agreement with the prediction of the thermodynamic theory, including adiabatic correction.[6,13] The value of C^+ is 3560°C. Present results on the behavior of ϵ near T_c seem to be somewhat different from those found by Craig.[9] We believe our sample might have fewer impurities or lattice defects.

V. SUMMARY AND DISCUSSION

The critical behavior of TGS has been studied through the temperature variation of spontaneous polarization and dielectric constant near T_c. This behavior can be accounted for by $P_s = \text{const}\times(\Delta T)^\beta$ and $1/\epsilon = \text{const}\times(\Delta T)^\gamma$, with $\beta = 0.51 \pm 0.05$ and $\gamma = 1.00 \pm 0.01$, in good agreement with the thermodynamic theory of ferroelectrics which predicts $\beta = \frac{1}{2}$ and $\gamma = 1$. The experimental values of β and γ from the present study do not appear to show parallelism with their analogs in second-order magnetic transitions.

It may be worthy of mention that the observed behavior of TGS in the critical region does not contradict the conclusions of the molecular field theory, in contrast with the experimental evidence for ferromagnetic transitions.

ACKNOWLEDGMENTS

Thanks are due to Dr. G. Shirane for helpful criticism and suggestions and to Dr. K. Okada for his aid and advice in preparing the experimental setup, and many useful discussions. The assistance and cooperation of J. M. Rivera and M. Ribott during he measurements is also appreciated.

PHYSICAL REVIEW VOLUME 144, NUMBER 2 15 APRIL 1966

Reflection and Transmission of Electromagnetic Waves by a Moving Dielectric Slab*

C. YEH
Electrical Engineering Department, University of Southern California, Los Angeles, California

AND

K. F. CASEY
Electrical Engineering Department, Air Force Institute of Technology, Wright Patterson Air Force Base, Ohio

(Received 1 November 1965)

The reflection and transmission of electromagnetic waves by a moving dielectric slab are investigated theoretically and the reflection and transmission coefficients are determined. Two cases of the movement are considered: (a) the dielectric slab moves parallel to the interface; (b) the dielectric slab moves perpendicular to the interface. Various interesting features concerning the variation of the reflection and transmission coefficients, angles of reflection and transmission, and the frequencies of the reflected and transmitted wave, as a function of the velocity of the moving medium, are discussed.

I. INTRODUCTION

THE effects of a perfectly reflecting moving boundary upon an incident plane electromagnetic wave were discussed many years ago by various authors.[1–3] The formula for the equivalent index of refraction of a dielectric medium moving at a uniform velocity with respect to a reference frame S, as viewed from the reference frame S, was first derived by Fresnel.[1–3] The well-known Fresnel formula was then verified experimentally by Fizeau. Sommerfeld gave a rather comprehensive treatment of these interesting problems in his book.[3] However, it is somewhat surprising to learn that the problem of the reflection and transmission of plane waves by a uniformly moving dielectric slab has not been treated.[4] The purpose of this paper is to present the solution to this important problem. The result shows that there exists no Doppler shift in frequency for the transmitted wave due to the movement of the slab. Furthermore, the sum of the reflection coefficient and the transmission coefficient is not unity in general. Discussion of these features as well as several other interesting features concerning the variation of the reflection and transmission coefficients, the angles of reflection and transmission, and the frequency of the reflected waves, as a function of the velocity of the moving medium will be given.

* Supported by the Naval Ordnance Test Station.

1 W. Pauli, *Theory of Relativity* (Pergamon Press, Inc., New York, 1958).

2 C. Møller, *The Theory of Relativity* (Oxford University Press, New York, 1952).

3 A. Sommerfeld, *Optik* (Akademische Verlagsgesellschaft, Leipzig, 1959), 2nd ed.

4 Most recently, Tai treated the problem of reflection by a dielectric half-space moving in a direction transverse to the direction of an incident wave. [C. T. Tai, Antenna Laboratory Report No. 1691-7, Ohio State University, 1964 (unpublished); oral presentation of the 1965 Spring URSI meeting in Washington, D.C.] The case in which the dielectric half-space is moving towards or away from an incident wave has been given by C. Yeh, J. Appl. Phys. **36**, 3513 (1965).

