PHYSICAL REVIEW VOLUME 144, NUMBER 4 29 APRIL 1966

Energy-Momentum Structure Form Factors of Particles

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(Received 24 November 1965)

By emphasizing the analogy between mass and charge as sources of fields, we are led to examine the mechanical structure of a particle in terms of the matrix elements $\langle \mathbf{p}'|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$, where $\theta_{\alpha\beta}(x)$ is the total energy-momentum tensor, just as the matrix elements $\langle \mathbf{p}'|j_\alpha(0)|\mathbf{p}\rangle$ of the current operator $j_\alpha(x)$ define its electromagnetic structure. Although the off-diagonal matrix elements $\langle \mathbf{p}'|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$ are not accessible to direct experimental observation, the diagonal element $\langle \mathbf{p}|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$ is just proportional to the total mass. Consequently, we can study the contributions to the total mass in terms of vertex functions instead of propagators and, using the techniques of dispersion theory, relate the contribution to the total mass to integrals over physical scattering processes. We examine electrodynamics and the pion-nucleon interaction in perturbation theory and show how the mass divergences emerge as a consequence of the high-energy behavior of the Coulomb amplitude and the nucleon-nucleon scattering amplitude. Finally, using elastic unitarity, we can relate mass splittings in a multiplet to integrals over the differences in S-wave phase shifts.

I. INTRODUCTION

MASS makes its appearance in physics in two fundamental ways. Inertial or dynamical mass is a measure of how a physical system responds to the influence of external forces and it is this mass that appropriately appears in Newton's equation of motion and the equations describing the interactions of quantum-mechanical systems. The second way that mass makes its appearance in physics is as a measure of the source strength of the gravitational field in analogy with the electric charge as a measure of the source strength of the electromagnetic field. The equivalence principle then asserts that for any physical system these two measures of the mass must be the same and that no experiment can be devised that distinguishes them.[1]

It is the concept of inertial mass that plays the central role in modern particle physics in relation to questions pertaining to mass renormalization or mass splittings in a multiplet. For example, it is the inertial mass that appears in the Lehmann representation of the propagator function or the dynamical equations of the S-matrix approach to strong interactions. Rather than emphasize the concept of mass as a dynamical term in the equations of motion, the approach we present here examines the role of gravitational mass and emphasizes the analogy between mass and electric charge as sources of fields. No new physics can emerge from this investigation which is not available from conventional approaches; however, techniques enabling one to simply relate mass splittings to physical scattering processes are presented in this on-the-mass-shell approach.

In examining the electromagnetic properties of a quantum mechanical system our interest is in the matrix elements of the total current operator $j_\alpha(x)$ which acts as a source for the electromagnetic field $A_\alpha(x)$. For example, the response of a single-particle state $|\mathbf{p}\rangle$ of momentum $\mathbf{p}$ to the probing electromagnetic potential $A_\alpha(x)$ is given in terms of the transition matrix elements $\langle \mathbf{p}'|j_\alpha(0)|\mathbf{p}\rangle$ which contain the information about its charge and possible magnetic structure in terms of form factors which are accessible to experimental study. Similarly, the mechanical structure of a particle is given in terms of the matrix elements of $\theta_{\alpha\beta}(x)$ the total symmetric stress-energy tensor taken between single-particle states. The transition matrix elements $\langle \mathbf{p}'|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$ describe the response of the system to a probing gravitational field, $g_{\alpha\beta}(x)$. Contrary to the case of electromagnetism, there is very little hope of learning anything about the detailed mechanical structure of a particle, because of the extreme weakness of the gravitational interaction. However, just as in the limit of zero-momentum transfer, $p'-p=0$, $\langle \mathbf{p}|j_\alpha(0)|\mathbf{p}\rangle$ is proportional to the totally renormalized charge of the system, so $\langle \mathbf{p}|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$ is proportional to the totally renormalized mass m. This is easily seen by going to the rest frame of the particle $p_\alpha=(m,0)$, where all components of $\langle \mathbf{p}=0|\theta_{\alpha\beta}(0)|\mathbf{p}=0\rangle$ vanish except $\langle \mathbf{p}=0|\theta_{00}(0)|\mathbf{p}=0\rangle$.[2] Since the Hamiltonian is given by

$$H=\int\theta_{00}(x)d^3x\,,$$

so that $\langle \mathbf{p}'|H|\mathbf{p}\rangle=(2\pi)^3\delta^3(\mathbf{p}'-\mathbf{p})\langle \mathbf{p}'|\theta_{00}(0)|\mathbf{p}\rangle$ and since $H|\mathbf{p}=0\rangle=m|\mathbf{p}=0\rangle$, we must have $\langle \mathbf{p}=0|\theta_{00}(0)|\mathbf{p}=0\rangle=m$.

In spite of the fact that the off-diagonal matrix element $\langle \mathbf{p}'|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$ has little opportunity to be experimentally investigated we can nonetheless make use of its analytic properties as a function of the invariant momentum transfer $q^2=(p'-p)^2$. The techniques of dispersion theory then enable us to express the contribution to the total mass in terms of integrals over physically measurable scattering amplitudes similar to the treatment of the matrix elements $\langle \mathbf{p}'|j_\alpha(0)|\mathbf{p}\rangle$.[3] Of course, the success of any such approach to determine the effect of interactions on the mass of a state depends

† Supported in part by U. S. Air Force Contract No. AFOSR-153-64.

[1] S. Weinberg, Phys. Rev. **138**, B988 (1965); **135**, B1049 (1964).

[2] This is implied by Lorentz invariance as we show in Sec. II.

[3] S. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, New York, 1961).

crucially on the asymptotic behavior of $\langle \mathbf{p}'|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$ as $|q^2|\to\infty$ just as does any dispersion-theory technique which purports to calculate physical constants rather than simply find relations between them.

Consequently, stressing the analogy between mass and charge as sources of fields suggests an examination of the problem of mass renormalization and mass splittings in terms of vertex functions rather than propagators. This has the advantage, as we will see in the sequel, that utilizing the analytic properties of this vertex it is possible to relate mass differences to differences in phase shifts of physically measurable scattering processes thus permitting a direct investigation of the relation between mass splittings and the forces responsible for the scattering process.

In the next section we will consider the implications of Lorentz invariance and energy-momentum conservation for restricting the form of the vertex $\langle \mathbf{p}'|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$. Here we also examine some the consequences of the fundamental Schwinger equal-time commutation relation $i[\theta_{00}(\mathbf{x}),\theta_{00}(\mathbf{y})]=[\theta_{0k}(\mathbf{x})+\theta_{0k}(\mathbf{y})]\partial_k\delta^3(\mathbf{x}-\mathbf{y})$ which is a general implication of Lorentz invariance.[4] In Sec. III, using quantum electrodynamics as a model, we compute the matrix elements $\langle \mathbf{p}'|\theta_{\alpha\beta}(0)|\mathbf{p}\rangle$, where $|\mathbf{p}\rangle$ is a single-electron state of momentum $\mathbf{p}$. Using the analytic properties of Feynman graphs we relate the renormalized mass to the bare mass in terms of an integral over the Coulomb scattering amplitude. Here the characteristic mass divergence is seen as a direct consequence of the high-energy behavior of the Coulomb amplitude as calculated in perturbation theory. We examine the pion-nucleon system in the same way. Here, the contribution to the nucleon mass is expressed as an integral over the pion-nucleon and nucleon-nucleon scattering amplitude. Finally in Sec. IV we examine the implication of the unitarity condition for the matrix elements of $\theta_{\alpha\beta}(x)$ taken between single-particle states. Here a relation between the total mass or mass splitting and the S-wave phase shift emerges. It is also possible to incorporate in a simple way the idea of octet enhancement into this approach to mass splitting. The implications of the tadpole hypothesis[5] as a mechanism for octet enhancement is briefly discussed.

In the following paper we use the ideas and techniques here presented to calculate the neutron-proton mass difference.

II. IMPLICATIONS OF LORENTZ INVARIANCE

Our purpose in this section is to examine the restrictions that invariance under the Lorentz group and the conservation of energy-momentum

$$\partial_\alpha\theta_{\alpha\beta}(x)=0 \tag{1}$$

impose on the general form of the matrix elements $\langle \mathbf{p}_1|\theta_{\alpha\beta}(0)|\mathbf{p}_2\rangle$, where $|\mathbf{p}_2\rangle$ and $\langle\mathbf{p}_1|$ are initial and final single particle states of momentum $\mathbf{p}_2$ and $\mathbf{p}_1$. We assume that $\theta_{\alpha\beta}(x)$ transforms like a tensor of the second rank under proper Lorentz transformations and spatial reflections and that the matrix elements are time-reversal invariant. Then, once the spin of the system is specified, the matrix elements can be written down in their most general form in terms of the invariant mechanical form factors $G_i(q^2)$ which are functions of the momentum transfer $q^2=(p_1-p_2)^2$. For illustrative purposes, we will examine only the case of systems of spin $\frac{1}{2}$ or spin 0.

