Electromagnetic Form Factor of the Neutrino

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(Received 25 June 1965; revised manuscript received 3 November 1965)

The electromagnetic form factors of the electron and muon neutrinos are evaluated using an intermediate-vector-boson theory. The vector bosons are treated by a Feynman-propagator theory in which all divergent integrals are renormalizable by conventional means. Implications of the renormalization of the weak by the electromagnetic interactions are discussed; it is pointed out that the usual procedure of treating such a theory by using regulators might not be consistent with its conservation laws.

I. INTRODUCTION

IN consequence of the speculation that the weak interactions are mediated by a charged, massive, vector boson,[1] and also that such bosons may exist,[2] numerous calculations have been made using this intermediate-vector-boson model.[3] One of the factors that inhibits the successful execution of a program of calculation using this model of weak interactions is the absence of a consistent and completely satisfactory theory of electromagnetic interactions of charged vector bosons.[4]

The Proca theory,[5] in which a subsidiary condition is used to eliminate the spin-0 component of the vector field, has fields, $\varphi_\alpha(x)$, which in the interaction picture obey the commutation rules

$$[\varphi_\alpha(x),\varphi_\beta^*(x')]=\left(\delta_{\alpha,\beta}-\frac{1}{M^2}\frac{\partial^2}{\partial x_\alpha\partial x_\beta}\right)\Delta(x-x')\,, \quad (1)$$

where * indicates Hermitian conjugation and $\varphi_\mu^* =\{+\varphi_j^\dagger, -\varphi_4^\dagger\}$. The derivative terms in this equation result in highly divergent boson propagators and a calculation based on this theory is not renormalizable by conventional means.

If, on the other hand, the subsidiary condition is eliminated, and both spin-0 and spin-1 components are accepted, then the energies of the two different spin components enter subtractively into the free field Hamiltonian and the latter is not positive semidefinite. A number of theoretical procedures, based upon the use of an indefinite metric, have been developed, in the past, to deal with this type of dilemma.[6] In an indefinite-metric theory, a Hermitian, unitary metric operator η is defined so that it commutes with spacelike and anticommutes with timelike field components. Matrix elements of operators Ω are defined as $\langle\Psi^*|\Omega|\Psi\rangle$, where $\langle\Psi^*|=\langle\Psi|\eta$, so that expectation values of $\varphi_\alpha^\dagger$ transform like four-vectors. This allows us to use a formalism which leads to a positive semidefinite Hamiltonian, and which yet provides us with expectation values of φ_α and $\varphi_\alpha^\dagger$ that transform like four-vectors.

One must then invent a rule for eliminating those state vectors that have negative norms, from the subspace of physically admissible state vectors. In electrodynamics, the condition $\partial A_\mu^{(+)}/\partial x_\mu|\Psi\rangle=0$ selects such a set of admissible states.[7] It then becomes important that the S matrix, which is necessarily unitary in the entire space, also be unitary in the subspace of physical states. This is only the case if the norm of the physical component of the state vector remains unity, i.e., if there is no "leaking" of probability out of the physical space. This is automatically true in electrodynamics, since the dynamical behavior of state vectors is such that they never move from the physical into the un-

* Supported by the National Science Foundation and by the University of Connecticut Research Foundation.
† Supported by the National Science Foundation and by the U. S. National Aeronautics and Space Administration.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960); this paper contains references to earlier work.

[2] Report on CERN neutrino experiment, presented by D. Perkins at the International Conference on Cosmic Ray Physics, 1963 (unpublished).

[3] J. Bernstein and T. D. Lee, Phys. Rev. Letters **11**, 512 (1963); Ph. Meyer and D. Schiff, Phys. Letters **8**, 217 (1964); W. K. Cheng and S. A. Bludman, Phys. Rev. **136**, B1787 (1965); F. Salzman and G. Salzman (unpublished); G. Feinberg, in *Astrophysics and the Many-Body Problem:* Brandeis Lectures, edited by K. W. Ford (Brandeis Summer Institute in Theoretical Physics, Waltham, Massachusetts, 1963), Vol. 2.

[4] C. N. Yang (private communication).