II. THE FORMAL SOLUTION

The geometry of this problem is shown in Fig. 1. A homogeneous dielectric slab having a permittivity of ϵ_1, a permeability of μ_0, and a conductivity of zero, is assumed to occupy the space $d \geq z' \geq 0$ in the S' system which is stationary with respect to the slab. The region outside the dielectric slab is filled by empty free space (ϵ_0, μ_0). It is assumed that the dielectric slab may move in the following directions: (a) The slab moves parallel to the interface in the x direction with a constant velocity v_x. (b) The slab moves perpendicular to the interface in the z direction with a constant velocity v_z. Finally, the incident wave in the free-space region is assumed to be plane with a harmonic time dependence. Only the case for an incident E wave will be analyzed in detail.

In the observer's system S which is stationary with respect to the free space, the incident plane wave takes the form

$$E_y^{(i)} = E_0 e^{i(k_x x - k_z z)} e^{-i\omega t}, \tag{1}$$

$$B_y^{(i)} = 0, \tag{2}$$

where E_0 and ω are, respectively, the amplitude and the frequency of the incident wave, $k_x = k_0 \sin\theta_0$, $k_z = k_0 \cos\theta_0$, and $k_0 = \omega(\mu_0\epsilon_0)^{1/2}$. θ_0 is the angle between the propagation vector and the positive z axis in the x-z plane.

In the moving system S' which is stationary with respect to the uniformly moving dielectric slab, the incident plane wave can be represented by the expressions

$$E_{y'}^{(i)\prime} = E_0' e^{i(k_x' x' - k_z' z')} e^{-i\omega' t'}, \tag{3}$$

$$B_{y'}^{(i)\prime} = 0, \tag{4}$$

where k_x', k_z', ω', and E_0' are related to combinations of k_x, k_z, ω, and E_0 by Eqs. (17) when the dielectric slab is moving in the positive x direction and by Eqs. (21) when the dielectric slab is moving in the positive z direction. x', z', and t' are related to combinations of x, z, and t by the Lorentz transformations. The reflected wave, the wave within the slab, and the transmitted wave must, respectively, have the form

$$E_{y'}^{(r)\prime} = A_r' e^{i(k_x' x' + k_z' z')} e^{-i\omega' t'}, \tag{5}$$

$$B_{y'}^{(r)\prime} = 0, \tag{6}$$

$$E_{y'}^{(p)\prime} = \{B_p' \exp[-i(\omega'^2\mu_0\epsilon_1 - k_x'^2)^{1/2} z'] + C_p' \exp[i(\omega'^2\mu_0\epsilon_1 - k_x'^2)^{1/2} z']\} \times e^{i' k_x' x'} e^{-i\omega' t'}, \tag{7}$$

$$B_{y'}^{(p)\prime} = 0, \tag{8}$$

and

$$E_{y'}^{(t)\prime} = G_t' e^{i(k_x' x' - k_z' z)} e^{-i\omega' t'}, \tag{9}$$

$$B_{y'}^{(t)\prime} = 0. \tag{10}$$

A_r', B_p', C_p', and G_t' are arbitrary constants to be determined by the boundary conditions. Of interest are the reflected-wave coefficient A_r' and the transmitted-

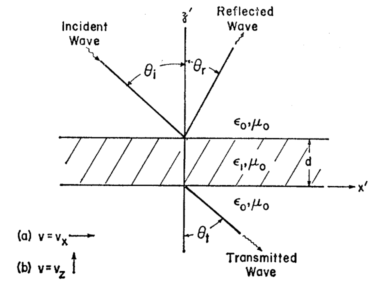

FIG. 1. The geometry of the problem.

wave coefficient G_t'. Matching the tangential electric and magnetic fields at the boundaries $z'=d$ and $z'=0$, one obtains

$$A_r' = \frac{iE_0'(\xi'^2 - k_z'^2) e^{-2ik_z'd} \sin\xi' d}{2\xi' k_z' \cos\xi' d - i(k_z'^2 + \xi'^2)\sin\xi' d}, \tag{11}$$

$$G_t' = \frac{2E_0' \xi' k_z' e^{-ik_z'd}}{2\xi' k_z' \cos\xi' d - i(k_z'^2 + \xi'^2)\sin\xi' d}, \tag{12}$$

where $\xi' = (\omega'^2\mu_0\epsilon_1 - k_x'^2)^{1/2}$. In the observer's system S, the reflected wave and the transmitted wave take the following forms:

For the reflected wave

$$E_y^{(r)} = A_r \exp[i(k_x^{(r)} x - k_z^{(r)} z)] \exp[-i\omega^{(r)} t], \tag{13}$$

$$B_y^{(r)} = 0; \tag{14}$$

for the transmitted wave

$$E_y^{(t)} = G_t \exp[i(k_x^{(t)} x - k_z^{(t)} z)] \exp[-i\omega^{(t)} t], \tag{15}$$

$$B_y^{(t)} = 0. \tag{16}$$

A_r, $k_x^{(r)}$, $k_z^{(r)}$, $\omega^{(r)}$, G_t, $k_x^{(t)}$, $k_z^{(t)}$, and $\omega^{(t)}$ are related to combinations of A_r', k_x', k_z', ω', and G_t' by Eqs. (18) when the dielectric slab is moving in the x direction, and by Eqs. (22) when the dielectric slab is moving in the z direction.

