Should $SU(3)$ Mass Formulas for Mesons Use the Masses of the Mesons or Their Squares?*

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The practice of using squares of masses in mass formulas for SU_3 meson multiplets, which arose originally in order to bring the Gell-Mann–Okubo mass formula for the 0^- meson octet more nearly into agreement with experiment, is criticized. The basis of the criticism is that $X^0-\eta$ mixing can make the Gell-Mann–Okubo formula compatible with the observed 0^- meson spectrum not only when one applies it to the squares of the masses, but also when one applies it to the masses themselves. Of course, the degree of the mixing is different in the two cases and this affects the prediction of decay rates. Accordingly, the decays of 2^+ mesons into two 0^- mesons are discussed in considerable detail as an example of a situation where experimental measurements may discriminate between the values of the mixing angle corresponding to linear and quadratic mass formulas.

1. INTRODUCTION

THE purposes of this paper are:

(a) to point out that existing experimental results give no clear answer to the question: Should particle masses or their squares be used in mass formulas for SU_3 meson multiplets?;

(b) to discuss situations where measurement of meson decay rates might yield the answer.

The procedure of using the squares of meson masses in SU_3 mass formulas was first employed when it was found that the Gell-Mann–Okubo mass formula for the 0^- mesons

$$4K=\pi+3\eta \qquad (1)$$

was better satisfied by the squares of the particle masses than by the masses themselves. (The η mass is predicted to within $3\frac{1}{2}\%$ and 12%, respectively.) Although the appropriate power of the mass might well vary with meson spin, the custom of using the squares of masses whenever mesons are involved seems to have fairly widespread acceptance. However, the meson families other than the 0^- one fail to provide any further justification of the custom. For the 1^- mesons, the Gell-Mann–Okubo mass formula fails for either of the possible simple-minded organizations of the ρ, K^*, ϕ, ω into an SU_3 octet and singlet. However, this can be explained[1] in terms of a mixing between the two $I=Y=0$ states, ϕ and ω, since they have the same G parity, so that we may say that the nine 1^- mesons can be associated with a reducible $1\oplus 8$ representation of SU_3. An identical situation obtains[2] for the nine 2^+ mesons A_2, $K^*(1410)$, $f'(1500)$, f. The argument in the case of the 1^- mesons may be presented as follows.

In the absence of the medium-strong interactions which induce the observed mass splittings, one has an SU_3 octet (ρ,K^*,ω_8) and an SU_3 singlet ω_1. Then, under the usual assumptions about the transformation properties of the symmetry-breaking interactions, the matrix of the mass-squared operator in the presence of symmetry breaking takes the form

$$\begin{pmatrix} \rho & & & \\ & K^* & & \\ & & \omega_8 & \beta \\ & & \beta & \omega_1 \end{pmatrix},$$

where β is the ω_8-ω_1 transition element. There is only one relationship, namely the Gell-Mann–Okubo formula

$$4K^*=\rho+3\omega_8,$$

between the five entries. When one demands that the physical particles ρ, K^*, ϕ, ω emerge as eigenstates of this matrix, so that the squares of their masses are its eigenvalues, the only mass formulas which emerge are the inequalities

$$(\rho+3\phi)>4K^*>(\rho+3\omega),$$

which are indeed satisfied. For the physical states ϕ and ω the diagonalization procedure yields

$$|\phi\rangle=\cos\theta_1|\omega_8\rangle+\sin\theta_1|\omega_1\rangle,$$
$$|\omega\rangle=-\sin\theta_1|\omega_8\rangle+\cos\theta_1|\omega_1\rangle,$$

where

$$\sin^2\theta_1=(\phi-\omega_8)/(\phi-\omega),$$

which yields $\theta_1=\pm 40.2°$. The argument is identical for the 2^+ case with the substitution of A_2, $K^*(1410)$, f', f, and θ_2 for ρ, K^*, ϕ, ω, and θ_1 and we find $\theta_2=\pm 31.3°$.

The mixing formalism is elevated from the status of a data-fitting procedure only because ϕ-ω and f'-f mixing turn out to be required[1,2] to explain the details of the decays of 1^- and 2^+ mesons. The experimental results may be said to be quantitatively explained when the above values of θ_1 and θ_2 are used.