The matrix elements of $\theta_{\alpha\beta}(x)$ between one-particle spin $\frac{1}{2}$ systems have already received intensive investigation in connection with the self-stress of the electron.[6–9] From this work we find that the most general form for the matrix elements $\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle$ consistent with the above requirements may be written as

$$\begin{aligned}\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle &= (m^2/p_0^1p_0^2)^{1/2}[\bar{u}(p_1,s_1)/4m][G_1(q^2)(l_\mu\gamma_\nu+l_\nu\gamma_\mu)\\ &\quad +G_2(q^2)l_\mu l_\nu/m+G_3(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)/m]u(p_2,s_2)\,, \end{aligned}\tag{2}$$

where $l_\mu=(p_1+p_2)_\mu$, $q_\mu=(p_1-p_2)_\mu$, and $m^2=p_1^2=p_2^2$ is the total mass of the particle. Here $u(p,s)$ denotes the Dirac spinor satisfying the Dirac equation $(p\!\!\!/-m)\times u(p,s)=0$ and normalized according to $\sum_s\bar{u}(p,s)u(p,s)=(p\!\!\!/+m)/2m$. The $G_i(q^2)$ appearing in Eq. (2) are the mechanical form factors describing the mechanical structure of the spin $\frac{1}{2}$ system. The condition of energy momentum conservation Eq. (1) implies that Eq. (2) must satisfy $q^\mu\langle\mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle=0$ which is easily verified using the Dirac equation. In fact it is a necessary requirement that the expression within the bracket of Eq. (2) be symmetric with respect to the interchange of p_1 and p_2 for conservation of energy momentum to hold.[7] In the limit of zero momentum transfer $q_\mu=0$, $p_1=p_2=p$ the general form of the diagonal matrix element is seen to be

$$\begin{aligned}\langle\mathbf{p}|\theta_{\mu\nu}(0)|\mathbf{p}\rangle &= (m^2/p_0^1p_0^2)^{1/2}\bar{u}(p,s)u(p,s)\\ &\quad\times(p_\mu p_\nu/m^2)[G_1(0)+G_2(0)]. \end{aligned}\tag{3}$$

Going to the rest frame of the particle $\mathbf{p}=0$, one sees from Eq. (3) that only the components $\langle\mathbf{p}=0|\theta_{00}(0)|\mathbf{p}=0\rangle$ survive, and consequently the self-stress automatically vanishes as is required by Lorentz invariance and the conservation of energy momentum. Since $\langle\mathbf{p}=0|\theta_{00}(0)|\mathbf{p}=0\rangle=m\bar{u}(p,s)u(p,s)$ we have from Eq. (3) the condition

$$G_1(0)+G_2(0)=m \tag{4}$$

in analogy to the condition $F_1(0)=e$ in the case of the electromagnetic form factors.[2]

Often it will be sufficient to examine the matrix elements of the trace $\theta(x)=\theta_\alpha{}^\alpha(x)$. Here the general form

[4] J. Schwinger, Phys. Rev. **130**, 406 (1963).

[5] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[6] A. Pais and S. T. Epstein, Rev. Mod. Phys. **21**, 445 (1949).

[7] F. Rohrlich, Phys. Rev. **77**, 357 (1950).

[8] F. Villars, Phys. Rev. **79**, 122 (1950).

[9] S. Borowitz and W. Kohn, Phys. Rev. **86**, 985 (1952).

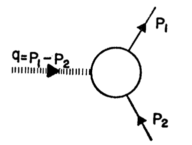

FIG. 1. Diagrammatic representation of the vertex $\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle$ or $\langle \mathbf{p}_1|\theta(0)|\mathbf{p}_2\rangle$.

is just

$$\langle \mathbf{p}_1|\theta(0)|\mathbf{p}_2\rangle=(m^2/p_0{}^1p_0{}^2)^{1/2}\bar{u}(p_1,s_1)u(p_2,s_2)G(q^2)\,, \quad (5)$$

where $G(q^2)$ is the linear combination

$$G(q^2)=G_1(q^2)+G_2(q^2)(1-q^2/4m^2)+G_3(q^2)3q^2/4m^2 \quad (6)$$

of the $G_i(q^2)$ appearing in Eq. (2). In the limit of zero-momentum transfer, and in the rest frame of the particle, since $\theta(0)=\theta_{00}(0)+\theta_i{}^i(0)$, and from the fact that the matrix elements of $\theta_i{}^i(0)$ vanish, we have $\langle \mathbf{p}=0|\theta(0)|\mathbf{p}=0\rangle=\langle \mathbf{p}=0|\theta_{00}(0)|\mathbf{p}=0\rangle=m\bar{u}(p,s)u(p,s)$, and consequently,

$$G(0)=m \quad (7)$$

as is also easily seen from Eqs. (4) and (6) with $q^2=0$. We will diagrammatically represent the vertices given by the matrix elements Eqs. (2) or (5) as shown in Fig. 1.

In the case of a spin-zero system like the pion the general form for the matrix element consistent with the requirements of Lorentz invariance and energy-momentum conservation is given by

$$\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle=(4p_0{}^1p_0{}^2)^{-1/2}[G_1(q^2)l_\mu l_\nu/4\mu^2 + G_2(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)/4\mu^2]\,, \quad (8)$$

where $l_\mu=(p_1+p_2)_\mu$, $q_\mu=(p_1-p_2)_\mu$, and $p_1{}^2=p_2{}^2=\mu^2$ the mass of the particle. In the limit of zero momentum transfer $q^2\to 0$, one obtains the requirement

$$G_1(0)=2\mu^2. \quad (9)$$

We note that for a free boson field $\phi(x)$ the stress tensor is given by $\theta_{\mu\nu}(x)=\partial_\mu\phi(x)\partial_\nu\phi(x)-\frac{1}{2}g_{\mu\nu}[\partial_\lambda\phi(x)\partial^\lambda\phi(x)-\mu^2\phi^2(x)]$ and it follows that in the absence of interactions $G_1(q^2)=G_2(q^2)=2\mu^2$. The trace $\theta(x)$ has the matrix elements

$$\langle \mathbf{p}_1|\theta(0)|\mathbf{p}_2\rangle=(4p_0{}^1p_0{}^2)^{-1/2}G(q^2) \quad (10)$$

with

$$G(q^2)=G_1(q^2)(1-q^2/4\mu^2)+G_2(q^2)3q^2/4\mu^2 \quad (11)$$

so that

$$G(0)=2\mu^2. \quad (12)$$

In the next section we will compute the form factors $G_i(q^2)$ appearing in Eq. (2) to first order in $\alpha=1/137$ for the electron interacting with the photon field and relate them to the Coulomb scattering amplitude and the pair annihilation amplitude. However, before engaging in any calculation we want to examine some additional implications of Lorentz invariance for our matrix elements. Schwinger[3] was able to show that if $\theta_{\alpha\beta}(x)$ was the symmetrical energy-momentum tensor, then for any theory of spin ≤ 1 particles, Lorentz invariance imposes the fundamental equal-time commutation rule

$$i[\theta_{00}(\mathbf{x}),\theta_{00}(\mathbf{y})]=[\theta^{0k}(\mathbf{x})+\theta^{0k}(\mathbf{y})]\partial_k\delta^3(\mathbf{x}-\mathbf{y})\,. \quad (13)$$

This relation will, in general, impose constraints on the form factors $G_i(q^2)$ which are best seen by examining Eq. (13) in the momentum representation.

Inserting both sides of Eq. (13) between one-particle states, placing a complete set of states $|n\rangle$ in the commutator and taking the Fourier transform, one obtains for the left-hand side

$$i\int d^3x\int d^3y\, e^{-i\mathbf{q}\cdot\mathbf{x}}e^{-i\mathbf{l}\cdot\mathbf{y}}\langle \mathbf{p}_1|[\theta_{00}(\mathbf{x}),\theta_{00}(\mathbf{y})]|\mathbf{p}_2\rangle$$
$$=i\int d^3x\int d^3y\, e^{-i\mathbf{q}\cdot\mathbf{x}}e^{-i\mathbf{l}\cdot\mathbf{y}}\sum_n[\langle \mathbf{p}_1|\theta_{00}(\mathbf{x})|n\rangle\langle n|\theta_{00}(\mathbf{y})|\mathbf{p}_2\rangle-\langle \mathbf{p}_1|\theta_{00}(\mathbf{y})|n\rangle\langle n|\theta_{00}(\mathbf{x})|\mathbf{p}_2\rangle]$$
$$=i(2\pi)6\sum_n\langle \mathbf{p}_1|\theta_{00}(0)|n\rangle\langle n|\theta_{00}(0)|\mathbf{p}_2\rangle\delta^3(\mathbf{p}_1-\mathbf{p}_2-\mathbf{l}-\mathbf{q})[\delta^3(\mathbf{p}_n-\mathbf{p}_2-\mathbf{l})-\delta^3(\mathbf{p}_n-\mathbf{p}_2-\mathbf{q})]\,,$$

and for the right-hand side

$$\int d^3x\int d^3y\, e^{-i\mathbf{q}\cdot\mathbf{x}}e^{-i\mathbf{l}\cdot\mathbf{y}}\langle \mathbf{p}_1|[\theta^{0k}(\mathbf{x})+\theta^{0k}(\mathbf{y})]\partial_k\delta^3(\mathbf{x}-\mathbf{y})|\mathbf{p}_2\rangle$$
$$=\int d^3y\, e^{-i\mathbf{l}\cdot\mathbf{y}}\langle \mathbf{p}_1|\partial_k[e^{-i\mathbf{q}\cdot\mathbf{y}}\theta^{0k}(\mathbf{y})]|\mathbf{p}_2\rangle-\int d^3x\, e^{-i\mathbf{q}\cdot\mathbf{x}}\langle \mathbf{p}_1|\partial_k[e^{-i\mathbf{l}\cdot\mathbf{x}}\theta^{0k}(\mathbf{x})]|\mathbf{p}_2\rangle$$
$$=\int d^3y\, e^{-i(\mathbf{l}+\mathbf{q})\cdot\mathbf{y}}-iq_k\langle \mathbf{p}_1|\theta^{0k}(\mathbf{y})|\mathbf{p}_2\rangle-\int d^3x\, e^{-i(\mathbf{l}+\mathbf{q})\cdot\mathbf{x}}-il_k\langle \mathbf{p}_1|\theta^{0k}(\mathbf{x})|\mathbf{p}_2\rangle$$
$$=(2\pi)^3\delta^3(\mathbf{p}_1-\mathbf{p}_2-\mathbf{l}-\mathbf{q})i(l_k-q_k)\langle \mathbf{p}_1|\theta^{0k}(0)|\mathbf{p}_2\rangle\,.$$

