\text{-}n \text{ or } p\text{-}p). \qquad (47)$$

We remark parenthetically that if the potential, Eq. (42), can indeed be used as a guide, then proton and neutron scattering will often excite the same level quite differently. For example, to the extent that the neutrons are responsible for the correlations present in the low states of the nickel isotopes proton scattering will hardly involve spin-flip transitions, while neutron scattering will [cf. Eqs. (45) and (47)].

3.3. Concerning the Approximations Used in the Calculations of Cross Sections

We have made two principal approximations that deserve comment. We neglect possible exchange scattering. (This is done implicitly in the macroscopic treatment.) The usual justification advanced for this involves an overlap argument which indicates that the exchange integral should be smaller than the direct when bound and scattering functions are involved together.[2] Presumably this approximation becomes better at energies sufficiently high that the wavelength of the scattered particle, in the region of overlap with the nucleus, is small compared to the nuclear radius.

Quite independent of the overlap argument, the exchange contribution to excitation of *collective* states must be small. The reason is that the direct integrals all interfere constructively [in Eq. (36)] for such states. The corresponding exchange integrals do not necessarily carry the same sign as the direct, and moreover the sign is a function of bombarding energy. So we are guaranteed by the constructive interference of the direct part that the exchange part will not be constructive.

The second approximation concerns the use of the distorted-wave method. We have no *a priori* way of expecting this approximation to be valid except for the collective states. The weaker states quite possibly can be fed by double excitation through a collective state in competition with their direct excitation from the ground. The only reliable way of handling such a situation involves solving the coupled equations and a program for this is in preparation. In view of this the cross sections reported here for the noncollective levels are not quantitatively reliable. However, we believe we can draw valid qualitative conclusions which are discussed in the next section.

3.4. Form Factors and Cross Sections

We have computed the form factors and cross sections for many levels of all the stable even nickel isotopes. The results for Ni^{60} and Ni^{62} are reported here as being typical of what was encountered. As previously noted, two form factors in general are needed to describe the scattering of nucleons from natural parity states of an even nucleus. These are the scalar and vector form

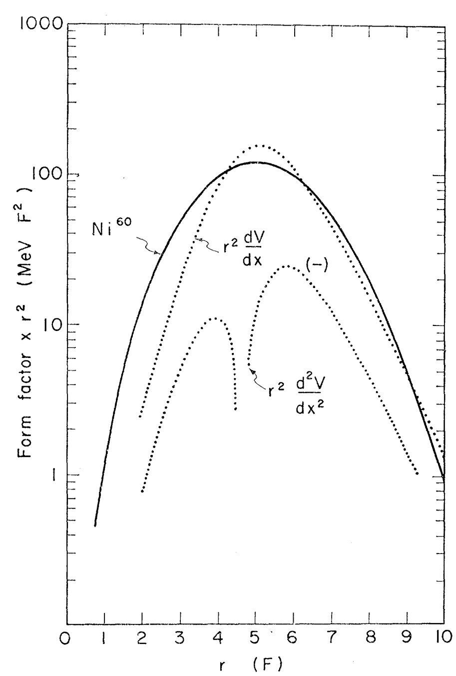

FIG. 3. The form factor of the collective 2^+ state of Ni^{60} computed from the microscopic model is compared with the derivative form factor of the macroscopic model. They are normalized to give the same integrated cross section. The second derivative form factor of the latter model is also shown. Form factors are here plotted on a logarithmic scale and changes in sign are shown by (−). We include the factor r^2 (from the volume element) in our figures as should be done to show the weighting given the distorted waves.

factors $\mathfrak{F}_{J0J}$ and $\mathfrak{F}_{J1J}$ [cf. Eq. (17)]. The vector form factor gives rise to spin-flip transitions. Although both have the same shape for a single-particle transition this is not true for a configuration mixed state. Consequently for the states of Ni we show both. We remark in passing that there is no precise counterpart of the vector form factor in the macroscopic model although if the spin-orbit term of the optical potential were assumed also to be deformed, a spin-flip mechanism would thus be introduced.

We have already emphasized that the form factor of any state, however complicated, must be a superposition of elementary form factors. Those that contribute to the first 2^+ state of Ni^{60} are shown in Fig. 2 multiplied by the phases and magnitudes dictated by Eq. (36) for the *scalar* form factor. Each contributes constructively to the nuclear form factor yielding the large single-peaked function shown. This is roughly analogous to, but broader than, the form factor of the macroscopic model which is compared in Fig. 3. (In the latter case, it is proportional to the first derivative of the optical potential). On the other hand the *vector* form factor of this collective state is smaller by a factor of about 5. These observations correspond to the fact that for the low-lying collective states of a nucleus the spin-independent part of the force is most important in building up the correlations. The vertex in the diagram corresponding to scattering of nuclear particles by a free particle may be different in details but is qualitatively similar to that entering the structure calculation.

