Pseudoscalar-Meson Universality Principle*

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It is pointed out that P-wave interactions of the pseudoscalar-octet (P_8) mesons provide a practical method of grouping hadrons together in supermultiplets. The hypothesis that P_8-meson-meson and P_8-baryon-baryon interactions in the limit of small momentum transfer are proportional to appropriate matrix elements of the axial-vector-octet generators of $SU(6)$ is found to agree fairly well with experiment. The P_8 universality principle provides a simple measure of the ω-φ mixing angle, and is useful also for identifying members of $SU(6)$ resonance multiplets. Identification of the $\Sigma_\gamma(1660 \text{ MeV})$ and the $\Lambda'(1405 \text{ MeV})$ with states of the 70-fold representation of $SU(6)$ leads to a predicted partial width of 14–19 MeV for the $\Sigma_\gamma \to \Lambda' + \pi$ decay. The application of various universality principles suggested by $SU(6)$ symmetry to meson-meson-meson interactions is investigated.

I. INTRODUCTION

THE $SU(3)$ symmetry is not a sufficiently powerful concept to enable one to write the various $BB\mu$ and $\mu\mu\mu$ interactions in terms of a very small number of parameters, (where μ and B denote mesons and baryons). Too many particle multiplets exist. Two simplifying principles proposed in recent years are particularly attractive. The first is the hypothesis that the vector-meson octet is coupled universally to the eight currents of $SU(3)$. The second proposal is that the static limit of the interaction of the 56-fold baryon supermultiplet and the 35-fold odd-parity meson supermultiplet is approximately $SU(6)$-symmetrix.[1] Since the pseudoscalar (P) and vector (V) mesons are members of the same $SU(6)$ multiplet, these two hypotheses suggest that the P mesons also satisfy a universality principle. The main purpose of this paper is to investigate this possibility.

In order to illustrate one of the reasons for this investigation, we consider the relations between various meson interactions and strong-interaction symmetries. An important practical question is: What criterion may be used to associate different particles of the same spin and parity with a common isotopic-spin multiplet? The usual criterion of similar masses is only half adequate, since there is no known dynamical principle that requires particles of different isospin multiplets to have very different masses. [This type of defect is important when one extends the argument to $SU(3)$ and $SU(6)$ symmetry.] On the other hand, if the ρ meson is coupled universally to the isotopic spin, as suggested by Sakurai, the ρ interactions provide a satisfactory criterion, in principle.[2] Two particles of I_z differing by one are members of the same isotopic-spin multiplet if and only if they are connected by large electric-type $\rho^\pm$ interactions at zero-momentum transfer. In this paper we assume the validity of the universal-ρ-coupling hypothesis.

If the group is enlarged to $SU(3)$, the hypothesis implies that the electric-type interactions of the V_8 (V-meson octet) are proportional to matrix elements of the eight generators of $SU(3)$. Since the generators corresponding to the K^* interactions do not commute with I^2, these interactions may be used as criteria for the assignment of particles of different isospin multiplets to $SU(3)$ multiplets. In practice, these principles are not very useful because few V-meson interactions can be measured accurately.

The extension to $SU(6)$ is clear. The P_8 (P octet) mesons are associated with generators of $SU(6)$ that do not commute with F^2, the $SU(3)$ spin.[3] Therefore, P-wave P_8 interactions may be used as a criterion for associating particles of different $SU(3)$ multiplets together in an $SU(6)$ supermultiplet.[4] This criterion may be very useful; because of the small pion mass and large $SU(6)$ mass splitting, many such interactions are measurable directly from particle decays.

It is well known that $SU(6)$ cannot be anexact symmetry of nature. Measurements of various P-wave pion interaction constants may provide valuable clues concerning the manner in which the symmetry is broken. It is interesting to note that one may specify the contents of the 35-fold meson supermultiplet (P_8, V_8, V_1) and the 56-fold baryon supermultiplet (B_8,B_{10}*) with no reference to $SU(6)$, simply by grouping together $SU(3)$ multiplets connected by known strong, P-wave P_8 interactions, such as the $\rho\pi\pi$, $\omega\rho\pi$, and $N^*N\pi$ interactions.

The P_8 universality principle is defined explicitly in Sec. II. In Sec. III, the principle is used to predict ratios of pion interaction constants that are known experimentally, or may be known in the near future. In Sec. IV, problems associated with application of $SU(6)$ symmetry and various universality principles to the meson-meson-meson interactions are examined.

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[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964).
[2] J. J. Sakurai, Ann. Phys. (N. Y.) 11, 1 (1960).
[3] R. H. Capps, Phys. Rev. 139, B421 (1965). This paper contains a discussion of the identification of the various mesons with $SU(6)$ generators.
[4] It is possible that the strong P-wave interactions of the P_8 mesons generate the approximate $SU(6)$ symmetry observed in nature. See R. H. Capps, in Proceedings of the Conference on Symmetry Principles at High Energy, Coral Gables, 1966 (to be published).