[5] A. Proca, J. Phys. Radium **7**, 347 (1963); see also G. Wentzel, *Quantum Theory of Fields* (Interscience Publishers Inc., New York, 1949), Chap. 3.

[6] R. Ascoli and E. Minardi, Nucl. Phys. **9**, 242 (1958); L. A. Maksimov, Zh. Eksperim. i Teor. Fiz. **36**, 465 (1959) [English transl: Soviet Phys.—JETP **9**, 324 (1959)]; K. L. Nagy, Nuovo Cimento Suppl. **17**, 760 (1960); L. K. Pandit, *ibid.* **11**, 157 (1959); A. Uhlmann, Nucl. Phys. **12**, 103 (1959); H. J. Schnitzer and E. C. G. Sudarshan, Phys. Rev. **123**, 2193 (1961); E. C. G. Sudarshan, Phys. Rev. **123**, 2183 (1961).

[7] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1955), Chap. 6.

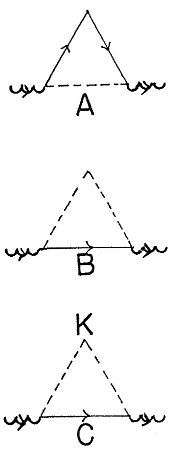

FIG. 1. Feynman graphs for the electromagnetic form factors of neutrinos. Wavy lines indicate neutrinos, solid lines massive leptons (e or μ), and dashed lines vector bosons. Diagram C represents the interaction of photons and bosons via a phenomenological magnetic moment not included in the Lagrangian that originates from the minimal-coupling rule.

physical part of the space.[8] The same effect is achieved in the theory of Lee and Yang[9] by making the unphysical states (in the limit $\xi=0$) energetically inaccessible. In other work,[10] it is presumed, without much further examination, that there is some appropriate rule that can be invoked to effect a proper separation of physical and unphysical subspaces.

It seems to us that the problem of designing a "leak-proof" physical subspace is not a simple one and one that we intend to discuss further elsewhere. In the present work, however, we also intend to take the same point of view as in other work based on this Lagrangian,[10] that at least as a Feynman-propagator theory, the full four-component theory can be used to calculate the form factor of the neutrino. None of the very substantial difficulties of the renormalizability of the Proca theory or of the limiting process in the ξ-limiting theory arise in that case. All diagrams, including the ones involving a phenomenological magnetic-moment term, give rise to renormalizable matrix elements.

II. CALCULATIONS AND RESULTS

The Lagrangian density for the vector boson interacting with the electromagnetic field, used in this work is given by

$$\mathcal{L}=-[(D_\nu^\dagger W_\mu^\dagger)(D_\nu W_\mu)+M^2W_\mu^\dagger W_\mu] +ie\kappa F_{\mu\nu}W_\nu^\dagger W_\mu, \quad (2)$$

where $D_\nu=\partial/\partial x_\nu-ieA_\nu$ and indicates the Hermitian adjoint for q-number quantities and complex conjugation for c-number quantities. κ denotes a phenomenological magnetic moment. This leads to a theory in which the usual Feynman rules apply with $\delta_{\mu\nu}[M^2+k^2-i\epsilon]^{-1}$ for the boson propagator. The interaction with the leptons will be taken to be the $V-A$ interaction term

$$\mathcal{L}_I=g_0[\bar{\psi}_l\gamma_\mu(1+\gamma_5)\psi_\nu W_\mu+\bar{\psi}_\nu\gamma_\mu(1+\gamma_5)\psi_l W_\mu^\dagger]. \quad (3)$$

It is, incidentally, of interest to note that this theory cannot possibly contribute any magnetic-moment terms aside from the ones included phenomenologically, since for the case of magnetic-moment interactions the two field components in the Lagrangian have differing subscripts, and the minimal coupling terms in this theory can never have this structure.

