Application of the Current Commutation Relations to the N^{***} Photoproduction*

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We derive three sum rules by taking the expectation values between proton states at rest of three commutation relations of electric quadrupole and magnetic octupole operators. If we assume that the sum rules are saturated by only one intermediate state, the $I=\frac{1}{2}$, $J=\frac{5}{2}^+$ pion-nucleon resonance N^{***} (1688), we obtain a consistency relation which appears to be satisfied by experimental numbers. We then derive the ratio of the E_{3-} and M_{3-} amplitudes in single-pion photoproduction and we get a result in good agreement with experiment.

I. INTRODUCTION

RECENTLY, the applications of the algebras generated by current components, whose importance was first emphasized by Gell-Mann,[1] have become more and more interesting. The basic idea is to take the expectation value of the commutator between physical states and then insert a complete set of intermediate states in order to get sum rules that can be compared with experiment. It is now clear that the question of whether the commutator is exhausted by a few intermediate states depends very much on the frame of reference of the physical states that one considers. In fact, there are essentially two ways for obtaining sum rules: the first one is to take the expectation values of the commutators between physical states in the limit of infinite momentum, while the second method amounts to taking expectation values between physical states at rest (static approach).

The first approach was introduced by Fubini *et al.*,[2] who also suggested a dispersion-theoretic treatment of the sum rules obtained. Perhaps the most interesting application of this method is the calculation of the renormalization of the axial-vector coupling constant in β decay done by Adler[3] and Weisberger.[4]

The second approach has been used, for example, in the derivation of the static sum rules for magnetic moments first described by Dashen and Gell-Mann[5] and Lee.[6] In the derivation, these authors use commutation relations between vector current densities.

This second method has the unpleasant feature that in general one has to deal with amplitudes involving time-like photons, while the Fubini method takes into account only amplitudes which are at least in principle directly connected with experimental quantities. On the other hand, with the Fubini method, one has to face problems of convergence of certain dispersion integrals.[7] Furthermore, in this method, intermediate states of all parities and angular momenta contribute while the static approach has the good feature of the parity and angular-momentum selection rules one can apply in the choice of the intermediate states.

We believe, therefore, that assuming a simple behavior of the amplitudes in the time-like region, the static approach can still give results of some interest.

We have recently applied[8] this method to some commutation relations, between electric dipole and magnetic quadrupole moments of the vector current, derived from commutation relations between current densities. Taking the expectation value of these commutators between proton states at rest, and assuming that the resulting sum rules are saturated including only one intermediate state [the $I=\frac{1}{2}$, $J=\frac{3}{2}^-$, N^{**}(1518)], we obtained a consistency relation in good agreement with experiment. The sum rules were then expressed in terms of the off-mass-shell electromagnetic transitions between the proton and the N^{**}. Assuming simple behavior for the form factors of these amplitudes, we were able to relate them to the photoproduction amplitudes and we found a remarkable agreement with experiment.

In this paper we want to use exactly the same approach, step by step, in order to get analogous results for the electromagnetic transitions between the proton and the $I=\frac{1}{2}$, $J=\frac{5}{2}^+$, N^{***}(1688) pion-nucleon resonance. Our starting point will be again the commutation relations between current densities, Eqs. (2)–(2″) of A, namely,

$$[J_4^\alpha(\mathbf{x}),J_4^\beta(\mathbf{x}')]=if_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_4^\gamma(\mathbf{x}'),\tag{1}$$

$$[J_i^\alpha(\mathbf{x}),J_4^\beta(\mathbf{x}')]=if_{\alpha\beta\gamma}\delta(\mathbf{x}-\mathbf{x}')J_i^\gamma(\mathbf{x}')+(\text{terms symmetric in }\alpha,\beta),\tag{1'}$$

$$[J_i^\alpha(\mathbf{x}),J_j^\beta(\mathbf{x}')]=if_{\alpha\beta\gamma}\delta_{ij}\delta(\mathbf{x}-\mathbf{x}')J_4^\gamma(\mathbf{x})+(\text{terms symmetric in }\alpha,\beta).\tag{1''}$$

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[2] S. Fubini and G. Furlan, Physics **1**, 229 (1965); S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, A1171 (1965).

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[4] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965).

[5] R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142, 145 (1965).

[6] B. W. Lee, Phys. Rev. Letters **14**, 676 (1965).