II. THE UNIVERSALITY PRINCIPLE

We are concerned with vertices $B_kA_jP_l$, for which the initial and final particles A_j and B_k are real. The subscript denotes the state within an $SU(3)$ multiplet. The $SU(3)$ multiplets A and B may be the same or different, but must have the same parity and belong to the same supermultiplet of $SU(6)$. The symbol P_l in the vertex represents either the octet P meson l in the initial state, or the final meson that is the antiparticle of the P meson l. The universality principle applies to the static limit, in which powers of the three-momenta of the particles A and B higher than the first are neglected. In this limit, the meson P_l is in a P state, and the interaction vertex may be written,

$$i[F(kjl)/M]\mathbf{S}\cdot\mathbf{q}\,, \qquad (1)$$

where $F(kjl)$ is a dimensionless interaction constant, M is a mass appropriate for the AB supermultiplet, $\mathbf{q}$ is the three-momentum transfer, and $\mathbf{S}$ is a vector operator, operating in the spin space of the particles A and B. It is assumed that the coupling constants are chosen to be real and the sping components S^x, S^y, and S^z are chosen to be Hermitian. These conventions imply the presence of the factor i in Eq. (1), as may be seen from the requirement that the residue of the pole corresponding to B_k in the P-wave elastic A_j-P_l scattering amplitude must be negative.

The universality principle may be written,

$$[F(kjl)/M]\langle B^n|S^i|A^m\rangle = C\langle B_k{}^n|J_l{}^i|A_j{}^m\rangle\,, \qquad (2)$$

where C is a universal constant, $A_j{}^m$ and $B_k{}^n$ denote states of an $SU(6)$ multiplet corresponding to the particles A_j and B_k in the spin states m and n, and A^m and B^n denote the corresponding states in spin space. The three-vector $\mathbf{J}_l$ represents the three generators of $SU(6)$ that correspond to the spin-one-octet state of $SU(3)$ index l, and i denotes a component of the vector operators $\mathbf{S}$ and $\mathbf{J}$. Because of the Wigner-Eckart theorem for $SU(2)$, any Hermitian spin operator whose matrix elements are not all zero may be used for $\mathbf{S}$. Thus, the universality condition may be thought of as a restriction only on the $SU(3)$ coupling constants $F(kjl)$.

Frequently we will refer to the meson associated with the generator as virtual. Actually, it is not necessary that this meson be virtual or that the particles A and B be real.[5] This distinction is used only in defining the static limit. In this limit the energies of the particles A and B are set equal to the heavier of the A and B masses, and the momenta $\mathbf{k}_A$ and $\mathbf{k}_B$ are assumed small. (If m_A and m_B are very different, this procedure is somewhat arbitrary, of course.) In order that Eq. (2) may be used to compare different types of interactions, a uniform method of normalizing the interaction vertices of Eq. (1) must be specified. A suitable prescription may be obtained by referring to the discussion of P-exchange potentials given previously by the author.[6] It is required that the potential for the process $A_1+A_2\rightarrow B_1+B_2$ resulting from the exchange of a particular P meson is given by

$$\frac{F_1F_2}{4\pi}\frac{1}{M_1M_2}(\mathbf{S}_1\cdot\mathbf{q}_1)(\mathbf{S}_2\cdot\mathbf{q}_2)Y\,, \qquad (3)$$

where Y is a Yukawa potential (in configuration space $Y=e^{-\mu r}/r$).

For completeness, we list here the corresponding V-octet universality principle. In the static limit, the electric-type $A_kA_jV_l$ interaction may be written in the form $G(kjl)e^0$, where e^0 is the fourth component of the V polarization vector.[3] The principle is expressed by the equation,

$$G(kjl)=C'\langle A_k|\mathcal{J}_l|A_j\rangle\,, \qquad (4)$$

where C' is a constant, and $\mathcal{J}_l$ is the appropriate spin-0-octet generator of $SU(6)$, [i.e., generator of $SU(3)$]. It may be shown that $SU(6)$ symmetry and the V universality condition imply the P universality condition in the form of Eq. (2), i.e., the quantities F/M (rather than F) are proportional to the matrix elements of the generators $\mathbf{J}$.

III. APPLICATION TO MEASUREABLE INTERACTIONS

A. *VPP*, *BBP*, and *BB*P* Interactions

In this section, Eq. (2) will be used to predict ratios of various interactions involving pi mesons. The pion interaction constants that are known most accurately from experiment are associated with VPP, BBP, and BB^*P interactions, where B and B^* denote the ground-state baryon octet and $J^P=\frac{3}{2}^+$ baryon decuplet. We consider first the interaction VP_aP_b, the subscripts representing $SU(3)$ indices. The particles V and P_a are assumed to belong to the 35-fold multiplet of $SU(6)$; according to Eq. (2) the interaction in the static limit is proportional to $\langle V|\mathbf{J}_b|P_a\rangle$. It should be noted that the real particles V and P_a are associated with spin-one and spin-zero states, respectively, but the virtual particle P_b is identified with a spin-one generator. This procedure is discussed more fully in Sec. IV. Consistency requires that the same answer be obtained if the roles of the two P mesons be reversed. This condition is satisfied, since both the VPP interactions and the matrix elements $\langle V|\mathbf{J}|P\rangle$ are of the antisymmetric "F" type. (The F-type property of the elements $\langle V|\mathbf{J}|P\rangle$ follows from lines 1 and 2 of Table I.)

[5] See R. H. Capps, Phys. Rev. Letters **14**, 31 (1965); J. G. Belinfante and R. E. Cutkosky, Phys. Rev. Letters **14**, 33 (1965). In the baryon-bootstrap model treated in these works, the meson is real and one of the baryons is virtual.

