Heat Currents in Liquid Helium II†

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Measurements on the deflection of a beam of negative ions due to a counterflow of superfluid and normal fluid in He II, show the existence of two thresholds for the velocity. The first one is interpreted as the critical velocity for creation of vorticity in the superfluid, while it is suggested that the second threshold corresponds to instability of the whole fluid with respect to turbulence. Measurements on the positive ions tell us the behavior of the normal-fluid velocity; when there is vorticity in the superfluid, these results indicate the existence of a normal-fluid velocity gradient in the flow direction. Parallel measurements on the temperature distribution show nonuniformity of the temperature gradient in the flow direction for large velocities. The assumptions on which the Gorter-Mellink expression for the mutual friction force is derived are not satisfied. The results are then interpreted as an indication that the higher order terms in the equations of motions may give instability at large velocities.

I. INTRODUCTION

It is well known that the linear hydrodynamical theory of liquid helium II does not hold for the high-velocity flow. The experimental results give evidence of a critical velocity much lower than that predicted by Landau for the creation of quantum excitations. An explanation of the magnitude of the critical velocity may be found in the creation of quantized vorticity.[1,2] In this picture, as soon as the velocity overcomes the critical value, the superfluid, being an ideal fluid, will exhibit turbulence. Scattering of rotons and phonons by the vortex lines will then produce a friction on the normal fluid and ultimately the fluid as a whole will become turbulent. One would then observe two threshold values, the first one corresponding to the creation of vorticity, and the second to the instability of the whole fluid with respect to turbulence. Therefore for the first threshold one would observe a critical velocity independent of temperature, while for the second one, using a Reynolds-number criterion (involving the total density of the fluid, since the normal fluid and superfluid are now strongly interacting), the critical velocity would show the same temperature dependence as the normal-fluid viscosity. There is some experimental evidence[3] for the occurrence of two thresholds; however, generally only one flow regime is considered at high velocities for which a number of expressions[4] have been given in the literature. However, their comparison with the experimental results is not easy. Therefore it has become usual to represent the nonlinearity by adding some extra terms to the linear equations of motion. Precisely, one writes the following equations:

$$\rho_s d\mathbf{v}_s/dt = -\frac{\rho_s}{\rho}\nabla p + \rho_s S\nabla T + \mathbf{F}_s + \mathbf{F}_{sn},$$
$$\rho_n d\mathbf{v}_n/dt = -\frac{\rho_n}{\rho}\nabla p - \rho_s S\nabla T + \mathbf{F}_n - \mathbf{F}_{sn} + \eta_n \nabla^2 \mathbf{v}_n, \tag{1}$$

where $\mathbf{F}_s$ and $\mathbf{F}_{sn}$ represent, respectively, the interaction of the superfluid with the boundaries and with the normal fluid, and $\mathbf{F}_n$ represents an additional friction on the normal fluid. $\mathbf{F}_s$, $\mathbf{F}_n$ are often assumed to be negligible with respect to $\mathbf{F}_{sn}$. To these equations the conservation equations are added:

$$\partial\rho/\partial t + \mathrm{div}\rho\mathbf{v} = 0, \quad \rho\mathbf{v} = \rho_s\mathbf{v}_s + \rho_n\mathbf{v}_n,$$
$$\partial(\rho S)/\partial t + \mathrm{div}(\rho S\mathbf{v}_n) = \dot{\sigma}_n + \dot{\sigma}_{sn}, \tag{2}$$

where $\dot{\sigma}_n$ is the entropy dissipation of the normal fluid, while $\dot{\sigma}_{sn}$ is the additional dissipation due to the mutual friction force $\mathbf{F}_{sn}$.

The experimental results are then used to get information on the form of the friction terms. In this way the so-called "Gorter-Mellink" expression[5] for the mutual friction force was derived from the experimental results on heat conduction. In this case, which in the two-fluid language is represented by a counterflow of superfluid and normal fluid

$$\rho_s\mathbf{v}_s + \rho_n\mathbf{v}_n = 0, \tag{3}$$

on the assumption that the velocities $\mathbf{v}_s$, $\mathbf{v}_n$ and the temperature gradient are uniform in the flow direction, the normal-fluid velocity is given in terms of the heat current density $\dot{q}$ by

$$|\mathbf{v}_n| = \dot{q}/\rho ST = v_n^0. \tag{4}$$

Then, from the dependence of the temperature difference between the ends of a channel (or capillary or

† Work supported by Consiglio Nazionale delle Ricerche.
[1] R. P. Feynman, in *Progress in Low Temperature Physics* (North-Holland Publishing Company, Amsterdam, 1955), Vol. I, p. 45.
[2] V. P. Peshkov, in *Progress in Low Temperature Physics* (North-Holland Publishing Company, Amsterdam, 1964), Vol. IV, p. 1.
[3] C. B. Benson and A. C. Hollis-Hallett, Can. J. Phys. **34**, 668 (1956); R. J. Donnelly and A. C. Hollis-Hallett, Ann. Phys. (N. Y.) **3**, 320 (1958).
[4] See, for instance, R. B. Dingle, Proc. Phys. Soc. (London) **A63**, 638 (1950); Advan. Phys. **1**, 111 (1952).
[5] C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).

slit) on $\dot{q}$, the following expression for $\mathbf{F}_{sn}$ is derived:

$$\mathbf{F}_{sn}=A\rho_s\rho_n[|\mathbf{v}_s-\mathbf{v}_n|^2-v_0^2](\mathbf{v}_s-\mathbf{v}_n). \tag{5}$$

(A and v_0 are constants depending on temperature.) A theoretical explanation of this expression has been given by Vinen[6] in terms of the scattering of vortex lines by the normal fluid. A number of experiments[7] do not agree with expression (5), and it is therefore interesting to look more closely at the assumptions on which its derivation has been made, namely,

(a) the same form of the equations describes the whole range of velocities higher than the critical value;

(b) the velocities and the temperature gradient are uniform in the flow direction.

For the first point we recall our previous analysis, which indicates that two types of supercritical flow may exist: a mutual-friction flow in a low range of velocity and a turbulent flow of the fluid as a whole for still larger velocities. On the other hand some measurements on the drift velocity of ions[8] due to the normal fluid have shown that, while for low velocity the relation[4] is satisfied, for high velocities the data indicate that the velocity may be less than v_n^0.