[7] A detailed discussion of the convergence problems in the commutators and on the two approaches, appears in R. F. Dashen and S. C. Frautschi, Phys. Rev. (to be published), in the framework of the bootstrap theory. These authors also consider the problem, which we do not discuss in this paper, of generating approximate symmetries from the commutators.

[8] A. Biette, Phys. Rev. **142**, 1258 (1966). This work will hereafter be referred to as A.

As in A, our results will involve only the terms in $f_{\alpha\beta\gamma}$ of these commutators. In Sec. II, from (1)–(1″), we will derive three commutation relations between electric-quadrupole and magnetic-octupole operators. Proceeding then in the same way as in A, we get three sum rules by taking the expectation value of the commutators between proton states at rest.

In particular, we get a consistency relation between the sum rules if we consider only one intermediate state, the N^{***} resonance. In Sec. III we express our sum rules in terms of the CGLN[9] electroproduction amplitudes, because the p-N^{***} transition is generated by a virtual photon. As in A, we then assume the same form factor for the electric and magnetic transitions so that we are able to compare the ratio of these transitions with the ratio that we get for the correspondent transitions in the photoproduction experiments, and we find a very good agreement.

The assumptions we have made about the form factors can be justified, as in A, by dominance of the ρ meson in the form factors.

II. SUM RULES AND CONSISTENCY RELATIONS

Let us consider the electric-quadrupole operator

$$E_{ij}{}^{\alpha}=\int d^3r\,(3r_ir_j-\delta_{ij}r^2)J_4{}^{\alpha}\,, \tag{2}$$

and the magnetic octupole

$$M_{ijk}{}^{\alpha}=\int d^3r\,(5r_ir_j-\delta_{ij}r^2)(\mathbf{r}\times\mathbf{J}^{\alpha})_k\,. \tag{2'}$$

From Eqs. (1) to (1″), we have

$$[E_{ij}{}^{\alpha},E_{i'j'}{}^{\beta}]=if_{\alpha\beta\gamma}\int d^3r\,(3r_ir_j-\delta_{ij}r^2)\times(3r_{i'}r_{j'}-\delta_{i'j'}r^2)J_4{}^{\alpha}\,, \tag{3}$$

$$[M_{ijk}{}^{\alpha},M_{i'j'k'}{}^{\beta}]=if_{\alpha\beta\gamma}\int d^3r\,(5r_ir_j-\delta_{ij}r^2)\times(5r_{i'}r_{j'}-\delta_{i'j'}r^2)(\delta_{kk'}r^2-r_kr_{k'})J_4{}^{\alpha}+(\text{terms symmetric in }\alpha,\beta)\,, \tag{3'}$$

$$[M_{klm}{}^{\alpha},E_{ij}{}^{\beta}]=if_{\alpha\beta\gamma}\int d^3r\,(5r_kr_l-\delta_{kl}r^2)\times(3r_ir_j-\delta_{ij}r^2)(\mathbf{r}\times\mathbf{J}^{\gamma})_m+(\text{terms symmetric in }\alpha,\beta)\,. \tag{3''}$$

If we now take in Eqs. (3)–(3″), $\alpha=1$, $\beta=2$, $i=j=i'=j'=k=k'=l=m=3$, we have

$$[E_{33}{}^1,E_{33}{}^2]=i\int(3z^2-r^2)^2J_4{}^3\,d^3r\,, \tag{4}$$

$$[M_{333}{}^1,M_{333}{}^2]=i\int(5z^2-r^2)^2(x^2+y^2)J_4{}^3\,d^3r\,, \tag{4'}$$

$$[M_{333}{}^1,E_{33}{}^2]=i\int(5z^2-r^2)(3z^2-r^2)(\mathbf{r}\times\mathbf{J}^3)_z\,d^3r\,. \tag{4''}$$

Proceeding in the same way as in A, we take the expectation values of Eqs. (4)–(4″) between proton states at rest. In the right-hand side we then obtain higher derivatives of the isovector electric and magnetic form factors of the nucleon. From parity and angular-momentum selection rules, as in A, we can see immediately which intermediate states can be present in Eqs. (4)–(4″).

In the commutator (4), it is possible to insert only $J=\frac{3}{2}^+$ and $J=\frac{5}{2}^+$ intermediate states, in (4′) only $J=\frac{5}{2}^+$ and $J=\frac{7}{2}^+$, while in (4″) only $J=\frac{5}{2}^+$ can appear.