The neutrino form factor is given by the matrix elements for the diagrams in Fig. 1; the matrix element is given by

$$\langle\nu'|j_\mu(0)|\nu\rangle=\bar{u}_{\nu'}^{(+)}\gamma_\mu(1+\gamma_5)u_\nu^{(+)}F(q^2),$$

where $q=(\nu'-\nu)$ and $u^\dagger u=1$. The form factor $F(q^2)$ is written

$$F(q^2)=F_A(q^2)+F_B(q^2)+\kappa F_C(q^2)$$

where the subscripts denote the diagram in Fig. 1. The individual terms are

$$\bar{u}_{\nu'}^{(+)}(1-\gamma_5)\gamma_\mu(1+\gamma_5)u_\nu^{(+)}F_A=\frac{-e_0g_0^2}{(2\pi)^4}\bar{u}_{\nu'}^{(+)} \times\int\frac{d_4k\,\gamma_\lambda(1+\gamma_5)[m_l-i\gamma\cdot(\nu'-k)]\gamma_\mu[m_l-i\gamma\cdot(\nu-k)]\gamma_\lambda(1+\gamma_5)}{(M^2+k^2)[m_l^2+(\nu'-k)^2][m_l^2+(\nu-k)^2]}u_\nu^{(+)}, \quad (4a)$$

$$\bar{u}_{\nu'}^{(+)}(1-\gamma_5)\gamma_\mu(1+\gamma_5)u_\nu^{(+)}F_B=\frac{-ie_0g_0^2}{(2\pi)^4}\bar{u}_{\nu'}^{(+)}\int\frac{d_4k\,\gamma_\lambda(1+\gamma_5)[m_l-i\gamma\cdot k]\gamma_\lambda(1+\gamma_5)(\nu+\nu'-2k)_\mu}{[M^2+(\nu'-k)^2][m_l^2+k^2][M^2+(\nu-k)^2]}u_\nu^{(+)}, \quad (4b)$$

$$\bar{u}_{\nu'}^{(+)}(1-\gamma_5)\gamma_\mu(1+\gamma_5)u_\nu^{(+)}F_C=\frac{-e_0g_0^2}{(2\pi)^4}\bar{u}_{\nu'}^{(+)}\int d_4k \times\{\gamma\cdot q(1+\gamma_5)[m_l-i\gamma\cdot k]\gamma_\mu(1+\gamma_5)+\gamma_\mu(1+\gamma_5)[m_l-i\gamma\cdot k]\gamma\cdot q(1+\gamma_5)\}u_\nu^{(+)} \times\{(m_l^2+k^2)[M^2+(\nu'-k)^2][M^2+(\nu-k)^2]\}^{-1}, \quad (4c)$$

[8] See Ref. 7.
[9] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962). T. D. Lee, Phys. Rev. **128**, 899 (1962).
[10] Z. Bialynicki-Birula, Nuovo Cimento **21**, 571 (1961); G. Dorman, *ibid.* **32**, 1226 (1964).

where e_0 and g_0 are the unrenormalized, rationalized semiweak charge and coupling constant, respectively, and where M and m_l are the boson and lepton mass, respectively (the latter is the electron mass in the case of ν_e and the muon mass in the case of ν_μ; κ is a phenomenological magnetic moment).

To obtain the form factor in terms of observable or "dressed" parameters, as well as in a form that contains convergent integrals only, we therefore define $\mathfrak{F}(q^2)=F(q^2)-F(0)$, where the subtraction is carried out under the integral sign. The charge and coupling constant are then taken to be their renormalized values. In the case of the charge, only the lowest order terms in e are taken and the change from e_0 to e is the only consequence of renormalizing. The weak interaction is renormalized by the electromagnetic one and that fact is reflected in the subtraction indicated above. The renormalized semiweak coupling constant g is the one given by low-energy β-decay data, because all weak-interaction vertices are modified by radiative corrections so that g_0 is not the coupling constant observed at low momentum transfer. There of course remains then the unanswered question of why the semiweak coupling strength is universal in view of the fact that the "weak vector" current is not conserved when the electromagnetic effects are included. There seems at the present not to be any satisfactory answer to that question.

