Dynamics of $J=\frac{3}{2}^+$ $\bar{K}\Xi$ and KN Systems in the 27-Fold Representation; Some Remarks on a Typical Feature of the Balázs-Type Procedure*

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We have studied the dynamics of the $I=1$, $J=\frac{3}{2}^+$ $\bar{K}\Xi$ and KN systems by the Balázs-type N/D method and have observed a typical feature of this procedure for treating the far left-hand singularities. The dynamical singularities of the partial-wave amplitude are assumed to arise mainly from the nearby cut (due to Σ and Λ exchange in the crossed channel) and the far left-hand cut ($-\infty<s\leq 0$). The contribution of the former is evaluated explicitly in terms of the relevant Yukawa coupling constants, and that of the latter by the method of Balázs, through the introduction of effective-range pole terms. We find that for a wide range of choice of the relevant Yukawa coupling constants, there exist self-consistent bound-state (or resonant) solutions for both $I=1$ $\bar{K}\Xi$ and $I=1$ KN systems. The self-consistent solution for the position in the case of the former is found to lie in the range 1650–1870 MeV, and for the latter in the range 1300–1600 MeV. The over-all conclusion is found to be rather insensitive to the choice of the relevant Yukawa coupling constants. On considering the general case of arbitrary baryon-meson systems, we find that at least in the Balázs-type procedure, one would obtain self-consistent bound-state or low-lying resonant solutions in almost any $J=\frac{3}{2}^+$ baryon-meson system, unless the Born terms are very strongly repulsive. Experimental confirmation of the existence or nonexistence of such systems would thus have strong implications for the dynamical methods such as, for example, those adopted in the present note. As one consequence of the present work, it appears that the qualitative features of any further Balázs-type calculations, especially in the $J=\frac{3}{2}^+$ baryon-meson system, could essentially be anticipated.

I. INTRODUCTION

THE discovery of the Ω^- hyperon at about 1680 MeV fits beautifully into the ten-fold representation of $SU(3)$. From the dynamical point of view it occurs as a pole (in this case a bound state) in the $I=0$, $J=\frac{3}{2}^+$ $\bar{K}\Xi$ scattering amplitude—a pole which presumably arises through the forces due to Σ and Λ exchange in the u channel. Explicit dynamical treatment of the above scattering via the N/D method with the specific purpose of studying whether one should expect a bound state (or a resonance) in the above system has been carried out by many authors.[1] One interesting feature that puts the dynamical treatment of the Ω^- on a somewhat better footing than those of its predecessors such as the Ξ^{*2} and the Y_1^{*3} is that to a fairly good approximation the Ω^- is essentially a one-channel problem; hence the virtues or defects of the dynamical methods may be attributed to sources other than the inherent multichannel problem (for example, to the treatment of the far left-hand-cut singularities, inelastic effects, etc.). The other feature (which is common to the Ξ^{*2} problem as well) is that in the method where one treats the far left-hand-cut singularities by the effective-range pole terms[4] (it may be hoped that this is better than just considering the explicit contribution of the Born singularities), the dependence on the relevant Yukawa couplings is not so marked, at least for the qualitative aspects of the conclusion. This may, at the outset, also be regarded as a virtue. (However, see the discussion in Sec. V.)

We note that there are three distinct systems in the 27-fold representation of $SU(3)$, which also have the characteristic of being essentially single-channel problems. They are the $I=1$ $\bar{K}\Xi$ system, the $I=1$ KN system, and the $I=2$ $\Sigma\pi$ system. In analogy with the Ω^- problem, we study in this note the dynamics of the $I=1$, $J=\frac{3}{2}^+$ $\bar{K}\Xi$ and KN systems with a view to finding out whether or not one should expect a bound state (or a resonance) in these systems, and if so at what energies and with what residues. The $I=2$ $\Sigma\pi$ system is studied similarly in a separate note.[5]

We find that for a wide range of choice of the relevant Yukawa coupling constants, there exist self-consistent bound-state (or resonant) solutions for both the $I=1$ $\bar{K}\Xi$ and $I=1$ KN systems. The results are presented in Sec. III and discussed in Sec. IV. In Sec. V we show

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[1] G. L. Kane, Phys. Rev. 135, B843 (1964). J. C. Pati (unpublished). In this work the Σ-Λ mass difference is retained, in contrast to Kane's work.

