nance[14] in every $J=\frac{3}{2}^{+}$ $(B+P)$ system will still be maintained, barring the situation where the Born terms are strongly repulsive.[12]

We note that the above arguments regarding self-consistency need not hold for resonant solutions $[(s_R)_{\rm in}>(B+P)^2]$, since the representation of the partial-wave amplitude by the fixed-energy dispersion relation is not expected to hold in the physical region. [In other words we cannot directly use Eq. (26) to judge the self-consistency in $\kappa$.] From this one might guess that the self-consistency may get worse as one goes sufficiently above the physical threshold $[(s_R)_{\rm in}>(B+P)^2+50m_\pi^2$, say]. This is found to be the case by actual calculation in various systems.

[14] In view of the role played by the nearby singularities $(N_{(n)})$ and $g_1^{+({\rm Born})}$ and the fact that the self-consistency cannot quite rigorously be judged on the basis of Eq. (A.2), it is quite possible to obtain a low-lying [low-lying compared to the physical threshold $(B+P)^2$] resonance rather than a bound-state solution. This is what happens in the case of the (3,3) $\pi N$ resonance, (Ref. 11) for example.

## VI. CONCLUDING REMARKS

In the present work we have carried out explicit calculations for $I=1$ $\bar{K}\Xi$ and $KN$ systems using the Balázs-type $N/D$ method, and found that self-consistent bound-state (or resonant) solutions exist for both systems. On considering the general case of arbitrary baryon-meson systems, we found a typical feature of this procedure that in almost any $J=\frac{3}{2}^{+}$ baryon–pseudoscalar-meson system, one would obtain self-consistent bound-state or low-lying resonant solutions. This is a remarkable and somewhat awkward result, it it corresponds to reality. It leads one to wonder about the physical implications of the results in such a scheme. At any rate the most interesting question is: Will experiments confirm such a conclusion?

## ACKNOWLEDGMENTS

We would like to thank Professor O. W. Greenberg for his interest in this work, and H. Y. Kim for assistance in programming and numerical computation.

PHYSICAL REVIEW VOLUME 144, NUMBER 4 29 APRIL 1966

# Lie Group of the Strong-Coupling Theory. I. Calculation of the Coupling-Constant Ratios and Magnetic Moments for Symmetric Pseudoscalar-Meson Theory

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All the isobar-pion coupling constants are calculated using the Lie algebra $[SU(2)\otimes SU(2)]\times T_9$ of the strong-coupling theory. The $N^*\rightarrow N\pi$ reduced width comes out to be in agreement with experiment. We also calculate isobar magnetic moments in terms of proton and neutron magnetic moments. The results obtained are also compared with $SU(6)$ and reciprocal-bootstrap predictions.

## I. INTRODUCTION

RECENTLY the Lie-group structure of the strong-coupling theory of baryon-meson scattering[1,2] has been deduced in the framework of the dispersion relations satisfied by the static models.[3] Various possible irreducible representations of this Lie group provide the possible isobar spectra. The number of isobars turns out to be infinite for any irreducible representation. Mathematically this is due to the group involved being noncompact, so that it has no finite-dimensional unitary representations. It is physically understandable that in the limit of very large baryon-meson coupling an infinite number of isobars would occur. More and more poles of the scattering amplitude, representing isobars, move onto the physical sheet as the coupling constants are increased to larger and larger values. In the physical case all the coupling constants are finite and only a few of these poles would have approached the physical sheet. So in the physical case one would observe only a few low-lying isobars. It should be emphasized that in this model only the scale of various isobar coupling constants tends to infinity; the ratios of these remain finite.

