Three-Particle Final States and Unitary Symmetry

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A large number of new predictions of $SU(3)$ are obtained for processes with three outgoing particles. All reactions involved in these relations are scattering of charged particles on protons with no more than one neutral particle in the final state. They can all be measured by the present experimental techniques. A detailed comparison between the available data and the predictions is presented, whenever possible. In a few cases, disagreement is found; but when we allow symmetry-breaking contributions within the matrix elements which describe the processes, the data are always consistent with the predictions. The general problem of comparing the predictions of a broken symmetry with scattering data is discussed, and various possibilities for performing such comparisons are suggested.

1. INTRODUCTION

In view of the large number of predictions which have been derived from the $SU(3)$ symmetry scheme concerning scattering processes, it is surprising that so few of them have already been compared with the available experimental data.[1] Going up and up in the ladder of higher symmetries, we always have to remember that the lowest of "higher symmetries" namely that of $SU(3)$, is broken, and its predictions are often found to be in contradiction with scattering data. This is not surprising at all, since we know that mass differences within the multiplets are quite large. However, the size of the symmetry-breaking effects in scattering processes is an extremely important physical quantity which is relevant, for example, to every prediction of $SU(6)$ or its various relativistic generalizations. We are not allowed to test any symmetry scheme which includes $SU(3)$ in a domain in which the exact $SU(3)$ fails to explain experimental facts. We are not attempting here to discredit $SU(3)$; on the contrary, we emphasize that there is no single known case in which an appropriately broken $SU(3)$ symmetry is in contradiction with experiment. However, if the exact $SU(3)$ is not a good approximation for certain processes, the higher symmetries should not be tested by these same processes before the effects of the $SU(3)$ symmetry-breaking interaction are included in the calculation.

It is therefore extremely important to continue comparing new and old $SU(3)$ predictions with the gathering experimental information and to indicate whether certain physical phenomena can be described, even approximately, by an exact $SU(3)$ symmetry.

It is our purpose in this paper to present a large number of new relations between cross sections of scattering processes with three-body final states, and to compare these predictions with the known data. We also make a few remarks concerning the question of taking into account the kinematical effects of mass splitting within the unitary multiplets, while comparing both the exact and the broken $SU(3)$ with experiment.

In Sec. 2 we present the predictions of the exact symmetry for meson-baryon reactions in which one meson is produced. The method of comparing these predictions with the data is discussed in Sec. 3, while in Sec. 4 we present the actual comparison. In Sec. 5 we discuss the possibility of introducing the symmetry-breaking effects into the matrix elements of the processes, and derive some relations that are satisfied by the broken symmetry. These are shown to be consistent with the data in all cases. Proton-proton collisions and other reactions with three outgoing particles are discussed in Sec. 6. Our results and conclusions are summarized in Sec. 7.

2. ONE-MESON PRODUCTION IN MESON-BARYON REACTIONS AND $SU(3)$

We denote the octets of pseudoscalar mesons, vector mesons, and baryons by M, V, and B, respectively, and focus our attention to one-meson production reactions of the form

$$M^{\pm}+p\rightarrow M+M+B\,, \tag{1}$$

$$M^{\pm}+p\rightarrow M+V+B\,, \tag{2}$$

$$M^{\pm}+p\rightarrow V+V+B\,. \tag{3}$$

We discuss reactions of type (1) in detail and then apply our results with some minor modifications to reactions (2) and (3). We shall use the U-spin technique[2] and the following convenient notations:

M^- is the U-spin doublet of negatively charged mesons (π^-, K^-);

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[1] For a review on this subject see, e.g., H. Harari, in Proceedings of the Seminar on High Energy Physics, Trieste, 1965 (to be published).

[2] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters 10, 361 (1963).

M^+ is the U-spin doublet of positively charged mesons (K^+, π^+);

M^0 is the U-spin *triplet* of neutral mesons $(K^0, m^0, \bar{K}^0)$, where m^0 is the $U=1$ combination of π^0 and η:

$$|m^0\rangle = \tfrac{1}{2}\sqrt{3}\,|\eta\rangle + \tfrac{1}{2}\,|\pi^0\rangle. \quad (4)$$

V^-, V^+, V^0 and B^-, B^+, B^0 are the appropriately charged U-spin multiplets in the V and B octets. The B^0 triplet is (n, b^0, Ξ^0) where

$$|b^0\rangle = \tfrac{1}{2}\sqrt{3}\,|\Lambda\rangle + \tfrac{1}{2}\,|\Sigma^0\rangle. \quad (5)$$

The reactions (1) may be classified into five different sets according to their U-spin properties:

$$M^- + p \to M^- + M^0 + B^+, \quad (6)$$

$$M^- + p \to M^- + M^+ + B^0, \quad (7)$$

$$M^- + p \to M^+ + M^0 + B^-, \quad (8)$$

$$M^+ + p \to M^+ + M^0 + B^+, \quad (9)$$

$$M^+ + p \to M^+ + M^+ + B^0. \quad (10)$$

Every set includes ten experimentally feasible processes. However, while using U-spin language these are reduced to seven processes where m^0 and b^0 are inserted in the equations instead of the physical particles.

The ten processes for each set are listed below:

(6a′) $\pi^- + p \to \pi^- + \pi^0 + p$ (6a″) $\pi^- + p \to \pi^- + \eta + p$	(6a) $\pi^- + p \to \pi^- + m^0 + p$,
(6b) $\pi^- + p \to K^- + K^0 + p$,	
(6c) $\pi^- + p \to \pi^- + K^0 + \Sigma^+$,	
(6d) $K^- + p \to \pi^- + \bar{K}^0 + p$,	
(6e′) $K^- + p \to \pi^- + \pi^0 + \Sigma^+$ (6e″) $K^- + p \to \pi^- + \eta + \Sigma^+$	(6e) $K^- + p \to \pi^- + m^0 + \Sigma^+$,
(6f′) $K^- + p \to K^- + \pi^0 + p$ (6f″) $K^- + p \to K^- + \eta + p$	(6f) $K^- + p \to K^- + m^0 + p$,
(6g) $K^- + p \to K^- + K^0 + \Sigma^+$;	
(7a′) $\pi^- + p \to \pi^- + K^+ + \Sigma^0$ (7a″) $\pi^- + p \to \pi^- + K^+ + \Lambda$	(7a) $\pi^- + p \to \pi^- + K^+ + b^0$,
(7b) $\pi^- + p \to K^- + K^+ + n$,	
(7c) $\pi^- + p \to \pi^- + \pi^+ + n$,	
(7d) $K^- + p \to \pi^- + K^+ + \Xi^0$,	
(7e′) $K^- + p \to \pi^- + \pi^+ + \Sigma^0$ (7e″) $K^- + p \to \pi^- + \pi^+ + \Lambda$	(7e) $K^- + p \to \pi^- + \pi^+ + b^0$,
(7f′) $K^- + p \to K^- + K^+ + \Sigma^0$ (7f″) $K^- + p \to K^- + K^+ + \Lambda$	(7f) $K^- + p \to K^- + K^+ + b^0$,
(7g) $K^- + p \to K^- + \pi^+ + n$;	
(8a′) $\pi^- + p \to K^+ + \pi^0 + \Sigma^-$ (8a″) $\pi^- + p \to K^+ + \eta + \Sigma^-$	(8a) $\pi^- + p \to K^+ + m^0 + \Sigma^-$,
(8b) $\pi^- + p \to \pi^+ + K^0 + \Sigma^-$,	
(8c) $\pi^- + p \to K^+ + K^0 + \Xi^-$,	
(8d) $K^- + p \to K^+ + \bar{K}^0 + \Sigma^-$,	
(8e′) $K^- + p \to K^+ + \pi^0 + \Xi^-$ (8e″) $K^- + p \to K^+ + \eta + \Xi^-$	(8e) $K^- + p \to K^+ + m^0 + \Xi^-$,
(8f′) $K^- + p \to \pi^+ + \pi^0 + \Sigma^-$ (8f″) $K^- + p \to \pi^+ + \eta + \Sigma^-$	(8f) $K^- + p \to \pi^+ + m^0 + \Sigma^-$,
(8g) $K^- + p \to \pi^+ + K^0 + \Xi^-$;	
(9a′) $K^+ + p \to K^+ + \pi^0 + p$ (9a″) $K^+ + p \to K^+ + \eta + p$	(9a) $K^+ + p \to K^+ + m^0 + p$,
(9b) $K^+ + p \to \pi^+ + K^0 + p$,	
(9c) $K^+ + p \to K^+ + K^0 + \Sigma^+$,	