A large number of experiments concerning the temperature difference at the ends of capillaries or slits in a counterflow process may be found in the literature.[7,9–11] In the majority of them the uniformity of the temperature gradient dT/dz is assumed to hold at all velocities without direct experimental evidence, and therefore dT/dz is evaluated by $\Delta T/L$ where ΔT is the temperature difference between the ends of a channel of length L. Since the existence of a normal-fluid jet outside of the channel has been demonstrated,[12] the temperature may be different from the bath temperature in the jet itself. This effect may be important when L is small and ΔT is measured as the difference of the hot-end temperature and the bath temperature. The experimental data obtained by Chase[9] indicate that the temperature gradient is not uniform. In fact Chase made a correction to the measured temperature difference in order to evaluate the gradient at the cold end of his channel, but this was an extremely empirical correction, since it was based on the experimental results themselves. Only two experiments[13,14] have dealt with the temperature distribution along a capillary. They were concerned with the growth of vorticity and therefore the range of heat-input values was restricted to a small interval around the critical region. We shall mention only the Peshkov-Tkachenko results since Mendelsshon used only three thermometers. Peshkov-Tkachenko had 12 thermometers more or less equally spaced along a capillary 1.4 mm in diameter and 8 m long. They report the results at one value of the bath temperature, $T=1.34$ °K. The temperature gradient is uniform and follows a cubic law in the heat input from the critical value up to 60 mW/cm² ($\simeq 4\ \dot{q}_{\mathrm{critical}}$). In this case, then, there is agreement with the derivation of expression (5) for $\mathbf{F}_{sn}$. Better yet, we may say that there is agreement with the Oliphant[15] calculations which show that the Gorter-Mellink hydrodynamical equations plus the conservation equations in the form (2) lead to uniformity in the flow direction of the temperature gradient as well as of the heat flux $\dot{q}=\rho STv_n$. Effectively, at this temperature and for this range of heat current density, one would have observed a true mutual-friction flow. No checks of the uniformity of dT/dz in a larger range of velocities and for different geometries have been made. Therefore a study of the normal-fluid velocity and temperature distribution along a channel for a large range of heat current density is interesting.

In a previous paper[16] it was shown that heat flow has no influence on the mobility μ of positive ions as long as the ions are dragged, with velocities less than 7.5 m/sec, by an electric field E which acts in a direction perpendicular to the flow direction. Therefore the drift velocity of the ions induced by the heat flow must always be equal to the normal-fluid velocity. The positive ion may then be considered as a probe for investigating the normal-fluid behavior as long as its drift velocity, induced by the electric field, is less than 5 m/sec. This restriction comes from the requirement of having a probe as simple as possible, whereas the known mobility discontinuity[17] at 5 m/sec may involve a complicated process.

On the other hand it is known, from experiments in rotating helium,[18] that the negative ions interact strongly with the vorticity. They may thus be used as probes for indicating the presence of vorticity as well as for determining the critical velocity value.

II. PRELIMINARY RESULTS

(A)

The normal-fluid velocity v_n may be derived from the measurement of the deflection α of the ionic beam due to

[6] W. F. Vinen, Proc. Roy. Soc. (London) **A240**, 114 (1957); **A242**, 493 (1957); **A243**, 400 (1957).

[7] See, for instance, P. Winkel and D. H. N. Wansik, in *Progress in Low Temperature Physics* (North-Holland Publishing Company, Amsterdam, 1957), Vol. II, p. 83; or K. R. Atkins, *Liquid Helium* (Cambridge University Press, New York, 1959).

[8] G. Careri, F. Scaramuzzi, and J. O. Thomson, Nuovo Cimento **13**, 186 (1959); **18**, 957 (1960).

[9] C. E. Chase, Phys. Rev. **127**, 361 (1962).

[10] W. E. Keller and E. F. Hammel, Ann. Phys. (N. Y.) **10**, 202 (1960).

[11] E. F. Hammel and W. E. Keller, Phys. Rev. **124**, 1641 (1961).

[12] P. L. Kapitza, Zh. Eksperim. i Teor. Fiz. **11**, 1 (1941).

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[14] V. P. Peshkov and V. W. Tkachenko, Zh. Eksperim. i Teor. Fiz. **41**, 1427 (1961) [English transl.: Soviet Phys.—JETP **14**, 1019 (1962)].

[15] T. A. Oliphant, Ann. Phys. (N. Y.) **23**, 38 (1963).

[16] G. Careri, S. Cunsolo, and M. Vicentini-Missoni, Phys. Rev. **136**, A311 (1964); referred to as I.

[17] G. Careri, S. Cunsolo, and P. Mazzoldi, Phys. Rev. **136**, A303 (1964).

[18] G. Careri, W. D. McCormick, and F. Scaramuzzi, Nuovo Cimento **22**, 215 (1962); Phys. Letters **2**, 61 (1962).

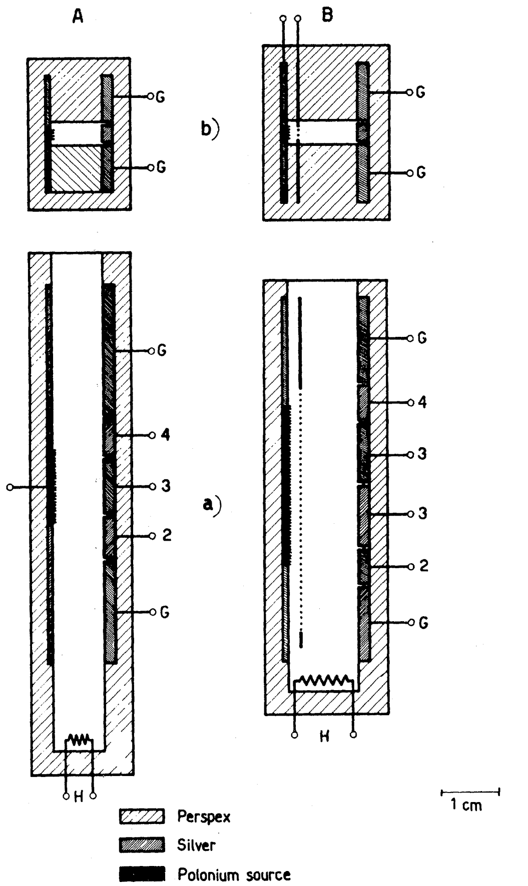

FIG. 1. Schematic view of the apparatus (same as Fig. 1 of Ref. 16).

a flow of normal fluid in a direction orthogonal to the beam itself: $\tan\alpha = v_n/\mu E$.[8,16] However, in order to obtain the absolute value of the velocity, we must know from independent measurements the ionic mobility. Since we want to study the hydrodynamics of the normal fluid we must be certain that the extracting electrode, necessary to get the time-of-flight measurement[19] of the mobility, does not induce any perturbation of the velocity field. Therefore we will compare measurements taken with different apparatus both with and without, and with various geometries of, this electrode. The apparatus and techniques are the same as used in I (see Fig. 1). In apparatus B the extracting electrode V_G was alternately made by a grid (B1), one slit 0.5 mm in width (B2), and two parallel slits 0.5 mm in width each (B3). In each case the linear dimensions were as in Table I. In apparatus B2 and B3 a carbon resistor (Allen-Bradley $10\Omega\pm0.5\%$, $\frac{1}{2}$-W size) was placed near the heater. The difference between its temperature and the bath temperature was then measured differentially with a sensitivity of about 0.01 m°K. The bath temperature was measured by means of a carbon resistor calibrated during each run,[20] and the accuracy in the absolute value of the temperature was of the order ±0.005°K. Different sources were used which provided ionic beams of variable density ranging from 1×10^5 to 2×10^6 ion/cm^3. Figure 2 shows the comparison between the results obtained with apparatus A and the ones obtained with apparatus B2. The slight disagreement may be explained by the difference in the temperature of the two runs. Since a better agreement was observed when changing the form of the electrode from one slit to two parallel slits, we conclude that the velocity field is indeed left unaltered and that measurements taken with the various apparatus are comparable and equally reliable.