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[1] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); S. Okubo, Phys. Letters **5**, 165 (1963); S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963).

[2] S. L. Glashow and R. Socolow, Phys. Rev. Letters **15**, 329 (1965), hereafter referred to as GS.

Now, the mixing formalism could be equally well applied to the mass matrix instead of to the mass-squared matrix. The necessary inequalities are still satisfied and the mixing angles become $\theta_1 = \pm 37.5°$ and $\theta_2 = \pm 29.9°$, smaller in magnitude by 2.7° and 1.4°, respectively. The relevant decay rates are not sufficiently accurately known to distinguish between the predictions of each mixing angle. Indeed, it may be argued that the calculational uncertainties in the treatment of phase space and symmetry breakdown must inevitably mask the effect of such a small variation in mixing angle.

In the case of the 0^- mesons, there exists, in addition to the (π, K, η) octet, a ninth 0^- meson X^0 with $I = Y = 0$ and the same G parity as η. Accordingly, it is often argued that the $3\frac{1}{2}\%$ error in the use of (1), with squares of masses, to predict the η mass can be accounted for by means of an X^0-η mixing, the effect of the presence of X^0 being to depress the η mass from the value predicted by (1). It is generally not realized, however, that such an approach removes the original motivation for preferring to use the squares of the meson masses rather than the masses themselves. This is because once X^0-η mixing is admitted, the X^0-η mixing formalism can be based on the mass matrix just as well as on the mass-squared matrix. Of course, the former involves a larger mixing effect ($\theta_0 = \pm 66.6°$) than the latter ($\theta_0 = \pm 79.6°$).[3] However, a large mixing effect is no harder to accept than a small one.[4] It is to be noted that the difference between the two values of θ_0 is 13.0° in magnitude, large compared with the corresponding differences for θ_1 and θ_2 in view of the relatively larger mass splittings in the 0^- reducible $1 \oplus 8$ multiplet. This is important, since the large difference may allow experimental discrimination between the predictions made using the two different values of θ_0, and this would imply a preference for either the use of masses or the use of their squares in 0^- mass formulas.

We now consider experimental situations which may yield a prediction of the value of θ_0.[5] One might attempt to fix θ_0 by measuring the absolute rates for particular decay modes, for example the 2γ modes, of X^0 and η. Measurements of such rates are extremely difficult, however, because the total widths of both particles lie in a range which is quite inaccessible with present techniques, although branching ratios are in many instances already known. Accordingly, we prefer to examine a case in which X^0 and η appear as final states in the decay of heavier mesons. A suitable heavier meson would be A_2 or perhaps K^* (1410). However, to discuss fully the use of either A_2 or K^* (1410) decays to determine θ_0, it is necessary to consider simultaneously all eleven decays of 2^+ into two 0^- mesons. This is described in detail in Sec. 2.

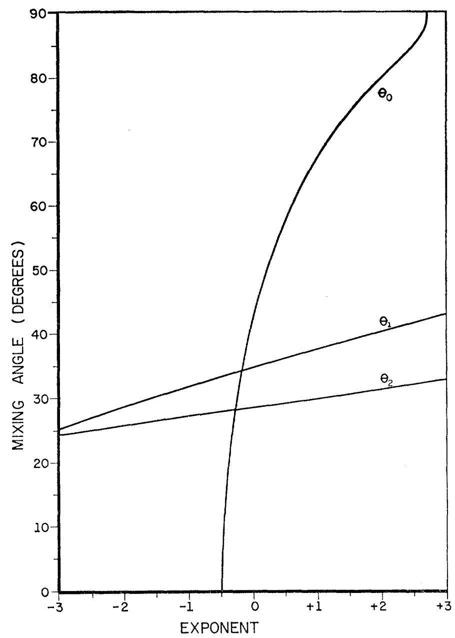

FIG. 1. The mixing angles θ_0, θ_1, and θ_2, describing the $J^P = 0^-$, 1^-, and 2^+ mesons, respectively, as functions of the power of the mass used in their determination.