We have used the translational invariance of the theory $\langle \mathbf{p}_1|\theta_{\mu\nu}(\mathbf{x})|\mathbf{p}_2\rangle = e^{i(\mathbf{p}_1-\mathbf{p}_2)\cdot\mathbf{x}}\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle$ and set $t=0$. Hence one obtains a sum rule for the matrix elements $\langle \mathbf{p}_1|\theta^{0k}(0)|\mathbf{p}_2\rangle$:

$$(2\pi)^3 \sum_n \langle \mathbf{p}_1|\theta_{00}(0)n\rangle\langle n|\theta_{00}(0)|\mathbf{p}_2\rangle \times[\delta^3(\mathbf{p}_n-\mathbf{p}_2-\mathbf{l})-\delta^3(\mathbf{p}_n-\mathbf{p}_2-\mathbf{q})] = (l_k-q_k)\langle \mathbf{p}_1|\theta^{0k}(0)|\mathbf{p}_2\rangle, \quad (14)$$

where $\mathbf{p}_1=\mathbf{p}_2+\mathbf{l}+\mathbf{q}$. In Eq. (14) we may set $\mathbf{q}=0$ and from the conservation of energy momentum, $(p_k{}^1-p_k{}^2)\times\langle \mathbf{p}_1|\theta^{0k}(0)|\mathbf{p}_2\rangle = (p^1-p^2)_0\langle \mathbf{p}_1|\theta_{00}(0)|\mathbf{p}_2\rangle$, one obtains

$$(2\pi)^3 \sum_n \langle \mathbf{p}_1|\theta_{00}(0)|n\rangle\langle n|\theta_{00}(0)|\mathbf{p}_2\rangle \times[\delta^3(\mathbf{p}_n-\mathbf{p}_1)-\delta^3(\mathbf{p}_n-\mathbf{p}_2)] = (p^1-p^2)_0\langle \mathbf{p}_1|\theta_{00}(0)|\mathbf{p}_2\rangle. \quad (15)$$

Equation (15) imposes a nonlinear constraint on the form factors $G_i(q^2)$ as a consequence of Lorentz invariance. We will not use this sum rule to calculate but simply note one consequence of Eq. (15) obtained by approximating the sum by including only the single-particle intermediate state. Inserting either the general form Eq. (2) for the spin-$\frac{1}{2}$ case or the general form Eq. (8) for the spin case one finds that Eq. (15) is an identity provided that $G_1(0)+G_2(0)=m$ in the spin-$\frac{1}{2}$ case or $G_1(0)=2\mu^2$ in the spin-0 case. So the commutation rule fixes the boundary values of the $G_i(q^2)$. Using this information on $G_i(0)$ we have

$$\sum_{n\neq 1} \langle \mathbf{p}_1|\theta_{00}(0)|n\rangle\langle n|\theta_{00}(0)|\mathbf{p}_2\rangle \times[\delta^3(\mathbf{p}_n-\mathbf{p}_1)-\delta^3(\mathbf{p}_n-\mathbf{p}_2)]=0, \quad (16)$$

where the sum does not include the single-particle state. Equation (16), which incorporates inelastic effects, then gives relations on the $G_i(q^2)$ for $q^2\neq 0$. Consequently, we see how the Schwinger commutation relation Eq. (13) serves to impose additional restrictions on the matrix elements $\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle$. We will not here investigate these implications further but rather, in the next section, relate the $G_i(q^2)$ to dispersion integrals over scattering amplitudes using quantum electrodynamics as a model.

III. ANALYTICAL PROPERTIES

In this section we will illustrate explicit methods for calculating the $G_i(q^2)$ from their analytic properties. We are motivated to use this approach, which leads to dispersion relations for the $G_i(q^2)$, since it enables us to relate the matrix elements $\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle$ to physical scattering processes. Thus, we will not be committed to a perturbation-theory technique when considering the strong interactions. For definiteness we examine the electrodynamics of electrons and photons as a model and will simply follow the already established methods for computing the electromagnetic form factors $F_i(q^2)$ of the electron.[10] Although we make use of dis-

[10] S. D. Drell and F. Zachariasen, Phys. Rev. **111**, 1727 (1958).

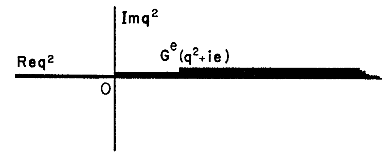

FIG. 2. Analytic properties of $G_i(q^2)$.

persion relations which presuppose certain analyticity properties of the $G_i(q^2)$ we will not attempt to rigorously prove these properties but rather choose to take first-order perturbation theory and the analytic properties of the corresponding Feynman graphs as our guide.

A. Electrodynamics

As an explicit illustration of our calculational methods we will compute the matrix element

$$\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle = (m^2/p_0{}^1p_0{}^2)^{1/2}[\bar{u}(p_1,s_1)/4m] \times[G_1{}^e(q^2)(l_\mu\gamma_\nu+l_\nu\gamma_\mu)+G_2{}^e(q^2)l_\mu l_\nu/4m +G_3{}^e(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)/m]u(p_2,s_2) \quad (17)$$

of the total stress-energy operator of the electron-photon system between single electron states of momentum $\mathbf{p}_1$ and $\mathbf{p}_2$. Here $l_\mu=(p_1+p_2)_\mu$, $q_\mu=(p_1-p_2)_\mu$, $p_1{}^2=p_2{}^2=m^2$, and the $G_i{}^e(q^2)$ are the mechanical form factors of the electron. The total stress energy operator is given by

$$\theta_{\mu\nu}(x)=\theta_{\mu\nu}{}^E(x)+\theta_{\mu\nu}{}^P(x)+\theta_{\mu\nu}{}^I(x), \quad (18)$$

where the electron field contributes

$$\theta_{\mu\nu}{}^E(x)=i\tfrac{1}{4}[\bar{\psi}(x),\gamma_{(\mu}\partial_{\nu)}\psi(x)], \quad (19)$$

the photon field

$$\theta_{\mu\nu}{}^P(x)=\tfrac{1}{2}(F_{\mu\lambda}F_\nu{}^\lambda+F_{\nu\lambda}F_\mu{}^\lambda-\tfrac{1}{2}g_{\mu\nu}F_{\sigma\tau}F^{\sigma\tau}) \quad (20)$$

and the interaction term is

$$\theta_{\mu\nu}{}^I(x)=\tfrac{1}{2}j_{(\mu}A_{\nu)}, \quad (21)$$

where

$$F_{\mu\nu}(x)=\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x), \quad j_\mu(x)=\Box A_\mu(x)$$

and we have introduced the notation $(\mu\nu)=\mu\nu+\nu\mu$. The self-stress problem of the electron, showing that the form of the diagonal matrix elements does not change when one takes the radiative corrections into account, has been extensively studied[5–8] and we have nothing to add to this investigation. We simply point out that this work showed that the net effect of the radiative corrections was to shift the mass.

Our aim here is to relate the $G_i{}^e(q^2)$ to scattering processes with the end in mind of examining the divergence of the electron self-mass $\delta m=m-m_0$ in terms of the asymptotic behavior of these scattering processes. We will proceed heuristically rather than with strict rigor and begin by assuming that the $G_i{}^e(q^2)$ are analytic functions of q^2 in the cut q^2 plane with a branch cut extending from $q^2=0$ to $q^2=\infty$ (see Fig. 2). This property of the $G_i{}^e(q^2)$ explicitly follows from first-order perturbation theory and the analytic properties of the corresponding Feynman graphs and we shall assume it as a property of the $G_i^e(q^2)$ independent of perturbation theory. Furthermore first-order perturbation theory reveals that as $|q^2|\to\infty$, $G_1^e(q^2)/q^2$, $G_2^e(q^2)$, and $G_3^e(q^2)$ all vanish. We may summarize the asymptotic behavior and analytic properties by the following dispersion relations:

$$G_1^e(q^2)=G_1^e(0)+\frac{q^2}{\pi}\int_0^\infty \frac{\mathrm{Im}G_1^e(q'^2)dq'^2}{q'^2(q'^2-q^2-i\epsilon)}, \tag{22}$$

$$G_i^e(q^2)=\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}G_i^e(q'^2)dq'^2}{(q'^2-q^2-i\epsilon)} \quad i=2,3, \tag{23}$$

where $2i\,\mathrm{Im}G_i^e(q^2)$ is the discontinuity of $G_i^e(q^2)$ across the cut. The dispersion relation for $G_1^e(q^2)$ required a subtraction in virtue of the prescribed asymptotic behavior as $|q^2|\to\infty$ which we have made at $q^2=0$. We note that the total mass m, the constant $G_2^e(0)$ which can be computed from (23) and the subtraction constant $G_1^e(0)$ are related by $G_1^e(0)+G_2^e(0)=m$, Eq. (4). In writing a subtracted dispersion relation for $G_1^e(q^2)$, we then give up all hope of calculating the mass m from a knowledge of the absorptive part $\mathrm{Im}G_i^e(q^2)$. Later we will return to the question of subtraction constants, for it is essential to any examination related to the calculation of masses in particle physics.