[2] J. C. Pati, Phys. Rev. 134, B387 (1964). This will be referred to as I in the present work.

[3] M. Der Sarkissian, Nuovo Cimento 30, 894 (1963).

[4] L. A. P. Baláazas, Phys. Rev. 126, 1220(1962).

[5] K. Vasavada, Nuovo Cimento 40, A1045 (1965).

that it appears to be a typical feature of the Balázs procedure for treating the far left singularities that in almost *any* $J=\frac{3}{2}^+$ *baryon–pseudoscalar-meson system, one would obtain a self-consistent bound-state or low-lying resonant solution*, except when the Born terms are very strongly repulsive. Thus experimental confirmation of the existence or nonexistence of such systems will have strong implications for dynamical methods such as the one adopted in the present note. As one consequence of the present work it appears that the qualitative results of any further Balázs-type calculations, especially in the $J=\frac{3}{2}^+$ baryon–pseudoscalar-meson system, could essentially be anticipated.

II. THE $J=\frac{3}{2}^+$, $I=1$, $\bar{K}\Xi$ AND KN SYSTEMS

$\bar{K}\Xi$ System

We will first discuss scattering in the $I=1$, $J=\frac{3}{2}^+$ $\bar{K}\Xi$ system and mention later what substitutions are needed for the corresponding KN system. We will follow the same notations and almost the same procedure as in I.[2] The reader is referred to I for details. The singularities of the partial-wave amplitude in the unphysical region arise from (i) Λ and Σ exchange in the u channel, (ii) higher mass exchanges in the u channel, (iii) vector-meson and higher mass exchanges in the t channel, and (iv) anomalous branch cuts arising from triangle diagrams. We neglect (ii), (iii), and (iv) in so far as they contribute to the singularities in the unphysical region in the right-half s plane. The latter are thus assumed to arise solely through (i). The contributions of (i), (ii), (iii), and (iv) to the singularities in the left-half s plane are represented by effective-range pole terms whose positions and residues are determined by the procedure suggested by Balázs.

Writing the partial-wave amplitude $g_{1^+}(s)$ as usual in the form $N(s)/D(s)$, where N contains all the unphysical singularities and D the physical right-hand cut, we have (using elastic unitarity)

$$D(s)=1-\frac{s-s_0}{\pi}\int_{(\bar{K}+\Xi)^2}^{\infty}\frac{q'^3N(s')}{s'(s'-s)(s'-s_0)}ds' \quad (1)$$

and

$$N(s)=N_{(f)}(s)+N_{(n)}(s)\,. \quad (2)$$

Here

$$N_f(s)=\frac{1}{\pi}\int_{-\infty}^{0}\frac{\{\operatorname{Im}g_{1^+}(s')\}D(s')}{s'-s}ds'\approx\frac{b_3}{s-s_3}+\frac{b_4}{s-s_4}, \quad (3)$$

where b_3 and b_4 are unknown constants independent of s and will be determined by the use of the fixed energy dispersion relation, and s_3 and s_4 are determined by drawing the Balázs curves and are found to be

$$s_3\simeq-22.6m_\pi^2,\quad s_4\simeq-625m_\pi^2. \quad (4)$$

$$N_{(n)}(s)=\frac{1}{\pi}\int_{L_1(\Sigma)}^{L_2(\Sigma)}ds'\frac{\operatorname{Im}g_{1^+}{}^{(\Sigma)}(s')}{s'-s}+\frac{1}{\pi}\int_{L_1(\Lambda)}^{L_2(\Lambda)}ds'\frac{\operatorname{Im}g_{1^+}{}^{(\Lambda)}(s')}{s'-s}, \quad (5)$$

where

$$g_{1^+}{}^{(Y)}(s)=(g_{Y\Xi K}{}^2/32\pi q^4)[\{(W+\Xi)^2-K^2\}\times(W+Y-2\Xi)Q_1(a)+\{(W-\Xi)^2-K^2\}\times(W+2\Xi-Y)Q_2(a)], \quad (6)$$