If one adopts the viewpoint that one should use the same power of the mass, linear or quadratic, for any meson multiplet, then of course a determination for the 0^- case is all that is required. However, as this viewpoint is subject to criticism, it seems desirable to find predictions that are sensitive to the small difference between the two values of θ_1 and the two values of θ_2 that correspond to the use of masses and their squares. Unfortunately, it is not so easy to suggest experimental situations favorable for discriminating between the two values of θ_1 or of θ_2 without going beyond basic SU_3 theory of meson decays. We discuss ways of determining θ_1 and θ_2 in Sec. 3.

While the preceding discussion has concerned itself with suggesting how one might make an experimental choice between the use of masses and the use of their squares in meson mass formulas, it is not being implied here either that these are the only powers of the mass to have been suggested or that they are the only powers worth considering. For example, the use of the inverse squares of masses has been proposed for 1^- mesons.[6]

[3] We define all mixing angles so that, for $\theta = 0$, the lighter $I = Y = 0$ meson would be a unitary singlet. Then the mixing angle as calculated with masses is always smaller than when calculated with their squares. All masses and decay rates are taken from A. H. Rosenfeld, A. Barbero-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

[4] We may point out that use of squares of masses introduces two anomalously small numbers, namely, the squared pion mass and the $X^0 - \eta$ mixing angle. The corresponding numbers are more nearly of the same magnitude as corresponding quantities for other multiplets if we deal instead with the masses themselves.

[5] $X^0 - \eta$ mixing has been used to estimate some electromagnetic decay rates by R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965); **38**, 1945 (1965), errata.

[6] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964). See also B. J. Bjørken and S. L. Glashow, Phys. Letters **11**, 255 (1964) and G. Segrè and J. Sucher, Nuovo Cimento **38**, 428 (1965).

We prefer, however, to take the point of view that the appropriate power of the mass to use is a question which should be settled by experiment if possible. While there are a number of dynamical models of symmetry breakdown in which sum rules for particle masses emerge most naturally in terms of squares of masses for mesons and in terms of the masses themselves for baryons,[7] it is not unreasonable to expect that other powers could be accommodated in suitable theories of symmetry breakdown. In Fig. 1, we display the variation in the mixing angles θ_0, θ_1, and θ_2 as a function of the power of the mass used in the computation of the mixing angle for values of that power in the range -3 to $+3$.[8] The power zero corresponds to the use of the logarithm of the mass in the calculation of the mixing angle. For the case of 1^- mesons, the mixing angle can be expressed in terms of the particle masses ρ, K^*, ϕ, ω according to $\sin^2\theta_1 = \ln(\phi^3\rho/K^{*4})/[\ln(\phi^3/\omega^3)]$. This yields $\sin^2\theta_1$ very close to $\frac{1}{3}$, a value which corresponds to the mass formula

$$(K^*)^4=\phi^2\rho\omega$$

which is exactly satisfied for $\rho=775$ MeV. Although we talk mainly about distinguishing between the two values, linear and quadratic, of mixing angle, the analysis described in Sec. 2 is evidently of more general applicability and could equally well admit the possibility of experimental selection of some other power.

2. ANALYSIS OF THE DECAYS OF 2^+ MESONS INTO TWO 0^- MESONS

We give here a slightly extended version of the GS analysis of the decays of 2^+ mesons into two 0^- mesons, and describe in particular the use of the branching ratios

$$\Gamma(A_2\to\pi\eta)/\Gamma(A_2\to K\bar{K}),$$
$$\Gamma(A_2\to\pi X^0)/\Gamma(A_2\to K\bar{K}),$$

to determine whether experiment prefers the linear mass or quadratic mass value of the $X^0-\eta$ mixing angle θ_0.