TABLE I. Linear dimensions of the various apparatus.

Apparatus	s(cm) distance grid-collecting electrodes	σ(cm^2)	L(cm)
A	0.8	0.38	8.0
B1	1.0	0.46	6.7
B2	0.8	0.34	6.7
B3	0.8	0.34	6.7

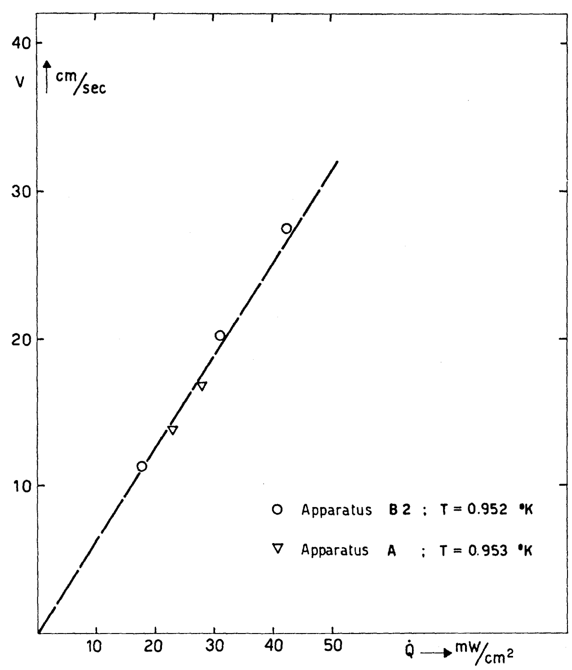

FIG. 2. Drift velocity of positive ions as a function of the heat current density, as detected in apparatus A (symbol ▽) and in apparatus B2 (symbol ○) at about the same temperature.

(B)

For low velocities, the linear theory must hold. Therefore the measured velocity must satisfy relation (4). This check could be more easily done, because of the

[19] S. Cunsolo, Nuovo Cimento **21**, 76 (1961).

[20] S. Cunsolo, M. Santini, and M. Vincentini-Missoni, Cryogenics **5**, 168 (1965).

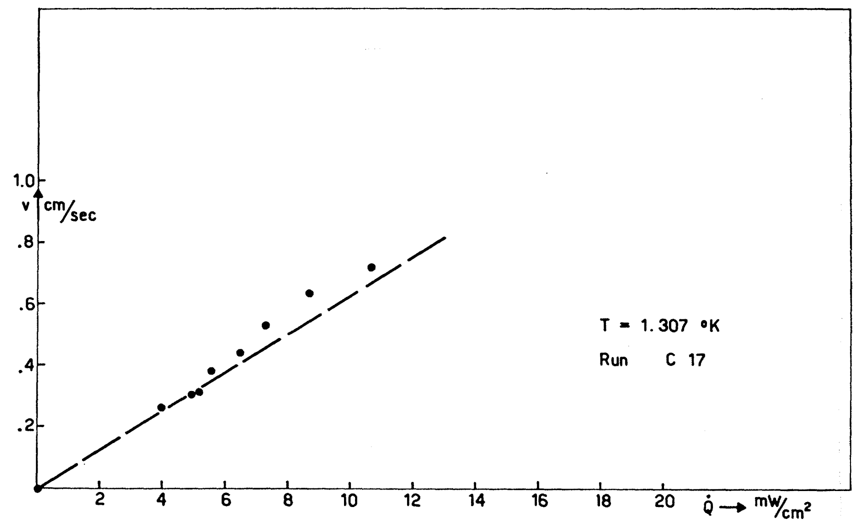

FIG. 3. Drift velocity of positive ions for low heat current densities at the temperature $T = 1.307$°K.

sensitivity of our technique, at relatively high temperature. A number of runs were made around 1.3°K and all agree in giving, for low heat input, the normal-fluid velocity according to relation (4). In Fig. 3 the results for one run are shown in detail. In contrast to the results of Ref. 8 there seems to be a tendency for the velocity to increase more than linearly with heat input when this becomes sufficiently high. To investigate this effect further, a number of runs were made in the temperature range from about 0.900 °K to about 1.310 °K for a large range of heat-input values. The temperature difference between the ends of the channel was determined, together with the velocity, since the knowledge of this quantity would allow a comparison with the flow regime studied in channels of comparable size and geometry by other investigators. In particular, Vinen[6] studied the difference in temperature between the ends of a channel which has a rectangular section 2.40×6.45 cm^2 and $L = 10$ cm, and Chase[9] did the same for one with a circular section of diameter 0.8 mm and $L = 5.16$ cm.

The velocity shows a nonlinear dependence on $\dot{q}$. As may be seen from Fig. 4, where the results of some runs are shown, it is therefore difficult to find an empirical relationship between v_n and $\dot{q}$. The comparison with the velocity value expected on the basis of the linear theory shows that the measured values are substantially higher than v_n^0. Moreover it is not possible to get a unified picture of the results for the whole temperature range investigated. The temperature-difference results are shown in a logarithmic plot in Fig. 5, from which one may see that the experimental values are well represented by a relation of the form used by Chase[9]:

$$\Delta T = D(\dot{q} - \dot{q}_0)^n; \tag{6}$$

$$(D,\ \dot{q}_0,\ n,\ \text{functions of } T).$$

In the common temperature and heat-input ranges, our

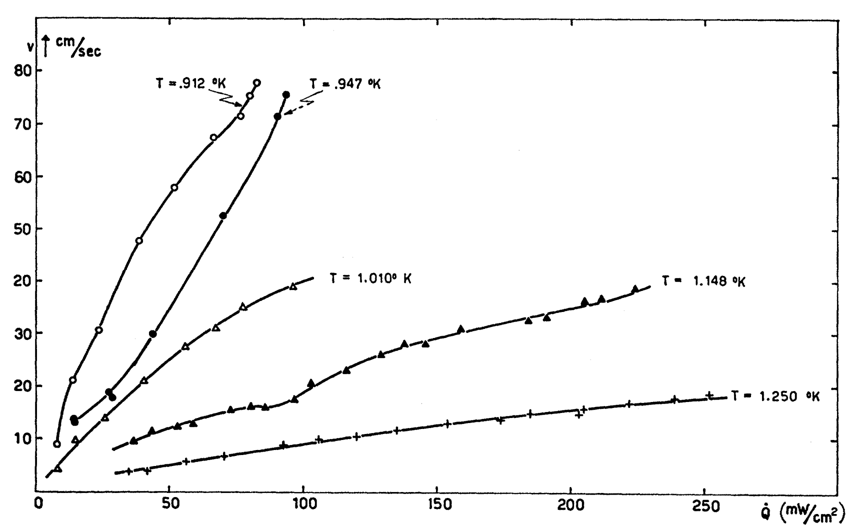

FIG. 4. Drift velocity of positive ions as a function of the heat current density at various temperatures.