(9d) $\pi^+ + p \to K^+ + \bar{K}^0 + p$,

(9e′) $\pi^+ + p \to K^+ + \pi^0 + \Sigma^+$
(9e″) $\pi^+ + p \to K^+ + \eta + \Sigma^+$ } (9e) $\pi^+ + p \to K^+ + m^0 + \Sigma^+$,

(9f′) $\pi^+ + p \to \pi^+ + \pi^0 + p$
(9f″) $\pi^+ + p \to \pi^+ + \eta + p$ } (9f) $\pi^+ + p \to \pi^+ + m^0 + p$,

(9g) $\pi^+ + p \to \pi^+ + K^0 + \Sigma^+$;

(10a′) $K^+ + p \to K^+ + K^+ + \Sigma^0$
(10a″) $K^+ + p \to K^+ + K^+ + \Lambda$ } (10a) $K^+ + p \to K^+ + K^+ + b^0$,

(10b) $K^+ + p \to \pi^+ + K^+ + n$,

(10c) $K^+ + p \to K^+ + \pi^+ + n$,

(10d) $\pi^+ + p \to K^+ + K^+ + \Xi^0$,

(10e′) $\pi^+ + p \to K^+ + \pi^+ + \Sigma^0$
(10e″) $\pi^+ + p \to K^+ + \pi^+ + \Lambda$ } (10e) $\pi^+ + p \to K^+ + \pi^+ + b^0$,

(10f′) $\pi^+ + p \to \pi^+ + K^+ + \Sigma^0$
(10f″) $\pi^+ + p \to \pi^+ + K^+ + \Lambda$ } (10f) $\pi^+ + p \to \pi^+ + K^+ + b^0$,

(10g) $\pi^+ + p \to \pi^+ + \pi^+ + n$.

In all these processes we have two $U=\frac{1}{2}$ objects in the initial state. These can react through two possible channels: $U=0$ and $U=1$. In the final state we always find a U-spin triplet and two U-spin doublets. These three multiplets can be coupled in two different ways to the $U=1$ channel and in a unique way to the $U=0$ state. The total number of independent amplitudes for each set is, hence, three, and we denote them by A_0, A_{01}, and A_{11}, where A_0 is the $U=0$ amplitude; A_{01} is the $U=1$ amplitude in which the two U-spin doublets in the final state are coupled to a $U=0$ state, and A_{11} is the $U=1$ amplitude for which the same U-spin doublets are coupled to a $U=1$ "intermediate state." From the U-spin point of view the four sets of processes (6)–(9) are completely equivalent. Set (10) is slightly different as the outgoing mesons belong to identical U-spin doublets and satisfy some additional symmetry properties. Notice that when we integrate over all angles, processes (10b) and (10c) and similarly (10e) and (10f) become equivalent. We shall come back to this point later.

We now express the amplitudes $A(\mathrm{a})$, $A(\mathrm{b})$, $\cdots$, $A(\mathrm{g})$ for any set of the processes (6)–(9), in terms of the three independent U-spin amplitudes A_0, A_{01}, A_{11}:

$$\begin{aligned} A(\mathrm{a}) &= (1/\sqrt{2})A_{11}, \\ A(\mathrm{b}) &= -\tfrac{1}{2}A_{11} - (1/\sqrt{2})A_{01}, \\ A(\mathrm{c}) &= -\tfrac{1}{2}A_{11} + (1/\sqrt{2})A_{01}, \\ A(\mathrm{d}) &= -(1/\sqrt{6})A_0 + \tfrac{1}{2}A_{11}, \\ A(\mathrm{e}) &= (1/2\sqrt{3})A_0 + \tfrac{1}{2}A_{01}, \\ A(\mathrm{f}) &= (1/2\sqrt{3})A_0 - \tfrac{1}{2}A_{01}, \\ A(\mathrm{g}) &= -(1/\sqrt{6})A_0 - \tfrac{1}{2}A_{11}. \end{aligned} \tag{11}$$

For each set we may obtain four independent equalities among the seven amplitudes. Unfortunately, these cannot be translated into equalities among cross sections since the arbitrary phases include not only the relative phases between A_0, A_{01}, and A_{11} but also some unknown relative Σ^0-Λ and π^0-η phases within the appropriate U-spin triplets. One possible set of equalities among the seven amplitudes is, for example,

$$\sqrt{2}A(\mathrm{a}) + A(\mathrm{b}) + A(\mathrm{c}) = 0, \tag{12}$$

$$\sqrt{2}A(\mathrm{a}) + A(\mathrm{g}) - A(\mathrm{d}) = 0, \tag{13}$$

$$A(\mathrm{d}) + A(\mathrm{g}) + \sqrt{2}A(\mathrm{e}) + \sqrt{2}A(\mathrm{f}) = 0, \tag{14}$$

$$A(\mathrm{c}) + A(\mathrm{d}) + \sqrt{2}A(\mathrm{f}) = 0. \tag{15}$$

Note that for $x=$ (a), (e), (f) we have

$$A(x) = \tfrac{1}{2}A(x') + \tfrac{1}{2}\sqrt{3}A(x''). \tag{16}$$

In Sec. 4 we present the explicit forms of some of these useful relations.

We now turn our attention to the reactions (10a)–(10g) where the two pseudoscalar mesons belong to identical U-spin doublets. Equations (11)–(16) are still satisfied by these processes. However, the required total symmetry of the wave functions of the two "identical" mesons gives us additional information: namely, there are no interference terms between the $U=0$ and $U=1$ amplitudes of the two mesons in the final state. This reduces the number of real independent parameters for the total cross sections from five to four. However, we are, again, unable to derive equalities among cross sections because of the unknown relative phases between the different contributions to amplitudes of the type (16). Moreover, it turns out that the present amount of experimental information for processes (10) is so small that we cannot test our predictions.