Case (a): $\mathbf{v} = v_x \mathbf{e}_x$

Let us consider the case in which the dielectric slab is moving at a uniform velocity v_x in the x direction. Making use of the covariance of Maxwell's equations and the phase invariance of a uniform plane wave, we have the following transformations [referring to Eq. (3)]:

$$\begin{aligned} k_x' &= \gamma_x[k_x - \omega v_x/c^2] = \gamma_x k_0[\sin\theta_0 - \beta_x], \\ k_z' &= k_z = k_0 \cos\theta_0, \\ \omega' &= \gamma_x(\omega - v_x k_x) = \gamma_x \omega[1 - \beta_x \sin\theta_0], \\ \xi' &= \gamma_x k_0[(\epsilon_1/\epsilon_0)(1 - \beta_z \sin\theta_0)^2 - (\sin\theta_0 - \beta_x)^2]^{1/2}, \\ E_0' &= \gamma_x E_0(1 - v_x k_x/\omega) = \gamma_x E_0[1 - \beta_x \sin\theta_0], \end{aligned} \tag{17}$$

where $\gamma_x = 1/(1-\beta_x^2)^{1/2}$, $\beta_x = v_x/c$, and c is the velocity of light. Viewing from the observer's system S, we have [referring to Eqs. (5)-(10)],

$$\begin{aligned} \omega^{(r)}&=\omega^{(t)}=\gamma_x(\omega'+v_xk_x'), \\ k_x^{(r)}&=k_x^{(t)}=\gamma_x(k_x'+v_x\omega'/c^2), \\ k_z^{(r)}&=k_z^{(t)}=k_z', \\ A_r&=\gamma_xA_r'(1+v_xk_x'/\omega'), \\ G_t&=\gamma_xG_x'(1+v_xk_x'/\omega'). \end{aligned} \tag{18}$$

Substituting Eq. (17) into (18) gives

$$\begin{aligned} &\omega^{(r)}=\omega^{(t)}=\omega, \\ &k_x^{(r)}=k_x^{(t)}=k_x=k_0\sin\theta_0, \\ &k_z^{(r)}=k_z^{(t)}=k_z=k_0\cos\theta_0, \\ &A_r=E_0\left\{\frac{i(\epsilon_1/\epsilon_0-1)(1-\beta_x\sin\theta_0)^2\gamma_x^2\sin(\eta_xk_0d)\exp(-2ik_0d\cos\theta_0)}{2\eta_x\cos\theta_0\cos(\eta_xk_0d)-i(\eta_x^2+\cos^2\theta_0)\sin(\eta_xk_0d)}\right\}, \\ &G_t=E_0\left\{\frac{2\eta_x\cos\theta_0\exp(-ik_0d\cos\theta_0)}{2\eta_x\cos\theta_0\cos(\eta_xk_0d)-i(\eta_x^2+\cos^2\theta_0)\sin(\eta_xk_0d)}\right\}, \end{aligned} \tag{19}$$

where

$$\eta_x=\gamma_x[(1-\beta_x\sin\theta_0)^2(\epsilon_1/\epsilon_0)-(\sin\theta_0-\beta_x)^2]^{1/2}. \tag{20}$$

According to Eqs. (19), we note that there exists no Doppler shift in frequency for the reflected and the transmitted waves. Furthermore, the familiar law concerning the equality of the angle of incidence and the angle of reflection is preserved. On the other hand, the coefficients for the reflected and the transmitted waves are affected by the transverse motion of the slab.