Some authors[11] apparently in an attempt to maintain a more satisfactory foundation for the universality of the weak interactions have chosen not to renormalize the form factors and have evaluated the matrix elements by a cutoff or regulator method. As we shall see, such a procedure, unless extreme caution is exercised, can at times lead to results which are sharply at variance with the conservation laws inherent in the theory. The present authors prefer the previously described procedure. The use of unrenormalized expressions is based, among other things, upon an improper identification of asymptotic wave functions for the interacting systems.[12] It has been pointed out, for example, that, in order to be identifiable with empirical data, field theories would require renormalization even if all integrations over internal lines were finite.[13]

The renormalized form factors are given by

$$\mathfrak{F}_A(q^2)=\frac{g^2e}{8\pi^2}\left\{-\tfrac{1}{2}-\frac{m_l^2}{M^2-m_l^2}+\frac{m_l^2M^2}{(M^2-m_l^2)^2}\ln\left(\frac{M^2}{m_l^2}\right)+\int_0^1 dx\frac{[(1-x/2)^2+D^2-m_l^2/q^2]}{D}\ln\left(\frac{D+x/2}{D-x/2}\right)\right\},\quad(5a)$$

$$\mathfrak{F}_B(q^2)=\frac{g^2e}{4\pi^2}\left\{-\tfrac{1}{2}+\int_0^1 dx\,\Delta\ln\left(\frac{\Delta+x/2}{\Delta-x/2}\right)\right\},\quad(5b)$$

$$\mathfrak{F}_C(q^2)=\frac{-g^2e}{16\pi^2}\int_0^1 dx\frac{x}{\Delta}\ln\left(\frac{\Delta+x/2}{\Delta-x/2}\right),\quad(5c)$$

where

$$D=\{[M^2(1-x)/q^2]+[m_l^2x/q^2]+x^2/4\}^{1/2},$$

and

$$\Delta=\{[m_l^2(1-x)/q^2]+[M^2x/q^2]+x^2/4\}^{1/2}.$$

The integrals in Eq. (4) have been evaluated on an electronic computer and are tabulated in Figs. 2 and 3. In the limit m_l^2/M^2 and $q^2/M^2\rightarrow 0$ but q^2/m_l^2 remains arbitrary, the expressions for the renormalized form factors can be evaluated in closed form and are given by

$$[\mathfrak{F}_A(q^2)]_{\text{lim}}=\frac{g^2e}{8\pi^2}\left\{\frac{q^2}{M^2}\left[\frac{11}{18}+\left\{\frac{\rho^2}{4}+\frac{1}{12}[(1+\rho^2)^{1/2}-1]^3\right\}\ln\left(\frac{(1+\rho^2)^{1/2}-1}{(1+\rho^2)^{1/2}+1}\right)-\frac{\rho^2}{12}-\tfrac{1}{3}\ln\left(\frac{q^2}{2M^2}\left[\frac{\rho^2}{2}+1+(1+\rho^2)^{1/2}\right]\right)\right]\right.$$
$$\left.+\frac{m^2}{M^2}\left[[(1+\rho^2)^{1/2}-1]\ln\left(\frac{(1+\rho^2)^{1/2}-1}{(1+\rho^2)^{1/2}+1}\right)+\ln\left(\frac{q^2}{2M^2}\left[\frac{\rho^2}{2}+1+(1+\rho^2)^{1/2}\right]\right)-1\right]\right\},\quad(5d)$$

$$[\mathfrak{F}_B(q^2)]_{\text{lim}}=\frac{g^2e}{8\pi^2}\left(\frac{q^2}{18M^2}\right),\quad(5e)$$

$$[\mathfrak{F}_C(q^2)]_{\text{lim}}=\frac{g^2e}{8\pi^2}\left(\frac{q^2}{4M^2}\right),\quad(5f)$$

[11] G. Dorman, Ref. 10.
[12] H. Gelman and K. Haller (to be published); L. Van Hove, Physica 21, 901 (1955).
[13] G. Chew, Phys. Rev. 94, 1749 (1954).