If we replace *both* pseudoscalar mesons in all final states of the processes (6)–(10) by pairs of vector mesons [reaction of type (3)] we find that all our arguments are still valid without any modifications and both the relations (11)–(16) among the amplitudes and the additional conditions for reactions (10) are still predicted. However, if we replace only one pseudoscalar by a vector [reactions of type (2)] we lose the additional symmetry of processes (10) and we obtain the same predictions for all five sets of reactions, with no *a priori* vanishing interference terms.

Experimentally, most reactions of types (2) and (3) have not yet been studied to an extent which enables us to compare the data with our predictions.

We remain, effectively, with type-(1) processes and in particular with the sets of reactions (6)–(9). We discuss these in detail in Sec. 4.

3. HOW TO COMPARE THE PREDICTIONS OF AN APPROXIMATE SYMMETRY WITH EXPERIMENTAL DATA

In the absence of mass differences within a given multiplet of a symmetry scheme, we can interpret the predictions of the symmetry either as relations between the squared matrix elements which describe the relevant processes, or as relations between the actual cross sections and decay rates as measured in the laboratory. The kinematical factors (including a phase-space factor for the final state and the momentum of the incoming particle) are, in this case, the same for all processes.

In most actual cases, however, we deal with multiplets in which large mass differences occur ($\eta-\pi$, $\Xi-N$, etc.) and the kinematical factors are capable of enhancing certain cross sections, while the appropriate matrix element remains relatively small. In addition, the matrix elements themselves may contain large contributions of a symmetry-breaking interaction. We will discuss this last possibility in Sec. 5. In the present section we assume that the matrix elements do satisfy the requirements of the exact symmetry and we analyze the other possible effects of the symmetry breaking.

It should be emphasized that every discussion of this problem must be model-dependent. In principle, we could multiply every physical quantity by $m(K)/m(\pi)$, for example, without being inconsistent with the symmetry. Clearly, we can get anything we want by using such a ridiculous procedure. What we try to do here, however, is to establish a plausible (though definitely not unique) procedure of comparing the data with the theoretical predictions.

The idea is, naturally, to try to remove from the experimental results all *obvious* effects of the mass differences within the unitary multiplets. While doing this we must take into account the following factors.

1. Phase-space corrections. For processes with two-body final states, all we have to do is to calculate a "corrected" cross section by dividing the experimental result by an appropriate phase-space factor. The "corrected" $\bar{\sigma}$ will be given by

$$\bar{\sigma}=E^2(P_{\rm in}/P_{\rm out})\sigma\,, \tag{17}$$

where σ is the measured cross section, $P_{\rm in}$, $P_{\rm out}$, and E are the initial momentum, final momentum, and total energy in the center-of-mass (c.m.) system, respectively, and $\bar{\sigma}$ is proportional (but not equal) to the squared matrix element for the process.

In the case of a three-body final state the situation is much more complicated. In the absence of two-body resonances in the final state, we may introduce a phase-space factor, analogous to the previous one, which is of the form

$$\bar{\sigma}=\frac{M_T P_{\rm lab}}{F_3}\sigma\,, \tag{18}$$

where

$$F_3=\int_0^{p_3(\max)}\frac{4\pi p_3^2}{2E_3}dp_3\frac{\{[E^2+m_3^2-2EE_3-(m_1-m_2)^2][E^2+m_3^2-2EE_3-(m_1+m_2)^2]\}^{1/2}}{2(E^2+m_3^2-2EE_3)}\,, \tag{19}$$

and

$$p_3(\max)=\frac{1}{2E}\{[E^2-(m_1+m_2-m_3)^2][E^2-(m_1+m_2+m_3)^2]\}^{1/2}. \tag{20}$$

m_1, m_2, m_3 are the masses of the three outgoing particles; p_3 and E_3 are the momentum and total energy of the particle with mass m_3 in the c.m. system, $P_{\rm lab}$ is the initial momentum in the laboratory system, and M_t is the target mass.

Usually, however, Eqs. (18)–(20) do not give the correct phase-space factor. This is due to the presence of resonances in the various possible two-body systems in the final three-body state. A correct procedure in this case would be either to remove all resonant events from the sample or to use a corrected version of Eq. (19) in which some Breit-Wigner–shaped resonance terms are taken into account. Both procedures require a clear experimental determination of the resonant events. Moreover, we may face the usual problems of resolving systems which resonate in competing channels, and of taking into account interference terms between resonances, etc. All this can presumably be solved when enough data are available. However, with our present amount of data for sets of processes such as reactions

(6)–(9), we are forced to follow a less accurate procedure. We calculate the phase-space corrections, in all cases, according to Eqs. (18)–(20), and estimate the error which could be introduced by such an approximation if, say, 50% of the events in a given $M+M+B$ channel are really events of the type $R+B$ where R is an M-M resonance. It turns out that for energies above 3 BeV the introduced errors are always of the order of less than 5%, whereas for lower energies they may be as large as 20% in a few cases. Our comparisons are, therefore, valid within 20% of the given values for the corrected cross sections.

2. We now turn our attention to another difficult problem: How should we compare these $\bar{\sigma}$ values? In principle, the results of the exact symmetry should be valid for any value of s and t (the usual Mandlestam variables) provided that we take the same s and t for all processes which are involved in a given relation. In the extreme case of complete mass degeneracy we may use any kinematical parameter (such as E, $P_{\rm in}$, $P_{\rm lab}$, Q, $E_{\rm kin}$, etc.) as the basis of our comparison and the results will be the same. However, in our actual problem we know that if we use, e.g., the same E values, $P_{\rm in}$ will be different and vice versa. We have to decide which kinematical parameter will be the basis for our comparison.

In order to do this we observe the following experimental facts:

(a) Various processes show threshold effects. This

TABLE I. Experimental data for processes (6): $M^-+B^+\rightarrow M^-+M^0+B^+$.