In the notation of GS, we describe the nonet of 2^+ mesons $[A_2,K^*(1410),f',f]$ by the 3×3 matrix T:

$$\begin{bmatrix} [f\cos(\theta_c-\theta_2)+f'\sin(\theta_c-\theta_2)+A_2{}^0]/\sqrt{2} & A_2{}^+ & K^{*+}(1410) \\ A_2{}^- & [f\cos(\theta_c-\theta_2)+f'\sin(\theta_c-\theta_2)-A_2{}^0]/\sqrt{2} & K^{*0}(1410) \\ K^{*-}(1410) & \bar{K}^{*0}(1410) & [f\sin(\theta_c-\theta_2)-f'\cos(\theta_c-\theta_2)] \end{bmatrix}, \tag{2}$$

where $\sin\theta_0=\sqrt{\frac{1}{3}}$, $\cos\theta_c=\sqrt{\frac{2}{3}}$. Similarly P, with π, K, X^0, η, and θ_0 in place of A_2, K^* (1410), f', f, and θ_2, describes the 0^- nonet. The most general SU_3-invariant coupling governing the decays of 2^+ mesons into two 0^- mesons can be expressed as

$$\sqrt{2}\alpha\,\mathrm{Tr}(T\{P,P\}) +(1/\sqrt{3})\beta\,\mathrm{Tr}T\,\mathrm{Tr}(PP)+(2/\sqrt{3})\gamma\,\mathrm{Tr}P\,\mathrm{Tr}(TP) +(1/\sqrt{27})\delta\,\mathrm{Tr}T\,\mathrm{Tr}P\,\mathrm{Tr}P, \tag{3}$$

where α, β, γ, δ are constant and radicals have been inserted in order to simplify the final results. The γ and δ terms are relevant only for decays involving X^0 and η, while the β and δ terms are relevant only for f and f' decays. If, as is usual at present in discussion of meson decays, one neglects symmetry-breaking effects except in the computation of phase space, then one can, by expansion of (3), evaluate the rate divided by phase space for each of the eleven allowed 2^+ decays. The results are displayed in Table I, which is essentially an extension of the upper portion of Table I of GS, in which X^0 was neglected and η regarded simply as the $I=Y=0$ state of the 0^- octet. (The earlier table is recovered by setting $\gamma=\delta=0$, $\theta_0=\pm90°$, $\alpha=\sqrt{3}F$, $\beta=G-2\sqrt{2}F$.) One proceeds to the prediction of decay rates by inclusion of the p^5/M^2 phase-space factor appropriate to d-wave decays.

The conclusion of GS was that all of the experimental data for decay rates could be accounted for qualitatively by setting $\beta=\gamma=\delta=0$ so that all decays are described in terms of a single coupling constant. These conclusions were based on $\theta_0=\pm90°$ and $\theta_2=30°$.[9]

TABLE I. Parametrized rates for decays of $J^P=2^+$ mesons into two $J^P=0^-$ mesons.

Decay mode[a]	Phase space[b]	Rate exclusive of phase space
$f\to\pi\pi$	53.6	$3[2\alpha\cos(\theta_c-\theta_2)+\beta\cos\theta_2]^2$
$f\to K\bar{K}$	5.3	$4[\sqrt{3}\alpha\sin(2\theta_c-\theta_2)+\beta\cos\theta_2]^2$
$f\to\eta\eta$	1.5	$\{\sqrt{3}\alpha[\sin\theta_2\cos(2\theta_c-2\theta_0)+\sin(2\theta_c-\theta_2)]+\beta\cos\theta_2+2\gamma\cos\theta_0\cos(\theta_0-\theta_2)+\delta\cos\theta_2\cos^2\theta_0\}^2$
$A_2\to\pi\eta$	25.2	$2[2\alpha\cos(\theta_c-\theta_0)+\gamma\cos\theta_0]^2$
$A_2\to K\bar{K}$	9.2	$4\alpha^2$
$A_2\to\pi X^0$	1.2	$2[2\alpha\sin(\theta_c-\theta_0)-\gamma\sin\theta_0]^2$
$K^*(1410)\to K\pi$	42.3	$6\alpha^2$
$K^*(1410)\to K\eta$	12.0	$2[\sqrt{3}\alpha\sin(2\theta_c-\theta_0)+\gamma\cos\theta_0]^2$
$f'(1500)\to\pi\pi$	96.6	$3[2\alpha\sin(\theta_c-\theta_2)-\beta\sin\theta_2]^2$
$f'(1500)\to K\bar{K}$	26.5	$4[\sqrt{3}\alpha\cos(2\theta_c-\theta_2)+\beta\sin\theta_2]^2$
$f'(1500)\to\eta\eta$	15.5	$\{\sqrt{3}\alpha[\cos\theta_2\cos(2\theta_c-2\theta_0)-\cos(2\theta_c-\theta_2)]-\beta\sin\theta_2+2\gamma\cos\theta_0\sin(\theta_0-\theta_2)-\delta\sin\theta_2\cos^2\theta_0\}^2$

[a] All charge states are included.
[b] Our phase space here is P^5/M^2, in units of 10^{-3} BeV^3.