We know from pion-nucleon scattering experiments and from photoproduction experiments that there are resonances in the pion-nucleon system with these quantum numbers.

More precisely, there is the well-known $I=\frac{3}{2}$, $J=\frac{3}{2}^+$, $N^*(1238)$ resonance, the $I=\frac{1}{2}$, $J=\frac{5}{2}^+$, $N^{***}(1688)$ resonance, and the less well-established $I=\frac{3}{2}$, $J=\frac{7}{2}^+$ resonance at $W=1920$ MeV. We know, however, from photoproduction analysis[10] that the electric quadrupole transition between the proton and the N^* is very small.

The relative importance of the N^* intermediate-state contribution to the commutator (4) is further reduced by an additional argument. This argument will also allow us to neglect the $J=\frac{7}{2}^+$ state contribution in the commutator (4′). It runs as follows: We have seen in A that the dominance of the $J=\frac{3}{2}^-$, $N^{***}(1518)$ intermediate state in the electric-dipole and magnetic-quadrupole sum rules can be easily explained if we assume the dominance of the ρ meson in the isovector form factor. In fact, for the proton-N^{**} electromagnetic transition, the "mass" of the virtual photon involved is given by $k_0=W_R-M\cong600$ MeV, where $W_R=1518$ MeV and M is the nucleon mass. This value is close to the ρ-meson mass, so that we have an enhancement in the form factor together with the resonance in the π-N system. If we now take into account the masses of the three resonances $J=\frac{3}{2}^+$, $J=\frac{5}{2}^+$, and $J=\frac{7}{2}^+$, we see that k_0 is, respectively, 300, 750, and 980 MeV. Therefore, if we assume ρ-meson dominance, the form factor squared is about equal to 30 for the $J=\frac{5}{2}^+$, $N^{***}(1688)$ and only 1 or 2 for the other two resonances.

We can then, with some confidence, assume that from the commutation relations (4)–(4″) we get sum rules

[9] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[10] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963); Ph. Salin, *ibid.* **28**, 1294 (1963).

which are saturated only by the $J=\frac{5}{2}^+, I=\frac{1}{2}$ intermediate state. In this approximation, we obtain

$$|\langle E_{25}\rangle|^2=12\left[\frac{d^2G_E{}^V(k^2)}{(dk^2)^2}\right]_{k=0}, \tag{5}$$

$$|\langle M_{35}\rangle|^2=-144\left[\frac{d^3G_E{}^V(k^2)}{(dk^2)^3}\right]_{k=0}, \tag{5$'$}$$

$$\langle E_{25}\rangle\langle M_{35}\rangle^*=\frac{-24i}{M}\left[\frac{d^2G_M{}^V(k^2)}{(dk^2)^2}\right]_{k=0}, \tag{5$''$}$$

where

$$\langle E_{25}\rangle=\langle N^{***}|E_{33}{}^3|P\rangle, \tag{6}$$

$$\langle M_{35}\rangle=\langle N^{***}|M_{333}{}^3|P\rangle, \tag{7}$$

and, as in A, $G_{E,M}{}^V(k^2)$ are defined in terms of the $G_{E,M}{}^{P,N}$ [11]

$$G_{E,M}{}^V=G_{E,M}{}^P-G_{E,M}{}^N.$$

From the sum rules (5)–(5$''$), we have the consistency relation

$$-3\left[\frac{d^2G_E{}^V(k^2)}{(dk^2)^2}\right]_{k=0}\left[\frac{d^3G_E{}^V(k^2)}{(kd^2)^3}\right]_{k=0}=\frac{1}{M^2}\left[\frac{d^2G_M{}^V(k^2)}{(dk^2)^2}\right]^2_{k=0}, \tag{8}$$

which is the analog of Eq. (12) of A. Computing the derivatives from the fit to the experimental values of the proton and neutron form factors,[11] one gets, for Eq. (8),

$$3.1\times10^{-5}\ \mathrm{F}^{10}=2.8\times10^{-5}\ \mathrm{F}^{10}.$$

In this case, the errors, in comparison with the analogous result in A, are certainly bigger and more difficult to estimate. Nevertheless, we feel that this result furnishes a very good support for the assumption of N^{***} dominance.