Process	$P_{\rm lab}$ (BeV/c)	E (BeV)	Q (BeV)	σ (mb)	F_3	$R=\sqrt{\bar{\sigma}}$	Ref.
(6a′) $\pi^-+p\rightarrow\pi^-+\pi^0+p$	0.48	1.35	0.14	0.31	0.01	4.1	4
	0.68	1.49	0.28	3.99 ±0.5	0.09	5.5 ±0.3	5
	0.74	1.52	0.31	4.98 ±0.54	0.12	5.5 ±0.2	6
	1.04	1.68	0.47	6.5 ±0.5	0.40	4.1 ±0.2	7
	1.105	1.74	0.53	4.3 ±0.3	0.44	3.3 ±0.1	7
	1.14	1.75	0.54	5.3 ±1.2	0.49	3.5 ±0.4	8
	1.45	1.90	0.69	6.2 ±0.9	0.80	3.3 ±0.3	9
	1.59	1.97	0.76	4.48 ±0.15	0.98	2.7 ±0.1	10
	2.75	2.45	1.24	2.8 ±0.1	2.65	1.70±0.04	11
	4	2.89	1.68	2.2 ±0.1	4.85	1.32±0.03	12
	10	4.43	3.22	0.47 ±0.09	16.8	0.52±0.05	13
(6a″) $\pi^-+p\rightarrow\pi^-+\eta+p$	1.275	1.81	0.18	0.005±0.002	0.06	0.3 ±0.1	14
	2.03	2.16	0.53	0.031±0.01	0.57	0.33±0.05	15
	2.12	2.21	0.58	0.037±0.015	0.64	0.35±0.07	16
	4	2.89	1.26	⩽0.16 ±0.07	3.15	⩽0.45±0.2	17
(6b) $\pi^-+p\rightarrow K^-+K^0+p$	2.7	2.43	0.50	0.081	0.6	0.6	18
	3	2.61	0.68	0.066±0.016	1.1	0.42±0.05	19
	4.65	3.09	1.16	0.02	3.1	0.16	20
	6.1	3.50	1.57	0.6 ±0.37	5.3	0.8 ±0.3	21
	10	4.43	2.50	0.2 ±0.03	12.6	0.40±0.03	22
	11.6	4.75	2.82	1.26 ±0.7	15.5	1.0 ±0.3	21
	18.1	5.9	3.97	1.06 ±0.39	25	0.88±0.13	21
(6c) $\pi^-+p\rightarrow\pi^-+K^0+\Sigma^+$	1.9	2.11	0.28	0.016±0.005	0.12	0.5 ±0.1	23
	2.01	2.16	0.33	0.03 ±0.009	0.2	0.56±0.09	24
	2.1	2.20	0.37	0.038±0.010	0.26	0.55±0.07	25
	2.7	2.43	0.60	0.051	0.67	0.45	18
	3	2.54	0.71	0.044±0.010	0.96	0.37±0.04	19
	4.65	3.09	1.26	0.03	3.1	0.66	20
	10	4.43	2.60	0	12.4	0	26
(6d) $K^-+p\rightarrow\pi^-+\bar{K}^0+p$	0.85	1.72	0.14	0.1 ±0.06	0.04	1.5 ±0.5	27
	1.05	1.79	0.21	0.78 ±0.11	0.08	3.2 ±0.2	28
	1.15	1.85	0.27	2 ±0.3	0.15	3.9 ±0.3	29
	1.32	1.93	0.35	1.44 ±0.12	0.26	2.7 ±0.2	28
	1.42	1.98	0.40	1.81 ±0.15	0.35	2.7 ±0.2	28
	1.50	2.02	0.44	1.73 ±0.12	0.41	2.5 ±0.1	28
	1.6	2.07	0.49	1.85 ±0.2	0.46	2.5 ±0.1	28
	1.695	2.1	0.52	2.24 ±0.25	0.58	2.6 ±0.1	28
	2.24	2.33	0.75	1.36	1.18	1.6	30
	3	2.61	1.03	1.49 ±0.3	2.1	1.5 ±0.2	31
(6e′) $K^-+p\rightarrow\pi^-+\pi^0+\Sigma^+$	0.62	1.61	0.15	0.1 ±0.2	0.02	$0\leqslant R\leqslant 3.5$	27
	0.76	1.67	0.21	0.8 ±0.2	0.06	3.2 ±0.4	27
	0.85	1.73	0.27	0.5 ±0.1	0.09	2.3 ±0.3	27
	1.15	1.85	0.39	1 ±0.2	0.23	2.2 ±0.2	29
	1.22	1.89	0.43	0.5 ±0.04	0.28	1.5 ±0.1	33
	1.47	2.00	0.54	1.2 ±0.2	0.45	2.0 ±0.2	32
	1.51	2.02	0.56	0.93 ±0.07	0.49	1.7 ±0.1	33
(6f′) $K^-+p\rightarrow K^-+\pi^0+p$	0.76	1.67	0.10	0.15 ±0.1	0.01	2.8 ±0.9	27
	0.85	1.73	0.16	0.3 ±0.1	0.035	2.7 ±0.5	27
	1.15	1.85	0.28	1 ±0.3	0.16	2.7 ±0.9	29
	1.47	2	0.43	2 ±0.3	0.4	2.7 ±0.2	32

suggests that we should try to compare different processes in a way which guarantees that all thresholds will coincide. We also prefer to compare different amplitudes when they are all in the physical region rather than comparing two processes while one of them is in the physical region and the other under threshold.

(b) Resonances in the s channel may greatly enhance the total cross sections of some processes. We prefer to find corresponding resonances for various processes in the same point on our graph. This will happen for the same values of E for processes with identical initial states, and will lead to a shift by an energy difference Δ for reactions with different initial states, where Δ is the mass difference between corresponding resonances in the two channels, which belong to the same $SU(3)$ representation.

(c) Various processes show forward or backward peaks in their angular distributions. While comparing angular distributions for such processes, we prefer to have these peaks at the same points for all processes, assuming that they are due to similar mechanisms in the compared reactions. This suggests that we shall compare such angular distributions on the basis of equal $\cos\theta$ rather than equal momentum transfer t, as the minimal t for different processes might have completely different values.

It is clearly impossible to satisfy requirements (a) and (b) simultaneously. In order to have all thresholds at the same point we have to choose either the Q value[3] (defined as $E-\sum_{\rm final}m_i$) or $P_{\rm out}$ as our parameter. Both have, however, certain disadvantages: Q vanishes at threshold only for reactions in which $\sum_{\rm final}m_i - \sum_{\rm initial}m_i \geqslant 0$. This is the case for most of our processes (6)–(10) and for most two-body final states, but it is not true in general. On the other hand, $P_{\rm out}$ which must vanish at threshold is uniquely defined only for two-body final states and can be generalized to the many-body case in several ways. We shall, therefore, prefer to use Q in our case, though not necessarily in general.

In order to satisfy requirement (b) we must use E or $E-\Delta$, as we have already pointed out. In the case of identical initial states there are no complications. The same resonance will appear in all channels. However, even in this case it is, of course, impossible to use E and Q simultaneously. It looks as if we should prefer E in cases in which strong resonances are involved while Q or $P_{\rm out}$ appears to be a better choice where strong threshold effects are present. In Sec. 4, we present both E and Q comparisons for our relevant predictions.

As for resonances in reactions with different initial states, we cannot say very much before we know how to classify all the relevant s-channel resonances into the representations of $SU(3)$. Only then we will be able to calculate Δ, the energy shift between different peaks which belong to the same representation.

[3] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

One fact is, however, completely clear: the only region in which all these ambiguities will be irrelevant is the region of high values of s and t, such that the mass differences are negligible with respect to any relevant kinematical parameter. Unfortunately, the only experimental information which is available for such s and t values indicates that all hitherto determined cross sections are extremely small.

4. THE REACTIONS $M+B\to M+M+B$. COMPARISON WITH EXPERIMENTAL DATA

A. $M^-+B^+\to M^-+M^0+B^+$

The available experimental data for reactions (6) are presented in Table I.[4–33] We define

$$R(ab/cde)=\sqrt{\bar\sigma}(a+b\to c+d+e)\,, \qquad (21)$$

R is, of course, proportional to the absolute value of the scattering amplitude.