[7] See, for example, G. Furlan, F. Lannoy, C. Rossetti, and G. Segrè (to be published).

[8] The curve for 0^- mesons is drawn only for exponents lying between -0.50 and 2.75. For exponents outside of this range the necessary inequalities are violated and the mixing theory cannot be applied.

[9] The value $\theta_2=30°$ was based on older mass values than those used now to give $\theta_2=31.3°$.

In Table II, we present the consequences of setting $\beta=\gamma=\delta=0$ and of using

(a) $\theta_0=-79.6°$, $\theta_2=31.3°$, corresponding to quadratic mass considerations, and
(b) $\theta_0=-66.6°$, $\theta_2=29.9°$, corresponding to linear mass considerations.

In either case, the signs of θ_0 and θ_2 are determined so as to suppress $f'\to\pi\pi$ and $A_2\to\pi\eta$. Although the experimental errors on the observed rates[10] are currently quite large, agreement between theory and experiment is good for both (a) and (b). It is our hope that as the experimental errors in the decay rates are reduced, it will be possible to accomplish three objectives:

(1) to establish the accuracy of (or if necessary modify) the prescription of setting $\beta=\gamma=\delta=0$;
(2) to generalize the phase-space factor, for example, by introducing a radius-of-interaction parameter X,[11] so that the phase-space factor becomes

$$(p^5/M^2)[X^2/(X^2+p^2)], \qquad (4)$$

and determining X [use of (p^5/M^2) corresponds to the limit $X\to\infty$];
(3) to identify θ_0 and θ_2.

To obtain these objectives, a fit of eleven decay rates in terms of seven parameters α, β, γ, δ, θ_0, θ_2, and X is required. If such a fit proves to be unobtainable, then one will probably conclude that neglect of symmetry-breaking effects in the calculation of decay matrix elements cannot be allowed. Another possible suggestion would be that use of relatively real coupling constants rather than complex reduced matrix elements is unjustified. However, at present one can still hope to achieve a satisfactory fit in terms of α, β, γ, δ, θ_0, θ_2, X.

TABLE II. Decay rates for restricted couplings ($\beta=\gamma=\delta=0$).

Decay mode	Predicted rates (MeV)[a] Linear mixing angles ($\theta_0=-66.6°$, $\theta_2=29.9°$)	Quadratic mixing angles ($\theta_0=-79.6°$, $\theta_2=31.3°$)	Observed rates (MeV)
$f\to\pi\pi$	100[b]	100[b]	100
$f\to K\bar{K}$	4.2	4.0	<4
$f\to\eta\eta$	0.03	0.06	<4
$A_2\to\pi\eta$	1.3	5.6	3.2±2.7
$A_2\to K\bar{K}$	5.8	5.7	5.0±1.5
$A_2\to\pi X^0$	1.4	1.2	<9
$K^*(1410)\to K\pi$	39.8	39.6	?
$K^*(1410)\to K\eta$	5.2	2.8	?
$f'(1500)\to\pi\pi$	1.6	0.9	?
$f'(1500)\to K\bar{K}$	28.8	29.8	?
$f'(1500)\to\eta\eta$	17.7	12.5	?

[b] We use the P^5/M^2 phase space given in Table I.
[b] Input.