III. APPLICATIONS IN PHOTOPRODUCTION

We proceed now in the same way as in A. We cannot compare our sum rules (5)–(5$''$) with the amplitudes that we have in single pion photoproduction in the region of the N^{***} resonance. In fact, as we already know, the integration over space in Eqs. (2) and (2$'$) makes the three-momenta of the proton and the N^{***} equal, so that with these two particles at rest the transition is induced by a virtual photon with $\mathbf{k}=0$ and mass $k_0=W_R-M=750$ MeV.

We then assume that the form factor is the same for the electric amplitude (6) and the magnetic (7). Therefore, as in A, we should be able to compare the ratios between the photoproduction amplitudes and those that we derive from Eqs. (5)–(5$''$). We have

$$\langle E_{25}\rangle_{ss'}=\chi_s(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}\times\left[-3\left(\frac{\partial^2}{\partial k_z{}^2}-\nabla_{\mathbf{k}}{}^2\right)j_4(\mathbf{k})\right]_{k=0}\chi_{s'},$$

$$\langle M_{35}\rangle_{ss'}=\chi_s(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}\times\left[-i\left(5\frac{\partial^2}{\partial k_z{}^2}-\nabla_{\mathbf{k}}{}^2\right)(\boldsymbol{\nabla}_{\mathbf{k}}\times\mathbf{j}(\mathbf{k}))_z\right]_{k=0}\chi_{s'},$$

where χ_s and $\chi_{s'}$ are two-component Pauli spinors; $(\mathbf{k},k_0)$ and $(\mathbf{q},\omega)$ are the photon and pion four-momenta in the c.m. system $E=(q^2+M^2)^{1/2}$, $\mathbf{j}(\mathbf{k})$, and $j_4(\mathbf{k})$ are given in A in terms of the CGLN[9] multipoles $M_{l\pm}$, $E_{l\pm}$, and $Y_{l\pm}$ of electroproduction.

We consider only states with $l=3$ and $J=\frac{5}{2}$ so that taking into account the threshold behavior of the multipoles

$$M_{3-}=m_{3-}k^3,\quad E_{3-}=e_{3-}(0)k+e_{3-}k^3,$$
$$Y_{3-}=y_{3-}(0)k+y_{3-}k^3,$$

we obtain

$$\langle E_{25}\rangle_{ss'}=(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}\frac{18e_{3-}(0)}{k_0}\chi_s\times\left[\frac{2\sigma_zq_z}{q}+\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})}{q}-\frac{5(\boldsymbol{\sigma}\cdot\mathbf{q})q_z{}^2}{q^3}\right]\chi_{s'}, \tag{9}$$

and

$$\langle M_{35}\rangle_{ss'}=(2\pi)^{-3/2}\left(\frac{M}{2\omega E}\right)^{1/2}180im_{3-}\chi_s\times\left[\frac{2q_z}{q}+\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})\sigma_z}{q}-\frac{5q_z{}^2(\boldsymbol{\sigma}\cdot\mathbf{q})\sigma_z}{q^3}\right]\chi_{s'}, \tag{10}$$

where we have used the "Feynman relation," Eq. (14) of A.

With the aid of Eqs. (9) and (10), our sum rules (5)–(5$''$) become, summing over the intermediate spin states,

$$\frac{9}{\pi^2}\int\frac{Mq\,dW}{W}\frac{|e_{3-}(0)|^2}{k_0{}^2}=\left[\frac{d^2G_E{}^V(k^2)}{(dk^2)^2}\right]_{k=0}, \tag{11}$$

$$\frac{75}{\pi^2}\int\frac{Mq\,dW}{W}|m_{3-}|^2=-\left[\frac{d^3G_E{}^V(k^2)}{(dk^2)^3}\right]_{k=0}, \tag{11$'$}$$

$$\frac{45}{\pi^2}\int\frac{Mq\,dW}{W}\frac{e_{3-}(0)}{k_0}m_{3-}{}^*=\frac{1}{M}\left[\frac{d^2G_M{}^V(k^2)}{(dk^2)^2}\right]_{k=0}, \tag{11$''$}$$

where the main contribution to the integrals comes

[11] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

when $W \simeq W_R = 1688$ MeV. The comparison with photoproduction proceeds now as in A.