For the higher lying states the vector form factor becomes relatively more important (cf. Fig. 5) and in some cases (not shown) significantly larger than the scalar part. This suggests the interesting speculation whether there might exist at higher excitation a new type of collective state whose correlations are built up by the spin-dependent part of the residual interaction. The importance of the vector part in scattering is in our example very minor because the spin part of the *direct* interaction is weak for protons scattered on neutrons [cf. Eq. (45) and last paragraph of that section]. If there were such a state, it seems that neutron scattering with measurement of the polarization or subsequent γ radiation with suitable geometry would be markedly different from the low-lying collective state. Similar states, if they exist, in nuclei whose excited state correlations depend considerably on the protons could be detected in the same fashion by proton scattering.

It is characteristic that the form factor of the collective 2^+ state in all the nickel (and tin) isotopes possesses one broad maximum near the nuclear surface as above. This is in contrast with the form factors of the higher 2^+ states which exhibit great variety as shown in Figs. 5 and 6 (where we have plotted the logarithm because of the wide range of magnitudes). The other 2^+ form factors are of course made up from the same elementary excitations as the collective state discussed above, but with different proportions and phases. It is to be expected therefore that the noncollective form factors will be characterized by variety rather than uniformity, in contrast with the predictions of the macroscopic model for the two phonon states. Probably this accounts for the arbitrary juggling of coupling constants typically required to obtain agreement with experiment when higher excited states are analyzed in terms of the macroscopic model.

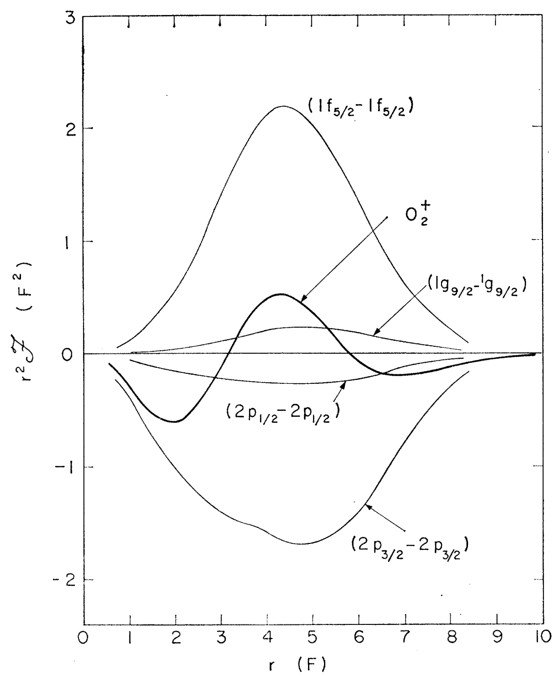

FIG. 4. The monopole single-particle form factors that contribute to the 0+ states of Ni^{60} are shown with magnitudes and signs appropriate to the first excited 0^+ state labeled 0_2. Here they interfere destructively to give the small form factor for the nuclear state shown by the heavy line labeled 0_2^+.

As an example of a noncollective transition we show in Fig. 4 the contributing elementary form factors for the 0_2^+ state of Ni^{60} (first excited 0^+). In this case the interference is destructive yielding the small form factor shown. For such an incoherent transition, the detailed shape of the form factor is of course very sensitive to the configuration mixing amplitudes. For this reason it is less certain than in the case of coherent transitions. The vector form factor vanishes identically for 0^+ states in an even nucleus.

The form factors for several 0^+, 2^+, and 4^+ states of Ni^{60} and Ni^{62} are shown in Figs. 5–10 together with the corresponding proton cross-sections at 11 and 40 MeV. The cross sections were computed in the distorted-wave approximation[21] and the optical model parame-

[21] For a recent review of the distorted wave method see W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, London, 1961), G. R. Satchler, Nucl. Phys. **55**, 1 (1964). The method was first derived as a first-order solution to the coupled equations for scattering by N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions*, 1st ed. (Clarendon Press, Oxford, 1933); in 2nd ed., see p. 144.