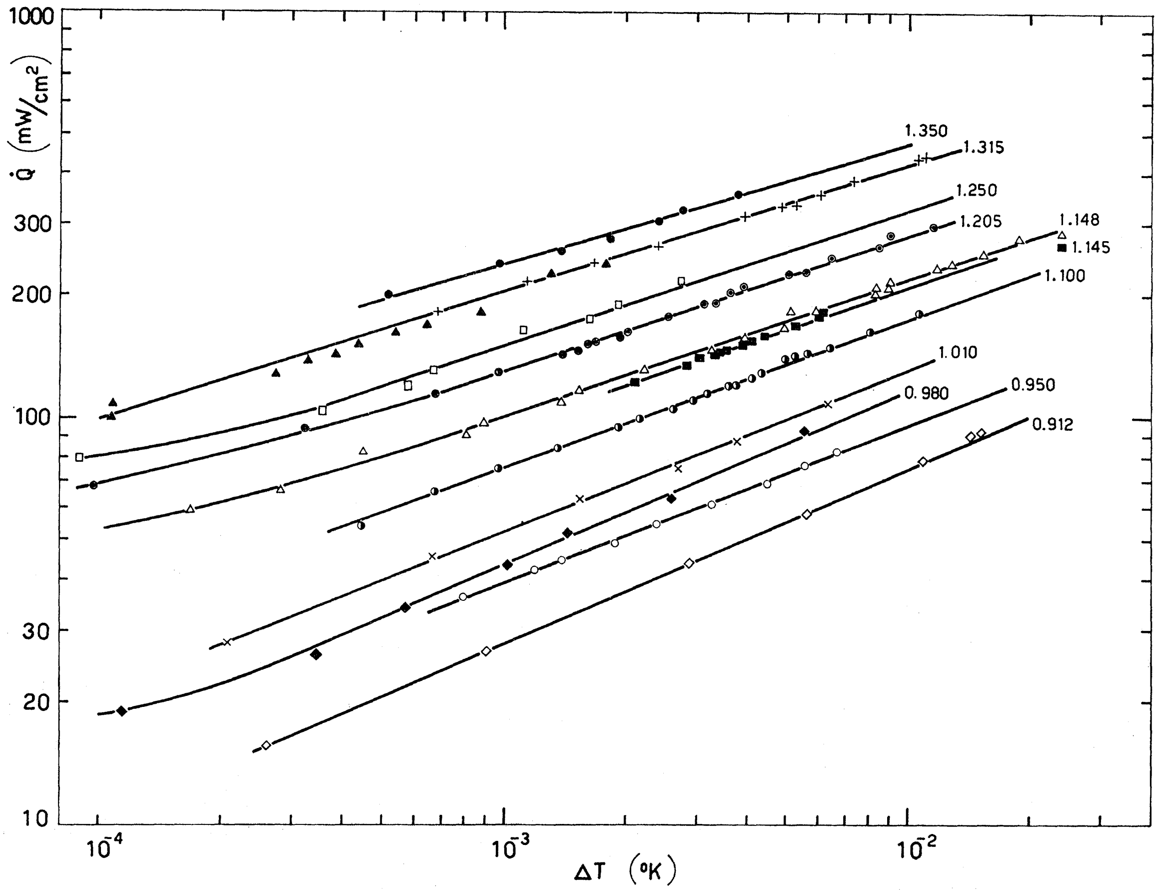

FIG. 5. Temperature difference between the ends of the channel as a function of the heat current density. Numbers attached to the curves are the temperatures in °K.

results may be compared with the results of Vinen and Chase. It is difficult to make a good quantitative check with either experiment because the geometry of the channels was slightly different, and one needs to know how the temperature gradient varies along the channel. In any case, there is a good qualitative agreement as

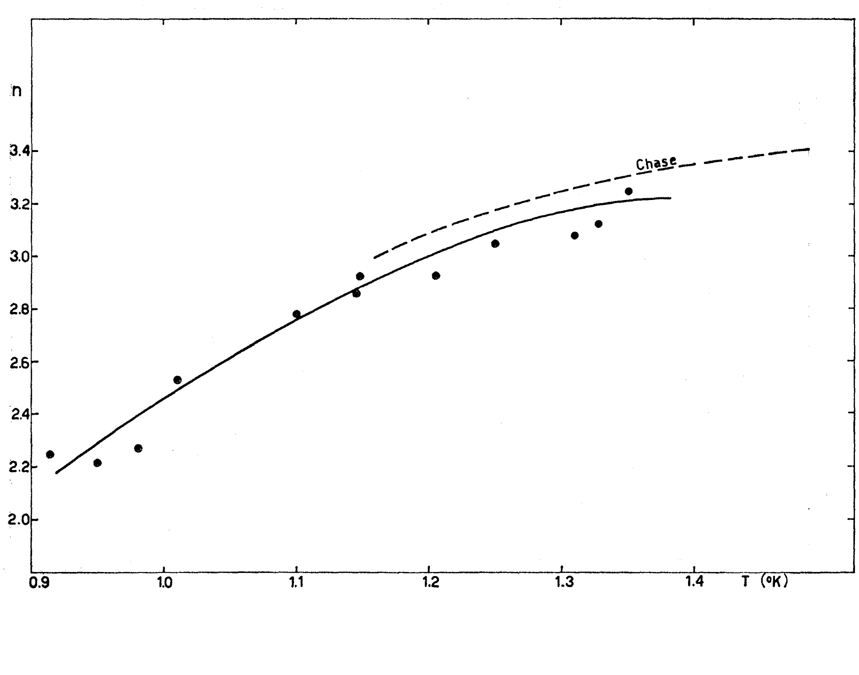

FIG. 6. Temperature dependence of the exponent n in the relation $\Delta T = D\dot{q}^n$. The broken line shows the temperature dependence given by Chase.

may be seen from Fig. 6 where the exponent n of relation (6) is plotted as a function of the temperature together with the Chase values. We may then conclude that we are dealing with the same type of flow encountered in all experiments on heat conductivity in helium II at high velocities.

We could then discuss our results along the same line as the Gorter and Mellink mutual-friction theory. But in this theory there is the assumption that, whatever the velocity, relation (4) is valid; and, if we may believe that the deflection of the ionic beam due to the heat flow gives the true normal-fluid velocity value, our results contradict this assertion. We must then investigate the possible sources of error in our measurements which may falsify the absolute value of the velocity. We recall that in order to evaluate the velocity, we had to make some assumptions (see I) concerning the uniformity of the ion source. While it is difficult to explain, by assuming a nonuniformity of the source, the discrepancy between the measured value of the velocity and the predicted one, it is desirable to examine the method in order to avoid any assumption which could be questionable. Therefore a new apparatus was designed in which more accurate deflection measurements could be made by introducing the possibility of a slight movement of the source electrode along the channel. Then the movement necessary to restore to each electrode, in a flow situation, the values of the current registered at rest, is directly related to the beam deflection. Furthermore, the apparatus was provided with two ionic sources and relative electrodes which allow the possibility of measuring the velocity at different positions in the channel. At the same time the temperature distribution along channels of various geometries was studied.