In photoproduction, in the neighborhood of the N^{***} resonance, the E_{3-} and M_{3-} multipoles can be written[12]

$$E_{3-}{}^r = \frac{\epsilon_{3-}{}^r(0)k - \mu_{3-}{}^r k^3}{W^2 - W_R{}^2 - i\Gamma_R W_R}, \quad M_{3-}{}^r = \frac{\mu_{3-}{}^r k^3}{W^2 - W_R{}^2 - i\Gamma_R W_R}, \tag{12}$$

and similarly, the amplitudes in Eqs. (11)–(11″) can be written

$$e_{3-}(0) = \frac{\epsilon_{3-}(0)}{W^2 - W_R{}^2 - i\Gamma_R W_R},$$

$$m_{3-} = \frac{\mu_{3-}}{W^2 - W_R{}^2 - i\Gamma_R W_R},$$

where the index r stands for real photon, $W_R = 1688$ MeV, and Γ_R ($\simeq 100$ MeV) is the width of the N^{***}.

From the behavior of the forward differential cross section in π^+ and π^0 photoproduction,[13] we have, for $W = W_R$,

$$\frac{E_{3-}{}^r}{M_{3-}{}^r} = 2, \quad \text{or} \quad \frac{\epsilon_{3-}{}^r(0)}{\mu_{3-}{}^r k^2} = 3. \tag{13}$$

Our assumption that the form factor is the same for E_{3-} and M_{3-} amounts to saying

$$\frac{\epsilon_{3-}(0)}{\mu_{3-}} = \frac{\epsilon_{3-}{}^r(0)}{\mu_{3-}}, \quad \text{for} \quad W \simeq W_R,$$

so that, from the ratio of Eqs. (11) and (11″), at $W \simeq W_R$ we obtain, with $k = 585$ MeV,

$$\frac{\epsilon_{3-}{}^r(0)}{\mu_{3-}{}^r k^2} = \frac{\epsilon_{3-}(0)}{\mu_{3-}{}^2 k^2} = \frac{5k_0 M}{k^2} \frac{\left[\dfrac{d^2 G_E{}^V(k^2)}{(dk^2)^2}\right]_{k=0}}{\left[\dfrac{d^2 G_M{}^V(k^2)}{(dk^2)^2}\right]_{k=0}} = 2.7, \tag{14}$$

in very good agreement with Eq. (13). We must, however, remember that the errors in the derivatives of the nucleon form factors are here certainly bigger and more difficult to estimate than those in Eq. (19) in A.

It should be emphasized that Eq. (14), like Eq. (19) in A, holds only for isovector amplitudes, while Eq. (13) holds for the π^+ and π^0 amplitudes; that is, combinations of isovector and isoscalar. Equation (14) therefore suggests isovector dominance.

We can obtain this result in another way, at least, for the electric amplitude, applying the same method used in A. In fact, taking in Eq. (3), $\alpha = 3, \beta = 8$, that is, commuting isovector and isoscalar electric quadrupole operators, and $i = 1, j = 3, i' = 2, j' = 3$, we have $f_{38\gamma} = 0$ for every γ and therefore

$$\int \frac{q dW}{W} \frac{e_{3-}{}^V(0)}{k_0{}^2} e_{3-}{}^*(0) = 0$$

so that the leading term of the electric amplitude is isovector.

Unfortunately, up to now, there are essentially no experiments on π^- photoproduction in the region of the N^{***} resonance, which, combined with the experiments on π^+ photoproduction, could confirm such isovector dominance. As a final result, we want to consider explicitly the ρ-meson dominance in the form factor; namely, we take

$$e_{3-}(0) = \frac{m_\rho{}^2}{(W-M)^2 - m_\rho{}^2 - i\Gamma_\rho m_\rho} e_{3-}{}^r(0).$$

With the experimental value[14] $(M/W \times 1/\sqrt{137}) E_{3-}{}^r \simeq (\sqrt{6\pi}) \times (0.4) \times 10^{-15}$ cm for $W = W_r$, and using Eq. (13), we can evaluate the integral in Eq. (11) over the peaks of the N^{***} resonance and of the form factor. Using the value given in Ref. 11 for the nucleon form factor, Eq. (11) then gives

$$5 \times 10^{-27}\ \text{cm}^2 \simeq 2 \times 10^{-27}\ \text{cm}^2. \tag{15}$$

Although the two sides of (15) disagree by a factor of 2.5, it is important to note that they would have disagreed by 12 if the form factor had not been included in Eq. (11), because, as we said before, we are right on the peak in the form factor.