We obtain the following relations which can be tested:

$$R(\pi^-p\,|\,\pi^-\pi^0p)\leqslant\sqrt{3}R(\pi^-p\,|\,\pi^-\eta p)+\sqrt{2}R(\pi^-p\,|\,K^-K^0p)+\sqrt{2}R(\pi^-p\,|\,\pi^-K^0\Sigma^+)\,, \qquad (22)$$

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[18] D. H. Miller *et al.* (unpublished).
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[26] A. Bigi *et al.*, Nuovo Cimento **33**, 1265 (1964).
[27] P. L. Bastien *et al.*, Phys. Rev. Letters **10**, 188 (1963).
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[30] L. Bertanza *et al.*, in *Proceedings of the International Conference on High Energy Physics at CERN, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 284.
[31] R. Barloutand *et al.*, Phys. Letters **12**, 352 (1964).
[32] W. A. Cooper *et al.*, in *Proceedings of the International Conference on High Energy Physics at CERN, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 298.
[33] M. H. Alston *et al.*, in *Proceedings of the International Conference on High Energy Physics at CERN, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 311.

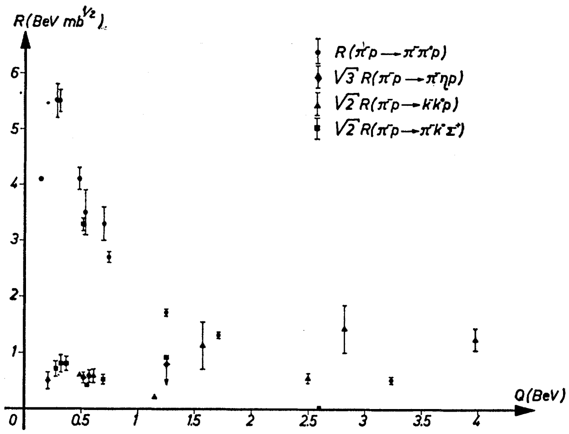

FIG. 1. Experimental values of $R(M^-B^+|M^-M^0B^+)$ for reactions (6a′), (6a″), (6b), (6c) as a function of Q.

$$R(\pi^-p|\pi^-\pi^0p)\leqslant\sqrt{2}R(K^-p|\pi^0\bar{K}^0p)+\sqrt{3}R(\pi^-p|\pi^-\eta p)+\sqrt{2}R(K^-p|K^-K^0\Sigma^+),\quad(23)$$

$$\sqrt{2}R(K^-p|\pi^-\bar{K}^0p)\leqslant R(K^-p|K^-\pi^0p)+R(K^-p|\pi^-\pi^0\Sigma^+)+\sqrt{3}R(K^-p|K^-\eta p)+\sqrt{3}R(K^-p|\pi^-\eta\Sigma^+)+\sqrt{2}R(K^-p|K^-K^0\Sigma^+),\quad(24)$$

$$R(K^-p|K^-\pi^0p)\leqslant\sqrt{2}R(\pi^-p|K^-K^0p)+R(K^-p|\pi^-\pi^0\Sigma^+)+\sqrt{3}R(K^-p|K^-\eta p)+\sqrt{3}R(K^-p|\pi^-\eta\Sigma^+)+\sqrt{2}R(\pi^-p|\pi^-K^0\Sigma^+).\quad(25)$$

Relation (22) is compared with the data in Figs. 1 and 2, in which we present Q and E plots of R. It is obvious that the Q comparison, in which all thresholds coincide but resonances do not, shows clear contradiction between the data and (22), at least for $Q\leq 1$ BeV. However, as illustrated in Fig. 2, where all resonances coincide but the thresholds do not, the discrepencies are mainly due to resonances in reaction (6a′) which are under threshold for (6b) and (6c) and are just above threshold for (6a″).

For $E>2$ BeV the situation is much improved and the disagreement disappears.

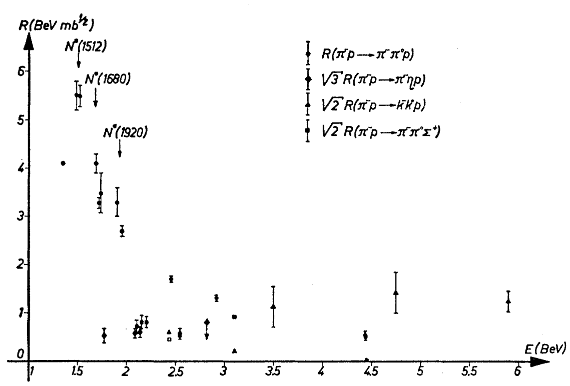

FIG. 2. Experimental values of $R(M^-B^+|M^-M^0B^+)$ for reactions (6a′), (6a″), (6b), (6c) as a function of the c.m. energy E.

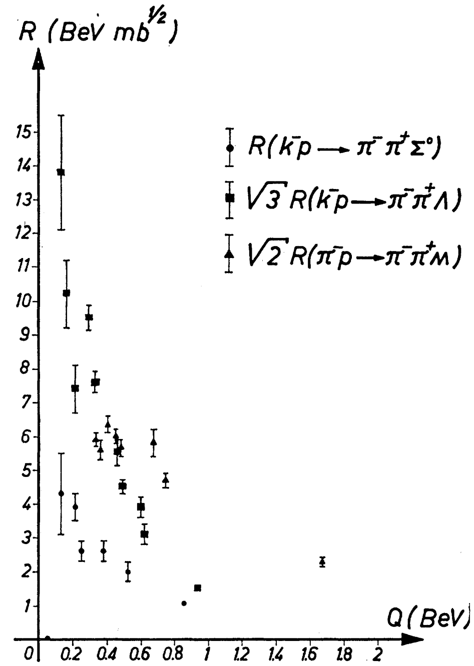

FIG. 3. Experimental values of $R(M^-B^+|M^-M^+B^0)$ for reactions (7c), (7e′), (7e″) as a function of Q.

Relations (23), (24), and (25) are clearly satisfied by the data in Table I. In all these cases, some of the processes involved have not yet been measured but the inequalities are satisfied for an arbitrary value of these still-to-be-measured cross sections.

B. $M^-+B^+\to M^-+M^++B^0$

The experimental data for reactions (7) are presented in Table II.[34–38] The predictions which can be tested at present are

$$\sqrt{3}R(K^-p|\pi^-\pi^+\Lambda)\leqslant R(K^-p|\pi^-\pi^+\Sigma^0)+\sqrt{2}R(\pi^-p|\pi^-\pi^+n)+\sqrt{2}R(\pi^-p|K^-K^+n)+R(K^-p|K^-K^+\Sigma^0)+\sqrt{3}R(K^-p|K^+K^-\Lambda),\quad(26)$$

$$R(\pi^-p|\pi^-\pi^+n)\leqslant R(\pi^-p|K^-K^+n)+(\tfrac{1}{2}\sqrt{6})R(\pi^-p|\pi^-K^+\Lambda)+\tfrac{1}{2}\sqrt{2}R(\pi^-p|\pi^-K^+\Sigma^0),\quad(27)$$

$$R(\pi^-p|\pi^-\pi^+n)\leqslant R(K^-p|\pi^-K^+\Xi^0)+\tfrac{1}{2}\sqrt{2}R(K^-p|K^-K^+\Sigma^0)+(\tfrac{1}{2}\sqrt{6})R(K^-p|K^-K^+\Lambda).\quad(28)$$

Equation (26) is satisfied by the data as is clear from the table and illustrated by Fig. 3.

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TABLE II. Experimental data for processes (7): $M^-+B^+ \to M^-+M^++B^0$.

