Proton Polarization and the Spin-Orbit Potential*

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A treatment of proton elastic scattering, valid to first order in the strength of the spin-orbit optical-model potential, shows that this quantity merely serves as a scaling factor for the polarization of the outgoing protons. The angular dependence of the polarization is then determined by the form factor of this potential together with the parameters of the central potential. At small scattering angles, however, only the spin-orbit form factor is important, aside from the well-known Coulomb interaction. Calculations of the proton polarization at various energies from 10 to 20 MeV support these conclusions and indicate that for medium-weight nuclei, the radius parameter of the spin-orbit potential $r_{so} \approx 1.1$ F instead of 1.3 F, the corresponding value used for the central potential.

I. INTRODUCTION

THE shape of the spin-orbit part of the optical-model potential has recieved little attention. Usually, the Thomas form is taken with geometrical parameters appropriate to the central potential and the strength is the only adjustable parameter. The extent and accuracy of published polarization measurements does not merit further parametrization, but clearly, with the development of polarized ion sources, more experimental data can be anticipated and closer examination will be required of the representation of this interaction.

The moderate success[1,2] that has so far been met in fitting polarization data with only one free parameter is largely due to the strong dependence of the polarization on the central terms of the optical-model potential. Thus, with a detailed fit to the differential cross section, which is almost entirely governed by the central part, it is found that one is guaranteed a reproduction of the gross features of the polarization. To this must be attributed the remarkable fits to the polarization obtained by Hufner and de-Shalit[3] who put forward a diffraction-model treatment in an attempt to avoid the parametrization associated with the optical model. However, the diffraction model gives very poor fits to the cross section, and to obtain their fits to the polarization the authors are obliged to incorporate the experimental data for the differential cross section.

A more logical procedure would be to learn about the central interaction by careful studies of the differential cross section, using some initial spin-orbit interaction and then to hope to learn more about the spin-orbit terms by polarization measurements.

The dominance of the central terms of the optical-model potential suggests the possibility of giving a first-order treatment to the spin-orbit part of the interaction. Such a procedure was indeed used by Rodberg,[4] who was led to a remarkably simple expression relating the polarization to the logarithmic derivative with respect to angle of the differential cross section. This result was, however, obtained only after the introduction of another assumption which is difficult to justify.

We intend in this paper to reapply the first-order treatment, but to avoid the additional assumption used by Rodberg. Putting the spin-orbit potential U_{so} in the form $V_{so}\boldsymbol{\sigma}\cdot\mathbf{l}h(r)$, one finds, within this approximation, that the angular dependence of the polarization is independent of V_{so}—the latter merely plays the role of a scaling factor. Information concerning the form factor $h(r)$ is to be obtained by a detailed study of the angular dependence of the polarization. We shall show that the forward-angle region is particularly favorable for future studies since there the information provided concerning $h(r)$ is to a good extent independent of a detailed knowledge of the central interaction, apart from the well-understood Coulomb amplitude.

Section 2 contains an outline of the perturbation approach and this is followed in the next section by a numerical justification of the validity of this approach. Detailed examination is then given to the polarization of 9.4-MeV protons, elastically scattered by medium-weight nuclei.

II. THEORY

We represent the solution to the scattering problem by eigenfunctions, $\psi^{\pm}(\mathbf{k},\mathbf{r})$, where the superscripts refer

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[2] G. W. Greenlees, U. Haznedaroglu, A. B. Robbins, P. M. Rolph, and J. Rosenblatt, Nucl. Phys. 49, 496 (1963).

[3] J. Hufner and A. de-Shalit, Phys. Letters 15, 52 (1965).