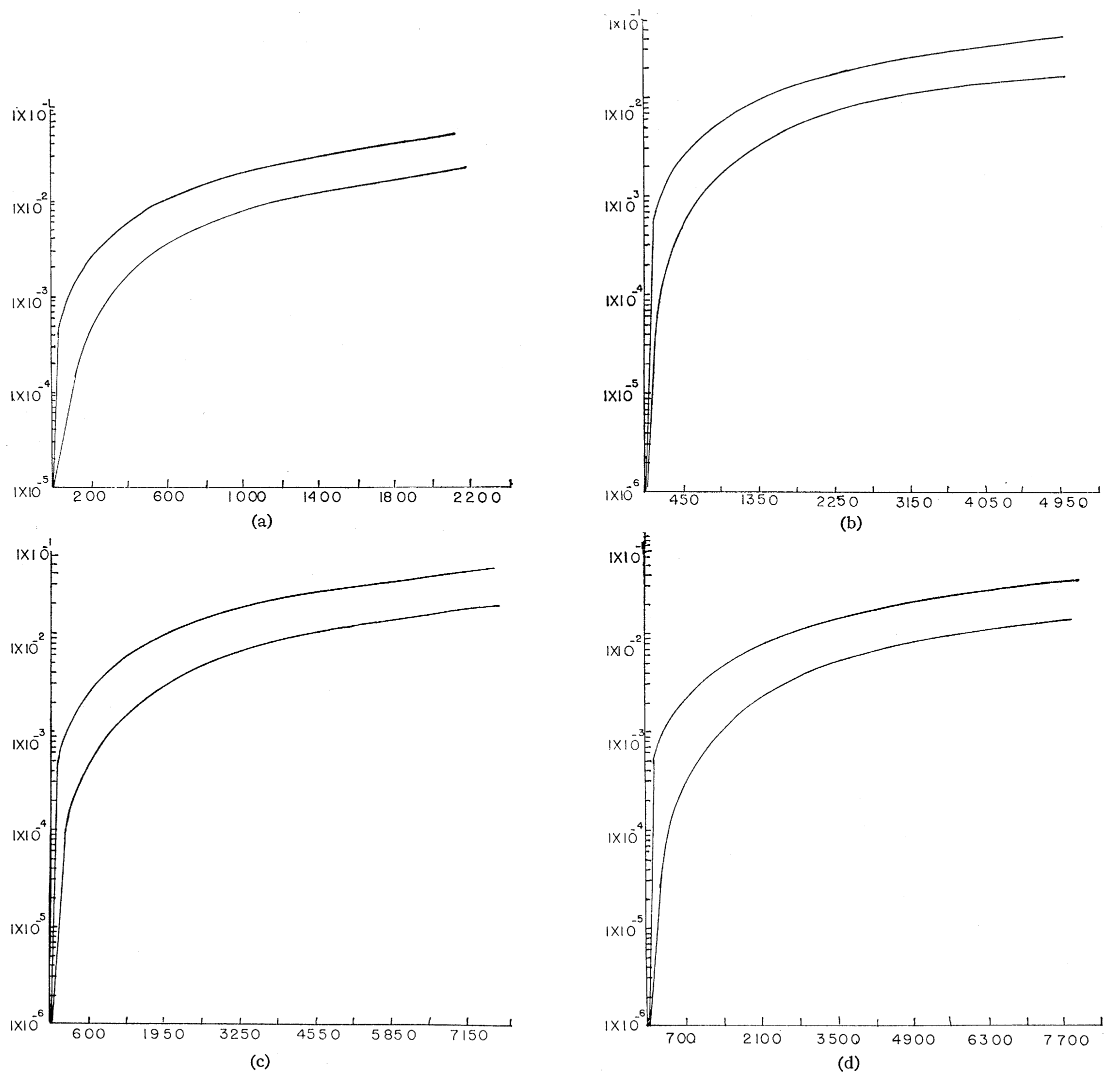

FIG. 2. Electromagnetic form factors of the electron neutrino for boson mass = (a) 500 MeV; (b) 1000 MeV; (c) 1500 MeV; (d) 2000 MeV. The form factor $\mathfrak{F}(q^2)$, where $\mathfrak{F}(q^2)=\mathfrak{F}_A(q^2)+\mathfrak{F}_B(q^2)$, is represented by the upper curve, $\mathfrak{F}_C(q^2)$ by the lower curve on graphs. The ordinate is the dimensionless quantity $\mathfrak{F}/(eg^2)$, where g is the semiweak coupling constant and e the electric charge. The abscissa is the momentum transfer in MeV/*c*.