III. VELOCITY MEASUREMENTS

Apparatus and Experimental Procedure

The experimental arrangement is the one used in the previous work: a counterflow of normal fluid and superfluid is established in a direction orthogonal to the direction of the electric field which acts on the ionic beam. Therefore the over-all design of the apparatus is again an ionization chamber with several detecting electrodes, wherein an ionic beam is produced by an α source and deflected by the heat current. However, the geometry is different in that we now have an apparatus with cylindrical symmetry in which the sources are placed on a central electrode leaving a channel of toroidal cross section ($R_{\text{int}}=2$ mm, $R_{\text{ext}}=9$ mm). The collecting electrodes are placed on the external cylinder. The reason for this geometry is found in the fact that we may be certain to measure the mean value of the velocity over the cross section of the channel, while in the rectangular channel this point could be questioned. A schematic view of the apparatus in shown in Fig. 7. The central electrode is directly

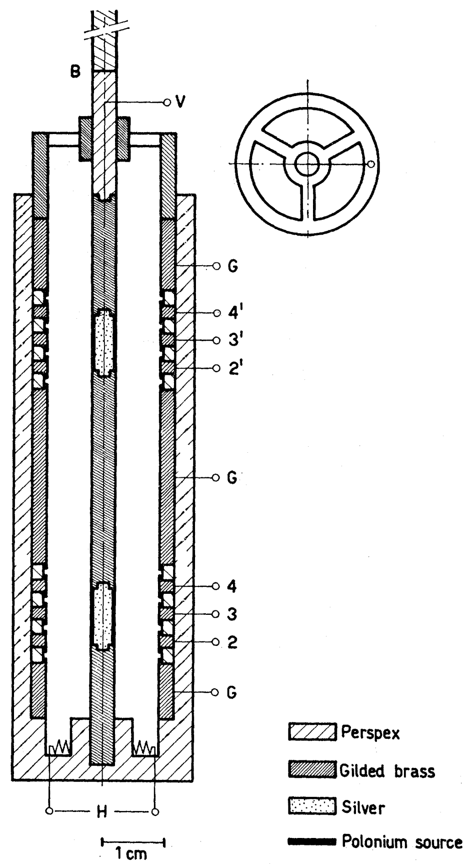

FIG. 7. Schematic view of the apparatus. The detail shows the cold end of the channel.

connected to a screw at the exterior of the Dewar, and its movements are registered by cathetometer reading of the mark at B with a precision of ± 0.02 mm.

The measurements are done while holding the potential difference V between the source and the collecting electrodes fixed. Then the ion moves in an electric field given by

$$E(r)=\frac{V}{\ln(R_2/R_1)}\frac{1}{r}, \tag{7}$$

R_1, R_2 being, respectively, the radii of the central and the collecting electrodes.

If the potential difference is such that the drift velocity of the ions never exceeds 5 m/sec, the mobility of the ions is constant and the time necessary for the ion to cross the channel is given by

$$t=\frac{R_2^2-R_1^2}{2\mu V}\ln\frac{R_2}{R_1}. \tag{8}$$

If, during this time, the ions are dragged by the normal fluid, the beam will suffer a displacement Δx:

$$\Delta x=tv_n=v_n\frac{R_2^2-R_1^2}{2\mu V}\ln\frac{R_2}{R_1}, \tag{9}$$

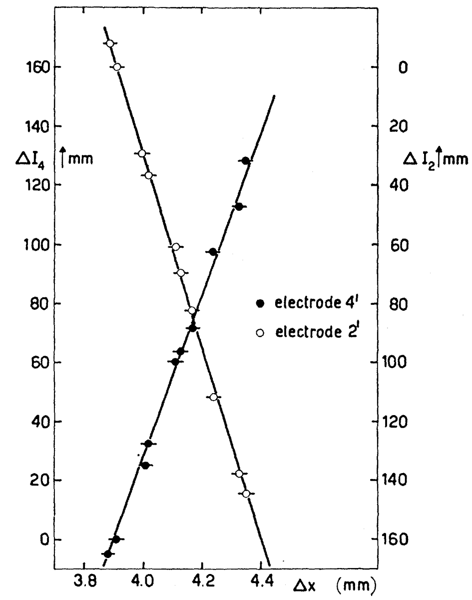

FIG. 8. Changes in the current of the external electrodes as a function of the source position. The lines are drawn according to relation (11).

and the external electrodes will register a change in current ΔI_i, which may be evaluated as in I, and is given by

$$\left(\frac{I_3}{I_i}\right)_\infty \frac{\Delta I_i}{I_3}=\frac{\Delta x}{d_i}=\frac{R_2^2-R_1^2}{2d_i}\frac{v_n}{\mu V}\ln\frac{R_2}{R_1}, \tag{10}$$

where I_3 is the current received by the central electrode, and d_i the length of the section of the external electrode i facing the source, which will be evaluated as in I.

Therefore we can check the validity of the method employed in the previous work for evaluating the beam displacement from the change in the currents of the external electrodes, simply by checking the validity of the relationship between ΔI_i and Δx, namely

$$\Delta I_i=\frac{I_3}{d_i}\left(\frac{I_i}{I_3}\right)_\infty \Delta x. \tag{11}$$

Experimental Results and Discussion

(a) *Positive ions*

In order to check relation (11) we register the change in the current on each electrode for a known displacement of the source at a given potential difference, and we compare the slope of the experimental curve with the one predicted by (11). This was done in a number of runs at different temperatures, for each source, and for various values of the potential difference. The agreement is very good, as may be seen from Fig. 8, where the data for one source are reported. For the same run, the velocity as a function of the heat input is shown in Fig. 9 where, for comparison, we draw the line expressing the relation (4). The results at the three positions of the channel are clearly different because the experimental error is smaller than the size of the symbols. The results for electrode 4 are in agreement with a continuous change of velocity along the channel. A number of runs, for different values of the bath temperature, agree in giving evidence of a velocity gradient in the flow direction. From Fig. 9 it is evident that in some positions the velocity may be higher than v_n^0.