[4] L. S. Rodberg, Nucl. Phys. 15, 72 (1960).

to the usual asymptotic boundary condtions set on these functions. In addition to this, we introduce auxiliary eigenfunctions $\chi^{\pm}(\mathbf{k},\mathbf{r})$ representing the solution if U_{so} is excluded. An exact solution for the transition amplitude $T(\theta)$ is then given by[5]

$$T(\theta)=\frac{-m}{2\pi\hbar^2}[\langle\chi_f^{(-)}|U_c(r)|\phi_i\rangle + V_{so}\langle\chi_f^{(-)}|h(r)\boldsymbol{\sigma}\cdot\mathbf{l}|\psi_i^{(+)}\rangle], \quad (1)$$

where the suffixes i and f designate the entrance and exit channel, respectively, ϕ_i is the free plane-wave eigenfunction, $U_c(r)$ is a combination of the Coulomb and central optical-model interactions, and m is the reduced nucleon mass. The first-order treatment, which consists of ignoring quantities depending on V_{so}^2 or higher powers, is then applied to the second term in (1) by replacing $\psi_i^{(+)}$ by $\chi_i^{(+)}$.

Choosing a z axis parallel to the outgoing direction, represented by $\mathbf{k}_f$, and a y axis normal to the scattering plane, i.e., along the unit vector $\mathbf{n}=\mathbf{k}_i\times\mathbf{k}_f/|\mathbf{k}_i\times\mathbf{k}_f|$, parity conservation shows that only the y component of $\boldsymbol{\sigma}$ enters into (1). The eigenfunctions are expanded into partial waves:

$$\chi_i^{(+)}=4\pi\sum_{l\lambda} f_l(k,r)i^l Y_{l\lambda}^*(\hat{k}_i)Y_{l\lambda}(\hat{r}),$$
$$\chi_f^{(-)*}=4\pi\sum_{l'\lambda'} f_{l'}(k,r)i^{-l'}Y_{l'\lambda'}(\hat{k}_f)Y_{l'\lambda'}^*(\hat{r}), \quad (2)$$

where $f_l(k,r)$ is the radial solution for the central (Coulomb and nuclear) interaction and $\hat{k}$ is a unit vector along $\mathbf{k}$. The transition amplitude takes on the form

$$T(\theta)=A(\theta)+i\sigma_y B(\theta), \quad (3)$$

where $A(\theta)$ is derived from the first term in (1), and

$$B(\theta)=\frac{4\pi m}{\hbar^2}V_{so}\sum_{ll'\lambda\lambda'} i^{l-l'}Y_{l\lambda}^*(\hat{k}_i)Y_{l'\lambda'}(\hat{k}_f)\int r^2dr\, f_l f_{l'}h(r) \times\int d\Omega\, Y_{l'\lambda'}^*(\hat{r})[(l_x+il_y)-(l_x-il_y)]Y_{l\lambda}(\hat{r}). \quad (4)$$

But

$$\int d\Omega\, Y_{l'\lambda'}^*(\hat{r})[(l_x+il_y)-(l_x-il_y)]Y_{l\lambda}(\hat{r}) = \delta_{ll'}\{[(l-\lambda)(l+\lambda+1)]^{1/2}\delta_{\lambda',\lambda+1} - [(l+\lambda)(l-\lambda+1)]^{1/2}\delta_{\lambda',\lambda-1}\} \quad (5)$$

and, since the z axis is along $\hat{k}_f$,

$$Y_{l'\lambda'}(\hat{k}_f)=\delta_{\lambda'0}\left(\frac{2l'+1}{4\pi}\right)^{1/2}. \quad (6)$$

It follows that

$$B(\theta)=\sum_l g_l P_l^{(1)}(\cos\theta), \quad (7)$$

where

$$g_l=\frac{2m}{\hbar^2}V_{so}(2l+1)\int_0^\infty r^2dr\, f_l^2(k,r)h(r) \quad (8)$$

and $P_l^{(1)}(\cos\theta)$ is as defined by Jahnke and Emde[6] so that $P_l^{(1)}(\cos\theta)=-(d/d\theta)P_l(\cos\theta)$.