[10] Preliminary measurements of relevant decay rates are reported in S. U. Chung, O. I. Dahl, L. M. Hardy, R. I. Hess, L. D. Jacobs, J. Kirz, and D. H. Miller, Phys. Rev. Letters **15**, 325 (1965). References to earlier results may be found here.
[11] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

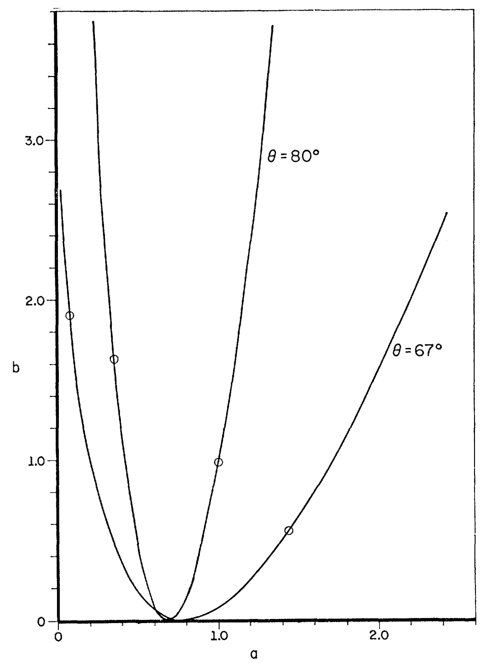

FIG. 2. The parabolas describing the locus of values of the branching ratios $\Gamma(A_2\to\pi\eta)/\Gamma(A_2\to K\bar{K})$ and $\Gamma(A_2\to\pi X^0)/\Gamma(A_2\to K\bar{K})$ allowed by the $\eta-X^0$ mixing angles $\theta_0=67°$ and $\theta_0=80°$. The axes represent a and b, which are defined in terms of the branching ratios in the text, [Eq. (5)].

Of course, the entire experimental analysis does not have to be carried out all at once. One can concentrate on the A_2 decays in order to obtain information about θ_0. Let us define the quantities

$$a=(L_a)\frac{\Gamma(A_2\to\pi\eta)}{\Gamma(A_2\to K\bar{K})}, \quad b=(L_b)\frac{\Gamma(A_2\to\pi X^0)}{\Gamma(A_2\to K\bar{K})}, \qquad (5)$$

where L_a is the ratio of the phase space for $A_2\to K\bar{K}$ to that for $A_2\to\pi\eta$, and L_b has a similar definition. For example, if one uses (p^5/M^2) phase space, then

$$L_a=(p_K/p_\eta)^5=0.36,$$
$$L_b=(p_K/p_{X^0})^5=8.1.$$

Alternatively, if one uses phase space (4) with X, for the purpose of illustration, put equal to the K-meson mass, then $L_a=0.44$ and $L_b=6.1$. If we allow for the possibility $\gamma\neq0$, then from Table I we obtain

$$2a=[2\cos(\theta_c-\theta_0)+(\gamma/\alpha)\cos\theta_0]^2,$$
$$2b=[2\sin(\theta_c-\theta_0)-(\gamma/\alpha)\sin\theta_0]^2.$$

Of course, β, δ, and θ_2 do not enter for A_2 decays. If we insert $\sin\theta_c=\sqrt{\frac{1}{3}}$, $\cos\theta_c=\sqrt{\frac{2}{3}}$, drop the subscript zero from θ_0, and eliminate (γ/α), we are led to

$$b^{1/2}\cos\theta + a^{1/2}\sin\theta = (\tfrac{2}{3})^{1/2},$$
$$9(b\cos^2\theta - a\sin^2\theta)^2 = 4(3b\cos^2\theta + 3a\sin^2\theta - 1), \quad (6)$$

or

$$9(b\cos\psi - a\sin\psi)^2 = 4(\cos\psi + \sin\psi) \times [(3b-1)\cos\psi + (3a-1)\sin\psi],$$

where $\tan\psi = \tan^2\theta$. For constant θ, Eq. (6) describes a parabola in the a-b plane.[12] The parabolas for $\theta = 66.6°$, 79.6° are displayed in Fig. 2, the points corresponding to $\gamma = 0$ being circled. The region of the a-b plane favored by experiment appears to satisfy $a < 0.5$; as yet, no good limit on b is available. Future measurements on A_2 decays may be expected to yield restrictions on the experimental region, and future measurement of the rates for the decays of other 2^+ mesons into two 0^- mesons will help define the proper phase space to use, thereby determining L_a and L_b. Then discrimination between $\theta = 79.6°$ and 66.6° should be possible.