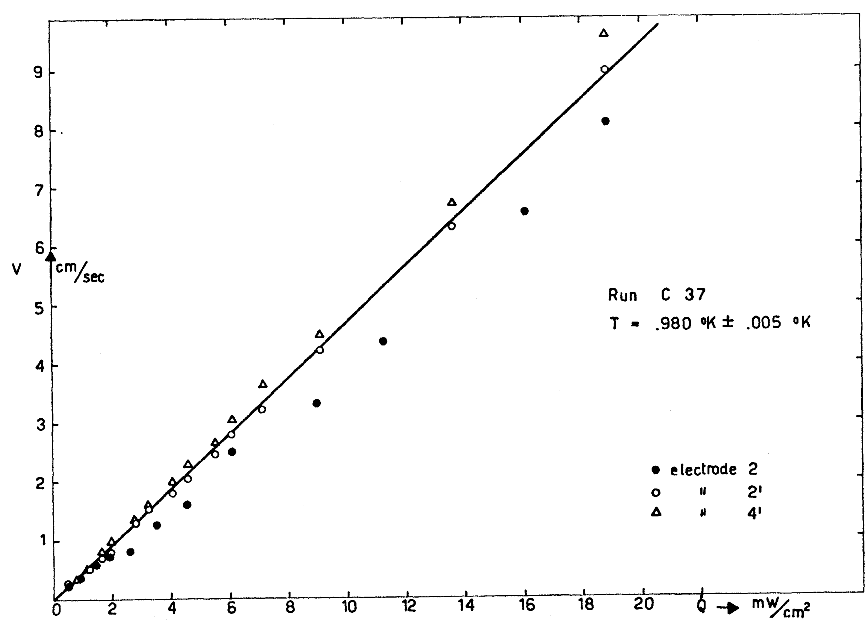

FIG. 9. Drift velocity of positive ions as a function of the heat current density at a temperature $T=0.980°K$ at different positions in the channel. The experimental error is smaller than the size of the symbol. The full line represents the relation $v_n=v_n^0=\dot{q}/\rho ST$.

(*b*) *Negative Ions*

Since these ions interact strongly with the vorticity, and the presence of vorticity may induce nonreproducibility, one must be very careful to start the experiment with a subcritical heat current. We will report two runs in detail:

Run *C*33: $T=1.086°\text{K}\pm0.005°\text{K}$. The heat input 0.77 mW/cm^2, is critical in the sense that for larger heat inputs an additional noise on the currents is registered which disappears when the helium is kept at rest for a while, or better still, in a subcritical flow. The corresponding normal-fluid velocity as detected by measurements on positive ions is 0.12 cm/sec and the superfluid velocity $v_s=1.7\times10^{-3}$ cm/sec. For lower velocities the 4 electrodes give, within the experimental errors, the same value of the velocity $v_n=v_n^0$.

Run *C*34: $T=1.012\pm0.005°\text{K}$ (see Fig. 10). For low heat input the 4 electrodes give $v_n=v_n^0$. At $\dot{q}=0.36$ mW/cm^2, $v_n=0.14$ cm/sec, $v_s=1.2\times10^{-3}$ cm/sec, the results for the various electrodes begin to be different, and on going back to low velocities, the first results are no longer reproducible. For larger velocities the currents on the electrodes varied continuously up to 2.8 mW/cm^2, and at this value there was a decrease in the total current. Such a decrease had been observed before[8] and was thought to be the critical velocity for a channel. Now we may say that there is another critical value, much lower, which is in rough agreement with the extrapolated value of the Meservey[21] curve. For this same run the positive ions gave for low heat input, on the 4 electrodes, $v_n=v_n^0$, but since this was not true for velocities higher than 0.14 cm/sec, we may also say that above this first critical velocity there is a gradient in the normal-fluid velocity. The gradient is such that, at some position in the channel, the velocity may be higher than v_n^0. We might try to ascribe the observed gradients to conduction of heat across the Perspex walls of the channel. However, on this basis we would expect, in the first place, the same qualitative effect at all temperatures and quantitatively a higher effect for higher heat input; in the second place we would nowhere have observed a velocity higher than v_n^0. Therefore, the gradient is indeed due to the hydrodynamics of the system.

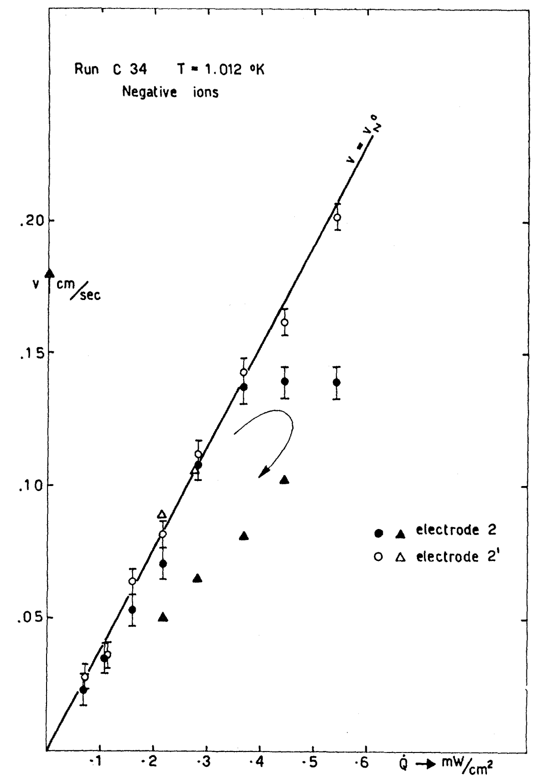

FIG. 10. Drift velocity of negative ions as a function of the heat current density at $T=1.012°$K. Symbols △, ▲ show measurements taken with diminishing heat current shortly after having observed the supercritical flow. The sequence of the measurements is shown by the arrow.

Since in every hydrodynamic problem the boundary conditions are very important, we may predict different effects due to variations not only of the geometry but also of the cross-sectional area and of the length of the channel. Thus the observed disagreement between the velocity values obtained with the different channels by our and others' experiments[8] may be due to either of two causes: that the boundary conditions are different, or that the velocity is measured at different positions relative to the heater.

TABLE II. Linear dimensions and number of thermometers in the channels used to detect the temperature distribution.

	σ(cm^2)	d_H(cm)[a]	L(cm)	No. of thermometers
R1	3.6×10^{-3}	0.06	12.3	7
C1	9.4×10^{-3}	0.11	11.6	7
R2	0.36	0.60	11.6	6
C2	0.36	0.68	11.6	8

[a] d_H is the hydraulic diameter.

[21] R. Meservey, Phys. Rev. **127**, 995 (1962).

IV. TEMPERATURE GRADIENTS

Experimental Apparatus

The apparatus used was of two geometries: square cross section (indicated by the symbol R) and circular cross section (indicated by the symbol C). In Table II we report the dimensions and number of thermometers used in each apparatus.

The channels were made of Perspex, whose walls were at least 2 mm thick. The heat conductivity of Perspex is 0.4 mW/°K cm; thus the heat flow across 1 mm of Perspex, for a temperature difference as large as 0.1°K, is equal to 0.4 mW/cm^2, which is essentially negligible compared with the heat input necessary to produce such a temperature difference. The thermometers were carbon resistors (Allen-Bradley 10 $\Omega \pm 0.5\%$, $\frac{1}{2}$-W size). They are held in the walls of the channel and communicate with it through small holes of 0.05-cm diam in order to introduce as small a perturbation as possible on the flow. If a transverse temperature gradient is established over the cross section of the channel, its influence on the longitudinal gradient may be considered as negligible. The temperature difference between each thermometer and the bath was measured differentially. A decade resistor could be placed in series with either of the internal thermometers, so that temperature differences could be measured directly. The sensitivity of the measurement was of the order of 0.01 m°K.