For the case of symmetric pseudoscalar-meson theory it was shown by Cook, Goebel, and Sakita that the Lie group $G$ of this theory is $G=[SU(2)\otimes SU(2)]\times T_9$. Further, using group contraction on the $SU(4)$ group with respect to its subgroup $SU(2)\otimes SU(2)$, it was shown that the only irreducible representations (IR) of group $G$ are given by the $SU(4)$ IR with Young-tableau characterization $(\infty,\lambda_2,\lambda_3)$. The mass spectrum was shown to be of the form

$$M(I,J)=M_0+M_1J(J+1)+M_2I(I+1),$$

[1] C. J. Goebel, *Proceedings of the International Conference on High Energy Physics, Dubna*, 1964 (Atomizdat, Moscow, 1965); Proceedings of the 1965 Midwest Conference on Theoretical Physics, Ohio State University (unpublished).
[2] T. Cook, C. J. Goebel, and B. Sakita, Phys. Rev. Letters **15**, 35 (1965) (to be referred to as CGS).
[3] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

where $M(I,J)$ is the mass of the isobar having isospin $I$ and spin $J$. Thus the low-lying isobars would be the ones with low isospin and spin. The simplest irreducible representation of $G$ is that characterized by the Young tableau $(\infty,0,0)$. We shall refer to this representation as "$B$." It has the isospin-spin content[2]

$$I=J=\tfrac{1}{2},\tfrac{3}{2},\tfrac{5}{2},\cdots,\infty\,.$$

The first two members of this series can be respectively identified with $N$(940 MeV) and $N^*$(1240 MeV). The status of the reported $I=J=\frac{5}{2}$ $p\pi^+\pi^+$ bump as a full-fledged isobar is rather uncertain. In any case it is probably a borderline case. Thus it would appear that this IR of $G$ is the physically interesting one. The others have a larger number of low-lying isobars and hence are physically less interesting. The purpose of the present paper is to calculate the coupling-constant ratios for this IR of $G$. This is necessary to calculate isobar widths.

It may be noted that the isobar series $I=J=\frac{1}{2},\frac{3}{2},\cdots$ was also obtained in the classical strong-coupling calculations.[4] There are, however, differences of principle between the classical and the present approach. Thus in the classical approach only the lowest mass baryon, i.e., the nucleon, is singled out and the Hamiltonian contains terms representing only its interaction with pions, while in the present Goebel version all the isobars are treated on the same footing. More recently the same series was obtained by Abers, Balázs, and Hara,[5] also in the dispersion theory framework, with the assumptions (i) that the denominator functions in the $N/D$ calculation are linear in energy, (ii) that the calculation of lower isobar states is not affected by higher isobars.

In Sec. II we give the commutation rules of the Lie algebra of $G$ in the convenient spherical basis. We then go on, in Sec. III, to determine the matrix elements of the generators of $G$ for the IR "$B$." The last two sections will be devoted to the physical applications. In Sec. IV, we calculate the $N^*\to N\pi$ decay width and also other $N^*$ coupling constants. The last section (V) deals with isobar magnetic moments. We also compare results obtained in the present approach with the results obtained from $SU(6)$ [6,7] and reciprocal-bootstrap calculations.[8]

## II. COMMUTATION RULES FOR THE LIE ALGEBRA OF THE GROUP $G$

The Lie algebra of the group $G$ for symmetric pseudoscalar-meson theory, as shown by CGS, is the semidirect product of the nine-parameter Abelian group $T_9$ and the invariance group of the problem, $SU(2)_I\otimes SU(2)_J$; i.e.,

$$G=[SU(2)_I\otimes SU(2)_J]\times T_9\,.$$

The $SU(2)_I$ and $SU(2)_J$ in the maximal compact subgroup of $G$ refer to isospin $SU(2)$ and angular-momentum $SU(2)$, respectively. The nine generators of $G$, which span $T_9$, correspond to pion current operators, the isospin 1 of pions together with the $p$-wave nature of interaction accounting for the nine components.