One might also seek a determination of θ_0 in the eventuality that α etc. were not simply coupling constants but complex reduced matrix elements. In this case, it would not be sufficient to use experimental measurements of a and b. However if, in addition to a and b, we use

$$c = (L_c)\Gamma(K^*(1410) \to K\pi)/\Gamma(K^*(1410) \to K\eta),$$

determination of three real parameters $|\gamma/\alpha|$, θ_0, and the relative α-γ phase again becomes possible. Alternatively, the measurement of the branching ratios a, b, and c affords a means of testing the assumption of the relative reality of α and γ.

3. THE 1^- AND 2^+ MIXING ANGLES

It was remarked in the Introduction that it will be difficult to discriminate between the linear and quadratic values of the 1^- and 2^+ mixing angles θ_1 and θ_2. We suggest here that the measurement of $\Gamma(\phi \to 3\pi)$ and $\Gamma(f' \to \pi\pi)$, both of which are experimentally anomalously small, might yield the desired information about θ_1 and θ_2, respectively.

Consider the mixing angle θ_2 first. Its value can be determined in principle from an accurate determination of $\Gamma(f \to \pi\pi)$, $\Gamma(f' \to \pi\pi)$, and $\Gamma(A_2 \to K\bar{K})$, since Table I reveals that the squared matrix elements for these decays depend on α, β, and θ_2 in precisely the same way as the decays $A_2 \to \pi\eta$, $A_2 \to \pi X^0$, and $A_2 \to K\bar{K}$ depend on α, γ, and θ_0. One can therefore adopt the method used to determine θ_0 in Sec. 2, drawing parabolas analogous to those in Fig. 2. Here, however, the parabolas corresponding to 29.9° (linear mixing angle) and 31.3° (quadratic mixing angle) will lie very close together. Nonetheless, there are indications that we may have a favorable situation for distinguishing the two values of θ_2. We have seen in GS and in the previous section that the experimental data for the entire family of 2^+ meson decays into two 0^- mesons appear to be consistent with $\beta = 0$. In that case, the rate for $f' \to \pi\pi$ is proportional to

$$\sin^2(\theta_c - \theta_2),$$

where $\theta_c = \sin^{-1}\sqrt{\tfrac{1}{3}}$, and it increases by a factor of 2 when one replaces the quadratic by the linear value of θ_2. Even if β is not zero, as long as it is known accurately (from the rates for $f \to \pi\pi$ and $A_2 \to K\bar{K}$ and a proper identification of the phase space) and is not large, a measurement of the rate for $f' \to \pi\pi$ may allow one to distinguish between the two values of θ_2. For example, the hypothetical input rates $\Gamma(f \to \pi\pi) = 80 \pm 5$ MeV and $\Gamma(A_2 \to K\bar{K}) = 5.0 \pm 0.2$ MeV, with (p^5/M^2) phase space, determine the coupling strengths $\alpha = 0.368 \pm 0.007$, $\beta = -0.032 \pm 0.026$. Then we find $\Gamma(f' \to \pi\pi) = 2.1 \pm 0.6$ MeV, using the linear mixing angle, but $\Gamma(f' \to \pi\pi) = 1.3 \pm 0.6$ MeV using the quadratic mixing angle. The example is illustrative only; at present, the rates for $f \to \pi\pi$ and $A_2 \to K\bar{K}$ have much larger experimental errors.

To determine the ϕ-ω mixing angle θ_1, we use the recent calculation by Yellin[13] of $\Gamma(\phi \to 3\pi)$ and $\Gamma(\omega \to 3\pi)$ on the basis of the model of Gell-Mann, Sharp, and Wagner,[14] in which the $\rho\pi$ intermediate states are assumed to dominate. Yellin's calculations, which take the ρ width into account,[15] relate $\Gamma(\phi \to 3\pi)$ to the $\phi\rho\pi$ coupling constant and $\Gamma(\omega \to 3\pi)$ to the $\omega\rho\pi$ coupling constant. If we use the coupling scheme of GS to relate these coupling constants and Yellin's phase-space evaluation, we are led to

$$\Gamma(\phi \to 3\pi) \approx (34 \pm 2)\tan^2(\theta_c - \theta_1)\Gamma(\omega \to 3\pi). \quad (7)$$

[The uncertainty in (7) reflects the uncertainty in the ρ mass.] Using $\Gamma(\omega \to 3\pi) = 10.6 \pm 1.5$ MeV, we get

$$\Gamma(\phi \to 3\pi) = 0.5 \pm 0.1 \text{ MeV},$$

using the linear mixing angle, and

$$\Gamma(\phi \to 3\pi) = 2.6 \pm 0.5 \text{ MeV},$$

using the quadratic mixing angle. The experimental result is 1.0 ± 0.5 MeV.