[6] R. H. Capps, Phys. Rev. Letters **14**, 842 (1965).

TABLE I. Some matrix elements of the $I=1$, axial-vector-octet (pion) generator. Subscripts and superscripts denote I_z and J_z, respectively. The notation for members of the **70** is that of Sec. IIIC.

$SU(6)$ multiplet	Initial state	Final state	Generator	Matrix element
35	$\rho_1{}^1$	π_0	$J_{-1}{}^{-1}$	2
35	$\rho_1{}^1$	η	$J_{-1}{}^{-1}$	0
35	$\rho_1{}^1$	$\rho_0{}^0$	$J_{-1}{}^{-1}$	0
35	$\rho_1{}^1$	$(\omega_8)^0$	$J_{-1}{}^{-1}$	$(4/3)^{1/2}$
35	$\rho_1{}^1$	$(\omega_1)^0$	$J_{-1}{}^{-1}$	$-(8/3)^{1/2}$
35	$\rho_1{}^0$	π_1	$J_0{}^0$	-2
56	$p^{1/2}$	$p^{1/2}$	$J_0{}^0$	5/3
56	$p^{1/2}$	$N^*{}_{1/2}{}^{1/2}$	$J_0{}^0$	$\frac{1}{3}(32)^{1/2}$
70	$(\Sigma_\gamma)_0{}^{1/2}$	$\Lambda'^{1/2}$	$J_0{}^0$	$(4/3)^{1/2}$
70	$\tilde{\Sigma}_0{}^{1/2}$	$\Lambda'^{1/2}$	$J_0{}^0$	$(4/3)^{1/2}$
70	$\tilde{N}^*{}_{1/2}{}^{1/2}$	$(N_\gamma)_{1/2}{}^{1/2}$	$J_0{}^0$	4/3
35	$(\omega_1)^0$	π_0	$J_0{}^0$	0

The relativistic VP_aP_b interaction is usually written in the form $fe\cdot(k_a-k_b)$, where e is the V four-polarization vector, and k_i is the four-momentum of the particle i. The momentum of the virtual particle is given by $\mathbf{k}_b=\mathbf{k}_V-\mathbf{k}_a$. We use the metric $A\cdot B=\mathbf{A}\cdot\mathbf{B}-A^0B^0$. In order that the interaction be normalized by the convention described in Sec. II, [Eq. (3)], we multiply by $1/(2m_V)$, where m_V is the mass of the V meson (the heavier of the two real particles). Thus, the normalized interaction may be written

$$f(VP_aP_b)e\cdot(2k_a-k_v)/(2m_V). \quad (5)$$

If the particles V, P_a, and P_b are the ρ^+, π^+, and π^0, the constant f is twice the interaction constant defined in Ref. 3 and by Gell-Mann, Sharp, and Wagner,[7] and is related to the width Γ_ρ of the ρ meson by the equation,

$$f^2/(4\pi)=\tfrac{3}{2}m_\rho{}^2\Gamma_\rho/k_\pi{}^3, \quad (6)$$

where k_π is the magnitude of the three-momentum of a decay pion in the ρ rest system. If the energies ω_a and ω_V are set equal to m_V, the components e^0 and $\mathbf{e}$ may be expressed in terms of the unit three-polarization vector $\mathbf{E}$ of the V rest system by the equations $e^0=\mathbf{E}\cdot\mathbf{k}_V/m_V$ and $\mathbf{e}=\mathbf{E}$. One may use these equations to rewrite the expression of Eq. (5) in the form (f/m_V) $\mathbf{E}\cdot(\mathbf{k}_a-\mathbf{k}_V)$. The quantity $\mathbf{k}_a-\mathbf{k}_V$ is equal to $\pm q$, depending on whether the V meson is in the initial or final state. However, if we redefine the phase of the state $|V\rangle$ by multiplying $|V\rangle$ by the phase factor $(-i)$, the interaction may be written in the form,

$$i(f/m_V)\mathbf{E}\cdot\mathbf{q}, \quad (7)$$

applicable whether the V is in the initial or final state. This form is appropriate for application of the universality principle, Eq. (2).

We first compare the $\rho\pi\pi$ and $K^*K\pi$ coupling constants. If the mass differences are neglected the ratio of these constants is given by $SU(3)$ symmetry alone. However, Eq. (2) requires that $f^2(\rho\pi\pi)/m_\rho{}^2$ and $f^2(K^*K\pi)/m_{K^*}{}^2$ be compared, rather than the coupling constants alone. This improves the agreement of theory and experiment, i.e., the experimental ρ and K^* widths of 106 and 50 MeV lead to the ratio $R=4f^2(K^{*+}K^+\pi^0)/f^2(\rho^+\pi^+\pi^0)=1.6$, while simple $SU(3)$ symmetry predicts $R=1$ and the P universality condition predicts $R=m_{K^*}{}^2/m_\rho{}^2=1.35$.[8,9]