Case (b): $\mathbf{v}=v_z\mathbf{e}_z$

It is assumed that the dielectric slab is moving at a uniform velocity v_z in the positive z direction. Again making use of the covariance of Maxwell's equations and the phase invariance of a uniform plane wave, we obtain the following transformations:

$$\begin{aligned} &\omega'=\gamma_z(\omega+v_zk_z)=\gamma_z\omega(1+\beta_z\cos\theta_0), && k_x'=k_x=k_0\sin\theta_0, \\ &k_z'=\gamma_z(k_z+\omega v_z/c^2)=\gamma_zk_0(\cos\theta_0+\beta_z), && \xi'=k_0[\gamma_z^2(\epsilon_1/\epsilon_0)(1+\beta_z\cos\theta_0)^2-\sin^2\theta_0]^{1/2}, \\ &E_0'=\gamma_zE_0(1+v_zk_z/\omega)=\gamma_zE_0(1+\beta_z\cos\theta_0), \end{aligned} \tag{21}$$

where $\gamma_z=1/(1-\beta_z^2)^{1/2}$ and $\beta_z=v_z/c$. Viewing from the observer's system S, we have

$$\begin{aligned} &\omega^{(r)}=\gamma_z(\omega'+v_zk_z'), \quad k_x^{(r)}=k_x^{(t)}=k_x', \quad k_z^{(r)}=\gamma_z(k_z'+\omega'v_z/c^2), \\ &\omega^{(t)}=\gamma_z(\omega'+v_zk_z'), \quad k_z^{(t)}=\gamma_z(k_z'-\omega'v_z/c^2), \quad A_r=\gamma_zA_r'(1+v_zk_z'/\omega'), \\ &G_t=\gamma_zG_t'(1+v_zk_z'/\omega'). \end{aligned} \tag{22}$$

Substituting Eq. (21) into (22) gives

$$\begin{aligned} &\omega^{(t)}=\omega, \\ &\omega^{(r)}=\omega\gamma_z^2[(1+\beta_z^2)+2\beta_z\cos\theta_0], \\ &k_x^{(r)}=k_x^{(t)}=k_0\sin\theta_0, \\ &k_z^{(r)}=k_0\gamma_z^2[2\beta_z+\cos\theta_0(1+\beta_z^2)], \\ &k_z^{(t)}=k_0\cos\theta_0, \\ &A_r=E_0\frac{i\gamma_z^4(\epsilon_1/\epsilon_0-1)(1+\beta_z\cos\theta_0)^2[(1+\beta_z^2)+2\beta_z\cos\theta_0]\sin(k_0\eta_zd)\exp[-2i\gamma_zk_0d(\cos\theta_0+\beta_z)]}{2\gamma_z\eta_z(\cos\theta_0+\beta_z)\cos(\eta_zk_0d)-i[\eta_z^2+\gamma_z^2(\cos\theta_0+\beta_z)^2]\sin(\eta_zk_0d)}, \\ &G_t=E_0\frac{2\gamma_z(\cos\theta_0+\beta_z)\eta_z\exp[-i\gamma_zk_0d(\cos\theta_0+\beta_z)]}{2\gamma_z\eta_z(\cos\theta_0+\beta_z)\cos(\eta_zk_0d)-i[\eta_z^2+\gamma_z^2(\cos\theta_0+\beta_z)^2]\sin(\eta_zk_0d)}, \end{aligned} \tag{23}$$

with

$$\eta_z=[\gamma_z^2(\epsilon_1/\epsilon_0)(1+\beta_z\cos\theta_0)^2-\sin^2\theta_0]^{1/2}. \tag{24}$$

Unlike case (a), there exists a Doppler shift in frequency for the reflected wave. But there is no frequency shift for the transmitted wave. The frequency shift for the reflected wave is independent of the permittivity of the slab and depends only on the slab velocity and the angle of incidence. The angle of reflection ($\theta_r=\tan^{-1}|k_x^{(r)}/k_z^{(r)}|$) is no longer equal to the angle of incidence However, the transmitted wave still propagates in the same direction as the incident wave The coefficients for the reflected and the transmitted waves are affected by the motion of the slab.

III. THE REFLECTION AND TRANSMISSION COEFFICIENTS

The reflection coefficient and the transmission coefficient are defined, respectively, by the relations

$$R=\mathbf{n}\cdot\mathbf{S}_r/\mathbf{n}\cdot\mathbf{S}_i, \tag{25}$$

and

$$T=\mathbf{n}\cdot\mathbf{S}_t/\mathbf{n}\cdot\mathbf{S}_i, \tag{26}$$

where $\mathbf{n}$ is the unit vector normal to the interface and

$$S_i=\tfrac{1}{2}(\mathbf{E}^{(i)}\times\mathbf{H}^{*(i)}), \tag{27}$$

$$S_r=\tfrac{1}{2}(\mathbf{E}^{(r)}\times\mathbf{H}^{*(r)}), \tag{28}$$

$$S_t=\tfrac{1}{2}(\mathbf{E}^{(t)}\times\mathbf{H}^{*(t)}). \tag{29}$$

The asterisk signifies the complex conjugate of the function. Simplifying Eqs. (25) and (26) gives