The commutation rules (CR) of the Lie algebra of $G$ in Cartesian basis for both $SU(2)$'s were given by CGS. We shall find it more convenient to use the spherical basis for both the $SU(2)$'s. The commutation rules can be specified as follows:

First there are the CR defining the subgroup $SU(2)\otimes SU(2)$. We take $I_+$, $I_-$, $I_z$ to define isospin $SU(2)$ and similarly $J_+$, $J_-$, $J_z$ for angular-momentum $SU(2)$. The CR are then given by

$$[J_z,J_\pm]=\pm J_\pm\,, \tag{2.1}$$

$$[J_+,J_-]=2J_z\,, \tag{2.2}$$

$$[I_z,I_\pm]=\pm I_\pm\,, \tag{2.3}$$

$$[I_+,I_-]=2I_z\,, \tag{2.4}$$

and

$$[J_z,I_\pm]=[J_z,I_z]=[J_\pm,I_\pm]=[J_\pm,I_\mp]=[J_\pm,I_z]=0\,. \tag{2.5}$$

Then there are CR expressing the fact that the meson currents $T_{\mu,\tau}$ transform like the regular representation under the $SU(2)_I\otimes SU(2)_J$. We shall take the convention that the first index of $T_{\mu,\tau}$ is the spin index while the second is the isospin index:

$$[J_\pm,T_{\mu,\tau}]=[(1\mp\mu)(2\pm\mu)]^{1/2}T_{\mu\pm1,\tau}\,, \tag{2.6}$$

$$[J_z,T_{\mu,\tau}]=\mu T_{\mu,\tau}\,, \tag{2.7}$$

$$[I_\pm,T_{\mu,\tau}]=[(1\mp\tau)(2\pm\tau)]^{1/2}T_{\mu,\tau\pm1}\,, \tag{2.8}$$

$$[I_z,T_{\mu,\tau}]=\tau T_{\mu,\tau} \tag{2.9}$$

$[\mu,\tau=\pm1,0]$.

The CR (2.1)–(2.9) are more or less "kinematical." The specific dynamics manifests itself in the form of CR between meson currents. In the strong-coupling theory one gets the following CR:

$$[T_{\mu,\tau},T_{\mu',\tau'}]=0\,. \tag{2.10}$$

This completes the specification of CR for $G$.

## III. DETERMINATION OF THE MATRIX ELEMENTS OF THE GENERATORS OF $G$

Our task is now to solve the CR (2.1)–(2.10) to obtain the explicit matrix representation of meson current operators $T_{\mu,\tau}$ for the case of IR "$B$," which has only the states with $I=J=\frac{1}{2},\frac{3}{2},\cdots$ each occurring

[4] G. Wentzel, Helv. Phys. Acta **13**, 269 (1940); W. Pauli and S. Dancoff, Phys. Rev. **62**, 85 (1942).
[5] E. S. Abers, L. A. P. Balázs, and Y. Hara, Phys. Rev. **136**, B1382 (1964).
[6] F. Gürsey, L. A. Radicati, and A. Pais, Phys. Rev. Letters **13**, 299 (1964).
[7] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).
[8] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962); See also Ref. 5 and L. A. P. Balázs, V. Singh, and B. M. Udgaonkar, Phys. Rev. **139**, B1313 (1965).

only once. We shall divide the discussion in two parts. The first part will be concerned with the "kinematical" CR, i.e., (2.1)–(2.9), while the second part will be devoted to the "dynamical" CR (2.10). The "kinematical" CR for group $G$ are the same as those for the $SU(4)$ symmetry group, i.e., the spin-isospin independence scheme. The $SU(4)$-group Lie algebra differs from Lie algebra (2.1)–(2.10) of $G$ only in the CR (2.10).

### A. "Kinematical" Commutation Rules

We shall denote the isobar states occurring in the IR "$B$" by specifying their spin and the third components of their spin and isospin. This constitutes a complete specification of the states in IR "$B$." For a general IR of $G$, this description of states would, of course, not be complete and one would need more quantum numbers to label the states. This is an essential simplification which occurs when one is interested in only the IR "$B$."