Experimental Results and Discussion

The distribution of temperature along the channel relative to the bath temperature is shown in Figs. 11 and 12 for the channel C1 for various values of the heat current density at a bath temperature $T=0.973$°K. This channel and the others have been examined at different values of the bath temperature in the range $T=0.865$–1.200°K for heat-current densities ranging from 20 mW/cm^2 to about 1 W/cm^2. The behavior is qualitatively the same as that shown for channel C_1. An example is given in Fig. 13 for one of the wider channels (R2). The temperature distribution seems to depend on the linear dimensions of the channel. The arrow on the distance coordinate in the figure indicates the length of the channel. Extrapolating linearly the temperatures measured by the two thermometers near the cold end of the channel, it may be seen that one obtains the bath value for a distance from the heater larger than the channel length. The difference decreases with increasing channel width and with increasing heat current. This effect is in agreement with the possibility of a temperature variation in the normal-fluid jet outside of the channel. But in that case all the experiments which determine the gradient by measuring the difference between the hot-end temperature and the bath temperature, and assuming grad$T=\Delta T/L$, are wrong by a factor which may not be independent of the heat input and which is large unless the length of the capillary is very large compared with the transversal dimensions. This last statement is satisfied in the experiments with very narrow capillaries or slits, in which the normal-fluid viscosity values, as determined from the linear dependence of the temperature gradient on $\dot{q}$, were found to be in agreement with the results ob-

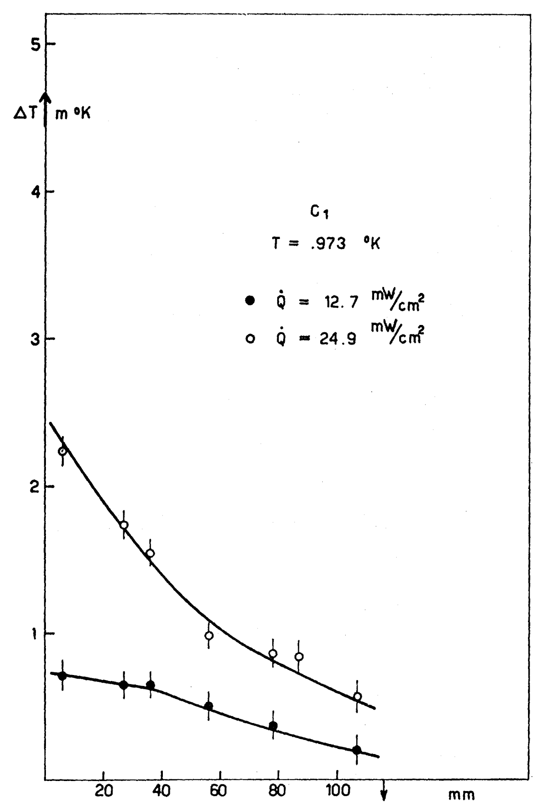

FIG. 11. Temperature distribution in channel C1 for different values of the heat current density. The lines are drawn simply to connect the experimental points and do not show any analytical relation. The arrow on the abscissa indicates the end of the channel.

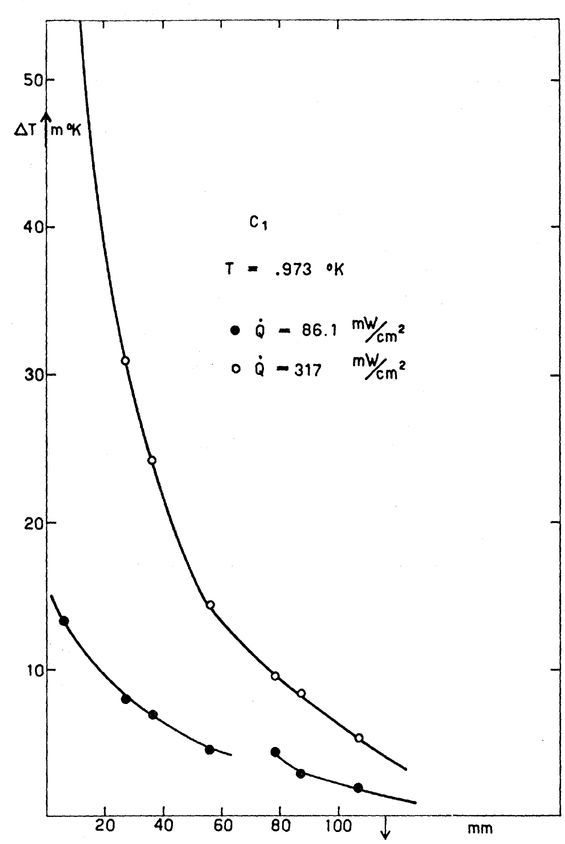

FIG. 12. Same as Fig. 10 for larger values of the heat current density.

tained by other methods.[10,11,22,23] We may compare our measurements with the results of other experiments[9,6] using the difference between the temperature of the thermometer nearer to the heater and the bath temperature. It is better to make this comparison with the values of Chase, which extend to lower temperature than those of Vinen; and for the channel C1, which is, by virtue of its shape and dimensions, the most similar of our channels to the one used by Chase. Chase gave his results in terms of (grad$T$)*, defined as the temperature gradient minus the contribution due to the normal-fluid viscosity, which for a cylindrical capillary is given by

$$\mathrm{grad}T = 8n_n\dot{q}/R^2(\rho S)^2T \tag{12}$$

(R is the radius of the capillary and the meaning of the other symbols is clear).

Thus we compare our results in terms of ΔT^*, defined in the same way as (grad$T$)*, with what we may estimate from the Chase plot. This comparison is shown in Fig. 14 for two values of the bath temperature which are very close to each other. The range of $\dot{q}$ is different in the two experiments but, for the common interval, the qualitative agreement is good and the slight discrepancy may be ascribed to the different temperature distribution caused by the difference in the geometry of the channels. All of our results show that, far from being uniform, the temperature gradient varies strongly along the flow direction. Only for the lowest heat current densities, that is for the lowest flow velocities, may the gradient be uniform within the experimental errors. On the basis of a mutual-friction flow one would expect a variation of the gradient along the channel due to the variation with temperature of the quantities entering the expression of the gradient in terms of $\dot{q}$.