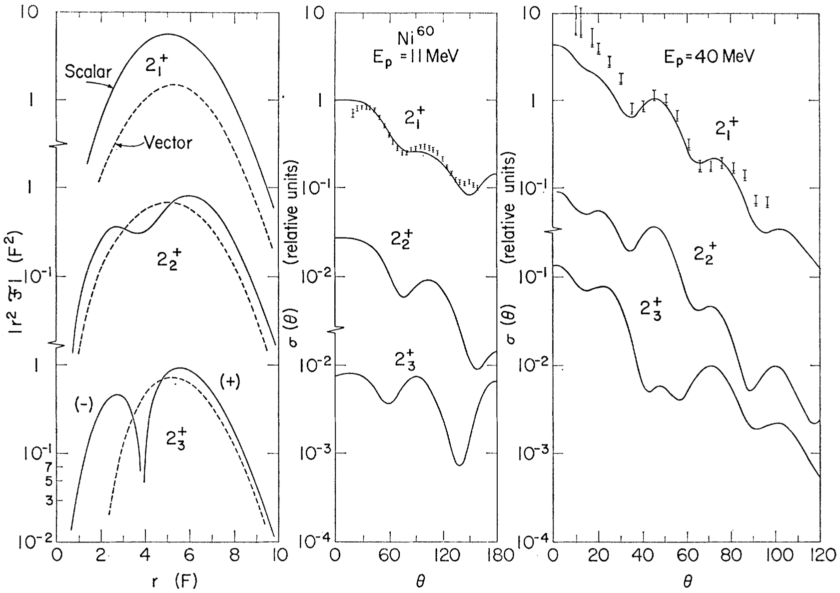

FIG. 5. The form factors for three 2^+ states in Ni^{60} together with the corresponding cross sections for 11 and 40-MeV protons. Scalar form factors are shown by solid lines and vector form factors by dashed lines. Changes in sign are indicated by (−) since the absolute values of form factors are plotted.

ters, taken from the literature,[7,22] are shown in Table I.

It is interesting to note that the characters of the two higher noncollective 2^+ states are interchanged in these two nuclei, according to the structure calculation, as revealed through their form factors. It is important to notice that these differences are indeed reflected in the calculated proton cross-sections, especially at the higher energy. This is of course in contrast with alpha cross-sections which are insensitive to details inside the nucleus as our earlier investigation indicated.[10]

TABLE I. Optical-model parameters for protons+nickel used in the cross-section calculation. The parametrization is detailed in Ref. 22.

E_p (MeV)	V (MeV)	W (MeV)	W_D (MeV)	r_0 (F)	r_0' (F)	a (F)	a' (F)	r_c (F)
11	50.84	0	10.21	1.25	1.25	0.65	0.47	1.25
40	44.7	0	9.9	1.184	1.056	0.707	0.653	1.2

We have acknowledged earlier that the distorted wave approximation may not be valid for all of the weakly excited states. However, the differences in nuclear structure which lead to the different form factors connecting the excited states to the ground state will also lead to different couplings to intermediate states,

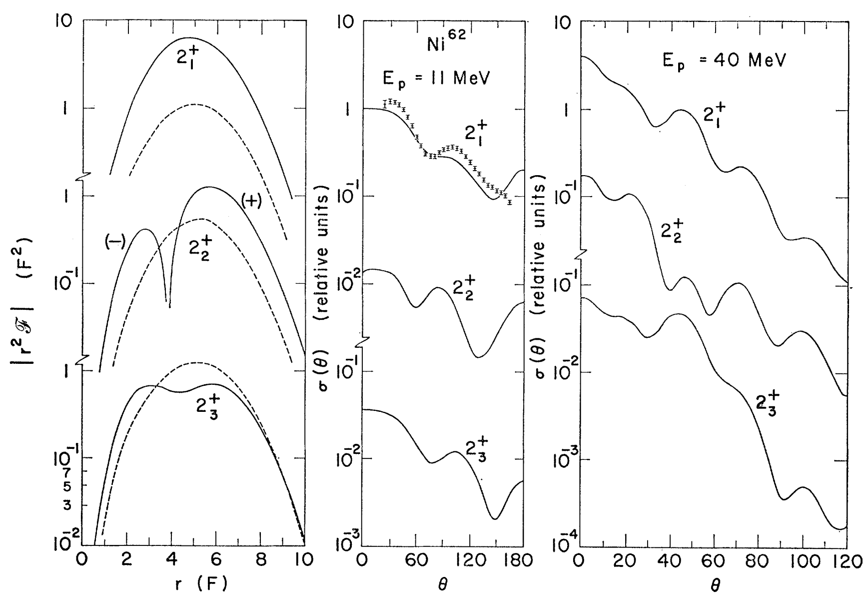

FIG. 6. The form factors for three 2^+ states in Ni^{62} together with the corresponding cross sections for 11 and 40 MeV protons. Scalar form factors are shown by solid lines and vector form factors by dashed lines. Changes in sign are indicated by (−) since the absolute values of form factors are plotted. Compare the 2_2 and 2_3 form factors with those of Ni^{60}, Fig. 5.

[22] M. P. Fricke and G. R. Satchler, Phys. Rev. **139**, B567 (1965).