The quantity $B(\theta)$ may also be expressed exactly (i.e., without making the first-order spin-orbit approximation) in terms of the phase shifts $\xi^{\pm}$ corresponding to elastic scattering by the full nuclear plus Coulomb potential for $j=l\pm\frac{1}{2}$, respectively. In this case

$$B(\theta)=\sum_l B_l P_l^{(1)}(\cos\theta), \quad (9)$$

where

$$B_l=\frac{-i}{2k}(e^{2i\xi_l^+}-e^{2i\xi_l^-}). \quad (10)$$

If the first-order treatment is valid, then $B_l=g_l$ and B_l is linearly proportional to V_{so}.

The polarization $P(\theta)$, for a transition amplitude in the form given by Eq. (3), is

$$P(\theta)=-\frac{2\,\mathrm{Im}[A^*(\theta)B(\theta)]}{|A(\theta)|^2+|B(\theta)|^2} \quad (11)$$
$$=-\frac{2\,\mathrm{Im}[A^*(\theta)B(\theta)]}{|A(\theta)|^2} \quad (12)$$

to first order in V_{so}. The differential cross section is given by the denominator in (11) and, to first order in V_{so} is simply

$$d\sigma/d\Omega=|A(\theta)|^2. \quad (13)$$

The polarization (in the first-order approximation) is then found to be

$$P(\theta)=2\left[\frac{d\sigma}{d\Omega}\right]^{-1/2}\sum_l|g_l|\frac{dP_l(\cos\theta)}{d\theta} \times\sin[\arg A^*(\theta)-\arg g_l]. \quad (14)$$

Rodberg[4] introduced quantities β_l defined as

$$\beta_l=-\frac{2\exp(-2i\xi_l)}{k}\int_0^\infty r^2dr\, f_l^2(k,r)h(r), \quad (15)$$

where ξ_l is the phase shift associated with $U_c(r)$. By making the conjecture that β_l is l-independent, Rodberg's derivative relation

$$P(\theta)\propto\frac{d}{d\theta}\left[\ln\frac{d\sigma}{d\Omega}\right]$$

[5] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **91**, 398 (1953).

[6] E. Jahnke and F. Emde, *Tables of Functions with Formulae and Curves* (Dover Publications, Inc., New York, 1945).

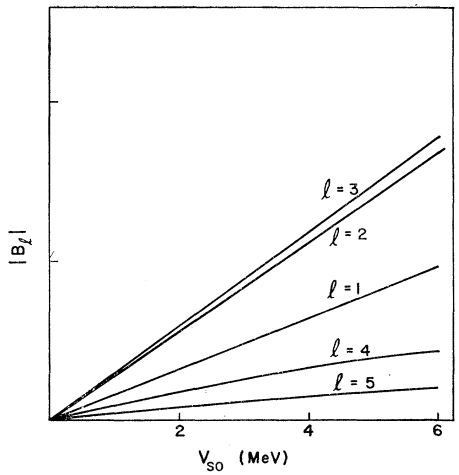

FIG. 1. The variation of $|B_l|$ [Eq. (10)] with the strength of the spin-orbit potential for 10-MeV protons on copper. Optical-model parameters: $V=47$ MeV (Woods-Saxon); $W=8.2$ MeV (surface derivative); $r=1.3A^{1/3}$ F, $a=0.65$ F for all potentials.

immediately follows. This supposition is, however, hardly borne out in practice because of the marked peaking of $h(r)$ near the nuclear surface. Although (14) is not as simple as the derivative relation, it involves only a discrete set of quantities g_l in addition to $A(\theta)$, and shows directly: (1) the proportionality of $P(\theta)$ with V_{so} and (2) the inverse correlation of $P(\theta)$ with $(d\sigma/d\Omega)^{1/2}$.