where ρ^2 is given by $\rho^2=4m_l^2/q^2$. These expressions can further be reduced to the case $q^2\to 0$. In that limit, the quantity $\lim_{(q^2\to 0)}[-12\mathfrak{F}(q^2)/q^2]$ is defined as the mean-square charge radius. The value for this quantity obtained in the present calculation is

$$\langle r^2\rangle=(-g^2e/2\pi^2M^2)[\ln(M^2/m_l^2)+\tfrac{1}{3}+\tfrac{3}{4}\kappa]. \quad (6)$$

It is of interest to compare this expression with those obtained by alternative procedures and reported in previously published work.[14] The results of Bernstein and Lee (BL) and those of Meyer and Schiff (MS) were obtained by applications of the ξ-limiting theory.[9] Cheng and Bludman (CB) use the conserved vector current theory with an $(e\nu)(e\nu)$ local interaction. By making a Fierz transformation the latter is rewritten in the form $(ee)(\nu\nu)$. The interaction with the electromagnetic field is then the same, to lowest order, as the vacuum polarization graph of quantum electrodynamics with $(\nu\nu)$ replacing one of the photons.

The results for the charge radii given by these calculations differ from each other in the following ways:

(1) A $\ln(\alpha)$ term appears in the results of BL, MS and CB; in BL and MS this term arises from discounting all but the dominant singularities in the limit $\xi\to 0$. In CB it is obtained by setting $\Lambda^2/m_l^2=\alpha$, in an essentially *ad hoc* fashion, where Λ is a parameter introduced by the regularization procedure; (for purposes of comparison we consider the results obtained in CB by the use of regulators).

[14] J. Bernstein and T. D. Lee, Phys. Rev. Letters **11**, 512 (1963); Ph. Meyer and D. Schiff, Phys. Letters **8**, 217 (1964); W. K. Cheng and S. A. Bludman, Phys. Rev. **136**, B1787 (1965).