Let the isobar state with spin $j$ and with third component of spin and isospin, respectively, equal to $m$ and $t$ be denoted by $\phi_{m,t}{}^j$. We have, using (2.1)–(2.5),

$$J_\pm\phi_{m,t}{}^j=[(j\mp m)(j\pm m+1)]^{1/2}\phi_{m\pm1,t}{}^j\,, \qquad (3.1)$$

$$J_z\phi_{m,t}{}^j=m\phi_{m,t}{}^j\,, \qquad (3.2)$$

$$I_\pm\phi_{m,t}{}^j=[(j\mp t)(j\pm t+1)]^{1/2}\phi_{m,t\pm1}{}^j\,, \qquad (3.3)$$

$$I_z\phi_{m,t}{}^j=t\phi_{m,t}{}^j\,. \qquad (3.4)$$

Let

$$T_{\mu,\tau}\phi_{m,t}{}^j=\sum_{j'm't'} S^{\mu,\tau}(j,m,t;j',m',t')\phi_{m',t'}{}^{j'}\,. \qquad (3.5)$$

Then using the commutation rules (2.7) and (2.9) it is easy to show that, on the right-hand side of (3.5), the summation of $m'$, $t'$ should be restricted to

$$m'=m+\mu \quad \text{and} \quad t'=t+\tau\,. \qquad (3.6)$$

Thus, we get

$$T_{\mu,\tau}\phi_{m,t}{}^j=\sum_{j'} S^{\mu,\tau}(j,m,t;j',m+\mu,t+\tau)\phi_{m+\mu,t+\tau}{}^{j'}\,. \qquad (3.7)$$

Furthermore since $T_{\mu,\tau}$ transforms like a vector operator under $SU(2)_I$ and $SU(2)_J$ and we have

$$\mathbf{j}\otimes\mathbf{1}=(\mathbf{j}-1)\oplus\mathbf{j}\oplus(\mathbf{j}+1)\,,$$

the summation over $j'$ in (3.7) has to be restricted to

$$j'=j-1,\ j,\ j+1\,.$$

We thus can write

$$T_{\mu,\tau}\phi_{m,t}{}^j=C_j{}^{\mu,\tau}(m,t)\phi_{m+\mu,t+\tau}{}^{j-1}+A_j{}^{\mu,\tau}(m,t)\phi_{m+\mu,t+\tau}{}^j +D_{j+1}{}^{\mu,\tau}(m,t)\phi_{m+\mu,t+\tau}{}^{j+1}\,. \qquad (3.8)$$

Let us now use the commutation rule (2.6) in the following form:

$$(J_\pm T_{\mu,\tau}-T_{\mu,\tau}J_\pm)\phi_{m,t}{}^j =[(1\mp\mu)(2\pm\mu)]^{1/2}T_{\mu\pm1,\tau}\phi_{m,t}{}^j\,. \qquad (3.9)$$

Using Eqs. (3.1) and (3.8) to evaluate the two sides of Eq. (3.9) and equating the coefficients of $\phi_{m+\mu\pm1,t+\tau}{}^{j-1}$, $\phi_{m+\mu\pm1,t+\tau}{}^{j},t$ and $\phi_{m+\mu\pm1,t+\tau}{}^{j+1}$ on both sides, we get the following equations:

$$[(j\mp m\mp\mu-1)(j\pm m\pm\mu)]^{1/2}C_j{}^{\mu,\tau}(m,t) -[(j\mp m)(j\pm m+1)]^{1/2}C_j{}^{\mu,\tau}(m\pm1,\,t) =[(1\mp\mu)(2\pm\mu)]^{1/2}C_j{}^{\mu\pm1,\tau}(m,t)\,, \qquad (3.10\text{a,b})$$

$$[(j\mp m\mp\mu)(j\pm m\pm\mu+1)]^{1/2}A_j{}^{\mu,\tau}(m,t) -[(j\mp m)(j\pm m+1)]^{1/2}A_j{}^{\mu,\tau}(m\pm1,\,t) =[(1\mp\mu)(2\pm\mu)]^{1/2}A_j{}^{\mu\pm1,\tau}(m,t)\,, \qquad (3.11\text{a,b})$$

$$[(j\mp m\mp\mu+1)(j\pm m\pm\mu+2)]^{1/2}D_{j+1}{}^{\mu,\tau}(m,t) -[(j\mp m)(j\pm m+1)]^{1/2}D_{j+1}{}^{\mu,\tau}(m\pm1,\,t) =[(1\mp\mu)(2\pm\mu)]^{1/2}D_{j+1}{}^{\mu\pm1,\tau}(m,t)\,. \qquad (3.12\text{a,b})$$