If we go on to consider effects of V_{so} to higher order, we see that we must add to $T(\theta)$

$$V_{so}\boldsymbol{\sigma}\langle\chi_f^{(-)}|h(r)\mathbf{l}|\psi_i^{(+)}-\chi_i^{(+)}\rangle = V_{so}^2\boldsymbol{\sigma}\langle\chi_f^{(-)}|h(r)\mathbf{l}G(\mathbf{r},\mathbf{r}')h(r')\boldsymbol{\sigma}\cdot\mathbf{l}|\chi_i^{(+)}\rangle, \quad (16)$$

where $G(\mathbf{r},\mathbf{r}')$ is the Green's function appropriate to the full interaction. The expression (16) contributes to both terms in (3), changing $A(\theta)$ to $[A(\theta)+A'(\theta)]$ and $B(\theta)$ to $[B(\theta)+B'(\theta)]$, where the added quantities are at least second in order in V_{so}. Thus $d\sigma/d\Omega$ involves second-order terms originating from both $A'(\theta)$ and $B(\theta)$.

III. NUMERICAL RESULTS

A. Comparison of Exact Calculations with First-Order Predictions

An attempt has been made to test the validity of the first-order treatment by considering the scattering of protons by copper at various energies and for several values of V_{so}.

The phase shifts $\xi_l^{\pm}$ for this reaction were computed numerically using an optical-model program, and the quantities B_l found, using (10). The form factor of the spin-orbit potential $h(r)$ was taken as the Thomas form, $a_0^2r^{-1}f'(r)$, where a_0 is the pion Compton wavelength, $\hbar/m_\pi c\sim\sqrt{2}$ F, $f(r)=\{1+\exp[(r-r_{so}A^{1/3})a_{so}^{-1}]\}^{-1}$, and r_{so} and a_{so} are equated to the corresponding quantities for the central nuclear potential.

Figures 1 and 2 show $|B_l|$ plotted against V_{so} for various l values and for energies of 10 and 20 MeV, respectively. The V_{so} dependence is seen to be remarkably linear, justifying the first-order spin-orbit expression (7), and indicating the unimportance of $B'(\theta)$.

The l dependence of $|B_l|$ for the two energies is shown in Fig. 3, and the pronounced peaking at l values close to the grazing value $l_0(\sim kR)$ is seen to be in sharp contrast to the constant values of β_l from 1 to ∞, as suggested by Rodberg, which to first order in V_{so}, implies a $(2l+1)$ dependence for B_l.

The effect of variation of V_{so} on the polarization has been computed for proton energies up to 20 MeV. The linearity exhibited by the B_l's is reproduced for energies up to about 15 MeV. This is illustrated for 10 MeV in Fig. 4, where there are plotted the magnitudes of $P(\theta)$ at the various turning points as a function of V_{so}. The nonlinearity at higher energies, and for large angles at lower energies, must be traced to the importance of the second-order terms associated with $A'(\theta)$ and $B(\theta)$. The step leading from (11) to (12) is no longer valid; however, (14) is still applicable. This is because the second-order contributions to the numerator of (11) can only involve $B'(\theta)$ in interference with $A(\theta)$ and $B'(\theta)$ has already been seen to be relatively unimportant for the examples considered here.

B. Comparison with Experiment

Inspection of the expression for the polarization in (14) shows the factors g_l which involve the form factor $h(r)$, through (8). If we are to learn more about $h(r)$, a detailed examination must be given to the shape of