$$R=|A_r/E_0|^2\cos\theta^{(r)}/\cos\theta_0, \tag{30}$$

$$T=|G_t/E_0|^2\cos\theta^{(t)}/\cos\theta_0, \tag{31}$$

where

$$\cos\theta^{(r)}=k_z^{(r)}/[k_z^{(r)2}+k_x^{(r)2}]^{1/2}, \tag{32}$$

$$\cos\theta^{(t)}=k_z^{(t)}/[k_z^{(t)2}+k_x^{(t)2}]^{1/2}, \tag{33}$$

$k_x^{(r)}$, $k_z^{(r)}$, $k_x^{(t)}$, and $k_z^{(t)}$ are given by Eqs. (19) when the dielectric slab is moving uniformly in the positive x direction, and they are given by Eqs. (23) when the dielectric slab is moving uniformly in the positive z direction. Making the appropriate substitutions into Eqs. (30) and (31), one has, for the case $\mathbf{v}=v_x\mathbf{e}_x$,

$$R_x=\frac{\gamma_x^4(\epsilon_1/\epsilon_0-1)^2(1-\beta_x\sin\theta_0)^4\sin^2(\eta_xk_0d)}{4\eta_x^2\cos^2\theta_0\cos^2(\eta_xk_0d)+(\eta_x^2+\cos^2\theta_0)^2\sin^2(\eta_xk_0d)}, \tag{34}$$

and

$$T_x=1-R_x; \tag{35}$$

for the case $\mathbf{v}=v_z\mathbf{e}_z$,

$$R_z=\frac{\chi\gamma_z^4(\epsilon_1/\epsilon_0-1)^2(1+\beta_z\cos\theta_0)^4\sin^2(\eta_zk_0d)}{4\gamma_z^2\eta_z^2(\cos\theta_0+\beta_z)^2\cos^2(\eta_zk_0d)+[\eta_z^2+\gamma_z^2(\cos\theta_0+\beta_z)^2]^2\sin^2(\eta_zk_0d)}, \tag{36}$$

and

$$T_z=\frac{4\gamma_z^2\eta_z^2(\cos\theta_0+\beta_z)^2}{4\gamma_z^2\eta_z^2(\cos\theta_0+\beta_z)^2\cos^2(\eta_zk_0d)+[\eta_z^2+\gamma_z^2(\cos\theta_0+\beta_z)^2]^2\sin^2(\eta_zk_0d)}, \tag{37}$$

where

$$\chi=\frac{\gamma_z^4[2\beta_z+(1+\beta_z^2)\cos\theta_0](1+2\beta_z\cos\theta_0+\beta_z^2)^2}{\cos\theta_0\{\sin^2\theta_0+\gamma_z^4[2\beta_z+(1+\beta_z^2)\cos\theta_0]^2\}^{1/2}}. \tag{38}$$

It is interesting to note that for the case $\mathbf{v}=v_z\mathbf{e}_z$, ($v_z\neq0$), $R_z+T_z\neq1$. This is because of energy transfer from the moving slab to the reflected wave. A similar situation also occurs in the case of a perfectly reflecting mirror moving normal to its surface. In that case, the power density of the reflected wave in the direction normal to the surface is greater than that of the incident wave.

It can be seen from Eqs. (34)–(38) that the reflection and the transmission coefficients are rather complicated functions of the angle of incidence, the velocity, the thickness, and the dielectric constant of the slab. To have a qualitative idea of how the reflection and the transmission coefficients vary as a function of the velocity of the

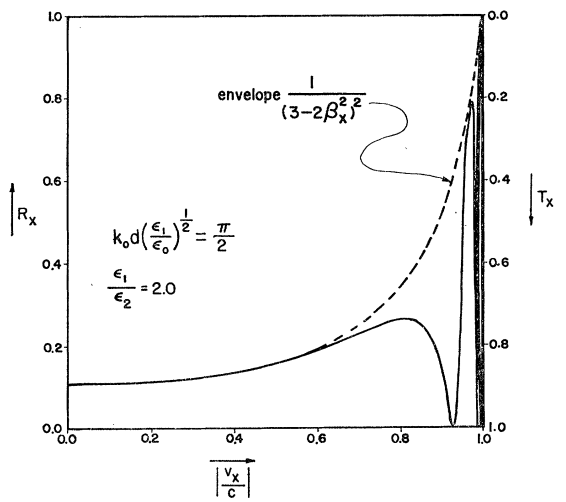

FIG. 2. The reflection and transmission coefficients as functions of $|v_x/c|$ for normal incidence.