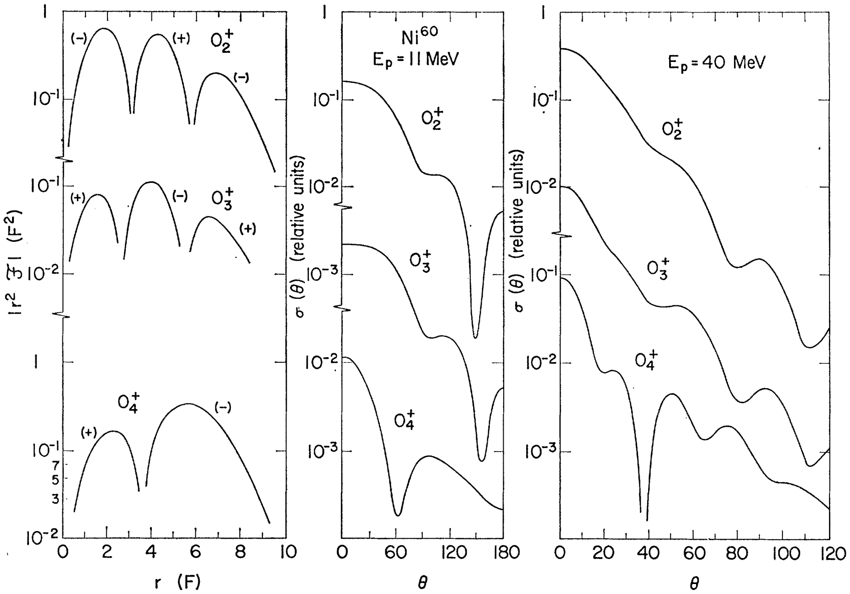

FIG. 7. Form factors for three 0^+ states in Ni^{60} together with corresponding cross-sections for 11 and 40 MeV protons. The form factors oscillate and changes in sign are shown by (−).

so that differences are likely to be further emphasized, not diminished by inclusion of higher order effects.

We turn now to the interesting question of the magnitude of the cross-sections. To achieve the agreement with the 40-MeV data[23] shown in Fig. 5 we had to use a value for V_0 of 41 MeV compared to the value of 23 MeV corresponding to a simple force which fits the low-energy *n-p* data. In view of our earlier discussion it is not surprising that we have had to use a more attractive potential. This has also been the case in earlier works.[1,2,24] With regard to the work of Funsten *et al.*, we remark that it seems not to have been fully appreciated that they have used a potential that is about three or four times stronger than the "vacuum" force. The enhancement of 1.8 that we have had to use seems at least plausible in view of our earlier discussion on this point. It will be interesting, when structure calculations that take into account the core as well as the cloud nucleons have been performed, to see by how much this factor is reduced.

With respect to the 11-MeV data,[7] we require a potential $V_0=72$ MeV or three times stronger than our assumed "vacuum" force in contrast to the situation above. Quite likely the residual interaction should

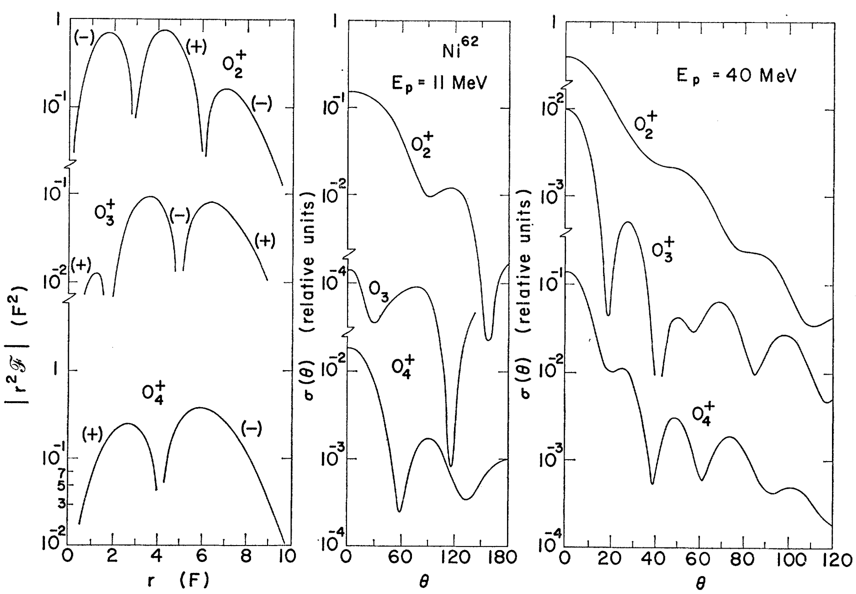

FIG. 8. Form factors for three 0^+ states in Ni^{62} together with corresponding cross sections for 11 and 40 MeV protons. The form factors oscillate and changes in sign are shown by (−).