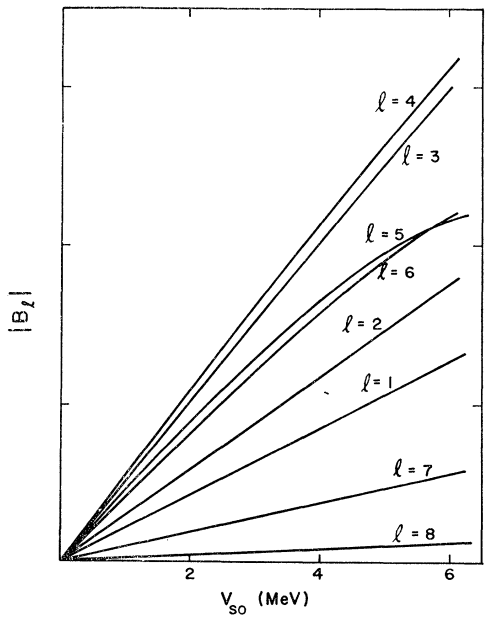

FIG. 2. The variation of $|B_l|$ [Eq. (10)] with the strength of the spin-orbit potential for 20-MeV protons on copper. Parameters as in Fig. 1.

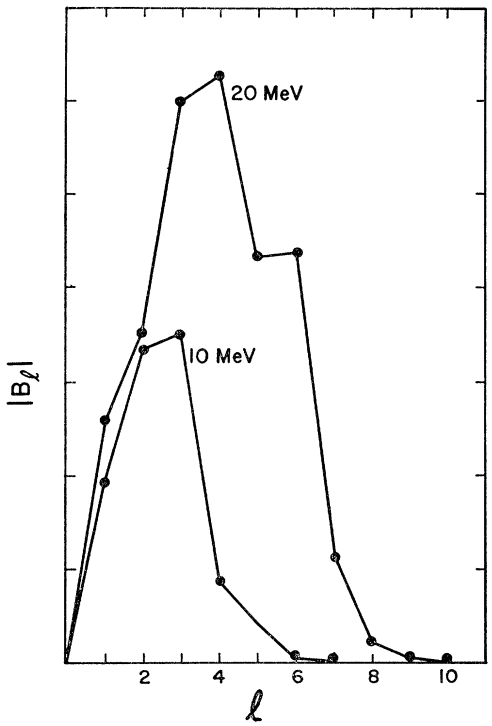

FIG. 3. Variation of $|B_l|$ [Eq. (10)] with l value for 10- and 20-MeV protons on copper. Corresponding to cases of $V_{so}=6$ MeV in Figs. 1 and 2.

$P(\theta)$; however, we need additional details concerning the central interaction since these affect the radial eigenfunctions, $f_l(k,r)$, $d\sigma/d\Omega$, and $\arg A(\theta)$.

One way to avoid this complication is to concentrate on the forward angle region. In this case, $A(\theta)$ is principally the Rutherford scattering amplitude and is relatively insensitive to the precise details given to the central interaction. Calculation of the quantities g_l shows that they also are relatively independent of this difficulty and the polarization thus relates to the form factor $h(r)$ in a clean and straightforward fashion. One anticipates small magnitudes of $P(\theta)$ in this angular region, but the cross sections are large and high accuracy is possible if careful attention is given to the geometrical factors of the experiment.

There is only a limited amount of information available to test these ideas since little attention has so far been given to this angular region. We have chosen for our study,[2] the scattering of 9.4-MeV protons by Cu, Ni^{58}, Ni^{60}, and Co^{59}. An optical-model analysis has already been given for these data. Using the Thomas form for $h(r)$ with parameters kept at the corresponding values for the central interaction ($r_0=1.3$ F and $a=0.65$ F), the experimental points from 20° to 40° were found to be noticeably below the theoretical predictions. Concentration in this angular region showed that considerable improvement was gained by lowering r_{so} to a value 1.1 F. Illustration of this is given in Fig. 5 for the case of Cu. Arbitrary changes were also made of the strengths of the central potential and confirmation was given of the insensitivity of the predictions in Fig. 5 to the central parameters.