$$a=\{2(\Xi^2+K^2)-W^2-N^2\}/2q^2+1\,. \quad (7)$$

$Q_l(x)$ stands for the Legendre function of the second kind, $g_{Y\Xi K}$ for the $Y\Xi K$ coupling constant and Y for Σ or Λ. The end points of the cuts are given by

$$L_1(Y)=(\Xi^2-K^2)^2/Y^2;\quad L_2(Y)=2(\Xi^2+K^2)-Y^2 \quad (8)$$

Thus:

$$L_1(\Lambda)=91m_\pi^2,\quad L_2(\Lambda)=139m_\pi^2,$$
$$L_1(\Sigma)=79m_\pi^2,\quad L_2(\Sigma)=129m_\pi^2. \quad (9)$$

As mentioned in I, the nearby cut contribution to the N function [i.e., $N_{(n)}(s)$] is well represented by a two-pole formula for s in the physical region. Thus for $s\gtrsim(\bar{K}+\Xi)^2$,

$$N_{(n)}(s)\simeq b_1/(s-s_1)+b_2/(s-s_2)\,(s\gtrsim(\bar{K}+\Xi)^2)\,, \quad (10)$$

where by explicit evaluation of $N_{(n)}(s)$ (as in I) for a few values of s, we find

$$s_1=95m_\pi^2;\quad s_2=123m_\pi^2. \quad (11)$$

The residues b_1 and b_2 are determined for a given choice of (i) the coupling constants $g_{YNK}{}^2/4\pi$, (ii) the subtraction point s_0, and (iii) the input value $(s_R)_{\text{in}}$ of the position of the bound state or resonance under examination.

For s below the physical region (i.e., for $s=sM_1$ or sM_2, to be introduced below) $N_n(s)$ is evaluated explicitly by numerical integration.

From Eqs. (1), (2), (3), and (10), the D function is given by

$$D(s)=1-\frac{s-s_0}{\pi}\sum_{i=1}^{4}b_iF(s,s_i,s_0)\,, \quad (12)$$

where

$$F(s,s_i,s_0)=\int_{(\bar{K}+\Xi)^2}^{\infty}\frac{(q'^3/s')}{(s'-s)(s'-s_0)(s'-s_i)}ds'. \quad (13)$$

The F functions were evaluated numerically by an IBM 7094 computer for various values of the argument s. Thus the partial-wave amplitude $N(s)/D(s)$ is determined except for the unknown constants b_3 and b_4. To determine b_3 and b_4 we use the fixed-s dispersion relation in a region where the partial-wave expansion is expected to be convergent. The fixed-energy dispersion relation for the invariant amplitude $A(s,t,u)$ is given by

$$A(s,t,u)=\frac{R_\Sigma}{u-m_\Sigma^2}+\frac{R_\Lambda}{u-m_\Lambda^2}+\frac{R_B}{s-s_R}+\frac{1}{\pi}\int dt'(\cdots)+\frac{1}{\pi}\int du'(\cdots). \quad (14)$$

A similar expression holds for $B(s,t,u)$. The third term corresponds to the contribution of the bound state or resonance in the direct channel with unknown position and residue. We assume that the contribution from the integral terms on the right side of Eq. (14) as well as of the vector-meson-exchange pole terms in the t channel are small. Thus our partial-wave amplitude approximated by the partial-wave projection of Eq. (14) in the appropriate region is given by

$$g_{1^+}(s)\simeq g_{1^+}^{(\Sigma)}(s)+g_{1^+}^{(\Lambda)}(s)+g_{1^+}^{(\text{Bound})}(s), \quad (15)$$

where

$$g_{1^+}^{(\text{Bound})}(s)=-(\kappa)\left\{\frac{(W+\Xi)^2-K^2}{(W_B+\Xi)^2-K^2}\right\}\frac{1}{W-W_B}. \quad (16)$$

W_B stands for $s_B^{1/2}$ and denotes the mass of the bound state (or resonance); κ is the corresponding residue.