We now consider the coupling of the P mesons with members of the 56-fold baryon supermultiplet, focusing attention on the $pp\pi^0$ and $N^{*+}p\pi^0$ interactions. In the static limit these interactions may be written in the respective forms,

$$i(G/2M_N)\boldsymbol{\sigma}\cdot\mathbf{q} \quad\text{and}\quad i(H/2M^*)\mathbf{S}\cdot\mathbf{q}, \quad (8)$$

where $\boldsymbol{\sigma}$ is the usual nucleon spin vector, and M_N and M^* are the masses of the N and N^*. The constant G is the usual π^0pp interaction constant, i.e., $(G^2/4\pi)\approx 14$. We normalize S by the requirement $\langle B^{*1/2}|S^z|B^{1/2}\rangle=1$. If follows from this definition that the magnitude of the interaction constant H is related to the measured N^* width by the relation $\Gamma=\frac{3}{8}(H^2/4\pi)k_\pi{}^3/M^{*2}$. One may use this relation and Table I to show that the P-universality condition leads to the relation

$$\Gamma=(12/25)(G^2/4\pi)(k_\pi{}^3/M_N{}^2). \quad (9)$$

This differs from the expression given by Gürsey, Pais, and Radicati only in the identification of the mass factors.[10] The value of Γ predicted from Eq. (9) is 95 MeV, as compared to the experimental value of about 125 MeV.

Finally, we compare the πNN and $\rho\pi\pi$ constants. One may use rows 6 and 7 of Table I to show that the predicted relation is

$$(f^2/4\pi)=(36/25)(m_\rho/2m_N)^2(G^2/4\pi). \quad (10)$$

This relation is identical, except for the identification of the mass factors, with that derived from μBB $SU(6)$ symmetry and V meson universality in Ref. 10. The fact that these two procedures lead to the same result is discussed further in Sec. IV; the point is not trivial, because of the difficulties involved in applying $SU(6)$ symmetry to meson-meson-meson vertices. The value of $f^2/(4\pi)$ predicted from Eq. (10) is ~ 3.35, as compared to the value 2.1 obtained from the experimental ρ width of 106 MeV. [See Eq. (6).] One cannot expect greater accuracy here, because of the large ρ-π mass difference.

Thus, application of the P_8-universality principle to the interactions best known experimentally leads to relations similar to those obtained previously by other

[7] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[8] Except where stated otherwise, the experimental masses and widths used here are taken from the compilation of A. H. Rosenfeld, *et al.*, Rev. Mod. Phys. **36**, 977 (1964).

[9] For tables of $SU(3)$ Clebsch-Gordan coefficients, see P. McNamee, S. J. Chilton, and Frank Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

[10] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964).

authors. These relations are satisfied fairly well. Clearly, one is justified in applying the principle to other interaction constants, that are not yet measured accurately. This is the program of Secs. IIIB and IIIC.

B. *VVP* Interactions

We now consider interactions of the type V_bV_aP, where the V are members of the $(\rho,K^*,\varphi,\omega)$ nonet. The interaction must be symmetric in V_b and $\bar{V}_a$ (where the bar denotes an antiparticle), since the wave function for two V mesons in a state of spin and parity 0^- is symmetric. This symmetry requirement is consistent with the P_8-universality principle. The $V_8V_8P_8$ interaction is of the symmetric "D type"; the fact that the matrix elements $\langle V_8|\mathbf{J}|V_8\rangle$ are of the D type follows from the relations $\langle\rho|\mathbf{J}|\rho\rangle=0$ and $\langle\omega_8|\mathbf{J}|\rho\rangle\neq0$, shown in Table I.

A common manner of writing the V_bV_aP interaction in relativistic form is $ge_a{}^\mu e_b{}^\nu k_a{}^\sigma k_b{}^\lambda\epsilon_{\mu\nu\sigma\lambda}$, where $\epsilon_{0123}=1$ and $\epsilon_{\mu\nu\sigma\lambda}$ is completely antisymmetric. The constant g is of dimension $(\text{mass})^{-1}$. In accordance with the normalization convention of Sec. II, we multiply by $(2m_V)^{-1}$, where m_V is the mass of the heavier V meson. The interaction for the process $V_a+P\rightarrow V_b$ is then written,

$$g(V_bV_aP)e_a{}^\mu e_b{}^\nu k_a{}^\sigma k_b{}^\lambda\epsilon_{\mu\nu\sigma\lambda}/(2m_V)\,. \tag{11}$$

The factor k_b may be replaced by k_a+q, where q is the four-momentum transfer. If this is done, the only terms of Eq. (11) that contribute in the static limit are those in which $\sigma=0$. The static limit of the expression is

$$i\tfrac{1}{2}g\mathfrak{S}\cdot\mathbf{q}\,, \tag{12}$$

where the matrix elements of the Hermitian spin operator $\mathfrak{S}$ between the V-polarization states are defined by the equation,

$$\langle V_b|\mathfrak{S}|V_a\rangle=i(\mathbf{E}_b\times\mathbf{E}_a)\,. \tag{13}$$

The VVP-interaction constants related most directly to experimental numbers are $f(\omega\rho^0\pi^0)$ and $f(\varphi\rho^0\pi^0)$. The symbols ω and φ represent the physical particles, assumed related to the octet and singlet mesons ω_8 and ω_1 by the usual equations

$$\varphi=(\sin\theta)\omega_1+(\cos\theta)\omega_8\,,$$
$$\omega=(\cos\theta)\omega_1-(\sin\theta)\omega_8\,.$$