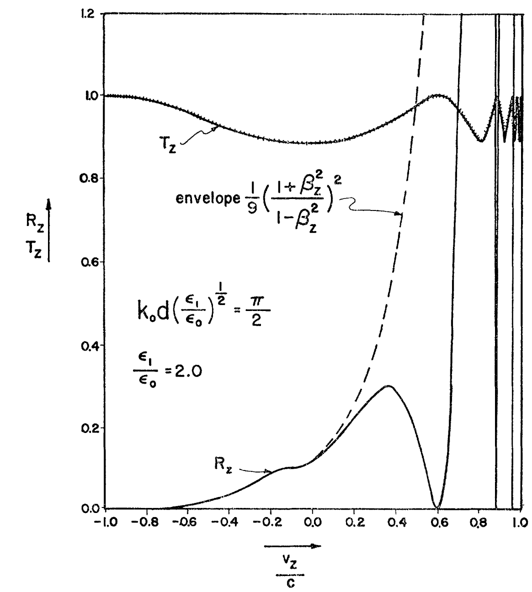

FIG. 3. The reflection and transmission coefficients as functions of v_z/c for normal incidence.

moving slab, we shall consider the limiting case of normal incidence. At normal incidence, i.e., $\theta_0=0$, Eqs. (34)–(38) reduce to

$$R_x=\frac{\gamma_x^4}{4\gamma_x^2(\epsilon_1/\epsilon_0-\beta_x^2)\cot^2[\gamma_x k_0 d(\epsilon_1/\epsilon_0-\beta_x^2)^{1/2}]+[\gamma_x^2(\epsilon_1/\epsilon_0-\beta_x^2)+1]^2}, \tag{39}$$

$$T_x=1-R_x, \tag{40}$$

$$R_z=\left(\frac{1+\beta_z}{1-\beta_z}\right)^2\frac{1}{4(\epsilon_1/\epsilon_0)\cot^2[\gamma_z k_0 d(\epsilon_1/\epsilon_0)^{1/2}(1+\beta_z)]+((\epsilon_1/\epsilon_0)+1))^2}, \tag{41}$$

$$T_z=\frac{4(\epsilon_1/\epsilon_0)/\sin^2[\gamma_z k_0 d(\epsilon_1/\epsilon_0)^{1/2}(1+\beta_z)]}{4(\epsilon_1/\epsilon_0)\cot^2[\gamma_z k_0 d(\epsilon_1/\epsilon_0)(1+\beta_z)]+((\epsilon_1/\epsilon_0)+1)^2}. \tag{42}$$

Equations (39)–(42) are plotted in Figs. 2 and 3. In these figures the reflection coefficient and the transmission coefficient are plotted as a function of the velocity of the moving slab. It is assumed that $\epsilon_1/\epsilon_0=2.0$ and $k_0d(\epsilon_1/\epsilon_0)^{1/2}=\pi/2$. Figure 2 shows that the reflection coefficient and the transmission coefficient oscillate more and more rapidly as β_x approaches unity. The oscillations are caused by the rapid change of the equivalent electrical thickness of the slab with velocity as viewed from the S system. Inspection of Eqs. (39) and (40) shows that R_x and T_x are even functions of β_x, as expected.

Figure 3 shows that the reflection coefficient also oscillates more and more rapidly as β_z varies from -1 and $+1$ and its envelope increases monotonically without bounds from zero. On the other hand, one notes from the same figure that the transmission coefficient oscillates between $4/(1+\epsilon_0/\epsilon_1)^2$ and 1 provided that $k_0d(\epsilon_1/\epsilon_0)^{1\,2}=\pi/2$, and the frequency of oscillation increases as β_z changes from -1 to 1. The fact that the reflection coefficient can be greater than 1 is worth noting. It means that the reflected energy can be more than the energy of the incident wave as far as the observer who is stationary with respect to the S system is concerned. Apparently, there is energy transfer from the moving slab to the reflected wave. It is also interesting to note that as far as an observer who is stationary with respect to the S system is concerned, the frequency of the transmitted wave suffers no frequency shift due to the constant motion of the dielectric slab.