In virtue of the prescribed analytic properties of the $G_i^e(q^2)$ we may analytically continue the scattering vertex $\langle \mathbf{p}_1|\theta_{\mu\nu}(0)|\mathbf{p}_2\rangle$ to the physical region for production of electron-positron pairs and the corresponding amplitude,

$$\begin{aligned}\langle \mathbf{p}_1\mathbf{p}_2{}^{(-)}|\theta_{\mu\nu}(0)|0\rangle &= (m^2/p_0{}^1p_0{}^2)^{1/2}[\bar{u}(p_1,s_1)/4m][G_1^e(q^2)(l_\mu\gamma_\nu+l_\nu\gamma_\mu)\\ &\quad +G_2^e(q^2)l_\mu l_\nu/m+G_3^e(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)/m]\\ &\quad \times v(p_2,s_2),\end{aligned} \tag{24}$$

where p_1 and p_2 are the four-momenta of the electron-positron pair and the total energy is q_0 with $q_\mu=(p_1+p_2)_\mu$ and $l_\mu=(p_1-p_2)_\mu$. The Dirac spinors of the electron and positron are denoted by $u(p_1,s_1)$ and $v(p_2,s_2)$ and satisfy $(p_2+m)v(p_2,s_2)=0$, $(p_1-m)u(p_1,s_1)=0$ and are normalized according to $\bar{u}(p)u(p)=(p+m)/2m$, $\bar{v}(p)v(p)=(p-m)/2m$. Here $\langle \mathbf{p}_1\mathbf{p}_2{}^{(-)}|$ denotes the state of the final electron-positron pair with outgoing boundary conditions. The $G_i^e(q^2)$ appearing in Eq. (24) are the same as those in Eq. (17), but now continued to the region $q^2\geq 0$.

We may rewrite the amplitude Eq. (24) in a different form using the LSZ reduction formalism[11] which reveals the branch cut and reality property $G_i^e(q^{2*})=G_i^{e*}(q^2)$ of the form factors required by the representations Eqs. (22) and (23). Contracting the electron state vector from $\langle \mathbf{p}_1\mathbf{p}_2{}^{(-)}|$, one obtains

$$\langle \mathbf{p}_1\mathbf{p}_2{}^{(-)}|\theta_{\mu\nu}(0)|0\rangle=-i\left(\frac{m}{p_0{}^1}\right)^{1/2}\int d^4x\, e^{ip_1\cdot x}\hat{\theta}(x_0)\langle \mathbf{p}_2|[\bar{u}(p_1)\eta(x),\theta_{\mu\nu}(0)]|0\rangle, \tag{25}$$

where $\eta(x)=(i\nabla-m)\psi_R(x)$ and $\hat{\theta}(x_0)=\frac{1}{2}(1+x_0/|x_0|)$. In writing Eq. (25), we have dropped the equal-time commutator $\delta(x_0)[\psi_R(x),\theta_{\mu\nu}(0)]$ of the renormalized Heisenberg operator $\psi_R(x)$ with $\theta_{\mu\nu}(0)$ since it does not effect the calculation of the absorptive part or the analytic properties. The equal-time commutator has a bearing on the issue of subtraction constants and we will return to it later. By inserting a complete set of states (which for convenience we chose to have incoming boundary conditions) in the commutator of Eq. (25), one can perform the above integral and thus obtain

$$\begin{aligned}\langle \mathbf{p}_1\mathbf{p}_2{}^{(-)}|\theta_{\mu\nu}(0)|0\rangle=-(m/p_0{}^1)^{1/2}\sum_n \Bigg\{&(2\pi)^3\delta^3(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_n)\frac{\langle \mathbf{p}_2|\bar{u}(p_1)\eta(0)|n^{(+)}\rangle\langle{}^{(+)}n|\theta_{\mu\nu}(0)|0\rangle}{p_0{}^n-p_0{}^1-p_0{}^2-i\epsilon}-(2\pi)^3\delta^3(\mathbf{p}_1+\mathbf{p}_n)\\ &\times\frac{\langle p_2|\theta_{\mu\nu}(0)|n^{(+)}\rangle\langle{}^{(+)}n|\bar{u}(p_1)\eta(0)|0\rangle}{p_0{}^n+p_0{}^1}\Bigg\}\end{aligned} \tag{26}$$

which shows the branch cut in the first term which is singular whenever $p_1+p_2=q=p_n$. This is satisfied for $q^2=0$ corresponding to a two-photon intermediate state. The second term has no singularity. The discontinuity across the branch cut can then be related to the absorptive parts $\mathrm{Im}G_i^e(q^2)$ using the reality condition for the $G_i^e(q^2)=G_i^{e*}(q^{2*})$,

$$\begin{aligned}G_{\mu\nu}&=[\bar{u}(p_1,s_1)/4m][\mathrm{Im}G_1^e(q^2)l_{(\mu}\gamma_{\nu)}+\mathrm{Im}G_2^e(q^2)l_\mu l_\nu/m\\ &\quad +\mathrm{Im}G_3^e(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)/m]v(p_2,s_2)\\ &=-[(2\pi)^4/2](p_0{}^2/m)^{1/2}\sum_n\langle \mathbf{p}_2|\bar{u}(p_1)\eta(0)|n^{(+)}\rangle\\ &\quad \times\langle{}^{(+)}n|\theta_{\mu\nu}(0)|0\rangle\delta^4(p_n-p_1-p_2),\\ &=\sum_n G_{\mu\nu}{}^n.\end{aligned} \tag{27}$$

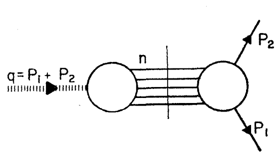

FIG. 3. Diagrammatic representation of the intermediate-state contribution to the absorptive part $\mathrm{Im}G_i(q^2)$.

[11] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento 1, 425 (1955).

We diagrammatically represent this expression for $\mathrm{Im}G_i^e(q^2)$ in terms of the scattering amplitude $\langle \mathbf{p}_2 | \bar{u}(p_1) \times\eta(0) | n^{(+)}\rangle$ and the production vertex $\langle n^{(+)} | \theta_{\mu\nu}(0) | 0\rangle$ in Fig. 3. The expression Eq. (27) for the absorptive part and the dispersion relations Eqs. (22) and (23) then provide us with a basis for calculation.

No approximations have been made so far. However, to proceed to actual calculations it is necessary to make a judicious choice of intermediate states $|n\rangle$ and truncate the sum on n. Usually such a choice of states is dictated by the recognition that the higher mass intermediate states will not contribute as much to the low q^2 values of $G_i(q^2)$ from the dispersion integral as the lower lying states because of the weighting denominator $q'^2-q^2-i\epsilon$ appearing in the dispersion integral. This conjecture depends on both the high-energy behavior of the scattering amplitude and the production vertex, something which is not at all known. Including only the lower lying states is making a virtue of a necessity since we cannot calculate with confidence in the high-energy region. Although this becomes a real problem in strong interactions, for our present considerations in electrodynamics we can appeal to the viability of perturbation theory and as a first approximation keep only those states which contribute to the absorptive part to order $\alpha=1/137$.

To lowest order in α we may approximate the sum in Eq. (27) by keeping only two intermediate states $n=\bar{e}e$ and $n=2\gamma$ (see Fig. 4). The state $n=\bar{e}e\gamma$ does not contribute to this order. We write the truncated sum

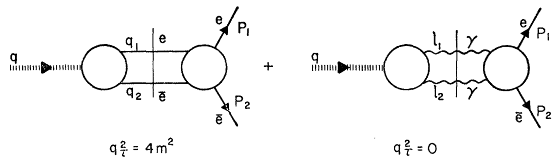

FIG. 4. Lowest order contribution to $\mathrm{Im}G_i^e(q^2)$.

as $G_{\mu\nu}=G_{\mu\nu}^{\bar{e}e}+G_{\mu\nu}^{2\gamma}$, where

$$G_{\mu\nu}^{\bar{e}e}=-\frac{(2\pi)^4}{2}\left(\frac{p_0^2}{m}\right)^{1/2}\sum_{\text{spin}}\int\frac{d^3\mathbf{q}_1}{(2\pi)^3}\frac{d^3\mathbf{q}_2}{(2\pi)^3}\times\delta^4(p_1+p_2-q_1-q_2)\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)\times|\mathbf{q}_1\mathbf{q}_2^{(+)}\rangle\langle^{(+)}\mathbf{q}_1\mathbf{q}_2|\theta_{\mu\nu}(0)|0\rangle,\quad(28)$$

$$G_{\mu\nu}^{2\gamma}=-\frac{(2\pi)^4}{2}\left(\frac{p_0^2}{m}\right)^{1/2}\sum_{\text{spin}}\int\frac{d^3\mathbf{l}_1}{(2\pi)^3}\frac{d^3\mathbf{l}_2}{(2\pi)^3}\times\delta^4(p_1+p_2-l_1-l_2)\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)\times|\mathbf{l}_1\mathbf{l}_2^{(+)}\rangle\langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta_{\mu\nu}(0)|0\rangle.\quad(29)$$