Similarly, using the CR (2.8), we get the following set of equations:

$$[(j\mp t\mp\tau-1)(j\pm t\pm\tau)]^{1/2}C_j{}^{\mu,\tau}(m,t) -[(j\mp t)(j\pm t+1)]^{1/2}C_j{}^{\mu,\tau}(m,\,t\pm1) =[(1\mp\tau)(2\pm\tau)]^{1/2}C_j{}^{\mu,\tau\pm1}(m,t)\,, \qquad (3.13\text{a,b})$$

$$[(j\mp t\mp\tau)(j\pm t\pm\tau+1)]^{1/2}A_j{}^{\mu,\tau}(m,t) -[(j\mp t)(j\pm t+1)]^{1/2}A_j{}^{\mu,\tau}(m,\,t\pm1) =[(1\mp\tau)(2\pm\tau)]^{1/2}A_j{}^{\mu,\tau\pm1}(m,t)\,, \qquad (3.14\text{a,b})$$

$$[(j\mp t\mp\tau+1)(j\pm t\pm\tau+2)]^{1/2}D_{j+1}{}^{\mu,\tau}(m,t) -[(j\mp t)(j\pm t+1)]^{1/2}D_{j+1}{}^{\mu,\tau}(m,\,t\pm1) =[(1\mp\tau)(2\pm\tau)]^{1/2}D_{j+1}{}^{\mu,\tau\pm1}(m,t)\,. \qquad (3.15\text{a,b})$$

Here "a, b" with the equation numbers refer to the equations obtained by taking only the upper or lower signs, respectively.

It is obvious from the structure of the Eqs. (3.10)–(3.15) that the solutions are of the form

$$C_j{}^{\mu,\tau}(m,t)=C_j{}^\mu(m)C_j{}^\tau(t)C_j\,, \qquad (3.16)$$

$$A_j{}^{\mu,\tau}(m,t)=A_j{}^\mu(m)A_j{}^\tau(t)A_j\,, \qquad (3.17)$$

$$D_j{}^{\mu,\tau}(m,t)=D_j{}^\mu(m)D_j{}^\tau(t)D_j\,. \qquad (3.18)$$

Using the expression (3.16) in Eqs. (3.10) and (3.13) we find they both reduce to

$$[(j\mp m\mp\mu-1)(j\pm m\pm\mu)]^{1/2}C_j{}^\mu(m) -[(j\mp m)(j\pm m+1)]^{1/2}C_j{}^\mu(m\pm1) =[(1\mp\mu)(2\pm\mu)]^{1/2}C_j{}^{\mu\pm1}\,. \qquad (3.19\text{a,b})$$

Substituting $\mu=1$ in (3.19a,b), we get

$$[(j-m-2)]^{1/2}C_j{}^1(m)=[(j-m)]^{1/2}C_j{}^1(m+1)\,, \qquad (3.20)$$

$$[(j+m)(j-m-1)]^{1/2}C_j{}^1(m) -[(j+m)(j-m+1)]^{1/2}C_j{}^1(m-1) =\sqrt{2}C_j{}^0(m)\,. \qquad (3.21)$$

Solving (3.20) and (3.21), we get

$$C_j{}^1(m)=[(j-m)(j-m-1)]^{1/2}\,, \qquad (3.22)$$

$$C_j{}^0(m)=-\sqrt{2}[j^2-m^2]^{1/2}\,. \qquad (3.23)$$

Substituting $\mu=0$ in (3.19b), we get

$$[(j+m-1)(j-m)]^{1/2}C_j^0(m) -[(j+m)(j-m+1)]^{1/2}C_j^0(m-1) =\sqrt{2}C_j^{-1}(m)\,, \quad (3.24)$$

which, using (3.23), leads to

$$C_j^{-1}(m)=[(j+m)(j+m-1)]^{1/2}. \quad (3.25)$$

Similarly one can solve for $A_j^\mu(m)$ and $D_j^\mu(m)$. We give the results in the tabular form below.