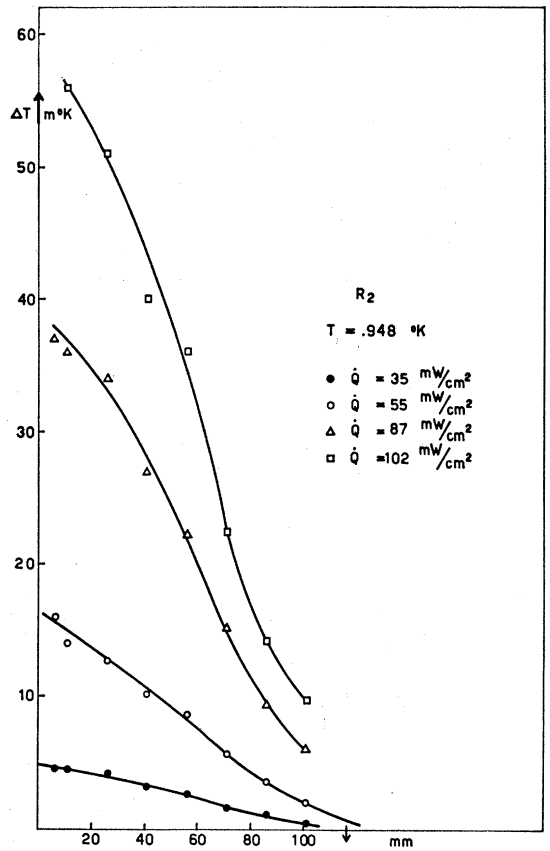

FIG. 13. Same as Fig. 10 for channel R2.

$$\mathrm{grad}T = \frac{A\rho_n}{(\rho ST)^3S}\left(\frac{\rho}{\rho_s}\right)^3 \dot{q}(\dot{q}-\dot{q}_0)^2. \tag{13}$$

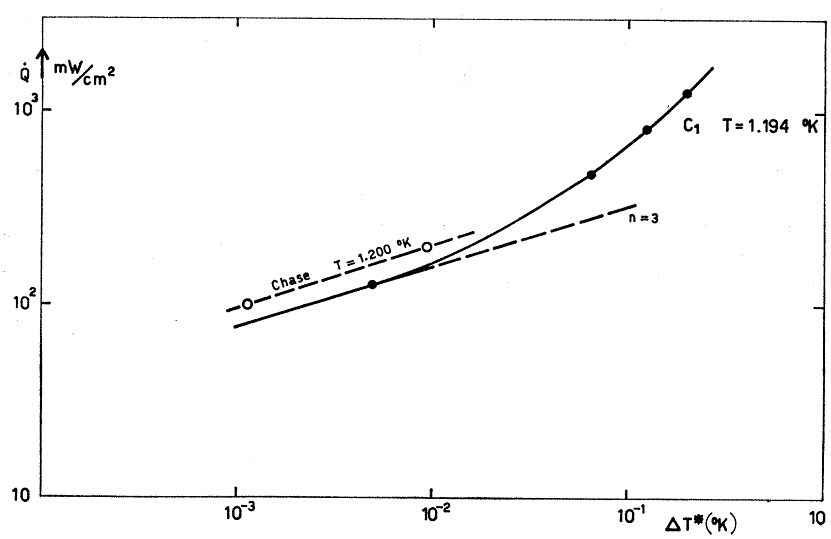

FIG. 14. Comparison of the total temperature difference between the ends of channel C1 with the Chase results for a channel of the same geometry and comparable transversal size.

[22] F. A. Staas, K. W. Taconis, and W. M. Van Alphen, Physica **27**, 893 (1961).
[23] D. F. Brewer and D. O. Edwards, Phil. Mag. **6**, 1173 (1961).

Qualitatively, this would correspond to a larger gradient nearer the heater, which is in agreement with what we found. However, there is no quantitative agreement, since the variation predicted by (13) is practically negligible unless the total temperature difference is of the order of 1°K, while the maximum measured value for this difference in our experiment is of the order of 0.1°K. We would then have observed a uniform gradient within the experimental errors. Also expression (13) does not indicate any dependence on the channel dimensions.

V. CONCLUSION

The measurements on the negative ions indicate that before the threshold, revealed by a decrease in the total current received by the collecting electrodes and which is explained by the capture of the ions by vortices,[8] another critical velocity exists. This first threshold is related to the appearance of various effects (larger noise in the currents, nonreproducibility of the data, hysteresis) which may be interpreted as indications of the presence of vorticity in the fluid. The fact that the superfluid velocity corresponding to this threshold ($v_s \simeq 1.5\times10^{-3}$ cm/sec) is in very good agreement with the critical value for the creation of a quantized vortex ring of radius equal to the external radius of the channel ($v_s = 1.7\times10^{-3}$ cm/sec) gives support to this interpretation. Since the second threshold has a temperature dependence which corresponds to a constant Reynolds number, one may suggest that it indicates a distribution of vorticity in the whole volume of the channel such that the two fluids, being strongly coupled through mutual friction, may show instability with respect to turbulence of the fluid as a whole. The results on the positive ions tell us the behavior of the normal fluid when there is vorticity. We do not observe any hysteresis phenomena in this case, and the results are completely reproducible. The deflection of the ionic beam gives the normal-fluid velocity directly. The results indicate the existence of a gradient in the flow direction. There is no clear evidence of the threshold for instability to turbulence but, as in the case of a classical fluid, this may be revealed by a change in the analytical dependence of the velocity on the heat input. In the case of mutual-friction flow, this dependence is of a complex nature and hence the inflection point, which would indicate the onset of turbulence in the whole fluid, could be obscured. The fact that the temperature gradient in the flow direction is not uniform is directly connected with the existence of a normal-fluid velocity gradient, since in He II the heat flows with the normal fluid, and may thus be used as an independent test of the behavior of the normal-fluid velocity. The assumptions on which the mutual-friction-force expression was derived by Gorter and Mellink and by Vinen (uniformity of dT/dz and v_n) are not satisfied except perhaps in a small velocity range. Since the normal-fluid velocity and the temperature gradient vary with the position along the channel, one could try to find the relationship between velocity and temperature gradient point by point in the hope of deriving the expression for the mutual-friction force. Some preliminary results along this line show that the temperature gradient is proportional to v_n^3 and the proportionality constant depends on the channel position. More work is in progress; however, this result may be interpreted as an indication that there is no empirical necessity for an additional term in the hydrodynamical equations describing the mutual friction, since in the presence of vorticity in the superfluid, the two velocity fields are no longer independent and the higher order terms in the equations (being no longer negligible) may explain the experimental results. For instance the thermohydrodynamical equations given by Galiasevich[24] contain some friction terms which may give instability at large velocities. This has been independently suggested by a Leyden[22] experiment which, indicating that the pressure gradient for large velocities follows Blasius's empirical relation for a classical turbulent fluid, has shown that the flow may be described better by new solutions of the equations rather than by additional terms. While the pressure gradient must be sensitive to the new energy dissipation which accompanies the creation of vorticity, the temperature gradient reacts to it only indirectly through the normal-fluid velocity. This fact may explain the disagreement found experimentally by various authors on the critical velocity values, since in order to observe the threshold for the creation of vorticity in the superfluid, one must be very careful which phenomenon is being observed.

ACKNOWLEDGMENTS

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[24] Z. Galiasevich, Institute for Nuclear Research, Dubna, 1961 (unpublished report).