Here $\mathbf{q}_1$ and $\mathbf{q}_2$ are the momenta of the intermediate electron and positron and $\mathbf{l}_1$ and $\mathbf{l}_2$ are the momenta of the intermediate photons. In Eqs. (28) and (29) we recognize $\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{q}_1\mathbf{q}_2^{(+)}\rangle$ as the Coulomb scattering amplitude for electron-positron pairs, $e+\bar{e}\rightarrow e+\bar{e}$, and $\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{l}_1\mathbf{l}_2^{(+)}\rangle$ as the production amplitude $2\gamma\rightarrow e+\bar{e}$. To lowest order in α, corresponding to Born approximation, these amplitudes are given by

$$\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{q}_1\mathbf{q}_2^{(+)}\rangle=e^2(m^3/p_0^2q_0^1q_0^2)^{1/2}\left[\frac{\bar{u}(p_1)\gamma_\nu u(q_1)\bar{v}(q_2)\gamma_\nu v(p_2)}{(p_1-q_1)^2}-\frac{\bar{u}(p_1)\gamma_\nu v(p_2)\bar{v}(q_2)\gamma_\nu u(q_1)}{(p_1+p_2)^2}\right],\quad(30)$$

$$\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{l}_1\mathbf{l}_2^{(+)}\rangle=e^2(m^3/4p_0^2l_0^1l_0^2)^{1/2}\bar{u}(p_1)\left[(\gamma\cdot\epsilon^2)\frac{1}{l_1-p_2-m}\gamma\cdot\epsilon^1+\gamma\cdot\epsilon^1\frac{1}{p_1-l_1-m}(\gamma\cdot\epsilon^2)\right]v(p_2),\quad(31)$$

corresponding to the diagrams of Fig. 5. Consequently the contribution to the $G_i^e(q^2)$ to first order in α arise directly from two processes, in one case the graviton probing the electron beneath the photon cloud and in the other case probing the photon cloud about the electron. The contribution to the absorptive part coming from the rescattering of $\bar{e}e$ pairs begins at $q^2=4m^2$ the threshold of this process while the 2γ intermediate state first contributes to the absorption at $q^2=0$.

There also appear in Eqs. (28) and (29) the vertex functions $\langle^{(+)}\mathbf{q}_1\mathbf{q}_2|\theta_{\mu\nu}(0)|0\rangle$ and $\langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta_{\mu\nu}(0)|0\rangle$. To lowest order in α these are given by

$$\langle^{(+)}\mathbf{q}_1\mathbf{q}_2|\theta_{\mu\nu}(0)|0\rangle=\langle^{(+)}\mathbf{q}_1\mathbf{q}_2|\theta_{\mu\nu}^E(0)|0\rangle=(m^2/q_0^1q_0^2)^{1/2}\tfrac{1}{4}\bar{u}(q_1)l'_{(\mu}\gamma_{\nu)}v(q_2),\quad(32)$$

$$l'_\mu=(q_1-q_2)_\mu,$$

$$\langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta_{\mu\nu}(0)|0\rangle=\langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta_{\mu\nu}^P(0)|0\rangle=(1/4l_0^1l_0^2)^{1/2}\times\tfrac{1}{4}\left[g_{\mu\nu}q^2\left(\epsilon^1\cdot\epsilon^2-\frac{2\epsilon^1\cdot q\epsilon^2\cdot q}{q^2}\right)-(q_\mu q_\nu-\bar{l}_\nu\bar{l}_\mu)\epsilon_1\cdot\epsilon_2-2\epsilon^1\cdot\bar{l}\epsilon^2{}_{(\mu}l^1{}_{\nu)}+2\epsilon^2\cdot\bar{l}\epsilon^1{}_{(\mu}l^2{}_{\nu)}-q^2\epsilon^1{}_{(\mu}\epsilon^2{}_{\nu)}\right],\quad(33)$$

$$\bar{l}_\mu=(l_1-l_2)_\mu,$$

$$q_\mu=(l_1+l_2)_\mu,$$

and are diagrammatically represented in Fig. 6. In Eq. (33) ϵ_μ^1 and ϵ_μ^2 refer to the polarization of the photons of momentum l_μ^1 and l_μ^2, respectively, and it is easily established that Eq. (33) has the property of invariance under the gauge transformation $\epsilon_1^\mu\rightarrow\epsilon_1^\mu+\lambda_1l_1^\mu$, $\epsilon_2^\mu\rightarrow$

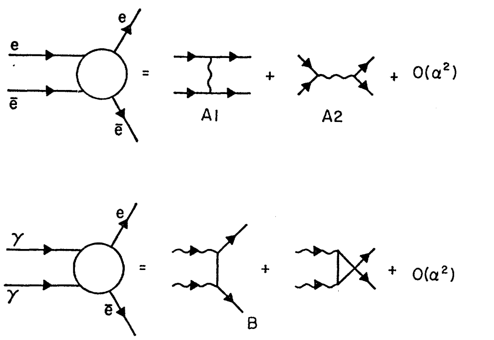

FIG. 5. Pole terms contributing to scattering amplitude.

$\epsilon_2^\mu+\lambda_2 l_2^\mu$, vanishing trace $\langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta^P(0)|0\rangle=0$, and the required conservation of energy $q^\mu\langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta_{\mu\nu}(0)|0\rangle=0$.

From these basic ingredients we may now calculate the $\mathrm{Im}G_i{}^e(q^2)$. Since the contribution of $G_{\mu\nu}{}^{e\bar{e}}$ and $G_{\mu\nu}{}^{2\gamma}$ to $G_{\mu\nu}$ separately conserve energy we may write our form factors as $G_i=G_i{}^E+G_i{}^P$ indicating the separate contributions from the $e\bar{e}$ and 2γ intermediate states. Inserting the scattering amplitudes Eqs. (30) and (31) and the vertex terms Eqs. (32) and (33) into Eqs. (28) and (29), performing the integration over intermediate scattering angles and summing on the intermediate spin states using standard trace techniques we may identify the contributions to the $\mathrm{Im}G_i{}^E(q^2)$ and $\mathrm{Im}G_i{}^P(q^2)$. The results of this calculation are

$$\mathrm{Im}G_1{}^E(q^2)=-\alpha\Phi(q^2)\left[\frac{17}{12}q^2-\frac{11}{3}m^2+\frac{1}{2}(2m^2-q^2)\ln\left(\frac{q^2-4m^2}{\lambda^2}\right)\right], \quad (34)$$

$$\mathrm{Im}G_2{}^E(q^2)=-\tfrac{2}{3}\alpha m^2\Phi(q^2),$$

$$\mathrm{Im}G_3{}^E(q^2)=(5/3)\alpha m^2[1-(4m^2/q^2)]\Phi(q^2),$$

$$\Phi(q^2)=m[q^2(q^2-4m^2)]^{-1/2},$$

and

$$\mathrm{Im}G_1{}^P(q^2)=\frac{\alpha q^2 m}{2(4m^2-q^2)}\left[a^2Q_1(a)-\frac{1}{a}Q_0(a)-\frac{1}{3}\right],$$

$$\mathrm{Im}G_2{}^P(q^2)=-\frac{\alpha q^2 m}{(4m^2-q^2)^2}\left[\frac{1}{a}Q_0(a)+(5a^2-6)Q_1(a)-\frac{5}{3}\right],$$

$$\mathrm{Im}G_3{}^P(q^2)=\frac{\alpha m^3}{(4m^2-q^2)}\left[\frac{1}{a}Q_0(a)-(2-a^2)Q_1(a)-\frac{1}{3}\right], \quad (35)$$

where $a^2=q^2/(q^2-4m^2)$ and the $Q_l(a)$ are Legendre functions of the second kind. In the expression for $\mathrm{Im}G_1{}^E(q^2)$ there appears the quantity $\lambda^2\ll q^2-4m^2$, a finite photon mass which we have placed in the pole term of Eq. (30) according to $(p_1-q_1)^2\to(p_1-q_1)^2-\lambda^2$. As $\lambda^2\to 0$, $\mathrm{Im}G_1{}^E(q^2)$ has an infrared divergence as a consequence of the divergence of the Coulomb amplitude in the forward direction. This divergence arises as a consequence of the fact that the electron is surrounded by a cloud of low-energy virtual photons the contribution of which we have not properly included. To further examine this divergence, we must examine the matrix elements $\langle e|\theta_{\mu\nu}(0)|e$, many photons$\rangle$ which takes us out of the scope of the present investigation.[12] We also note that the annihilation diagram Fig. 5A2 can not contribute to $\mathrm{Im}G_i{}^E(q^2)$ since the operator $\theta_{\mu\nu}$ can not connect the vacuum with a state of $J=1$. Finally we note that as $q^2\to\infty$ $\mathrm{Im}G_{2,3}{}^E(q^2)$ and $\mathrm{Im}G_{2,3}{}^P(q^2)$ vanish while $\mathrm{Im}G_1{}^E(q^2)\to\ln q^2$ and $\mathrm{Im}G_1{}^P(q^2)\to$ constant and the dispersion integrals Eqs. (22) and (23) are well defined.