One may use Eq. (12), the corresponding equation for the VPP interactions, [Eq. (7)] and Table I to show that the P_8 universality principle leads to the predictions,

$$\frac{g^2(\omega\rho^0\pi^0)m_\rho{}^2}{f^2(\rho^+\pi^+\pi^0)}=4\cos^2(\theta-\beta)\,, \tag{14}$$

$$\frac{g^2(\varphi\rho^0\pi^0)m_\rho{}^2}{f^2(\rho^+\pi^+\pi^0)}=4\sin^2(\theta-\beta)\,, \tag{15}$$

where β is the value of θ that follows from the assumption that the mass splitting of the V part of the meson supermultiplet transforms as the 35-fold representation, $[\beta=\tan^{-1}(2)^{-1/2}\approx35°]$.[11]

If the P_8-universality principle is valid, the $\varphi\rightarrow\rho+\pi$ partial width is a measure of the magnitude of the angle $\theta-\beta$. This partial width is related to $g^2(\varphi\rho^0\pi^0)$ by the equation, $\Gamma_{\varphi\to\rho\pi}=[g^2(\varphi\rho^0\pi^0)/(4\pi)]k_\pi{}^3$. If Eq. (15), and the experimental value of $k_\pi{}^3$ are substituted into this relation, the result is

$$\Gamma_{\varphi\to\rho\pi}=(48\text{ MeV})\sin^2(\theta-\beta)f^2(\rho^+\pi^+\pi^0)/(4\pi)\,. \tag{16}$$

Since the large $\rho-\pi$ mass difference leads to some arbitrariness when the universality principle is applied to the $\rho\pi\pi$ interaction, a more accurate value of $\sin^2(\theta-\beta)$ may be obtained from a comparison with the πNN constants. We use the equivalent procedure of substituting the value $f^2/(4\pi)=3.35$ [calculated from Eq. (10)] into Eq. (16). The experimental data concerning the partial width is given by $\Gamma_{\text{total}}=(3.1\pm1.0)$ MeV, and $\Gamma_{\rho\pi}/\Gamma_{K\bar{K}}=0.35\pm0.15$.[12,13] If $\Gamma_{\rho\pi}=0.8$ MeV, the value of $|\theta-\beta|$ obtained from Eq. (16) is about 4°. It is amusing that the assumption that the octet mass formula is satisfied exactly for the pure V octet leads to a calculated value $\theta\approx39.8\pm1°$, or $\theta-\beta\approx4\tfrac{1}{2}°\pm1°$.[14]

One may then set $\cos^2(\theta-\beta)=1$ and use Eq. (14) to predict $g^2(\omega\rho^0\pi^0)$.[15] The result is, $g^2(\omega)m_\rho{}^2/(4\pi)=13.4$. This constant is not measured experimentally. However, Gell-Mann, Sharp, and Wagner have developed a simple model in which the $\omega\rightarrow3\pi$ decay width is proportional to the product $g^2(\omega)f^2(\rho\pi\pi)$.[7] This formula, with $g^2m_\rho{}^2/(4\pi)=13.4$ and $f^2/(4\pi)=2.1$ (the value of f^2 obtained from the experimental ρ width), leads to the result $\Gamma_\omega=8.8$ MeV, while the experimental width is 9.4 MeV.[8,12] The closeness of the agreement is probably fortuitous, but certainly no contradiction with universality is indicated.

C. Interactions of the Odd-Parity Baryons

Experimental data favors a spin-parity assignment of $\tfrac{3}{2}^-$ for the 1660-MeV Σ_γ resonance, and an assignment of $\tfrac{1}{2}^-$ for the 1405-MeV Λ'.[16] It has been postulated that the Λ' is an $SU(3)$ singlet, the Σ_γ is an octet member,

[11] The value of β is derived by F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964).

[12] N. Gelfand *et al.*, Phys. Rev. Letters **11**, 438 (1963).

[13] J. Leitner, in talk at 1965 Autumn Meeting of the American Physical Society at Chicago (unpublished). See also P. L. Connolly *et al.*, Phys. Rev. Letters **10**, 371 (1963).

[14] This result and the errors quoted are computed from the masses and experimental errors listed in Ref. 8. However, it should be emphasized that there is no reason to expect such precision in the validity of the octet mass formula. A simple method of calculating the mixing angle required by the mass formula is given in Sec. IV of R. H. Capps, Phys. Rev. **137**, B1545 (1965).

[15] If $\theta=\beta$, the relation [Eq. (14)] is identical to the relation derived by B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965), and Phys. Rev. **139**, B1355 (1965).

[16] Our notation for the odd-parity baryon resonances is based on the assignments to the $SU(6)$ representation **70**, and is taken from M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 509 (1964).

and that they are both members of a 70-fold $SU(6)$ multiplet.[17] In this section, we calculate some of the values of $B^*B^*\pi$ interaction constants that follow from this postulate and the universality principle. For reasons discussed at the end of the section, no mixing is assumed for the particles of the **70**, i.e., the observed particles are taken as eigenvalues of F^2, the quadratic Casimir operator for $SU(3)$. The $SU(3)\otimes SU(2)$ structure of the **70** is $(1,2)\oplus(8,4)\oplus(8,2)\oplus(10,2)$.