Process	P_{lab} (BeV/c)	E (BeV)	Q (BeV)	σ (mb)	F_3	$R=\sqrt{\sigma}$	Ref.
(7a′) $\pi^-+p\to\pi^-+K^++\Sigma^0$	2.7	2.43	0.60	0.075	0.7	0.55	18
(7a″) $\pi^-+p\to\pi^-+K^++\Lambda$	2.01	2.16	0.41	0.072±0.012	0.31	0.68±0.06	24
	2.7	2.43	0.68	0.097	0.9	0.3	18
(7b) $\pi^-+p\to K^-+K^++n$	2.7	2.43	0.50	0.084	0.57	0.65	18
	4.65	3.09	1.16	0.11	3.13	0.40	20
(7c) $\pi^-+p\to\pi^-+\pi^++n$	0.82	1.56	0.34	3.8 ±0.2	0.18	4.15±0.10	34
	0.87	1.59	0.37	4 ±0.3	0.22	4.0 ±0.2	34
	0.94	1.63	0.41	5.8 ±0.3	0.27	4.5 ±0.1	34
	1.00	1.68	0.46	6.1 ±0.4	0.34	4.2 ±0.1	34
	1.06	1.71	0.49	6.1 ±0.3	0.4	4.0 ±0.1	34
	1.14	1.75	0.53	10.4 ±1.8	0.5	4.9 ±0.4	8
	1.45	1.90	0.68	9.2 ±1.4	0.8	4.1 ±0.3	9
	1.59	1.97	0.75	6.45 ±0.17	0.97	3.3 ±0.1	13
	4	2.89	1.67	3.16 ±0.13	4.8	1.63±0.04	11
(7e′) $K^-+p\to\pi^-+\pi^++\Sigma^0$	0.434	1.53	0.06	0	···	0	35
	0.62	1.61	0.14	0.3 ±0.15	0.01	4.3 ±1.2	27
	0.76	1.69	0.22	0.8 ±0.15	0.04	3.9 ±0.4	27
	0.85	1.73	0.26	0.7 ±0.15	0.09	2.6 ±0.3	27
	1.15	1.86	0.39	1 ±0.2	0.22	2.6 ±0.3	29
	1.47	2.00	0.53	1.2 ±0.3	0.43	2.0 ±0.3	32
	2.24	2.33	0.86	0.46	0.95	1.04	30
(7e″) $K^-+p\to\pi^-+\pi^++\Lambda$	0.434	1.53	0.14	1.5 ±0.4	0.01	8. ±1	35
	0.513	1.56	0.17	2 ±0.4	0.03	5.9 ±0.6	35
	0.62	1.61	0.22	1.7 ±0.3	0.06	4.3 ±0.4	27
	0.76	1.69	0.30	4.3 ±0.3	0.11	5.5 ±0.2	27
	0.85	1.73	0.34	3.5 ±0.3	0.15	4.4 ±0.2	27
	1.15	1.86	0.47	3.1 ±0.4	0.35	3.2 ±0.2	29
	1.22	1.89	0.50	2.2 ±0.2	0.40	2.6 ±0.1	36
	1.47	2.00	0.61	2 ±0.3	0.59	2.25±0.15	32
	1.51	2.02	0.63	1.4 ±0.3	0.64	1.8 ±0.2	37
	2.24	2.33	0.94	0.52	1.5	0.87	30
(7f″) $K^-+p\to K^-+K^++\Lambda$	2.24	2.33	0.22	0.076±0.017	0.14	1.1 ±0.1	30
	3	2.61	0.50	0.051±0.006	0.63	0.50±0.03	38
(7g) $K^-+p\to K^-+\pi^++n$	0.76	1.69	0.12	0	···	0	27
	0.85	1.73	0.16	0.2 ±0.1	0.03	2.4 ±0.6	27
	1.15	1.86	0.29	2.1 ±0.4	0.16	3.9 ±0.4	29

TABLE III. Experimental data for processes (8): $M^-+B^+ \to M^++M^0+B^-$.

Process	P_{lab} (BeV/c)	E (BeV)	Q (BeV)	σ (mb)	F_3	$R=\sqrt{\sigma}$	Ref.
(8a′) $\pi^-+p\to K^++\pi^0+\Sigma^-$	1.69	2.02	0.19	0.017±0.004	0.05	0.8 ±0.1	39
	2.05	2.17	0.34	0.045±0.008	0.19	0.7 ±0.1	39
	2.36	2.32	0.49	0.037±0.005	0.44	0.45±0.05	39
	2.7	2.43	0.60	0.040	0.67	0.4	18
	4.65	3.09	1.26	0.06	3.08	0.3	20
(8b) $\pi^-+p\to\pi^++K^0+\Sigma^-$	1.69	2.02	0.18	0.036±0.005	0.05	1.1 ±0.1	39
	2.05	2.17	0.33	0.085±0.010	0.19	0.95±0.05	39
	2.36	2.32	0.48	0.095±0.014	0.44	0.75±0.05	39
	2.7	2.43	0.59	0.118	0.67	0.69	18
	4.56	3.09	1.25	0.04	3.05	0.25	20
	10	4.43	2.59	0.009	12.3	0.08	26
(8c) $\pi^-+p\to K^++K^0+\Xi^-$	6.8	3.69	1.38	0.0036	4.7	0.08	40
	8	3.96	1.65	0.011	5.8	0.12	40
(8e′) $K^-+p\to K^++\pi^0+\Xi^-$	1.8	2.15	0.2	0.03 ±0.008	0.06	0.95±0.15	41
	2.24	2.33	0.38	0.037±0.008	0.27	0.55±0.1	30
	3	2.61	0.66	0.02 ±0.003	0.8	0.28±0.02	38
(8f′) $K^-+p\to\pi^++\pi^0+\Sigma^-$	0.62	1.61	0.14	0.3 ±0.2	0.01	4.3 ±1.2	27
	0.76	1.69	0.22	0.8 ±0.1	0.05	3.5 ±0.3	27
	0.85	1.73	0.26	0.7 ±0.1	0.085	2.65±0.2	27
	1.15	1.86	0.39	0.8 ±0.2	0.22	2.05±0.25	29
	1.22	1.89	0.42	0.45 ±0.04	0.26	1.45±0.06	33
	1.47	2	0.53	0.9 ±0.2	0.42	1.8 ±0.2	32
	1.51	2.02	0.55	0.93 ±0.07	0.46	1.75±0.1	33
(8g) $K^-+p\to\pi^++K^0+\Xi^-$	1.8	2.15	0.19	0.075±0.013	0.06	1.5 ±0.3	41
	2.24	2.33	0.37	0.084±0.012	0.26	0.83±0.06	30
	3	2.61	0.65	0.04 ±0.005	0.8	0.39±0.02	38

Although for several processes which are included in relations (27), (28) only few experimental results are available, it is indicated that these predictions contradict the data. The one-pion production amplitude which represents the left-hand side in these two inequalities is much bigger than the strange-particle production amplitudes in the right-hand sides. This provides us with additional evidence to the already well-known fact that the low-production rates of strange particles are incompatible with the exact $SU(3)$.