The form factors for all values of q^2 may now be obtained from the expressions for the absorptive parts Eqs. (34) and (35) and the dispersion integrals Eqs. (22) and (23). In particular, we may calculate the static values $G_2{}^e(0)$ and $G_3{}^e(0)$ from the representations

$$G_2{}^e(0)=\frac{1}{\pi}\int_{4m^2}^\infty \mathrm{Im}G_2{}^E(q^2)\frac{dq^2}{q^2}+\frac{1}{\pi}\int_0^\infty \mathrm{Im}G_2{}^P(q^2)\frac{dq^2}{q^2}, \quad (36)$$

$$G_3{}^e(0)=\frac{1}{\pi}\int_{4m^2}^\infty \mathrm{Im}G_3{}^E(q^2)\frac{dq^2}{q^2}+\frac{1}{\pi}\int_{4\lambda^2}^\infty \mathrm{Im}G_3{}^P(q^2)\frac{dq^2}{q^2}. \quad (37)$$

In writing the expression for $G_3{}^e(0)$ we have inserted a threshold of $4\lambda^2$ in the second term corresponding to the threshold to produce two mass$=\lambda$ photons. We are required to do this since $\mathrm{Im}G_3{}^P(0)=\alpha m/6$ does not vanish. Hence $G_3{}^e(0)$ is an infrared divergent quantity; however, it does not effect the diagonal matrix elements $\langle\mathbf{p}|\theta_{\mu\nu}|\mathbf{p}\rangle$ since $G_3{}^e(0)$ has a coefficient proportional to the momentum transfer. Computing $G_2{}^e(0)$ from Eq. (36), we find $G_2{}^e(0)=-\alpha m/3\pi+\alpha m/3\pi=0$ so that $G_2{}^e(0)$ vanishes to this order.

Considerable simplification is achieved if instead of considering the matrix elements of the full stress tensor $\theta_{\mu\nu}(x)$ we restrict our attention to just the trace $\theta(x)$. The diagonal matrix elements are all that are experimentally accessible and since $\langle\mathbf{p}|\theta_{\mu\nu}(0)|\mathbf{p}\rangle=(p_\mu p_\nu/2m)\times\langle\mathbf{p}|\theta(0)|\mathbf{p}\rangle$ we need consider only the properties of a single analytic function $G(q^2)$ with

$$\langle\mathbf{p}_1|\theta(0)|\mathbf{p}_2\rangle=(m^2/p_0{}^1p_0{}^2)^{1/2}\bar{u}(p_1)u(p_2)G(q^2),$$

where $G(0)=m$, the total mass. Applying the reduction formalism to the production amplitude $\langle\mathbf{p}_1\mathbf{p}_2{}^{(-)}|\theta(0)|0\rangle$ by contracting out the electron state vector we obtain

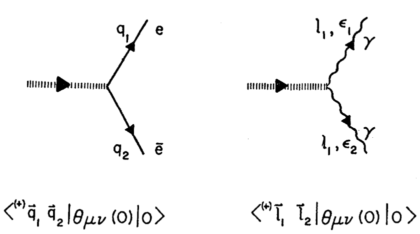

FIG. 6. Lowest order vertex for electrodynamics.

[12] D. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. 13, 379 (1961).

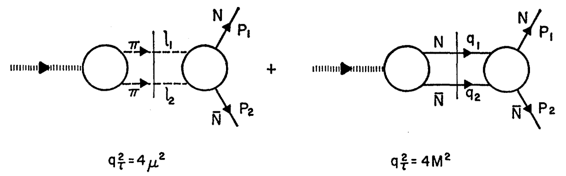

FIG. 7. Contribution to $\mathrm{Im}G^N(q^2)$ from 2π and $N\bar{N}$ intermediate states.

just the trace of Eq. (25)

$$\langle \mathbf{p}_1\mathbf{p}_2^{(-)}|\theta(0)|0\rangle = -i\left(\frac{m}{p_0^1}\right)^{1/2}\int d^4x\, e^{ip_1\cdot x}\hat{\theta}(x_0)\langle \mathbf{p}_2|[\bar{u}(p_1)\eta(x),\theta(0)]|0\rangle + (m^2/p_0^1p_0^2)^{1/2}m_0\bar{u}(p_1)v(p_2)\,, \quad (38)$$

where we have included the term arising from the equal time commutator.[13] Here m_0 is the bare mass of the electron.

Various alternatives suggest themselves with respect to the issue of subtraction constants in the dispersion relation for $G^e(q^2)$. If, as is suggested by perturbation theory, the integral in Eq. (38) implies only that $G(q^2)/q^2 \to 0$ as $q^2\to\infty$, then we must use a subtracted relation and give up all hope of calculating the mass $G^e(0)$ unless we can fix in advance the value $G^e(q^2)$ at some other value of $q^2\neq 0$. It is also a possibility that the integral has the property that as $q^2\to\infty$ it vanishes. Then we may write for $G^e(q^2)$ the representation

$$G^e(q^2)=m_0+\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}G^e(q'^2)dq'^2}{q'^2-q^2-i\epsilon} \quad (39)$$

and we could calculate the mass shift $\delta m=G^e(0)-m_0 = m-m_0$ due to interactions. Finally, there is the possibility that $G^e(q^2)\to 0$ as $q^2\to\infty$. Then we may calculate the mass from[14]

$$G^e(q^2)=\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}G^e(q'^2)dq'^2}{q'^2-q^2-i\epsilon}. \quad (40)$$

This dispersion relation could be interpreted as implying all the mass arises as a consequence of interactions.

For our example of electrodynamics to compute $\mathrm{Im}G^e(q^2)$, we simply have to take the linear combinations of the $\mathrm{Im}G_i^E(q^2)$ given by Eq. (34) according to Eq. (6) with the result

$$\mathrm{Im}G^e(q^2) = -\alpha\Phi(q^2)\{2m^2+\tfrac{1}{2}(2m^2-q^2)\ln[(q^2-4m^2)/\lambda^2]\} \to \tfrac{1}{2}\alpha m \ln q^2 \quad q^2\to\infty\,. \quad (41)$$

[13] The trace operator is $\theta(x)=m_0Z_2\bar{\psi}_R(x)\psi_R(x)$, where the renormalized Heisenberg fields obey the equal-time relation $[\bar{\psi}_R(x'),\psi_R(x)]_+=\gamma_0Z_2^{-1}\delta^3(x'-x)$ so that $\delta(x_0)[\psi_R(x),\theta(0)]=m_0\gamma_0 \times\psi_R(x)\delta^4(x)$, which gives rise to the second term of Eq. (38).

[14] This can be obtained from Eq. (39) by setting $m_0=0$. In the case of electrodynamics only one parameter with the dimensions of a mass appears and we may set $m=1$ to define the scale. Then Eq. (40) with $q^2=0$ is a sum rule.

There is no contribution from the 2γ intermediate state since in this perturbation approximation $\langle \mathbf{l}_1\mathbf{l}_2|\theta(0)|0\rangle =0$, a consequence of the vanishing of the trace of the photon stress tensor. So $\mathrm{Im}G^e(q^2)$ depends only on the Coulomb amplitude and in particular only on the $J=0$ channel. The high-energy behavior of $\mathrm{Im}G^e(q^2)$ is a direct consequence of the high-energy behavior of the Coulomb amplitude. Of course, substitution of $\mathrm{Im}G^e(q^2)$ given by Eq. (41) into either Eqs. (39) or (40) will lead to a divergence in the quantity $G^e(0)=m$. This is simply another way of expressing the failure of quantum electrodynamics to yield a finite self-energy of the electron.

B. Pion-Nucleon Interaction

As a final example we consider the pion-nucleon interaction in perturbation theory. We will assume the validity of an expansion in the pion nucleon coupling $g_{\pi N}$ and will calculate the matrix elements of $\theta(x)$ between one-nucleon states $\langle\mathbf{p}_1|\theta(0)|\mathbf{p}_2\rangle$, where $p_1^2=p_2^2 =M^2$ and M is the mass of the nucleon. The general form for this matrix element is as before

$$\langle\mathbf{p}_1|\theta(0)|\mathbf{p}_2\rangle=(M^2/p_0^1p_0^2)^{1/2}\bar{u}(p_1)u(p_2)G^N(q^2) \quad (42)$$

with $q^2=(p_1-p_2)^2$ and $G^N(0)=M$. Furthermore, we assume that $G^N(q^2)$ has a representation

$$G^N(q^2)=M_0+\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}G^N(q'^2)dq'^2}{q'^2-q^2-i\epsilon} \quad (43)$$

similar to Eq. (39) for the electron form factor. Here M_0 is the bare mass of the nucleon.