We consider first the decay $\Sigma_\gamma\to\Lambda'+\pi$. The interaction vertex may be written in the same form as that used in describing the 1238-MeV N^* decay, Eq. (8). It follows from lines 8 and 9 of Table I, and from ordinary isotopic-spin Clebsch-Gordan coefficients, that the $\Sigma_\gamma\to\Lambda'+\pi$ partial width Γ' is related to the N^* width by the equation,

$$\Gamma'/\Gamma_{N^*}=\tfrac{1}{4}k_{\Lambda'}{}^3/k_N{}^3. \quad (17)$$

The ratio of decay momenta $k_{\Lambda'}/k_N$ is about 0.84. The range 95–125 MeV for Γ_{N^*} (the first number is the value predicted from the observed πNN interaction strength) leads to a predicted range 14–19 MeV for Γ'. This partial width has not been measured. However, a $\Sigma+2\pi$ partial width of the order of 15 MeV is listed in Ref. 8, while the measurement of Eberhard *et al.*, shows that the $\Sigma+2\pi$ mode may be dominated almost completely by $\Lambda'+\pi$ events.[18] It is important that a more accurate measurement of Γ' be made, in order to test the P-meson universality principle together with the assignments of the Λ' and Σ_γ to the 70-fold representation of $SU(6)$.

Two other decay interactions that may be useful in the future for identifying odd-parity baryon resonances with states of the **70** are the interactions $\tilde{\Sigma}(8,2)\to\Lambda'+\pi$ and $\tilde{N}^*(10,2)\to N_\gamma(8,4)+\pi$. We list below the partial widths for these decays that follow from the universality principle. These widths, determined from the numbers in Table I, are compared to the N^*(1238-MeV) width.

$$\Gamma(\tilde{\Sigma}\to\Lambda')/\Gamma_{N^*}=\tfrac{1}{2}k_{\Lambda'}{}^3/k_N{}^3,$$
$$\Gamma(\tilde{N}^*\to N_\gamma)/\Gamma_{N^*}=k_{N(\gamma)}{}^3/k_N{}^3.$$

If the $I=\frac{3}{2}$ resonance postulated recently at a total mass of $\sim$1700 MeV is the $\tilde{N}^*$, the $N^*\to N_\gamma+\pi$ energy is so small that the branching ratio for the decay is expected to be small except for N^* energies in the upper portion of the resonance peak.[19] In this case, measurement of this decay amplitude would be very difficult, and probably would necessitate sufficient data so that different regions of the $\tilde{N}^*$ energy could be analyzed separately.

We now return to the question of particle mixing. No mixing is possible for the $j=\frac{3}{2}$ or $I=\frac{3}{2}$ parts of the **70**, but mixing between the Λ' and the $\tilde{\Lambda}(8,2)$, and between the $\tilde{\Sigma}$ and the $\tilde{Y}^*(10,2)$, may occur. The following argument indicates that these mixing effects are probably not large. If one considers only mass-splitting terms that transform as $SU(3)$ octets, it is well-known that the predominant splitting terms for the meson and baryon multiplets **35** and **56** transform as the $SU(6)$ representation **35**.[20] This approximate 35-rule is expected to apply also to the multiplet **70**, if these states are essentially "bound states" of the meson-baryon type. The 35-rule for the **70** leads to two mass-splitting parameters.[20] These parameters may be estimated from the three known members of the multiplet (8,4), a multiplet convenient for this purpose since it cannot be involved with mixing. The experimental N_γ, Σ_γ, and Ξ_γ masses lead to the conclusion that the only large [$SU(3)$-octet type] splitting term is the term proportional to the hypercharge.[21] The simple hypercharge-splitting term leads to no particle mixing. Therefore, since the splitting of the Λ' and the average mass of the (8,2) multiplet [induced by a splitting term that transforms as an $SU(3)$ singlet] is known not to be small, little mixing is expected for the Λ'.[22] A similar conclusion applies to the $\tilde{\Sigma}$, unless the averages masses of the (8,2) and (10,2) multiplets are nearly the same. Baryon and meson multiplets are quite different with respect to the importance of particle mixing.

IV. MESON-MESON-MESON INTERACTIONS

We consider $AA\mu$ interactions, where the mesons μ are members of the P_8, V_8, V_1, and P_1 (the 959 MeV X^0) multiplets. In the treatment of $SU(6)$ appropriate for P-wave mesons, the P_8 and P_1 are identified with the spin-one octet and singlet of the representation **35**. The V_8 and V_1, in states of total angular momentum zero, are identified with the spin-zero octet of the **35** and with the representation **1**.[3,5] Four universality principles are suggested, in which the P_8, P_1, and electric-type V_8 interactions are proportional to the matrix elements of the appropriate generators, and the electric-type V_1 interactions are proportional to the baryon numbers of the A multiplets. However, if the mesons are themselves to be identified with two (35-fold and one-fold) A multiplets, the $\mu\mu\mu$ interactions are restricted by the

[17] A. Pais, Phys. Rev. Letters **13**, 175 (1964); I. P. Gyuk and S. F. Tuan, Phys. Rev. **140**, B164 (1965). A dynamical model of this multiplet is given in Ref. 6.

[18] P. Eberhard, F. T. Shively, R. R. Ross, and D. M. Siegel, Phys. Rev. Letters **14**, 466 (1965).