[23] T. Stovall and N. M. Hintz, Phys. Rev. **135**, B330 (1964). We are indebted to Professor Hintz for providing tables of the data.
[24] H. O. Funsten, N. R. Robertson, and E. Rost, Phys. Rev. **134**, B117 (1964).

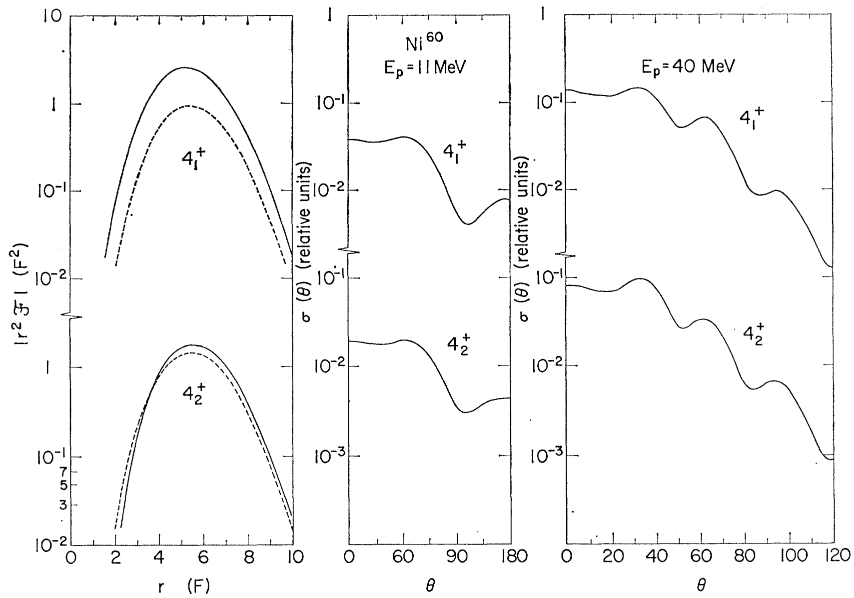

FIG. 9. Form factors for two 4^+ states of Ni^{60} are shown together with corresponding cross sections for protons. Scalar form factors are shown by solid line and vector by dashed.

be momentum as well as density-dependent. The second dependence would act in such a way as to reduce this discrepancy but we do not know about the first. However, part of the discrepancy may be due to the optical model parameters. Their differences (especially the geometry, see Table I) suggest that they do not evolve one into the other as a function of energy and if this so, a spurious energy dependence of the inelastic cross-section would be introduced.

The data for the collective 2^+ state have also been analyzed using the macroscopic model.[7,22] The agreement as far as the angular distributions are concerned are of the same quality as shown by the microscopic description in Fig. 5 and 6, which reflects the similarity of the form factors. The greatest difference between the two descriptions of the nucleus are expected in the higher excited states. These, we emphasize, are characterized by variety of coupling form factors in the microscopic description, as contrasted with the macroscopic model where the couplings are all interrelated and the same for one nucleus as for another, aside from their over-all strength. That the variety exists in nature is attested by the fact that the existant analyses on the basis of the macroscopic model require that almost every level have a different quadrupole parameter β.

Although the microscopic description of the collective 2_1^+ states seems to be satisfactory, we cannot make any such statement, at the moment, concerning the

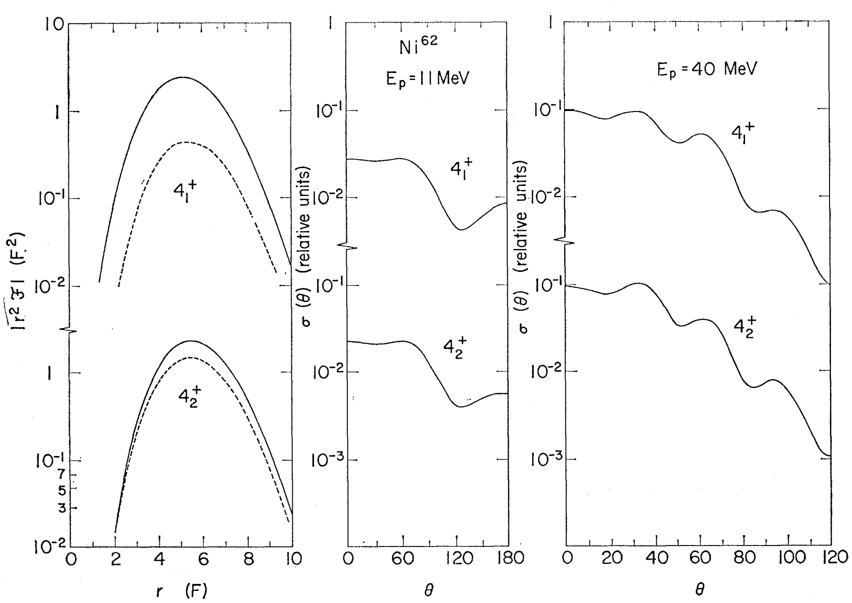

FIG. 10. Form factors for two 4^+ states of Ni^{62} and corresponding cross-sections for 11 and 40 MeV protons. Scalar and vector form factors are shown by solid and dashed lines, respectively.