| $\mu$ | $+1$ | $0$ | $-1$ |
|---|---|---|---|
| $C_j^\mu(m)$ | $[(j-m)(j-m-1)]^{1/2}$ | $-\sqrt{2}[(j^2-m^2)]^{1/2}$ | $+[(j+m)(j+m-1)]^{1/2}$ |
| $A_j^\mu(m)$ | $-[(j+m+1)(j-m)]^{1/2}$ | $\sqrt{2}m$ | $+[(j-m+1)(j+m)]^{1/2}$ (3.26) |
| $D_{j+1}^\mu(m)$ | $[(j+m+1)(j+m+2)]^{1/2}$ | $\sqrt{2}[(j-m+1)(j+m+1)]^{1/2}$ | $+[(j-m+1)(j-m+2)]^{1/2}$. |

### B. "Dynamical" Commutation Rules

We shall now work out the implications of the CR (2.10). We have, as one of these CR,

$$[T_{+0},T_{00}]=0. \quad (3.27)$$

Hence we must have

$$(T_{+0}T_{00}-T_{00}T_{+0})\phi_{m,t}{}^j=0. \quad (3.28)$$

Using (3.8), (3.16)–(3.18), and (3.26), we get

$$T_{+0}\phi_{m,t}{}^j=-\sqrt{2}[(j-m)(j-m-1)(j^2-t^2)]^{1/2}C_j\phi_{m+1,t}{}^{j-1}-\sqrt{2}[(j+m+1)(j-m)t^2]^{1/2}A_j\phi_{m+1,t}{}^j +\sqrt{2}[(j+m+1)(j+m+2)(j+t+1)(j-t+1)]^{1/2}D_{j+1}\phi_{m+1,t}{}^{j+1} \quad (3.29)$$

and

$$T_{00}\phi_{m,t}{}^j=2[(j^2-m^2)(j^2-t^2)]^{1/2}C_j\phi_{m,t}{}^{j-1}+2mtA_j\phi_{m,t}{}^j+2[(j+1)^2-m^2]^{1/2}[(j+1)^2-t^2]^{1/2}D_{j+1}\phi_{m,t}{}^{j+1}. \quad (3.30)$$

Using (3.29) and (3.30), we can evaluate the left-hand side of Eq. (3.28). The result of a tedious calculation is

$$(1/2\sqrt{2})(T_{+0}T_{00}-T_{00}T_{+0})\phi_{m,t}{}^j =\phi_{m+1,t}{}^{j-1}[t^2(j^2-t^2)(j-m)(j-m-1)]^{1/2}[(j-1)A_{j-1}-(j+1)A_j]C_j+\phi_{m+1,t}{}^j[(j-m)(j+m+1)]^{1/2} \times\{[-j^2(2j-1)C_jD_j+(j+1)^2(2j+3)C_{j+1}D_{j+1}]+t^2[(2j-1)C_jD_j-(2j+3)C_{j+1}D_{j+1}-A_j^2]\} +\phi_{m+1,t}{}^{j+1}[(j+m)(j+m+2)t^2(j+t+1)(j-t+1)]^{1/2}[-jA_j+(j+2)A_{j+1}]D_{j+1}. \quad (3.31)$$

Hence, in order to satisfy (3.28) we must have

$$[(j-1)A_{j-1}-(j+1)A_j]C_j=0\,, \quad (3.32)$$

$$[jA_j-(j+2)A_{j+1}]D_{j+1}=0\,, \quad (3.33)$$

$$[j^2(2j-1)C_jD_j-(j+1)^2(2j+3)C_{j+1}D_{j+1}]=0\,, \quad (3.34)$$

$$(2j-1)C_jD_j-(2j+3)C_{j+1}D_{j+1}-A_j^2=0. \quad (3.35)$$

These equations are easy to solve and the solution is

$$A_j=a/j(j+1)\,, \quad (3.36)$$

$$C_jD_j=a^2/j^2(4j^2-1). \quad (3.37)$$

This is all the information that can be gotten out of the CR of $G$. The use of other commutation rules does not give any further information. So far we have not imposed the condition that IR "$B$" be unitary. Thus, we must have