C. $M^- + B^+ \to M^+ + M^0 + B^-$

The data for processes (8) are presented in Table III.[39–41] The small amount of available data does not allow us to draw definite conclusions. However, for completeness we list a few predictions concerning processes which will probably be measured in the near future:

$$R(\pi^-p|K^+\pi^0\Sigma^-) \leqslant \sqrt{3}R(\pi^-p|K^+\eta\Sigma^-) + \sqrt{2}R(K^-p|K^+\bar{K}^0\Sigma^-) + \sqrt{2}R(K^-p|\pi^+K^0\Xi^-), \quad (29)$$

$$R(K^-p|\pi^+\pi^0\Sigma^-) \leqslant R(K^-p|K^+\pi^0\Xi^-) + \sqrt{3}R(K^-p|K^+\eta\Xi^-) + \sqrt{2}R(K^-p|K^+\bar{K}^0\Sigma^-) + \sqrt{2}R(K^-p|\pi^+K^0\Xi^-), \quad (30)$$

$$R(\pi^-p|K^+\pi^0\Sigma^-) \leqslant \sqrt{3}R(\pi^-p|K^+\eta\Sigma^-) + \sqrt{2}R(\pi^-p|\pi^+K^0\Sigma^-) + \sqrt{2}R(\pi^-p|K^+K^0\Xi^-). \quad (31)$$

The preliminary results, which are available now, indicate that the process (8f′) is much stronger than any other process in this set of reactions. Consequently, we expect relation (30) to be the strongest predicted inequality. Whether it is satisfied or not has still to be determined.

D. $M^+ + B^+ \to M^+ + M^0 + B^+$

The available data for reactions (9) are presented in Table IV.[42–56] It is, again, not possible to arrive at definite conclusions, for a wide energy range. The most significant predictions which can be derived are

$$R(\pi^+p|\pi^+\pi^0p) \leqslant \sqrt{3}R(\pi^+p|\pi^+\eta p) + \sqrt{2}R(K^+p|K^+K^0\Sigma^+) + \sqrt{2}R(\pi^+p|K^+\bar{K}^0p), \quad (32)$$

$$|R(K^+p|K^+\pi^0p) - \sqrt{2}R(K^+p|\pi^+K^0p)| \leqslant \sqrt{3}R(K^+p|K^+\eta p) + \sqrt{2}R(K^+p|K^+K^0\Sigma^+), \quad (33)$$

$$R(\pi^+p|\pi^+\pi^0p) \leqslant R(\pi^+p|K^+\pi^0\Sigma^+) + \sqrt{3}R(\pi^+p|K^+\eta\Sigma^+) + \sqrt{3}R(\pi^+p|\pi^+\eta p) + \sqrt{2}R(\pi^+p|K^+\bar{K}^0p) + \sqrt{2}R(\pi^+p|\pi^+K^0\Sigma^+). \quad (34)$$

5. PREDICTIONS OF THE BROKEN $SU(3)$

Both the size of the mass splitting within the unitary multiplets and the disagreement between various $SU(3)$ predictions and the experimental data indicate that a significant symmetry-breaking interaction is present in most processes and has to be taken into account. The success of the Gell-Mann–Okubo[57] mass formula suggests that a term transforming like the $I=Y=0$ component of the octet will be introduced into our S-matrix elements for the discussed processes. This approach was previously applied to various processes,[1,58] and in all cases no discrepancies between the predictions of the broken symmetry and the data were found. As we have seen in the previous section, there are some indications that the exact symmetry cannot explain, e.g., the large production rate of the $\pi^+\pi^-n$ system as compared with K^+K^-n, $K^+\pi^-\Lambda$, and $K^+\pi^-\Sigma^0$ [relations (27), (28)]. It is hence necessary to introduce the symmetry breaking into the matrix elements, at least for some of our processes.

In order to do so we note that the octet symmetry-breaking term transforms like a combination of a U-spin scalar and a U-spin vector. Following the method of Sec. 2, all we have to do is to introduce an additional $U=1$ "spurion"[59] into all the reactions (6)–(10) and recalculate them in terms of a larger number of independent amplitudes which include the new amplitudes of the $\Delta U=1$ transition. There are six independent amplitudes for each set of processes and we obtain one equality among amplitudes which can be translated into an inequality among the directly measured quantities. Using the notation of Sec. 2 we find, for reactions (6)–(10),

$$\sqrt{2}A(\mathrm{a}) + A(\mathrm{b}) + A(\mathrm{c}) = A(\mathrm{d}) + \sqrt{2}A(\mathrm{e}) + \sqrt{2}A(\mathrm{f}) + A(\mathrm{g}), \quad (35)$$

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[59] This method was first used by Meshkov *et al.*, Ref. 58. See also, H. Harari, Ref. 1.

TABLE IV. Experimental data for processes (9): $M^+ + B \to M^+ + M^0 + B^+$.

Process	P_{lab} (BeV/c)	E (BeV)	Q (BeV)	σ (mb)	F_3	$R=\sqrt{\sigma}$	Ref.
(9a′) $K^+ + p \to K^+ + \pi^0 + p$	0.81	1.7	0.13	0.22 ±0.05	0.02	3 ±0.4	42
	1.45	2	0.43	1.95 ±0.2	0.33	2.9 ±0.2	43
(9b) $K^+ + p \to K^0 + \pi^+ + p$	0.81	1.7	0.12	0.64 ±0.08	0.015	6 ±0.4	42
	0.91	1.75	0.17	2.1 ±0.2	0.046	6.5 ±0.4	44
	1.14	1.85	0.27	4.6 ±0.3	0.15	5.9 ±0.2	45
	1.45	2	0.42	5.6 ±0.6	0.37	4.7 ±0.2	46
	2.965	2.6	1.02	2.1 ±0.2	2	1.75±0.1	47
(9d) $\pi^+ + p \to K^+ + \bar{K}^0 + p$	2.77	2.47	0.54	0.059±0.013	0.7	0.48±0.04	48
	2.35, 2.62, 2.90			0.024±0.011		~0.3	49
(9e′) $\pi^+ + p \to K^+ + \pi^0 + \Sigma^+$	2.77	2.47	0.65	0.12 ±0.021	0.75	0.67±0.05	48
(9f′) $\pi^+ + p \to \pi^+ + \pi^0 + p$	0.55	1.4	0.19	0	0.01	0	50
	0.75	1.53	0.32	3	0.15	3.9	50
	0.85	1.58	0.37	6.5	0.21	5.1	50
	1	1.67	0.46	10	0.32	5.6	50
	1.15	1.74	0.53	8	0.47	4.4	50
	1.23	1.8	0.59	10.8 ±1	0.6	4.7 ±0.2	51
	1.3	1.83	0.62	10	0.65	4.5	50
	1.41	1.88	0.67	11.9 ±1.2	0.78	6.5 ±0.4	51
	2.75	2.46	1.25	2.79 ±0.1	2.71	1.69±0.04	52
	4	2.88	1.67	2.31	4.76	1.41	53
(9g) $\pi^+ + p \to \pi^+ + K^0 + \Sigma^+$	1.04	1.7	0.07	<0.04	···	···	54
	1.17	1.76	0.13	0.069±0.01	0.02	2 ±0.15	54
	1.27	1.82	0.19	0.3	0.053	2.6	55
	1.42	1.89	0.26	0.13 ±0.45	0.145	1.15±0.15	54
	2.08	2.2	0.57	0.6 ±0.2	0.61	1.4 ±0.3	56
	2.34	2.3	0.67	0.75 ±0.15	0.8	1.5 ±0.1	56
	2.64	2.42	0.79	0.75 ±0.18	1.17	1.3 ±0.15	56
	2.9[illegible]	2.51	0.88	0.8 ±0.15	1.5	1.2 ±0.15	56
	3.43	2.69	1.06	0.3 ±0.13	2.33	0.66±0.15	56
	3.54	2.73	1.10	0.23 ±0.06	2.45	0.58±0.06	56
	4	2.88	1.25	0.04	3.15	0.23	53

or

$$A(\mathrm{a}')+\sqrt{3}A(\mathrm{a}'')+\sqrt{2}A(\mathrm{b})+\sqrt{2}A(\mathrm{c}) =\sqrt{2}A(\mathrm{d})+A(\mathrm{e}')+\sqrt{3}A(\mathrm{e}'')+A(\mathrm{f}') +\sqrt{3}A(\mathrm{f}'')+\sqrt{2}A(\mathrm{g}). \quad (36)$$