description of the weaker states, since their proper analysis requires in general the solution of the coupled equations for scattering. These have not yet been solved with the microscopic form factors. Moreover, there are very few data available on nucleon scattering from the higher lying states of single-closed-shell nuclei.

4. SUMMARY

The scattering of light nuclides from nuclei has been formulated in terms of the two-nucleon interaction and the detailed shell-model wave functions of the nucleus. The theory was applied in an earlier work to alpha particle scattering[10] from the nickel isotopes, some of whose levels are collective. In the present work we have applied it to proton scattering. We have used the nuclear structure calculations of Arvieu, Salusti and Veneroni for these isotopes.[17] In their work, all levels, including the collective ones, are treated in terms of their underlying nucleon structure. The constructive coherence in the transition to the 2_1^+ state of all the nickel (and tin) isotopes leads to a *scalar* form factor having the general shape used in the macroscopic description of collective motion. The *vector* form factor is much smaller, reflecting the dominant role of the spin-independent part of the nuclear force in inducing the collective motion in the lowest collective states. We have speculated on the existence of states at higher energy for which enhanced transitions proceed through correlations induced by the spin-dependent part.

In contrast to the qualitative agreement between the form factors predicted by the microscopic and macroscopic descriptions of the collective motion in the 2_1^+ state, the microscopic model predicts a much greater variety in shape and magnitude of the form factors for the unenhanced transitions. The details constituting this variety are reflected in the calculated cross-sections for nucleon scattering but not for composite particle scattering.

Concerning comparisons with experiment, we find that the microscopic description given by Arvieu *et al.*[17] for the collective 2_1^+ states yields good agreement with the differential cross sections for *proton* scattering. As has been observed by us, and by Madson and Tobocman,[10] the calculated differential cross section for *alpha* scattering is shifted by several degrees to smaller angles compared to experiment. This suggests that the slope, or the position of the form factor outside the nucleus is somewhat in error. Presumably, a small error here would not effect the *proton* scattering so much since the interior contributes an important fraction of the cross section. In any case there do not seem to be any fundamental difficulties either with the description of the nucleus or the scattering process. It would however be interesting to have structure calculations in which the inner nucleons play a part in the correlations so that one aspect of the ambiguity connected with the effective direct interaction could be removed.

Now we summarize our impression of the uses of the several types of projectiles employed in inelastic-scattering experiments. The discussion of course divides into two parts dealing with strongly and moderately absorbed particles.

Composite particles like deuterons and alphas are strongly absorbed. Therefore direct reactions involving them take place predominantly in the nuclear surface. For an *ideal* surface reaction involving only one L transfer it can be proven[2] that the angular distribution corresponding to the direct excitation from the ground state does not depend upon the mechanism by which the transfer is effected. It is independent of the nuclear structure or the nature of the direct interaction. In practice the transfer may take place throughout the *surface* region, but even then the angular distribution is largely insensitive to the details mentioned. The theorem does not apply to levels fed principally through some other excited state (double excitation). We therefore divide levels into two types, those whose direct coupling to the ground state dominates over the coupling through an intermediate level and those for which the two couplings are competitive or for which the indirect route dominates.

Levels of type I are the enhanced collective states like the 2_1^+ and 3_1^- as well possibly as some weak levels. Levels of type II include such higher excited 2^+ states for which the direct E_2 transition to ground is weak compared to the stopover transitions to the 2_1^+.

The consequences of these statements are the following. For a level of type I, the angular distribution is a simple meter of its spin and it can be deduced by applications of any convenient means of calculating a surface transfer of angular momentum. The Blair-Drozdov model and its more sophisticated variants would be suitable for this purpose, or any distorted wave calculation employing a surface-peaked form factor. We re-emphasize that success in making spin assignments for this type of level does not reflect on the merits of the nuclear model.

For levels of type II which are fed through an intermediate level as well as, or instead of, the direct transition, no general statement about the phase of the angular distribution can be made. The relative importance of the two couplings is a nuclear property which may change from level to level and from nucleus to nucleus. The details of these couplings are of course interesting nuclear properties.