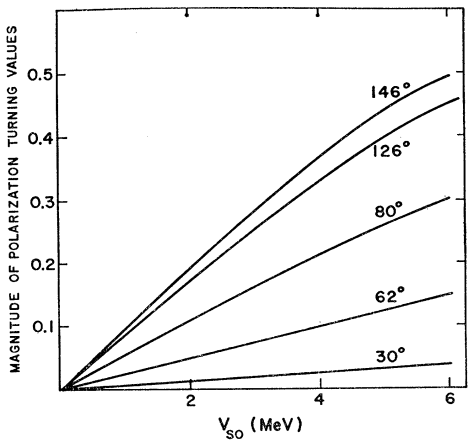

FIG. 4. The magnitudes of the polarization at the turning values plotted against V_{so}. The parameters and energy are the same as for Fig. 1.

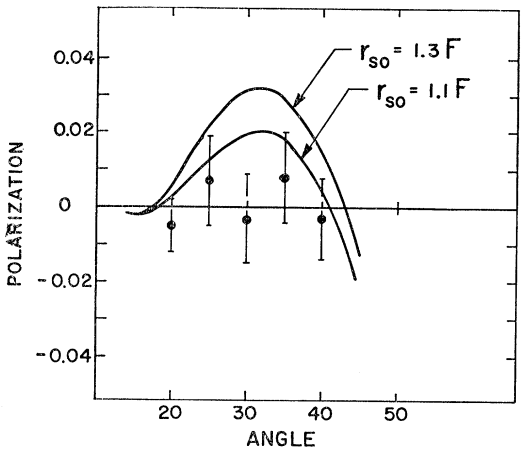

FIG. 5. The predicted polarization for different spin-orbit radii compared with experiment for copper at 9.4 MeV. Spin-orbit strength 6 MeV and all parameters, except spin-orbit radius, as in Fig. 1. Comparison with experimental data from Ref. 2.

This change in geometry of $h(r)$ did not affect $P(\theta)$ at larger angles if a minor adjustment was made of the scaling factor V_{so}. To confirm the conclusion based on the small-angle analysis, all the data were re-analyzed using a search program and allowing variation of both the central and spin-orbit parameters. The geometry for the central potential was, however, kept fixed at the values used in the earlier analysis. The search resulted in only minor changes to the strengths of the central potential; however, in all cases, the χ^2 values were improved by a reduced value for the spin-orbit radius.

The procedure was then to obtain the best choice for the central real and imaginary potentials and then to

search for best values of r_{so} and V_{so} for a range of values of a_{so}. The goodness of fit was found to be insensitive to the choice of a_{so}, as is illustrated in Fig. 6, and it was, therefore, decided to fix a_{so} at the value 0.65 F, that was used for the central potential. The best values of r_{so} and V_{so} are displayed in Table I for the different nuclei,

TABLE I. Comparison of polarization χ_P^2 values for different spin-orbit radii. Columns 2–5 with the spin-orbit geometry the same as the central geometry. Columns 6–9 with the spin-orbit strength and radius allowed to vary. A search was made for the minimum χ^2 on the differential cross section and polarization data of Ref. 2.

	Spin-orbit parameters				Spin-orbit parameters			
Element	r (F)	a (F)	V (MeV)	χ_P^2	r (F)	a (F)	V (MeV)	χ_P^2
Ni^{58}	1.3	0.65	6.0	9.8	1.18	0.65	5.0	6.4
Ni^{60}	1.3	0.65	6.0	9.5	1.10	0.65	4.8	7.3
Cu	1.3	0.65	6.0	3.4	1.12	0.65	6.4	3.1
Co^{59}	1.3	0.65	6.0	3.1	1.08	0.65	4.7	1.6

together with the χ^2 values. Also shown for comparison are the corresponding values using spin-orbit parameters fixed at the values used for the central interaction.

One notes that a slightly larger spin-orbit radius is obtained for Ni^{58}. This is probably a simple reflection of an unsatisfactory choice for the central parameters. The proton energy is below the (p,n) threshold; consequently, one expects appreciable compound-elastic contributions and these should be most important at large angles. This further points to the desirability of the small-angle analysis.