If we equate the right side of Eq. (15) with $N(s)/D(s)$ at two points sM_1 and sM_2 [chosen appropriately[2] so that both the representation (15) and the approximate representation for $N(s)/D(s)$ outlined above may hold at sM_1 and sM_2], we can evaluate b_3 and b_4 in terms of the input values of s_R and κ [called $(s_R)_{\text{in}}$ and $(\kappa)_{\text{in}}$] for a given choice of the $Y\Xi K$ coupling constants. We choose, subject to the criterion discussed in I,

$$sM_1=75m_\pi^2, \quad sM_2=142m_\pi^2. \quad (17)$$

Once b_3 and b_4 are determined, as mentioned above, one can compute the D function, look for the zero of the real part of the D function, which will be identified as the output value $(s_R)_{\text{out}}$ of s_R, and the corresponding output value of the residue is given by

$$(\kappa)_{\text{out}}=-\frac{N[(s_R)_{\text{out}}]}{\text{Re}D'[(s_R)_{\text{out}}]}\left[\frac{1}{2(W_R)_{\text{out}}}\right]. \quad (18)$$

Solutions for s_R and κ are to be regarded as acceptable if their output values are consistent (to within say 5%) with the input values. It is hoped that these solutions, if they exist, will lie within a narrow range and will determine the question of existence of the physical bound state (or resonance).

$I=1$, $J=\frac{3}{2}^+$, KN System

The treatment of the $I=1$, $J=\frac{3}{2}^+$ KN system is very similar to that of the $I=1$, $J=\frac{3}{2}^+$ $\bar{K}\Xi$ system. The singularities of the partial-wave amplitudes of both systems arise from similar exchanges in the crossed channels, such as, for example, Λ and Σ exchange in the u channel. Thus the treatment of the KN system is simply obtained from that of the $\bar{K}\Xi$ system by the following substitutions:

$$\bar{K}\to K,\ \Xi\to N,\ g_{\Lambda\Xi K}\to g_{\Lambda NK}, \text{ and } g_{\Sigma\Xi K}\to g_{\Sigma NK}. \quad (19)$$

For the KN system, essentially due to the Ξ-N mass difference, we find

$$L_1(\Lambda)\simeq 16.6m_\pi^2, L_2(\Lambda)\simeq 51.6m_\pi^2, L_1(\Sigma)\simeq 14.5m_\pi^2, L_2(\Sigma)\simeq 42m_\pi^2,$$
$$s_1\simeq 18.8m_\pi^2,\quad s_2\simeq 39.4m_\pi^2,$$
$$s_3\simeq -17m_\pi^2,\quad s_4\simeq -400m_\pi^2. \quad (20)$$

We choose

$$sM_1=60m_\pi^2,\quad sM_2=80m_\pi^2,$$

and

$$s_0=sM_1. \quad (21)$$

The matching procedure for the KN system and the testing for the existence of self-consistent solutions for bound states or resonances are done in the same way as for the $\bar{K}\Xi$ system. The results for both systems are summarized in the following section.

III. RESULTS

In the following we first summarize the results for the $J=\frac{3}{2}^+$, $I=1$ $\bar{K}\Xi$ system and then do the same for the corresponding KN system.

$J=\frac{3}{2}^+$, $I=1$, $\bar{K}\Xi$ System

(1) First of all, irrespective of the choice of the relevant Yukawa coupling constants (confined within a reasonable range) we find that there does exist a self-consistent solution for position and residue. This indicates the existence of a bound state (or resonance) in the $I=1$ $\bar{K}\Xi$ system (call the corresponding particles z^{--}, z^-, and z^0).