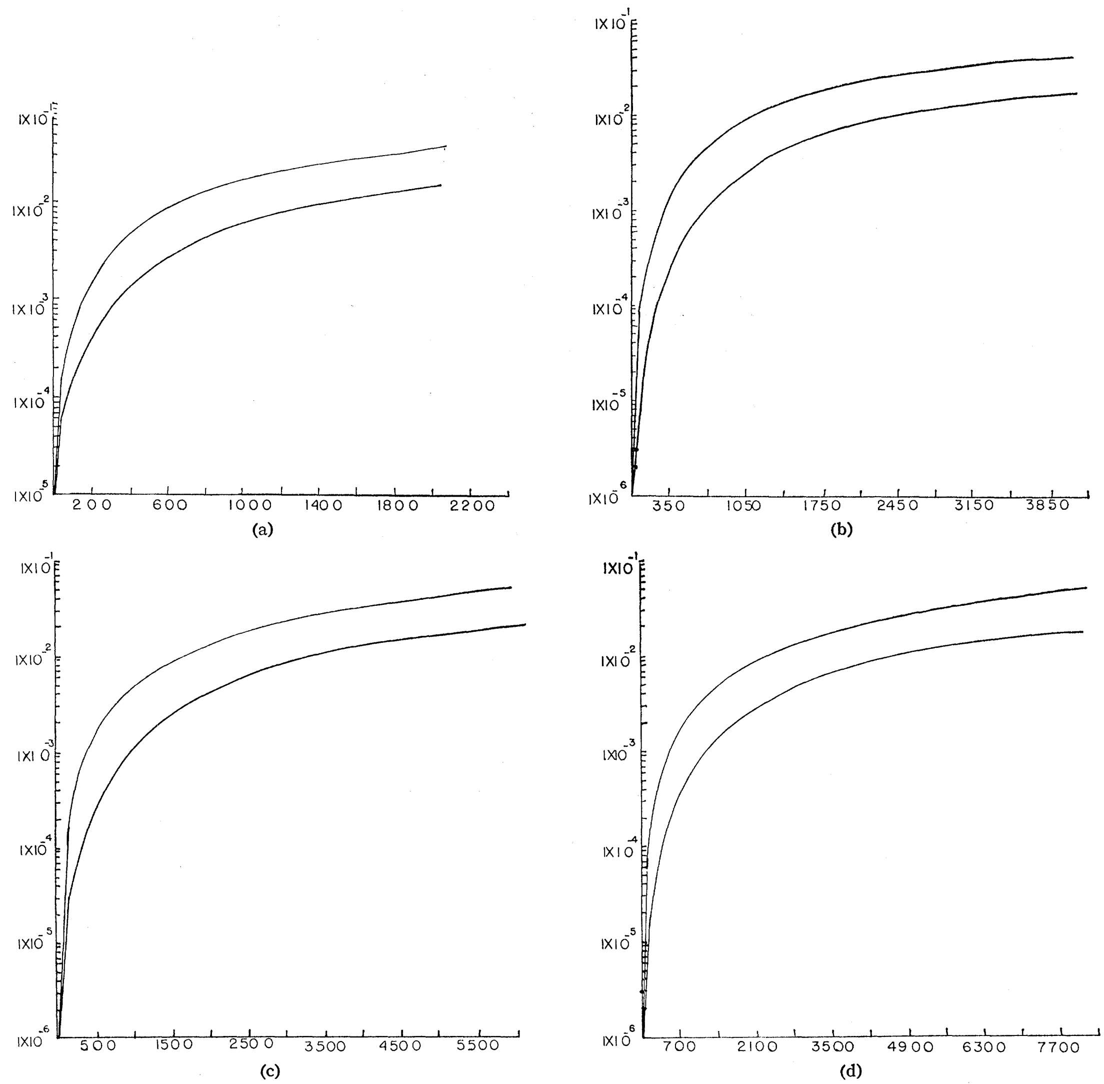

FIG. 3. Electromagnetic form factors of the muon neutrino for boson mass = (a) 500 MeV; (b) 1000 MeV; (c) 1500 MeV; (d) 2000 MeV. The upper and lower curves and the axes are as described in the caption to Fig. 2.

In our own work, which is completely renormalizable in the customary sense, there is no $\ln(\alpha)$ term.

(2) A term proportional to $\ln(M^2/m_l^2)$ appears in BL, MS, CB as well as in our work, and is identical in all of these four results.

(3) A term independent of the masses (except for the trivial appearance of $g^2/2M$ in the role of the weak-coupling constant) arises in all of these calculations and is different in each.

III. DISCUSSION

In the renormalized expressions given above, the quantities $F_A(0)$ and $F_B(0)$ have been subtracted in the renormalization process; $F_C(0)$ disappears primitively. It might be argued that it is irrelevant whether the renormalized or unrenormalized expressions for $F(q^2)$ are used, since $F_A(0)+F_B(0)=0$ in order that the total charge of the neutrino vanishes. In addition to the fact that this point bears on the previously discussed universality of the weak coupling constant, however, there is a computational consequence of the use of unrenormalized values of F. If one computes by the usual techniques of Feynman integration, in which one uses

$$\frac{1}{a_1a_2\cdots a_n}=(n-1)!\int_0^1 dx_1\int_0^{x_1} dx_2\cdots\int_0^{x_{n-2}} dx_{n-1} \times\frac{1}{[a_1x_{n-1}+a_2(x_{n-2}-x_{n-1})+\cdots a_n(1-x_1)]^n}$$

and interchanges the orders of integration over the x_i and over the internal lines in the diagrams, then one obtains

$$F_A(0)+F_B(0)=(2\pi)^{-4}\tfrac{1}{2}g_0^2.$$

The discrepancy between this result and the proper one, namely,

$$F_A(0)+F_B(0)=0$$

is due to an error in the zero-momentum-transfer parts that arises from the usual Feynman integration techniques, as can readily be seen by examining the neutron form factor for a pseudoscalar nucleon-pion interaction.[15] To lowest order in e and in the nucleon-pion coupling constant, f_0, $F_N(0)$ defined by $F_N(0)=[F_N(0)]_A+[F_N(0)]_B$, when evaluated as the $q^2\to 0$ limit of the integral evaluated by Feynman techniques, is given by $F_N(0)=(2\pi)^{-4}\frac{1}{2}f_0^2$. However, when evaluated directly as the matrix element taken between two nucleons at rest it is correctly given by $F_N(0)=0$.