This method of determining θ_1 is particularly model-dependent. Quite clearly we should prefer a method which required fewer assumptions. Perhaps such a method will be provided by the analysis of decay modes,

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[12] The parabola has its axis inclined at angle ψ degrees to the a axis and the lines $a=0$, $b=0$, and $a+b=\tfrac{2}{3}$ as tangents. The region $a+b<\tfrac{2}{3}$ of the ab plane is not accessible for any value of θ.

[13] J. Yellin (to be published).

[14] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

[15] In GS, a zero-width approximation was made. It is to be noted that taking account of the ρ width apparently increases $\Gamma(\phi \to 3\pi)$ by a factor of 2.

involving members of the 1^- meson nonet of some as yet undiscovered heavy multiplet of baryons or mesons.

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Wide-Angle Electron-Pair Production*†

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An experiment testing quantum electrodynamics at high energies and small distances is described. The photoproduction from carbon of electron-positron pairs was measured at laboratory angles of 4.60°, 6.23°, and 7.46°. Symmetrical electron-positron pairs in the energy range from 1 to 5 BeV were detected with a magnet-counter system which consisted of two mirror-image arms. Extensive internal checks of the apparatus were made and the results were reproducible. The theoretical values for the electron-pair yield were calculated by integrating the differential pair-production cross section over the acceptance of the apparatus using a Monte Carlo technique. The ratio $R=$ (experimental yield)/(theoretical yield) was not 1.0. R was approximately given by

$$R=0.62\{(1.00\pm0.05)+k^2/(4.31\pm0.17)^2\},$$

where k is the energy in BeV of the photon which produced the pair, and by

$$R=0.67\{(1.00\pm0.04)-Q_f^2/(313\pm13)^2\},$$

where Q_f^2 is the four-momentum of the virtual fermion in (MeV)^2. The apparatus studies and a comparison of the measured single-electron yields with the theoretical yields suggest that an error exists in the absolute normalization of the results. There are no indications that the observed variation of the electron-pair yields with momentum or the large excess of wide-angle electron pairs at high energies is due to any systematic error. The experimental results do not agree with the predictions of quantum electrodynamics; they indicate a breakdown of the theory or the presence of other processes.

I. INTRODUCTION

QUANTUM electrodynamics is one of the most firmly established theories of modern physics. This theory describes the electromagnetic interactions of electrons, muons, and photons, and, as far as is known, it also describes correctly the structure of the electron and the muon. The best evidence for the correctness of this theory comes from high-precision measurements of the energy levels of simple atoms and of the anomalous magnetic moments of the electron and muon. However, these measurements are relatively insensitive to the behavior of the theory at very small distances and high momentum transfers. It is conceivable that the theory correctly describes low-energy phenomena such as the Lamb shift and the anomalous moment of the electron but fails to describe correctly the structure of the electron or electron-electron scattering at high momentum transfers. It is also not clear whether the present theory of quantum electrodynamics (QED) arrived at by a renormalization procedure is a final theory or whether it is a temporary solution to a more involved problem. For these reasons, it is important to look for deviations from quantum electrodynamics in situations where the experiments are sensitive to the behavior of the theory at high momentum transfers or small distances.

This paper reports an experiment performed to study the behavior of the electron propagator for large spacelike virtual momenta. This experiment studies the photoproduction of electron-positron pairs at large angles, and was first proposed by Drell as a technique for studying the behavior of quantum electrodynamics at small distances.[1] In the first part of the paper, the theory of the experiment is discussed; later sections describe the apparatus, the mode of analysis, and the results.

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[1] S. D. Drell, Ann. Phys. 4, 75 (1958).