(2) In the $SU(3)$ limit, $g_{\Sigma\Xi K}^2/4\pi\simeq 15$, and for an f/d ratio $\simeq\frac{1}{3}$, $g_{\Lambda\Xi K}^2/4\pi\simeq 0$. Since it is not clear how well these predictions are fulfilled by the physical coupling constants apart from what is a reasonable value for the f/d ratio, we tried a wide range of values for these coupling constants. We chose $g_{\Sigma\Xi K}^2/4\pi=16$, 8, 4, 1, 0 and independently $g_{\Lambda\Xi K}^2/4\pi=16$, 8, 4, 1, 0. We find, as in the Ξ^* problem, that the results are rather insensitive to the choice of the Yukawa coupling constants Of course, as is expected, there is found to be a gradual increase in the self-consistent value of the residue κ with an increase of the effective[6] coupling constant. Except for this, it is found that there is no very marked dependence of the position of the bound state (or resonance) on the choice of the coupling constants. In other words, if we require that the self-consistency in

[6] By "effective" we mean the combined effect of the Σ and Λ terms.

position and residue be good to say 5–10%, then for any choice of coupling constants in the above range and for input values of the position in a reasonable range [say $(140\text{–}180)m_\pi^2$], one can pick an input value of κ for which the output values of s_R and κ are consistent with the corresponding input values. The input values of s_R that give rise to self-consistent output values are found to lie within the range

$$s_R \simeq (140\text{–}175)m_\pi^2. \tag{22}$$

Above[7] this range the self-consistency becomes poorer. Depending upon the choice of the coupling constants the corresponding self-consistent solution for κ is found to lie within the range

$$\kappa \simeq 8\text{–}16. \tag{23}$$

Of course, if one demands a better degree of self-consistency (say, better than 1% in both residue and position), the self-consistent solution for a given choice of coupling constants gets restricted to a narrower region. This is part of the reason for the quantitative discrepancy between Kane's results[8] and ours. Qualitatively the results agree. We feel that it is hard to judge *a priori* how good a self-consistency one should really expect in such an approximate method. So it may not be proper to disregard solutions which are not exactly self-consistent but are so within, say, 5 to 10%. The other reason for the discrepancy is the neglect of the Σ-Λ mass difference in Kane's work.

(3) Again as in the Ξ^* problem, with the self-consistent value of κ and $g^2/4\pi < 5$ (say), the effective-range pole terms giving the far-left-hand-cut contribution ($N_{(f)}$) are found to be larger than the nearby-cut contribution ($N_{(n)}$) by at least an order of magnitude. Similarly, in the fixed-energy dispersion relation the Σ-Λ contribution to g_1^+ is found to be smaller than that of the bound-state term ($g_1^{+(\text{Bound})}$).

$J=\frac{3}{2}^+$, $I=1$, KN System

The qualitative aspects of the results (1), (2), and (3) mentioned above also hold for the KN system. The self-consistent solutions for the position and residue are found to lie in the range

$$s_R \simeq (85\text{–}135)m_\pi^2,$$
$$\kappa \simeq 7.5\text{–}11. \tag{24}$$

Above $135m_\pi^2$ the self-consistency becomes considerably poorer.

Thus one should expect not only a bound state (or resonance) in the $J=\frac{3}{2}^+$, $I=0$ $\bar{K}\Xi$ system (i.e., Ω^-) and $I=1$ $\bar{K}\Xi$ system (i.e., z^{--}, z^-, z^0), but also in the $J=\frac{3}{2}^+$ K^+p system. The latter two are yet to be found Their exact locations cannot be predicted very accurately in the present framework. However, z^{--} is predicted to lie roughly in the range 1650–1870 MeV while the K^+p bound state (or resonance) is predicted to lie in the range 1300–1600 MeV.

IV. DISCUSSION

The $I=1$ $\bar{K}\Xi$ system (z^{--},z^-,z^0) and the $I=1$ KN system (K^+p, etc.) belong to the 27-fold representation of $SU(3)$. One would expect, on the basis of the present calculation and from the point of view of unitary symmetry, to observe a host of $J=\frac{3}{2}^+$ *27-fold bound states (or resonances)* in the baryon-meson system in addition to the already observed ten-fold representation. So far there is some indication of the existence of only one system which belongs to the 27-fold representation, i.e., the resonance in the $\Sigma^-\pi^-$ system around 1415 MeV.[9] From the experimental point of view the detection of z^{--} ($I=1$, $Y=-2$), if it were produced in the K^-p experiment, would be considerably easier than that of Ω^-. Since it has not been detected as yet, one would guess that it lies, if anywhere, quite a bit higher than Ω^-. As regards the K^+p system, there are already strong experimental indications[10] of the absence of any bound state or resonance in this system. At this stage we only note that if experiments were to confirm the absence of any bound state or resonance in the $J=\frac{3}{2}^+$, $I=1$ $\bar{K}\Xi$ and/or $\bar{K}N$ system, one would seriously question the success of the methods and the results in the previous dynamical calculations. These remarks are related to a general feature of the Balázs-type bootstrap procedure, which we note in the following section.