To calculate the absorptive part $\mathrm{Im}G^N(q^2)$ we follow a procedure completely similar to that for the electron. Here to lowest order the contributing intermediate states will be the 2π state with a threshold at $q^2=4\mu^2$ with μ the mass of pion and the $N\bar{N}$ state with a threshold at $q^2=4M^2$. We diagrammatically represent this contribution to the absorption in Fig. 7. In this approximation, we then obtain

$$\bar{u}(p_1)v(p_2)\,\mathrm{Im}G^N(q^2)=G^{2\pi}(q^2)+G^{N\bar{N}}(q^2) \quad (44)$$

with

$$G^{2\pi}(q^2)=-\frac{(2\pi)^4}{2}(p_0^2/M)^{1/2} \times\sum_{\text{spins}}\int\frac{d^3\mathbf{l}_1}{(2\pi)^3}\frac{d^3\mathbf{l}_2}{(2\pi)^3}\delta^4(p_1+p_2-l_1-l_2) \times\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{l}_1\mathbf{l}_2^{(+)}\rangle\langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta(0)|0\rangle, \quad (45)$$

$$G^{N\bar{N}}(q^2)=-\frac{(2\pi)^4}{2}(p_0^2/M)^{1/2} \times\sum_{\text{spins}}\int\frac{d^3\mathbf{q}_1}{(2\pi)^3}\frac{d^3\mathbf{q}_2}{(2\pi)^3}\delta^4(p_1+p_2-q_1-q_2) \times\langle\mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{q}_1\mathbf{q}_2^{(+)}\rangle\langle^{(+)}\mathbf{q}_1\mathbf{q}_2|\theta(0)|0\rangle, \quad (46)$$

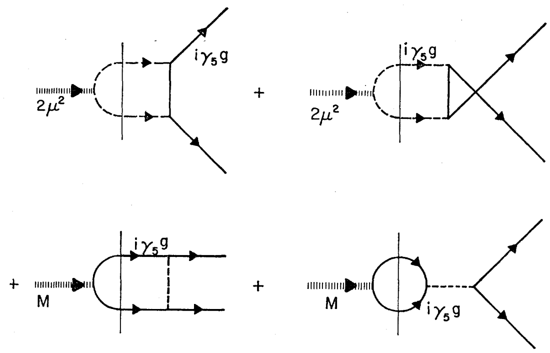

FIG. 8. First-order contribution to $\mathrm{Im}G^N(q^2)$.

where $q^2=(p_1+p_2)^2$ and l_1, l_2 are the momenta of the intermediate π pair and q_1, q_2 are the momenta of the intermediate $N\bar{N}$ pair. The sum indicates a sum on both spin and isotopic-spin states. From these expressions we see that $\mathrm{Im}G_N(q^2)$ depends directly on the $2\pi\to N\bar{N}$ amplitude $\langle \mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{l}_1\mathbf{l}_2^{(+)}\rangle$ and the $NN\to N\bar{N}$ amplitude $\langle \mathbf{p}_2|\bar{u}(p_1)\eta(0)|\mathbf{q}_1\mathbf{q}_2^{(+)}\rangle$. We will approximate these scattering amplitudes with their first-order perturbation theory values which are diagrammatically represented in Fig. 8. Also appearing in Eqs. (45) and (46) are the vertices[15]

$$\begin{aligned}\langle^{(+)}\mathbf{q}_1\mathbf{q}_2|\theta(0)|0\rangle&=(M^2/q_0^1q_0^2)^{1/2}\bar{u}(q_1)v(q_2)G^N(q^2)^*,\\ \langle^{(+)}\mathbf{l}_1\mathbf{l}_2|\theta(0)|0\rangle&=(1/4l_0^1l_0^2)^{1/2}G^\pi(q^2)^*,\end{aligned}\qquad(47)$$

which in perturbation theory are given correctly to lowest order by setting $G^N(q^2)^*=M$, $G^\pi(q^2)^*=2\mu^2$.

Inserting these expressions for the vertices and scattering amplitudes one finds from Eqs. (45) and (46)

$$\mathrm{Im}G_N(q^2)=\mathrm{Im}G^{2\pi}(q^2)\hat{\theta}(q^2-4\mu^2)+\mathrm{Im}G^{N\bar{N}}(q^2)\hat{\theta}(q^2-4M^2),\qquad(48)$$

with

$$\mathrm{Im}G^{2\pi}(q^2)=-6M\left(\frac{\mu}{M}\right)^2\left(\frac{g^2}{4\pi}\right)\left(\frac{q^2-4\mu^2}{q^2}\right)^{1/2}\frac{Q_1(z)}{q^2-4M^2},$$

$$\mathrm{Im}G^{N\bar{N}}(q^2)=\frac{3M}{4}\left(\frac{g^2}{4\pi}\right)\left(\frac{q^2-4M^2}{q^2}\right)^{1/2}\times\left[1+\frac{\mu^2}{q^2-4M^2}\ln\frac{\mu^2}{\mu^2+q^2-4M^2}\right],\qquad(49)$$

where $z=(q^2-2\mu^2)^2/(q^2-4M^2)(q^2-4\mu^2)$. Setting $q^2=0$ in Eq. (43) and using Eq. (48) one has

$$M-M_0=\frac{1}{\pi}\int_{4\mu^2}^\infty \mathrm{Im}G^{2\pi}(q^2)\frac{dq^2}{q^2}+\frac{1}{\pi}\int_{4M^2}^\infty \mathrm{Im}G^{N\bar{N}}(q^2)\frac{dq^2}{q^2}.\qquad(50)$$

The first integral represents the contribution to $\delta M=M-M_0$ from the pions in the cloud surrounding the nucleon. This integral is finite with the value

$$\frac{1}{\pi}\int_{4\mu^2}^\infty \mathrm{Im}G^{2\pi}(q^2)\frac{dq^2}{q^2}=\frac{3M}{2\pi}\left(\frac{g^2}{4\pi}\right)\left(\frac{\mu}{M}\right)^2\times[(\ln(\mu^2/M^2)+2\ln(2)-\tfrac{1}{2}+O(\mu/M)].\qquad(51)$$

The second integral represents the contributions to δM from the nucleon under the pion cloud and this integral gives rise to the divergence in the nucleon self-energy. Cutting this integral off at $\lambda^2 4M^2$ one finds as $\lambda^2\to\infty$ that $\delta M\to(g^2/4\pi)(3M/4\pi)\ln\lambda^2$. We do note however that in this approach it is possible to make a well-defined separation between the finite and divergent contributions to δM and they are seen to emerge from distinct physical processes. The divergent contribution is seen to arise from the high-energy behavior of the $N\bar{N}\to N\bar{N}$ amplitude as calculated in this model perturbation theory.

In a more realistic approach to calculating the nucleon mass shift δM, one would of course not use perturbation theory to calculate $\mathrm{Im}G^N(q^2)$. In such an approach one might suppose that the effects of intermediate $N\bar{N}$ and higher mass states could be lumped into the unknown constant M_0 and then calculate the contribution to the mass from the lighter mesons in the nucleon cloud. Assuming that the constant M_0 was approximately the same for all the baryons, the mass splitting between the baryons could then be seen as arising from the mass splittings of the various mesons in the clouds surrounding the baryons. We have seen that in perturbation theory, which is equivalent to keeping just the pole terms, the contribution from the pseudoscalar mesons is finite and this suggests convergence in a more exact approach.

There is, however, a danger inherent in all approaches to calculating mass splittings and that is the neglect of higher mass states. Since one is calculating δM, which is a quantity having the dimensions of a mass, it is easily seen that the inclusion of higher mass states may well dominate since there will be terms contributing to δM proportional to the highest available mass. Usually one neglects higher mass states since their contribution under the dispersion integral is damped by the factors in the denominator. This argument appears to work for the electromagnetic form factors $F_i(q^2)$ which are dimensionless. But for the mechanical form factor $G(q^2)$ such arguments for neglecting higher mass states are by no means so clear.

IV. UNITARITY CONDITION

In this section we will examine several simple consequences of the unitarity condition for the mechanical form factors and no longer restrict ourselves to perturbation theory. For simplicity we consider an interacting scalar field $\phi(x)$ of mass μ so that the matrix elements of the total stress-energy tensor $\theta(x)$ are given by $\langle\mathbf{p}_1|\theta(x)|\mathbf{p}_2\rangle=(4p_0^1p_0^2)^{-1/2}G(s)$, $s=(p_1-p_2)^2$ with

[15] Here the complex conjugate appears because we have used outgoing boundary conditions.

$G(0)=2\mu^2$. Furthermore, we shall first assume $G(s)$ has the representation

$$G(s)=2\mu_0^2+\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}G(s')ds'}{s'-s-i\epsilon}, \tag{52}$$

where μ_0 is the mass of the particle in the absence of all interactions. The absorptive part $\mathrm{Im}G(s)$ appearing in Eq. (52) may be obtained from the unitary condition which implies

$$\mathrm{Im}G(s)=-\tfrac{1}{2}(2\pi)^4(2p_0^2)^{1/2}\sum_n\langle \mathbf{p}_2|\tilde{J}(0)|n^{(+)}\rangle \times\langle^{(+)}n|\theta(0)|0\rangle\delta^4(p_n-p_1-p_2), \tag{53}$$

where $\tilde{J}(x)=(\Box+\mu^2)\phi(x)$ and the sum is over a complete set of intermediate states. Here $\langle \mathbf{p}_2|\tilde{J}(0)|n^{(+)}\rangle$ is the amplitude for the process $2\phi\rightarrow n$, where n is any allowed state.

In the approximation of elastic unitarity, we keep only the 2ϕ intermediate state. Then Eq. (53) yields for the absorptive part

$$\mathrm{Im}G(s)=\rho(s)G^*(s)A_0(s)\hat{\theta}(s-4\mu^2), \tag{54}$$
$$A_0(s)=e^{i\delta_0(s)}\sin\delta_0(s)/\rho(s),$$

where $A_0(s)$ is the S-wave $2\phi\rightarrow 2\phi$ scattering amplitude and $\delta_0(s)$ the S-wave phase shift. Here $\rho(s)=(s-4\mu^2/s)^{1/2}$, the phase shace of the intermediate state. Equation (54) implies that for $s>4\mu^2$ that $G(s)$ has the phase $\delta_0(s)$. This information along with the assumed dispersion relation (52) then allows us to write the Omnès representation[16] for $G(s)$

$$G(s)=2\mu_0\exp\left[\frac{1}{\pi}\int_{4\mu^2}^\infty \frac{\delta_0(s')ds'}{s'-s}\right] \tag{55}$$

where we have assumed that $\delta_0(s)\rightarrow 0$ as $s\rightarrow\infty$. In particular Eq. (55) implies that in this approximation of elastic unitarity

$$\frac{\mu^2}{\mu_0^2}=\exp\left[\frac{1}{\pi}\int_{4\mu^2}^\infty \delta_0(s)\frac{ds}{s}\right], \tag{56}$$

so that the ratio μ^2/μ_0^2 is directly related to an integral over the S-wave phase shift (see Fig. 9).