$$(\phi_{m,t}{}^j,T_{00}\phi_{m,t}{}^j)=(T_{00}\phi_{m,t}{}^j,\phi_{m,t}{}^j) \quad (3.38)$$

and

$$(\phi_{m,t}{}^{j-1},T_{00}\phi_{m,t}{}^j)=(T_{00}\phi_{m,t}{}^{j-1},\phi_{m,t}{}^j)\,, \quad (3.39)$$

which leads to

$$A_j=A_j^*\,, \quad (3.40)$$

$$C_j=D_j^*. \quad (3.41)$$

Combining (3.36) and (3.40) we see that $a$ must be real. We can choose arbitrary phases such that we finally have

$$A_j=a/j(j+1) \quad (3.42)$$

$$C_j=D_j=a/j[4j^2-1]^{1/2} \quad (3.43)$$

($a$ real). This completes the determination of the matrix elements of generators of the group $G$ for IR "$B$."

## IV. $N^*\rightarrow N\pi$ DECAY WIDTH AND OTHER $N^*$ COUPLING CONSTANTS

In the last section we were able to determine the matrix elements of meson currents between isobar states, which is the same thing as determining isobar coupling constants. Since there occurs an arbitrary parameter $a$, all possible isobar coupling constants are determined in terms of this parameter $a$, which fixes the scale of coupling constants.

Let us calculate $N^*(1240)\rightarrow N(940)+\pi$ decay width and other $N^*$ coupling constants. Using (3.30), we obtain

$$T_{00}\phi_{1/2,1/2}{}^{1/2}=\tfrac{1}{2}A_{1/2}\phi_{1/2,1/2}{}^{1/2}+4D_{3/2}\phi_{1/2,1/2}{}^{3/2}\,, \quad (4.1)$$

$$T_{00}\phi_{1/2,1/2}{}^{3/2}=4C_{3/2}\phi_{1/2,1/2}{}^{1/2}+\tfrac{1}{2}A_{3/2}\phi_{1/2,1/2}{}^{3/2} +12D_{5/2}\phi_{1/2,1/2}{}^{5/2}. \quad (4.2)$$

We have from (3.42)–(3.43)

$$A_{1/2}=4a/3,\quad A_{3/2}=4a/15,$$
$$C_{3/2}=D_{3/2}=a/3\sqrt{2},\quad C_{5/2}=D_{5/2}=a/5\sqrt{6},$$

which leads to the following $\pi^0$ coupling constants, in an obvious notation:

$$\langle N^{*+}, J_z=\tfrac{1}{2}|\pi^0|p, J_z=\tfrac{1}{2}\rangle =\sqrt{2}\langle p, J_z=\tfrac{1}{2}|\pi^0|p, J_z=\tfrac{1}{2}\rangle, \quad (4.3)$$

$$\langle N^{*+}, J_z=\tfrac{1}{2}|\pi^0|N^{*+}, J_z=\tfrac{1}{2}\rangle =\tfrac{1}{5}\langle p, J_z=\tfrac{1}{2}|\pi^0|p, J_z=\tfrac{1}{2}\rangle, \quad (4.4)$$

and

$$\langle N^{**+}, J_z=\tfrac{1}{2}|\pi^0|N^{*+}, J_z=\tfrac{1}{2}\rangle = (3\sqrt{6}/5)\langle p, J_z=\tfrac{1}{2}|\pi^0|p, J_z=\tfrac{1}{2}\rangle. \quad (4.5)$$

Here we have denoted by $N^{**}$ the possible $I=J=\frac{5}{2}$ isobar state.