V. A TYPICAL FEATURE OF THE BALÁZS-TYPE PROCEDURE

We observe that various types of $J=\frac{3}{2}^+$ systems, such as $N_{3/2}^*$,[11] Y_1^*,[3] Ξ^*,[2] Ω^-,[1] z^{--}, K^+p and $\Sigma^-\pi^-$,[5] etc., subject to the bootstrap procedure as outlined in the present note, have all yielded self-consistent bound-state (or resonant) solutions. From this one might guess that perhaps the analysis is not so sensitive to the choice of the system, its strangeness, isospin, and $SU(3)$ respresentation, and that it will lead to a self-consistent bound-state or resonance solution in any $J=\frac{3}{2}^+$ baryon–pseudoscalar-meson (BP) system, except when the Born terms may be very strongly repulsive.[12] We show

[7] It is hard to investigate what happens below this range because of the singularity structure.

[8] G. L. Kane, Phys. Rev. **135**, B843 (1964). The $I=1$ $\bar{K}\Xi$ system is also considered in this paper. But it drops the Σ-Λ mass difference, which leads to some quantitative discrepancy between the results of Kane and ours.

[9] Y. L. Pan and R. P. Ely, Phys. Rev. Letters **13**, 277 (1964).

[10] L. Lyons and O. I. Dahl, Phys. Letters **14**, 225 (1965). Earlier references may be found here.

[11] V. Singh and B. M. Udgaonkar, Phys. Rev. **130**, 1117 (1963).

[12] Even if one considers a system with repulsive Born terms it is quite possible to obtain a self-consistent bound-state (or resonant) solution in the Balázs-type procedure. For example we tried, just as a test, negative values of $g_{Y\Xi K}^2$ (i.e., $g^2/4\pi \approx -4$, say) in the present calculation and still obtained self-consistent solutions. Kane (see Ref. 1) also noted a similar situation. Of course, for large negative values of $g_{Y\Xi K}^2/4\pi$ it is not possible to obtain a self-consistent solution.

in this section that this situation is typical of at least the Balázs-type bootstrap procedure, in which the Yukawa coupling constants play a role in a rather inconspicuous way.

Typically in a problem involving $(B+P)$ scattering one has: $B\approx(6.5\text{–}10)m_\pi$, $P\approx(1\text{–}4)m_\pi$; one is interested in studying the occurrence of bound state or resonance in the range of $(B+P)^2\pm50m_\pi^2$ (say); thus $(B+P)^2-50m_\pi^2<(s_R)_{\text{in}}<(B+P)^2+50m_\pi^2$; sM_1 and sM_2 (the two matching points) are roughly $(20\text{–}60)m_\pi^2$ below threshold. Under these circumstances one can check quite generally that irrespective of the choice of the Yukawa coupling constants (confined within a reasonable range), one has to take in the first place a value of κ_{in} which is at least 6–12 to have self-consistency in the position [i.e., $(s_R)_{\text{out}}=(s_R)_{\text{in}}$]. For such value of κ_{in} it is found by explicit calculations in various systems that the over-all conclusion on the existence (and perhaps even the location) is hardly affected by varying the Yukawa coupling constants by more than an order of magnitude ($g^2/4\pi=0\text{–}16$, say). Furthermore, at least when the Yukawa coupling constants are not very large ($g^2/4\pi\approx1$, say) one finds that in general $N_{(f)}$ and $g_1^{+(\text{Bound})}$ are bigger than $N_{(n)}$ and $g_1^{+(\text{Born})}$ respectively by an order of magnitude or more. Given this, one is led to ask what would happen if one had, to start with, set

$$b_1=b_2=0\,,\quad\text{and}\quad g_1^{+(\text{Born})}=0\,. \tag{25}$$

One then needs to solve for b_3 and b_4 from the matching equations given by

$$\frac{b_3/(s-s_3)+b_4/(s-s_4)}{1-[(s-s_0)/\pi]\sum_{i=3}^{4}b_iF(s,s_i,s_0)}\approx-\kappa_{\text{in}}\frac{(W+B)^2-P^2}{[(W_R)_{\text{in}}+B]^2-P^2}\frac{1}{W-(W_R)_{\text{in}}}\,. \tag{26}$$