Since one does not know in advance to which category a level belongs (except for the strongly enhanced levels) it will be necessary to determine this *before* reliable spin and parity assignments can be made. Since the direct and indirect routes to an excited state very likely have probabilities that vary with energy in different manners, a study of the phase of the angular distribution as a function of energy as compared with a known direct excitation such as the 2_1^+, may reveal to which category it belongs.[25]

Turning now to the scattering of nucleons, the situation is quite different from that described above. Because they are not so strongly absorbed, the differential cross section reflects details of the nuclear structure well within the nuclear radius. Compare for example the form factors and their corresponding differential cross-sections of the 0^+ states shown in Fig. 8. Just because of this sensitivity they are not as useful in determining spins and parities as alpha particles. But they do afford quite a good glimpse of the interior, which the alphas do not. For this reason nucleon scattering should provide a valuable means of putting microscopic descriptions of nuclear structure to very detailed tests.

APPENDIX A. CONVENTIONS

A shell-model calculation involves a choice of phases and conventions which are not standard and must be specified if the wave functions are subsequently used to calculate other properties. For the calculations reported here the following conventions hold: (1) Condon-Shortley phases for spherical harmonics. (2) Order of spin-orbit coupling is $\mathbf{l}+\mathbf{s}=\mathbf{j}$. (3) Radial functions have positive slope at origin. (4) Conventions above and the way in which the quasiparticle and particles are connected, Eq. (32), imply that the lowest energy solution of the BCS method is such that $UV(-)^l$ is positive.

APPENDIX B. EXPLICIT EXPRESSION OF FORM FACTOR

The integral defining the form factors, Eq. (22), can be evaluated as a closed expansion,[2] having the structure shown in Eq. (25), when the potential shape is Gaussian and harmonic oscillator functions are used for the radial functions of the bound single-particle state. We give here the relevant formulas which are convenient for computer calculation or by hand. The harmonic oscillator functions are

$$u_{nl}(r)=\mathfrak{N}_{n}\nu^{3/4}(\nu^{1/2}r)^{l}e^{-1/2(\nu r^2)}F(1-n\,|\,l+\tfrac{3}{2}\,|\,\nu r^2)\,, \quad \text{(B1)}$$

$$\mathfrak{N}_{nl}=\left\{\frac{2\Gamma(n+l+\frac{1}{2})}{\Gamma(n)}\right\}^{1/2}\frac{1}{\Gamma(l+\frac{3}{2})}\,,\quad n=1,2,\cdots. \quad \text{(B2)}$$

The function F above is the confluent hypergeometric function[26] and $\Gamma(z+1)=z\Gamma(z)$, $\Gamma(\frac{1}{2})=\pi^{1/2}$. The product of two such radial functions appears in the integral and we write it

$$u_{nl}u_{n'l'}=\mathfrak{N}_{nl}\mathfrak{N}_{n'l'}\nu^{3/2}(\nu^{\frac{1}{2}}r)^{l+l'}e^{-\nu r^2}\sum_{m=1}^{n+n'-1}\alpha_m(\nu r^2)^{m-1}\,, \quad \text{(B3)}$$

where

$$\alpha_m\equiv\alpha_m(nl;n'l')=(-)^{m-1}\sum_{k=0}^{m-1}\frac{\binom{n+l-\frac{1}{2}}{n-k-1}}{\binom{n+l-\frac{1}{2}}{n-1}}\frac{\binom{n'+l'-\frac{1}{2}}{n'+k-m}}{\binom{n'+l'-\frac{1}{2}}{n'-1}}\times\frac{1}{k!(m-k-1)!}\,. \quad \text{(B4)}$$

Then the coefficients G in Eq. (25) are

$$G_m{}^L(a,b)=(\tfrac{1}{4}\sqrt{\pi})\mathfrak{N}_a\mathfrak{N}_b[\nu/(\beta+\nu)]^{L+3/2}\times\sum_{k=\max(g,m)}^{\bar{m}}\alpha_{k-g+1}(a,b)B(k-m,k,L)\left(\frac{\beta+\nu}{\nu}\right)^{k-m}\quad\text{for}\quad m\leq\bar{m} \quad \text{(B5)}$$

and are zero for $m>\bar{m}$ where

$$2\bar{m}=N_a+N_b-L\quad\text{and}\quad 2g=l_a+l_b-L\,. \quad \text{(B6)}$$

As mentioned earlier in connection with Eq. (25) N is the oscillator quantum number.