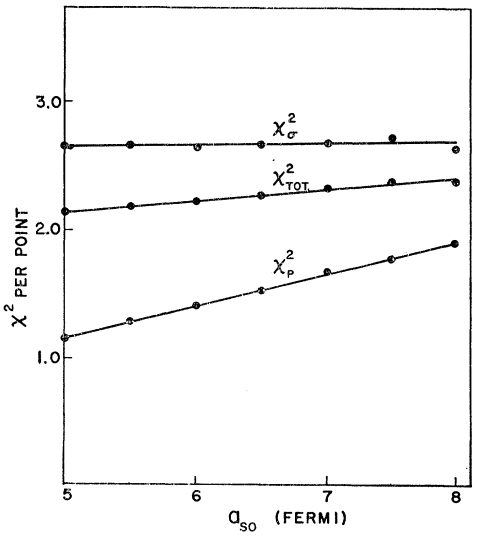

FIG. 6. Cobalt, 9.4-MeV protons, variation of the best χ^2 value obtained for various values of spin-orbit diffuseness. Both differential cross section (χ_σ^2) and polarization (χ_P^2) data of Ref. 2 were used to search on the central real and imaginary strengths followed by a search on the spin-orbit strength and radius for minimum total χ^2 (χ_{tot}^2).

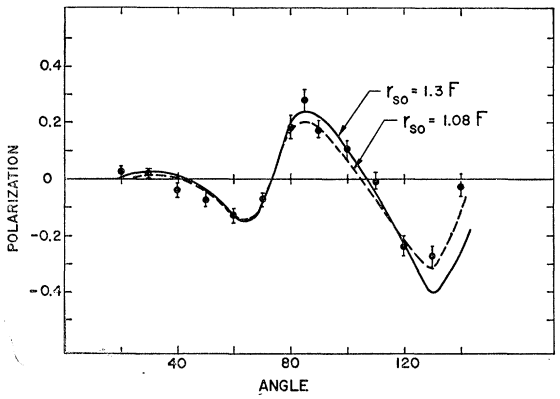

FIG. 7. Cobalt 9.4-MeV proton elastic polarization. Comparison of best fits obtained with: (a) geometrical parameters of the spin-orbit potential the same as the corresponding values of the central potential with the potential strengths allowed to vary, and (b) the spin-orbit radius as well as the potential strengths allowed to vary. Data from Ref. 2.

The predictions for two values of spin-orbit radius together with the experimental points are shown in Fig. 7 for the case of Co, which is typical of the results that were obtained. One notes that the fractional change of $P(\theta)$ at small angles is very much larger than at larger angles, which again favors the small-angle region in determining $h(r)$.

Preliminary calculations were also made with a Woods-Saxon (volume) form factor for $h(r)$. The fits to the polarization with parameters fixed at the original central potential values ($r_0=1.3$ F, $a=0.65$ F) were very similar to those with a Thomas form factor. However, if the parameters were allowed to vary, the Woods-Saxon spin-orbit potential gave generally inferior fits and moreover its optimum values for r_{so} showed much larger variation among the four different nuclei.

IV. CONCLUSIONS

As more polarization data become available, a closer examination of the spin-orbit potential should be possible. What we have attempted to do here is to stress the specific and very different roles played by the strength and shape of U_{so}. Thus, we find for medium-weight nuclei, that V_{so} serves as a scaling factor up to energies ≈ 15 MeV, to a good approximation. In general, the shape of $P(\theta)$ depends on $h(r)$ and on the specific details of the central interaction; however, the small-angle region presents the attractive possibility of eliminating the uncertainties associated with the central interaction.

Analysis at 9.4 MeV shows that $r_{so}\approx 1.1$ F, in contrast to the central value, ≈ 1.3 F. This is corroborated by study over the whole angular region. More accurate data is needed before more specific statements can be made concerning a_{so}, or a fuller investigation of alternative form factors is worthwhile.