The above equality is expected to hold in an *appropriate region* (see discussion on this in I), within which one chooses the two matching points sM_1 and sM_2. By matching the right and left sides of the above equation, we can evaluate b_3 and b_4 in terms of κ_{in} and $(s_R)_{\text{in}}$. We can then compute the D function, the zero of whose real part gives $(s_R)_{\text{out}}$; the corresponding value of κ_{out} is given by [see Eq. (18)]

$$\kappa_{\text{out}}\simeq-\frac{b_3/[(s_R)_{\text{out}}-s_3]+b_4/[(s_R)_{\text{out}}-s_4]}{\text{Re}D'[(s_R)_{\text{out}}]}\frac{1}{2(W_R)_{\text{out}}}\,. \tag{27}$$

Choosing B, P, and $(s_R)_{\text{in}}$ in the range mentioned above, one can now check quite generally the following:

(A) First, starting from Eq. (26) one can solve for the value of κ_{in} that leads to $(s_R)_{\text{out}}=(s_R)_{\text{in}}$. It is found that there *always exists* a value of κ_{in}, typically in the range of 8–12, which yields $(s_R)_{\text{out}}=(s_R)_{\text{in}}$, chosen in the range mentioned above.

(B) If one next asks what is the value of κ_{out} corresponding to κ_{in} so chosen as to yield $(s_R)_{\text{out}}=(s_R)_{\text{in}}$ [as mentioned in (A)], one finds [as may have been expected from Eqs. (26) and (27)] that if $(s_R)_{\text{in}}$ is chosen in a certain range below the physical threshold (corresponding to a bound-state solution), where the fixed-energy dispersion relation is expected to hold to the same extent that we used it to determine b_3 and b_4, then κ_{out} *is identically equal to* κ_{in}. This may be seen as follows. For values of $(s_R)_{\text{in}}$ as mentioned above we may put

$$N(s_R)_{\text{out}}\big|_{(s_R)_{\text{out}}\to(s_R)_{\text{in}}}=\left[\frac{b_3}{(s_R)_{\text{out}}-s_3}+\frac{b_4}{(s_R)_{\text{out}}-s_4}\right]_{(s_R)_{\text{out}}\to(s_R)_{\text{in}}}$$
$$\approx-\kappa_{\text{in}}\left\{\frac{[(W_R)_{\text{out}}+B]^2-P^2}{[(W_R)_{\text{in}}+B]^2-P^2}\frac{D(s_R)_{\text{out}}}{(W_R)_{\text{out}}-(W_R)_{\text{in}}}\right\}_{(s_R)_{\text{out}}\to(s_R)_{\text{in}}}\simeq-\kappa_{\text{in}}(2W_R)_{\text{out}}D'(s_R)_{\text{out}}\,. \tag{28}$$

Using Eq. (27) it follows that $\kappa_{\text{out}}=\kappa_{\text{in}}$. This would imply that in the Balázs procedure, insofar as the Born terms and $N_{(n)}$ are much smaller than $g_1^{+(\text{Bound})}$ and $N_{(f)}$, respectively, one is almost *guaranteed* to obtain at least bound-state self-consistent solutions for any $J=\frac{3}{2}^+$ $(B+P)$ system.[13] In some cases (especially when the relevant Yukawa coupling constants are large) the Born terms and $N_{(n)}$ are appreciable. These may make some quantitative difference in the results. However, in actual practice, it is found by explicit calculation in various systems (Refs. 1, 2, 5 and the present note, etc.) that there do exist very good self-consistent solutions even for a very wide range of values of the Yukawa coupling constants ($g^2/4\pi$ 0–20, say). Thus it appears that even with the inclusion of the Born terms ($g_1^{+(\text{Born})}$ and $N_{(n)}$), the over-all qualitative conclusion regarding the existence of self-consistent solutions corresponding to either a bound state or a low-lying reso-