[15] B. D. Fried, Phys. Rev. 88, 1142 (1952).

Therefore, unrenormalized expressions for the neutrino form factor, evaluated by Feynman techniques, incorrectly impute an electric charge to the neutrino.

ACKNOWLEDGMENTS

The computational part of this work was carried out in the Computer Center of the University of Connecticut which is supported in part by Grant No. GP-1819 of the National Science Foundation. We would like to thank Professor Spindel for helping with the computations.

PHYSICAL REVIEW VOLUME 144, NUMBER 4 29 APRIL 1966

Dynamics of the Y_1^* Branching Ratio*

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(Received 29 September 1965; revised manuscript received 20 December 1965)

The $\pi\Sigma/\pi\Lambda$ branching ratio of the $Y_1^*(1385)$ is significantly smaller than the phase space estimate. The fact that in the $\frac{3}{2}^+$, $I=1$ $\pi\Sigma$ state the force is repulsive, while in the corresponding $\pi\Lambda$ state it is attractive, is shown to be important in bringing this about. A general analysis of n-channel diagonalizable systems, and a number of specific diagonalizable and nondiagonalizable models, are presented to establish that this importance is not model-dependent.

I. INTRODUCTION

A NOTABLE feature of the Y_1^*(1385 MeV) resonance, which decays into $\pi+\Lambda$ and $\pi+\Sigma$, is its small branching ratio. Experimental measurements of $\Gamma(Y_1^*\to\pi\Sigma)/\Gamma(Y_1^*\to\pi\Lambda)$ have yielded values of 0.04 ± 0.04,[1] 0.09 ± 0.04,[2] and 0.16 ± 0.04.[3] While these results are not in complete agreement, they all indicate that the branching ratio is smaller than the phase space ratio of 0.25. [We use the phase space factor $q^3/(q^2+X^2)$ of S. Glashow and A. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963). q is the c.m. momentum of the decay products and X was determined in their paper to be 350 MeV.]

This deviation from phase space can be interpreted in terms of unbroken $SU(3)$ symmetry, which predicts a branching ratio of $\frac{2}{3}\times$(phase space)$=0.17$. However, it is desirable to be able to understand it *dynamically* as well.[4] In this paper we note that while the force in the $3/2^+$, $I=1$ state of the $\pi\Lambda$ channel is attractive, that in the corresponding state of the $\pi\Sigma$ channel is repulsive. We try to show that this circumstance is one of the important reasons why the Y_1^* branching ratio is less than the phase space estimate.

* This work was supported in part by the U. S. Air Force Office of Scientific Research, Grant No. AF-AFOSR-232-65, and in part by the U. S. Atomic Energy Commission.

[1] P. Bastien, M. Ferro-Luzzi, and A. Rosenfeld, Phys. Rev. Letters **6**, 702 (1961).

[2] D. Huwe, Lawrence Radiation Laboratory Report UCRL-11291 (unpublished).

[3] R. Armenteros *et al.*, Phys. Letters **19**, 75 (1965).

II. FORCES IN THE $\pi\Lambda$ AND $\pi\Sigma$ CHANNELS

The forces in the $J^P=3/2^+$, $I=1$ states of the $\pi\Lambda$ and $\pi\Sigma$ channels are estimated on the basis of the Born amplitudes for exchange of all possible stable particles and low-energy (below 1500 MeV) resonances, namely

[4] Dynamical predictions for the Y_1^* branching ratio have been given by A. Martin and K. Wali, Phys. Rev. **130**, 2455 (1963); P. Tarjanne and R. Cutkosky, *ibid.* **133**, B1292 (1964); K. Wali and R. Warnock, *ibid.* **135**, B1358 (1964); E. Johnson and E. McCliment, *ibid.* **139**, B951 (1965); M. Swiecki, Phys. Letters **19**, 333 (1965); and R. Dashen, Y. Dothan, S. Frautschi, and D. Sharp, Phys. Rev. **143**, 1185 (1966). All these authors except M. Swiecki start with an $SU(3)$ symmetric situation and then introduce symmetry breaking. They all find that the breaking causes the branching ratio to be reduced substantially below the unbroken $SU(3)$ prediction of 0.17.