Let us consider the existence of a second scalar field $\phi'(x)$ which has the mass μ'. In the absence of interactions we assume that its bare mass is μ_0 the same as that of the previous particle. If $\phi(x)$ and $\phi'(x)$ are not identical then we may consider the problem of relating their mass difference to differences in their interactions. If we assume elastic unitarity, then we will have an

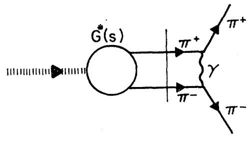

FIG. 9. Application of elastic unitarity.

[16] R. Omnès, Nuovo Cimento 8, 316 (1958).

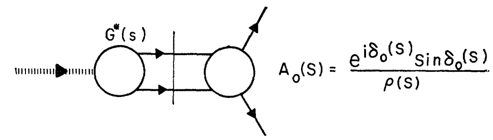

FIG. 10. Contribution of Coulomb scattering to $\bar{\mu}_+^2/\mu_0^2$.

equation similar to Eq. (56) for the ratio μ'^2/μ_0^2 in terms of the S-wave phase shift $\delta_0'(s)$ for the process $2\phi'\rightarrow 2\phi'$. Then it follows from this equation and Eq. (56) that the ratio μ'^2/μ^2 is given by

$$\frac{\mu'^2}{\mu^2}=\exp\left[\frac{1}{\pi}\int\frac{ds}{s}[\delta_0'(s)-\delta_0(s)]\right]. \tag{57}$$

To obtain Eq. (57) for the mass ratio μ'^2/μ^2 it is not necessary to assume the representation Eq. (52) for $G(s)$ and a similar representation for $G'(s)$. Instead one may make the weaker assumption that $J(s)=G'(s)/G(s)\rightarrow 1$ as $s\rightarrow\infty$ so that either $G(s)$ or $G'(s)$ may diverge as $s\rightarrow\infty$ in which case we can not write the representation Eq. (52). If $G(s)$ has no zeros then $J(s)$ is analytic except along the cuts of $G(s)$ and $G'(s)$. Elastic unitarity implies the phase of $J(s)$ along the cut is just $\delta_0'(s)-\delta_0(s)$ so that the asymptotic behavior of $J(s)$ and the fact that $J(0)=\mu'^2/\mu^2$ together imply Eq. (57).

Let us identify ϕ' with a charged pion π^+ and ϕ with the uncharged π^0 and denote the π^+ mass by $\bar{\mu}_+$ and that of the π^0 by $\bar{\mu}_0$. Neglecting all but the Coulomb interaction as a possible source for the pion electromagnetic mass difference the phase-shift difference, $\delta_0^{\pi^+}(s)-\delta_0^{\pi^0}(s)=\delta_0^{\mathrm{Coul}}(s)$ is just the S-wave Coulomb phase shift (see Fig. 10). Then Eq. (57) implies

$$\frac{\bar{\mu}_+^2}{\bar{\mu}_0^2}\simeq\exp\left[\frac{1}{\pi}\int_{4\mu^2}^\infty \delta_0^{\mathrm{Coul}}(s)\frac{ds}{s}\right]. \tag{58}$$

Because of the attractive nature of Coulomb forces $\delta_0^{\mathrm{Coul}}(s)>0$ and hence we conclude that $\bar{\mu}_+^2>\bar{\mu}_0^2$ as is observed.

The purpose of this example was to illustrate how mass splittings are related to differences in the forces in the rescattering amplitude. Similar considerations can be expected to apply to the splittings among members of $SU(3)$ multiplets. In general, in the approximation of elastic unitarity the mass splittings of particles may be so related to the S-wave phase shift. This phase shift is not well known for strongly interacting systems since it contains information about the core of the particles. In calculating mass splitting among members of a multiplet one need know only the *difference* of the S-wave phase shifts, and to a first approximation the effects of a common core do not enter. Here one would have to assume that in the high-energy limit the difference $\Delta\delta_0(s)$ of the S-wave phase shifts would vanish. The major feature of the present approach is that it enables one to relate mass shifts to experimental observables.

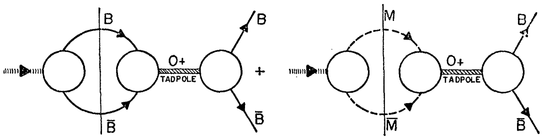

FIG. 11. Tadpole contribution to the absorption $\mathrm{Im}G_i^B(s)$.

As a final example we consider the mass splittings among the members of the baryon and pseudoscalar octets. For each of the eight members of the baryon octet we define a form factor $G_i^B(s)$ so that $G_i^B(0)=M_i$, the masses of the baryons and similarly for the eight pseudoscalar mesons we have the form factors $G_i^M(s)$ with $G_i^M(0)=2\mu_i^2$. Then unitarity implies in the approximation of keeping only the $B\bar{B}$ and $M\bar{M}$ intermediate states

$$\begin{aligned}\mathrm{Im}G_i^B(s)&=\sum_{j=1}^{8}\left[\rho_j^B(s)G_j^{*B}(s)A_{ji}^{B\bar{B}}(s)\right.\\&\qquad\left.+\rho_j^M(s)G_j^{*M}(s)A_{ji}^{MB}(s)\right],\\\mathrm{Im}G_i^M(s)&=\sum_{j=1}^{8}\left[\rho_j^B(s)G_j^{*B}(s)A_{ji}^{BM}(s)\right.\\&\qquad\left.+\rho_j^M(s)G_j^{*M}(s)A_{ji}^{M\bar{M}}(s)\right],\end{aligned}\tag{59}$$

where $A_{ij}(s)$ are the S-wave scattering amplitudes for $BB\to B\bar{B}$, $M\bar{M}\to B\bar{B}$, $BB\to M\bar{M}$, $M\bar{M}\to M\bar{M}$ and they have the property $A_{ij}^{BM}=(A_{ij}^{MB})^T$ as follows from time-reversal invariance of the strong interactions and $\rho(s)$ is the phase space of the intermediate state. We also assume the existence of convergent dispersion relations

$$\begin{aligned}G_i^B(s)&=M_0+\frac{1}{\pi}\int\frac{\mathrm{Im}G_i^B(s')ds'}{s'-s-i\epsilon},\\G_i^M(s)&=2\mu_0^2+\frac{1}{\pi}\int\frac{\mathrm{Im}G_i^M(s')ds'}{s'-s-i\epsilon}.\end{aligned}\tag{60}$$

Once the $A_{ij}(s)$ are specified, Eqs. (60) and (59) constitute a set of coupled linear integral equations which can be solved for the $G_i(s)$ by standard methods.[17,18] From the solutions we may obtain the mass splittings $G_i(0)-G_j(0)$ which do not depend on M_0 or μ_0. We do not engage in any such calculation here but rather examine how some of the models of octet enhancement[19] may be incorporated into this method.

The S-wave scattering amplitudes $A_{ij}(s)$ will in general transform like all the irreducible representations contained in the product $8\otimes8$ and these transformation properties are then reflected in the mass splittings. Various mechanisms have been proposed that would imply the dominance of the 8 dimensional representation in the product $8\otimes8$.[20–22] One of the simplest is the tadpole mechanism[4] which assumes the existence of a 0^+ octet of scalar mesons. If we write our matrix amplitude in the form $A(s)=N(s)D^{-1}(s)$, then the tadpole hypothesis asserts that $D(s)$ will have its zeros at the masses of the tadpoles. The contribution of the tadpoles to the absorptive part is shown in Fig. 11. The absorption will be very large in the neighborhood of energies close to the tadpole mass. Approximating the scattering amplitude with the dominant pole terms, the solutions to our integral equations will then have the characteristic octet transformation properties. Unfortunately there is little experimental evidence for the 0^+ octet.

A likely candidate for the octet enhancement mechanism is provided by the recently discovered nonet of 2^+ mesons. Assuming that the $Q=0$, $Y=0$ members of the nonet couple universally to the gravitational field $g_{\alpha,\beta}(x)$ and a pole dominance model, then the matrix elements of the full stress tensor, which transforms like 2^+, and in particular the space integral of $\theta_{0,0}(x)$ which gives the Hamiltonian, would have the desired octet transformation properties.

Another suggestion to account for octet dominance has been that of spontaneous breakdown of $SU(3)$ symmetry.[21,23] Here we note that the amplitudes $A_{ij}(s)$ depend on the baryon and meson masses and coupling constants in some complicated way which could presumably be calculated using some model for the strong interactions. Then there may be two solutions to Eqs. (59) and (60), one corresponding to the symmetric solution, the other a nondegenerate solution corresponding to the observed splittings.

V. CONCLUSION AND SUMMARY

By emphasizing the analogy between charge and mass as sources of fields we see how it is possible to define mechanical form factors $G_i(q^2)$ in analogy with the electromagnetic form factors $F_i(q^2)$. By exploiting the analytic properties of the $G_i(q^2)$, the contributions to the total mass may be related to dispersion integrals over on the mass shell scattering amplitudes. In this way, for example, the self-mass divergence of the electron is directly related to the high-energy behavior of the Coulomb amplitude. Furthermore, the magnitude and sign of dynamical contributions to the mass can be seen as consequences of the forces in the rescattering amplitude. In the following paper[24] we shall use these ideas and techniques to suggest a simple picture of the origin of the proton-neutron mass difference.

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