[13] We have not checked yet whether a similar situation holds for other values of angular momentum and parity.

nance[14] in every $J=\frac{3}{2}^+$ $(B+P)$ system will still be maintained, barring the situation where the Born terms are strongly repulsive.[12]

We note that the above arguments regarding self-consistency need not hold for resonant solutions [$(s_R)_{\text{in}}>(B+P)^2$], since the representation of the partial-wave amplitude by the fixed-energy dispersion relation is not expected to hold in the physical region. [In other words we cannot directly use Eq. (26) to judge the self-consistency in κ.] From this one might guess that the self-consistency may get worse as one goes sufficiently above the physical threshold [$(s_R)_{\text{in}}>(B+P)^2+50m_\pi^2$, say]. This is found to be the case by actual calculation in various systems.

[14] In view of the role played by the nearby singularities ($N_{(n)}$) and $g_1^{+(\text{Born})}$ and the fact that the self-consistency cannot quite rigorously be judged on the basis of Eq. (A.2), it is quite possible to obtain a low-lying [low-lying compared to the physical threshold $(B+P)^2$] resonance rather than a bound-state solution. This is what happens in the case of the (3,3) πN resonance, (Ref. 11) for example.

VI. CONCLUDING REMARKS

In the present work we have carried out explicit calculations for $I=1$ $\bar{K}\Xi$ and KN systems using the Balázs-type N/D method, and found that self-consistent bound-state (or resonant) solutions exist for both systems. On considering the general case of arbitrary baryon-meson systems, we found a typical feature of this procedure that in almost any $J=\frac{3}{2}^+$ baryon–pseudoscalar-meson system, one would obtain self-consistent bound-state or low-lying resonant solutions. This is a remarkable and somewhat awkward result, it it corresponds to reality. It leads one to wonder about the physical implications of the results in such a scheme. At any rate the most interesting question is: Will experiments confirm such a conclusion?

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Lie Group of the Strong-Coupling Theory. I. Calculation of the Coupling-Constant Ratios and Magnetic Moments for Symmetric Pseudoscalar-Meson Theory

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All the isobar-pion coupling constants are calculated using the Lie algebra $[SU(2)\otimes SU(2)]\times T_9$ of the strong-coupling theory. The $N^*\to N\pi$ reduced width comes out to be in agreement with experiment. We also calculate isobar magnetic moments in terms of proton and neutron magnetic moments. The results obtained are also compared with $SU(6)$ and reciprocal-bootstrap predictions.

I. INTRODUCTION

RECENTLY the Lie-group structure of the strong-coupling theory of baryon-meson scattering[1,2] has been deduced in the framework of the dispersion relations satisfied by the static models.[3] Various possible irreducible representations of this Lie group provide the possible isobar spectra. The number of isobars turns out to be infinite for any irreducible representation. Mathematically this is due to the group involved being noncompact, so that it has no finite-dimensional unitary representations. It is physically understandable that in the limit of very large baryon-meson coupling an infinite number of isobars would occur. More and more poles of the scattering amplitude, representing isobars, move onto the physical sheet as the coupling constants are increased to larger and larger values. In the physical case all the coupling constants are finite and only a few of these poles would have approached the physical sheet. So in the physical case one would observe only a few low-lying isobars. It should be emphasized that in this model only the scale of various isobar coupling constants tends to infinity; the ratios of these remain finite.

For the case of symmetric pseudoscalar-meson theory it was shown by Cook, Goebel, and Sakita that the Lie group G of this theory is $G=[SU(2)\otimes SU(2)]\times T_9$. Further, using group contraction on the $SU(4)$ group with respect to its subgroup $SU(2)\otimes SU(2)$, it was shown that the only irreducible representations (IR) of group G are given by the $SU(4)$ IR with Young-tableau characterization $(\infty,\lambda_2,\lambda_3)$. The mass spectrum was shown to be of the form

$$M(I,J)=M_0+M_1J(J+1)+M_2I(I+1),$$

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