[19] A. Donnachie, A. T. Lea, and C. Lovelace (preprint); P. Bareyre, C. Brickman, A. V. Stirling, and G. Villet, Phys. Letters **18**, 342 (1965).

[20] T. K. Kuo and Tsu Yao, Phys. Rev. Letters **13**, 415 (1964); M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1965).

[21] This conclusion is reached in the detailed analysis of Gyuk and Tuan (Ref. 17), as may be seen from the large size of b_1 in Eq. (33) of this reference.

[22] The mixing angles of 45° predicted by Kuo and Yao (Ref. 20) result from the fact that no term corresponding to an $SU(3)$ singlet is included in the splitting for particles of the same spin. Hence, the masses corresponding to the "pure $SU(3)$" states of $j=\frac{1}{2}$ and $Y=0$ are all the same in this model, so that off-diagonal elements of any magnitude lead to 45° mixing. In reality, it is clear that the Λ' is much lighter than the $Y=0$ states of the other $J=\frac{1}{2}$ multiplets of the **70**.

requirement of permutation symmetry. The purpose of this section is to investigate the extent that the various universality principles and $SU(6)$ symmetry may be made consistent for the $\mu\mu\mu$ interactions.

Two of the mesons at any vertex are called real, and the third, to be identified with a generator, is called virtual. (As discussed in Sec. II this distinction is made in order to define the static limit.) In order that the theory make sense, the real mesons must be identified with representations **35** and **1** in the *usual way*, (P mesons associated with spin-0 states, etc.), rather than in the way used for the virtual mesons. The situation is rather complicated, because of the dual roles of the mesons. However, we will attempt to construct a $\mu\mu\mu$ interaction consistent with $SU(6)$ and the various universality principles in the following manner. For each set of mesons, a relativistically invariant interaction is written. Each of the mesons in turn is taken as the virtual meson, and the limit of small momenta of the real mesons is obtained. If the coefficients of the relativistic interactions may be so chosen that the static-limit interactions are proportional to the appropriate matrix elements of generators, the interaction is consistent. The arguments of Sec. III show that consistency with respect to the P_8-universality principle may be obtained; we enlarge the argument to full $SU(6)$ symmetry here. If this full consistency were obtained, it would be implied that S-wave, $SU(6)$-symmetric meson-exchange potentials of the type postualted in Ref. 6 could be constructed for states involving mesons.

Mass differences between mesons will be neglected in this section. Interactions of the type PPP are not allowed because of parity and angular-momentum conservation. This is consistent with the $SU(6)$ and universality principles, because matrix elements of axial-vector generators between spin-zero states all vanish. We must consider further VVV, VVP, and VPP interactions.

(*i*) *VVV Interactions*

The VVV interactions are assumed to be of the completely antisymmetric type suggested by Cutkosky.[23] The relativistic interaction for the process $a+c\rightarrow b$ may be written in the form[3]

$$\gamma(\mathrm{bac})[e_a\cdot e_b(k_a+k_b)\cdot e_c-e_b\cdot e_c(k_b+k_c)\cdot e_a +e_c\cdot e_a(k_c-k_a)\cdot e_b]. \quad (18)$$

If c is the virtual meson, the limit of the interaction corresponding to the small three-momenta $\mathbf{k}_a$ and $\mathbf{k}_b$ is given by

$$-\gamma(\mathrm{bac})\mathbf{E}_a\cdot\mathbf{E}_b e_c{}^0,$$

where a factor $(2m)^{-1}$ has been included in accordance with the normalization requirement specified in Sec. II. The vectors $\mathbf{E}$ are the V rest system polarization vectors defined in Sec. IIIA. The V-meson invariance principles imply that the coefficient of $e_c{}^0$ must be proportional to the appropriate matrix element of one of the $SU(3)$ generators $\mathcal{G}_8$, or of the baryon number operator (if the meson c is the V_1). Consistency with respect to the antisymmetric (F type) $V_8V_8V_8$ interactions may be obtained, since the matrix elements $\langle V_8|\mathcal{G}_8|V_8\rangle$ are also of the F type. We assume that no $V_1V_8V_8$ or $V_1V_1V_1$ interaction exists. This is consistent with $SU(6)$ and all the universality principles, since the virtual V_1 is assumed not coupled to mesons, and since the $SU(3)$ generators associated with the virtual V_8 particles annihilate all $SU(3)$ singlets.

(*ii*) *VVP Interactions*

The relativistic form of the V_bV_aP vertex is

$$ge_a{}^\mu e_b{}^\nu k_a{}^\sigma k_b{}^\lambda\epsilon_{\mu\nu\sigma\lambda},$$

as given in Sec. IIIB. If the P meson is virtual, the static limit is given by Eq. (12). On the other hand, if the vector meson V_a is virtual, the static limit of the electric interaction vanishes, since the coefficient of $e_a{}^0$ is of second order in the three-momenta $\mathbf{k}_b$ and $\mathbf{k}_P$. This corresponds to the fact that the baryon number operator and $SU(3)$ generators do not connect states of different spin.