For reactions (7), for example, we obtain

$$\sqrt{2}R(\pi^-p|\pi^-\pi^+n)\leqslant R(\pi^-p|\pi^-K^+\Sigma^0) +\sqrt{3}R(\pi^-p|\pi^-K^+\Lambda)+\sqrt{2}R(\pi^-p|K^-K^+n) +\sqrt{2}R(K^-p|\pi^-K^+\Xi^0)+R(K^-p|\pi^-\pi^+\Sigma^0) +\sqrt{3}R(K^-p|\pi^-\pi^+\Lambda)+R(K^-p|K^-K^+\Sigma^0) +\sqrt{3}R(K^-p|K^-K^+\Lambda)+\sqrt{2}(K^-p|K^-\pi^+n). \quad (37)$$

This is clearly satisfied by the known data.

Relations similar to (37) for other sets of reactions are usually even weaker and they are in agreement with the data in all known cases.

6. SOME OTHER THREE-PARTICLE FINAL STATES

The methods of Sec. 2 can be applied without any modifications to processes such as one-meson production in $\bar{p}p$ and pp scattering and $\bar{p}p$ annihilation into three mesons. The following predictions are obtained for $\bar{B}BM$ final states in $\bar{p}p$ scattering:

$$R(\bar{p}p|\bar{p}p\pi^0)\leqslant\sqrt{2}R(\bar{p}p|\bar{\Sigma}^-p\bar{K}^0) +\sqrt{2}R(\bar{p}p|\bar{p}\Sigma^+K^0)+R(\bar{p}p|\bar{\Sigma}^-\Sigma^+\pi^0) +\sqrt{3}R(\bar{p}p|\bar{\Sigma}^-\Sigma^+\eta)+\sqrt{3}R(\bar{p}p|\bar{p}p\eta), \quad (38)$$

$$\sqrt{2}R(\bar{p}p|\bar{n}p\pi^-)\leqslant R(\bar{p}p|\bar{\Sigma}^0\Sigma^+\pi^-) +\sqrt{3}R(\bar{p}p|\bar{\Lambda}\Sigma^+\pi^-)+R(\bar{p}p|\bar{\Sigma}^0pK^-) +\sqrt{3}R(\bar{p}p|\bar{\Lambda}pK^-)+\sqrt{2}R(\bar{p}p|\bar{\Xi}^0\Sigma^+K^-), \quad (39)$$

$$\sqrt{2}R(\bar{p}p|\bar{p}n\pi^+)\leqslant R(\bar{p}p|\bar{\Sigma}^-\Sigma^0\pi^+) +\sqrt{3}R(\bar{p}p|\bar{\Sigma}^-\Lambda\pi^+)+R(\bar{p}p|\bar{p}\Sigma^0K^+) +\sqrt{3}R(\bar{p}p|\bar{p}\Lambda K^+)+\sqrt{2}R(\bar{p}p|\bar{\Sigma}^-\Xi^0K^+). \quad (40)$$

In all these cases we find a large one-pion-production amplitude on the left-hand side and a sum of small strange-particle production amplitudes on the right-hand side. These relations may be regarded as the three-body version of the well-known relation for $\bar{p}p\to\bar{B}B$ processes[60]:

$$R(\bar{p}p|\bar{n}n)\leqslant\tfrac{3}{2}R(\bar{p}p|\bar{\Lambda}\Lambda) +\tfrac{1}{2}\sqrt{3}R(\bar{p}p|\bar{\Lambda}\Sigma^0)+\tfrac{1}{2}\sqrt{3}R(\bar{p}p|\bar{\Sigma}^0\Lambda) +\tfrac{1}{2}R(\bar{p}p|\bar{\Sigma}^0\Sigma^0)+\tfrac{1}{2}R(\bar{p}p|\bar{\Xi}^0\Xi^0), \quad (41)$$

[60] M. Konuma and K. Tomozawa, Phys. Rev. Letters **12**, 425 (1964).

Equation (41) is in clear disagreement with the data.[60] It is impossible to draw definite conclusions with respect to relations (38)–(40). However, it is clear that a detailed comparison with the data will be possible in the near future as all the relevant reactions can be easily detected. Relations similar to (38)–(40) can be obtained by substituting:

$$\bar{p} \to \Xi^-,\ p \to \bar{\Xi}^+,\ \bar{\Sigma}^- \to \Sigma^-,\ \Sigma^+ \to \bar{\Sigma}^+,\ \bar{\Lambda} \leftrightarrow \Lambda,\ \bar{\Sigma}^0 \leftrightarrow \Sigma^0,\ n \leftrightarrow \bar{\Xi}^0,\ \bar{n} \leftrightarrow \Xi^0$$

in all *final* states.

For the annihilation case we obtain

$$R(\bar{p}p|\pi^+\pi^-\pi^0) \leqslant \sqrt{3}R(\bar{p}p|\pi^+\pi^-\eta) + \sqrt{3}R(\bar{p}p|K^+K^-\eta) + R(\bar{p}p|K^+K^-\pi^0) + \sqrt{2}R(\bar{p}p|\pi^+K^-K^0) + \sqrt{2}R(\bar{p}p|K^+\pi^-\bar{K}^0). \quad (42)$$

For pp scattering we obtain the following prediction:

$$\sqrt{2}R(pp|pn\pi^+) \leqslant \sqrt{3}R(pp|p\Lambda K^+) + R(pp|p\Sigma^0K^+) + \sqrt{2}R(pp|\Sigma^+nK^+). \quad (43)$$

No predictions are obtained for these processes from a broken $SU(3)$.

7. DISCUSSION OF THE RESULTS

We found a large number of new predictions of both exact and broken $SU(3)$ symmetry for processes with three outgoing particles. It turns out that most of these predictions agree with the available data, although many more data are needed in order to arrive at final conclusions. It is, however, clear that in some cases the exact $SU(3)$ fails and the symmetry has to be broken in order to explain the data. In our present work, all possible contradictions appear in cases where strange-particle production amplitudes are compared with one-pion production reactions.

The data do not allow us to test the various alternative ways of comparing the predictions with experiment. In the only case where we have been able to test it, E plotting works better than Q plotting. It is expected that this will be the case in all resonant regions.

Our results once more emphasize, among other things, that $SU(3)$ symmetry breaking is a crucial factor for many scattering processes, and it has to be included in any discussion of reactions in the framework of any symmetry which includes $SU(3)$.

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