In conclusion, the study of the commutation relations between magnetic-octupole and electric-quadrupole operators gives rise to results for the N^{***} resonance completely analogous to those obtained in A for the N^{***} resonance from the commutators of electric dipole and magnetic quadrupole operators.

As we have already pointed out, the results obtained here are more uncertain, in the sense that the errors on the higher derivatives of the nucleon form factors are certainly bigger.

The question naturally arises if one can continue this program commuting higher and higher multipole moments, obtaining results for higher resonances and therefore more and more relations like Eq. (8) and, in A, Eq. (12).

Generally speaking, we do not believe that this is possible. In fact, the success of our approach for the N^{**} and the N^{***} and the consistency relations, as we have already pointed out, depends on the circumstance that both these resonances have a mass such that k_0, the mass of the virtual photon involved in the transition, is equal or very close to the mass of the ρ meson.

[12] Ph. Salin, Nuovo Cimento **28**, 1294 (1963).

[13] D. S. Beder, Nuovo Cimento **33**, 94 (1964); R. L. Walker (private communication).

[14] R. L. Walker (private communication).

It is evident that, in principle, this situation cannot be reproduced for other resonances.

A typical example is the magnetic-moment sum rule, where in addition to the nucleon and N^* intermediate states, the ρ-meson pole in the form factor makes also continuum states very important.[15]

It is also clear that a relation like Eq. (8) cannot be obtained for the $W_R=1920$, $J=\frac{7}{2}^+$, $I=\frac{3}{2}$ resonance. In fact, as we have seen in the commutation relation for the magnetic octupole moments (4′), the N^{***} strongly dominates the other intermediate states.

[15] A. Bietti, Phys. Rev. 140, B908 (1965).

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Saturation of $SU(3)$-Symmetric Hadron Forces*

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It is pointed out that the problem of saturation exists not only for nuclear forces, but also for other forces among hadrons. It is shown that the coupling of the vector-meson octet to the eight currents of $SU(3)$ provides a mechanism that can help produce saturation in states of many hadrons. Two theorems are proved within the context of a simple model of an arbitrary number of mutually interacting mesons and either zero or one baryon. The first theorem shows that very few multimeson bound states may exist, and that nearly every possible baryon bound state corresponds to a representation smaller than that of the fundamental baryon. The second theorem shows that a saturation mechanism provides some justification for neglecting states of more than two particles in composite models. In the course of the investigation, a simple formula is developed for listing the representations contained in the direct product of an irreducible representation of $SU(n)$ and an arbitrary number of regular representations.

I. INTRODUCTION

IN recent years, many authors have constructed models of the hadrons (strongly interacting particles), in which many or all the particles are composites, the forces being supplied by simple particle-exchange mechanisms. At best these models are incomplete representations of reality, since many known particles and interactions are neglected. This type of incompleteness is necessary, of course, since one of the main features of strong interactions is that each hadron interacts with very many others. Unfortunately, there is no theoretical principle that may be used to give a convincing justification of the neglect of so many interactions.

We propose here a general principle that may be used to help evaluate models of composite particles. The principle is that *attraction tends to dominate repulsion, and therefore is more dangerous*. In order to illustrate the meaning of the principle, we consider two nonrelativistic particles interacting by means of a simple potential that is attractive in some regions of space and repulsive in others. The attraction dominates in the sense that it leads to a bound state if the over-all strength of the potential is great enough. This follows simply from the variational principle; the wave function tends to nestle in the attractive region. The principle holds also if several two-particle channels are coupled together (a situation characteristic of strong interactions). In this case the potential may be considered to be a nondiagonal matrix, and the wave function a vector. It follows from the variational principle that, if the potential in any one or more of the channels is sufficiently attractive to produce binding, the coupling of additional channels cannot decrease the binding energy, even if the diagonal forces in the added states are repulsive.

Because of this principle, bound states or resonances predicted in an incomplete model of the hadrons generally survive the inclusion of new forces and particles. Thus, models in which known particles are produced as composites by known forces possess a creditability that is not reduced greatly by the fact that many known interactions are neglected. Furthermore, it is not a serious defect of the model if mechanisms do not exist for producing some of the known higher mass states as composites; the inclusion of other interactions would be expected to lead to more predicted bound states and resonances. On the other hand, it is a serious defect if strong attractions exist for channels in which it is known experimentally that no bound state or resonance exists.

It is not easy to design a simple model universe in

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