Thus, the coefficients of the PPV interactions are not related to the V-universality principles, and may be chosen in accordance with the P principles. The $P_8V_8V_8$ and $P_8V_8V_1$ interactions are discussed in Sec. IIIB. The other possible PVV interactions are of the types $P_1V_1V_1$ and $P_1V_8V_8$. The P_1-universality principle requires that the coefficients of these two interactions are equal and nonzero, since the three-component generator corresponding to P_1 is proportional to the spin angular momentum operator.[3]

It should be pointed out that the magnetic interaction (interaction proportional to the space components of the polarization vector) of a virtual V meson associated with a VVP vertex does not vanish in the static limit, but is of the same order as the interaction associated with a virtual P meson. These magnetic interactions cannot be proportional to the generators $\mathbf{J}$, since the $V_8V_8P_8$ interaction is of the D type. Hence, magnetic V-exchange interactions destroy the $SU(6)$ symmetry of a V-exchange potential in a state involving a meson, but they do not destroy any of the universality principles postulated here.

(*iii*) *PPV Interactions*

The relativistic vertex for the P_aP_bV interaction is $e\cdot(k_a+k_b)$. If a P meson is virtual, the static limit is given by Eq. (7) of Sec. IIIA. If the V meson is virtual, the corresponding expression is $-fe^0$ (a factor of $2m$ being divided out in accordance with the normalization convention of Sec. II).

[23] R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963).

We consider first interactions involving at least one $SU(3)$ singlet. The conceivable interactions of this type are $P_1P_1V_1$, $P_8P_8V_1$, and $P_1P_8V_8$ interactions. The first two are ruled out by the antisymmetry requirement on the $SU(3)$ indices of the P mesons. We assume further that the coefficient of the $P_1P_8V_8$ interaction is zero. In order to demonstrate that the absence of these three interactions is consistent with the various universality principles and $SU(6)$ symmetry, we adopt the notation $AB(C)$ to refer to the case in which A and B represent real mesons and C represents the virtual meson associated with a generator. The matrix elements $P_1V_1(P_1)$, $P_1P_8(V_8)$, and $P_1V_8(P_8)$ are zero because the real meson P_1 is in a different $SU(6)$ multiplet from the other mesons, and is not connected to them by the generators. The elements $P_1P_1(V_1)$ and $P_8P_8(V_1)$ are zero since the (V_1) is associated with the baryon number. The elements $P_8V_8(P_1)$ are zero because the spin angular-momentum generator associated with P_1 does not connect states of different spin. The fact that the elements $V_1P_8(P_8)$ are zero follows from the last line of Table I.

The only PPV interaction not yet discussed is the $P_8P_8V_8$ interaction. As discussed in Sec. IIIA, this interaction, and the matrix elements $V_8P_8(P_8)$ are of F type. The elements $P_8P_8(V_8)$ are also of F type, so consistency with both the P_8 and V_8-universality principles may be obtained.

We have exhausted the possible types of $\mu\mu\mu$ interactions, and have found that the four universality principles may be satisfied simultaneously. Because of the dual roles of the P and V mesons, these universality principles are rather complicated. However, it is not clear how nature could tolerate a simpler principle of this type. If the mesons were scalar and axial-vector mesons, emitted and absorbed in S waves, the interaction of the scalar mesons with pairs of scalar or pairs of axial-vector mesons would be of the D type, so the interactions could not be proportional to the matrix elements of the generators.

An alternate approach to the $\mu\mu\mu$ interaction problem involves using the group $SU(6)_W$, the subgroup of $SU(12)$ that is invariant to Lorentz transformations along the z axis.[24] A convenient method of applying $SU(6)_W$ is to classify the mesons as in Ref. 24, and then examine the $\mu\to\mu\mu$ "decay" amplitudes that could occur at imaginary values of the final meson momenta. The rest system of the original meson is taken as the Lorentz system, and the z direction as the decay direction. It can be shown that the requirement that the amplitudes are proportional to the appropriate Clebsch-Gordan coefficients of $SU(6)_W$ leads to the same set of relativistic $\mu\mu\mu$ interactions as that obtained from the four universality principles, provided that the small F-wave part of the VVV interaction of Eq. (18) is neglected.

It is pointed out in Sec. III that P_8 universality leads to the same $f^2(\rho\pi\pi)/G^2(\pi NN)$ ratio as that obtained from the requirements of V universality and relativistic $SU(6)$;[10] on the other hand, if one uses the $SU(6)$ symmetry of the baryon-bootstrap model or of the meson exchange-force model of Ref. 6, the predicted f^2/G^2 ratio is a factor of three smaller.[3,6] One may use $SU(6)_W$ to resolve this difficulty. Magnetic-type V interactions are neglected in Ref. 6. However, $SU(6)_W$ requires that this coupling is responsible for $\frac{2}{3}$ of the spin-dependent S-wave meson-exchange potential, P exchange accounting for the other $\frac{1}{3}$. An $SU(6)_W$-invariant baryon bootstrap model may be constructed from the simple model of Ref. 5 by replacing the P states by states of the type $(\frac{1}{3})^{1/2}P+(\frac{2}{3})^{1/2}V_m$, where V_m denotes a meson in a state of total angular momentum one. However, it must be emphasized that since a P-wave scattering amplitude is not a linear process, one cannot expect $SU(6)_W$ to lead to a simple bootstrap model of the mesons as meson-meson composites.

In conclusion, measurement of the interactions of P-wave P_8 mesons with pairs of hadrons may provide valuable clues to the manner in which $SU(6)$ symmetry applies to the hadrons, and the extent to which the symmetry is broken.

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[24] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965).