The coupling-constant relation (4.3) leads to the following ratio between the $N^*$ reduced width $\gamma_{33}$ and $N$ reduced coupling constant $\gamma_{11}(=3f^2)$,

$$\gamma_{33}/\gamma_{11}=\tfrac{1}{2}. \quad (4.6)$$

Surprisingly this agrees exactly with the value of $\gamma_{33}/\gamma_{11}$ obtained in Chew's reciprocal bootstrap scheme between $N$ and $N^*$.[8] It is interesting to compare it with $SU(6)$ prediction,[6] which is

$$\gamma_{33}/\gamma_{11}=8/25. \quad (4.7)$$

Experimental results are in agreement with Chew's reciprocal bootstrap value and not with the $SU(6)$ value.

We do not have any experimental information on the $\bar{N}^*N^*\pi$ coupling constant. The result (4.4) given by the present calculation is in agreement with the $SU(6)$ prediction. The $SU(6)$ has, however, nothing to say about higher isobar coupling constants, so it does not have any prediction like (4.5).

## V. ISOBAR MAGNETIC MOMENTS

We now wish to make some comments about the isobar magnetic moments. If we assume that the isovector-magnetic-moment current transforms like $T_{\mu,\tau}$, then we can get all isobar isovector magnetic moments in terms of just one parameter.

Let $\mu_V(j,I_3)$ denote the isovector magnetic moment of isobar with spin $j$, isospin $j$ and electric charge equal to $I_3+\frac{1}{2}$. We get from (3.30) and (3.42)

$$\langle j,J_z,I_3|T_{00}|j,J_z,I_3\rangle=2J_zI_3a/j(j+1). \quad (5.1)$$

Here we have denoted the eigenvalues of the third components of $J_z$ and $I_3$ by the same symbols. Hence, making the above assumption about the transformation properties of isovector magnetic moment operator, we get

$$\mu_V(j,I_3)=[I_3/(j+1)]\mu, \quad (5.2)$$

where $\mu$ is some constant. Note that the magnetic moments are defined with $J_z=j$.

The result (5.2) follows in the old strong-coupling theory for the *total magnetic moments* and was first derived by Pauli and Dancoff.[4] In the present approach it, however, refers only to the *isovector part of the total magnetic moment*. We thus do not get, as in the old strong-coupling theory, $\mu(p)=-\mu(n)$.

Similarly, if the isoscalar part of the magnetic moment transforms like a generator of $G$, then it must transform like **J**.[9] This leads for the isoscalar magnetic moment $\mu_s(j)$ of isobar with spin $j$.

$$\mu_s(j)=\text{const}\times j, \quad (5.3)$$

again remembering that magnetic moments are defined with $J_z=j$ for spin-$j$ particles.

If one regards the total magnetic moments as composed of only isoscalar and isovector parts in accordance with the minimal-electromagnetic-interaction hypothesis, then one can predict *all* the isobar magnetic moments in terms of proton and neutron magnetic moments $\mu(p)$ and $\mu(n)$ only.

For example, using (5.2), one gets

$$\mu(N^{*+})-\mu(N^{*0})=(3/5)[\mu(p)-\mu(n)], \quad (5.4)$$

and using (5.3), one gets

$$\mu(N^{*+})+\mu(N^{*0})=3[\mu(p)+\mu(n)]. \quad (5.5)$$

Therefore, we get

$$\mu(N^{*+})=(9/5)\mu(p)+(6/5)\mu(n), \quad (5.6)$$

$$\mu(N^{*0})=(6/5)\mu(p)+(9/5)\mu(n). \quad (5.7)$$

Let us compare these results with $SU(6)$ results. In $SU(6)$ we have

$$\mu(p)=-(\tfrac{3}{2})\mu(n). \quad (5.8)$$

Using this $\mu(p)/\mu(n)$ ratio in (5.6) and (5.7), we get

$$\mu(N^{*+})=\mu(p), \quad (5.9)$$

$$\mu(N^{*0})=0, \quad (5.10)$$

which agrees with $SU(6)$ results for the $N^*$ magnetic moments.[7] It should be emphasized that the extra predictive power of $SU(6)$ comes from the inclusion of strange particles in it. Thus the $SU(4)$ subgroup of $SU(6)$ would not lead to any more predictions than (5.6) and (5.7).

[9] The author is grateful to Dr. P. Babu for a discussion about the isoscalar part of magnetic moment.