Finally, the coefficients B above are given by

$$B(0,s,L)=[\nu/(\beta+\nu)]^{2s}\,, \quad \text{(B7)}$$

$$B(p,s,L)=\frac{(2L+3)!}{(L+1)!}[\nu/(\beta+\nu)]^{2s}\times\sum_{m=0}^{p}(1-\delta_{m0}\delta_{ps})\binom{s}{p-m}\binom{s+m-p}{m}\frac{(s-1)!}{(s+m-p-1)!}\times\begin{cases}\dfrac{1}{2^{2m}}\dfrac{(L-m+1)!}{(2L-2m+3)!}\,, & m<L+2\\[2ex] \dfrac{1}{2^{2L+3}}\,, & m=L+2\\[2ex] \dfrac{(-)^{m-L}}{2^{2(m-1)}}\dfrac{(2m-2L-5)!}{(m-L-3)!}\,, & m>L+2\,.\end{cases} \quad \text{(B8)}$$

APPENDIX C. EFFECTIVE POTENTIAL FOR COMPOSITE SCATTERED PARTICLES

Our results are derived in terms of a direct interaction between the nucleons of the nucleus and the center of mass of the scattered particle. The form of the interaction is given by Eq. (4) which implies that for scattered nucleons, the expectation value with respect to i spin of the full interaction, Eq. (43), has been taken. We want now to relate the parameter of Eq. (4) to those of the nucleon-nucleon potential of Eq. (43), when the scattered particle is a light nuclide. For this purpose we use a very simple wave function for the

[25] D. J. Horen, J. R. Meriwether, B. G. Harvey, A. Bussiere de Nercy, and J. Mahoney, Nucl. Phys. 72, 97 (1965).

[26] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 552.

nuclide, with radial part

$$u = \mathfrak{N}\exp(-\eta^2 \sum r_{ij}^2), \tag{C1}$$

where the sum is over the relative distances between nucleons in the nuclide. This will at least give us a rough guide in selecting the parameters of Eq. (4).

(a) Deuteron

The space part according to Eq. (C1) is

$$u_{10}(2\eta^2 r^2) = (2\eta^2/\pi)^{3/4} e^{-\eta^2 r^2} \tag{C2}$$

and the spin and isospin parts are, respectively, triplet and singlet. We want the expectation value, with respect to the isospin part and the internal coordinate of the deuteron, of the interaction between the deuteron and some other nucleon whose coordinates are $\mathbf{r}_i$, $\boldsymbol{\sigma}_i \cdots$. This expectation value is our pseudopotential which is a function of spins and the distance between the center of mass of the deuteron R, and the nucleon. Denote this distance vector by $\boldsymbol{\xi} = \mathbf{R} - \mathbf{r}_i$. Then we have

$$V'(\xi, \mathbf{S}\cdot\boldsymbol{\sigma}_i) = (V_{00} + V_{10}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_i) I_+(\xi) + (V_{00} + V_{10}\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_i) I_-(\xi), \tag{C3}$$

where

$$I_\pm(\xi) = \int u_{10}^2 e^{-\beta|\xi \pm r/2|^2} d\mathbf{r}. \tag{C4}$$

The integrals are equal to each other and can be evaluated by use of

$$\int_0^\infty d\mathbf{r} \exp(\mathbf{k}\cdot\mathbf{r} - \alpha r^2) = \left(\frac{\pi}{\alpha}\right)^{3/2} e^{k^2/4\alpha}. \tag{C5}$$

We find in this way that the parameters of Eq. (4) are related to those of the nucleon-nucleon potential Eq. (43) by

$$V_0' = 2x^{3/2} V_{00},\ V_1' = 2x^{3/2} V_{10},\ \beta' = x\beta,\quad \mathbf{S} = (\boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p)/2, \tag{C6}$$

where

$$x = \frac{8\eta^2}{8\eta^2 + \beta}. \tag{C7}$$

(b) Alpha

A convenient coordinate system consists of the centers of mass of the alpha and of the two neutrons and two protons, which we denote by $\mathbf{R}$, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, and the distance between the last two, $\boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$ (the Jacobian of the transformation from the nucleon coordinates to these is unity). The function Eq. (C1) in terms of these is

$$u = u_{10}(4\eta^2\rho_1^2) u_{10}(4\eta^2\rho_2^2) u_{10}(8\eta^2\rho^2) \tag{C8}$$

again the integrals involved in evaluating the pseudopotential are of the form Eq. (C5). We find

$$V'(\xi) = 4x^{3/2} V_{00} e^{-\beta'\xi^2},\quad \beta' = x\beta,\quad x = 32\eta^2/(32\eta^2 + 3\beta). \tag{C9}$$

In terms of the assumed vacuum interaction, Eq. (42) this potential for nucleon-alpha interaction is

$$V' = 34 e^{-(r/2.26)^2} \quad (\text{MeV},) \tag{C10}$$

where for the alpha size parameter we used $\eta = 0.233$, consistent with electron scattering. That is, the alpha nucleon interaction has a range considerably larger than the nucleon-nucleon interaction, but a well depth of only about twice the V_{00} part of the latter interaction.