Can Nearby Interaction Singularities Generate Observed Resonances?*

L. F. Cook† and C. E. Jones
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 7 July 1965; revised manuscript received 6 January 1966)

The question of whether simple nearby singularities together with unitarity are generally capable of reproducing the masses and widths of observed low-energy resonances is studied. As a specific case, we study the ρ meson. We replace the left-hand cut by one or two nearby poles and ask if the interaction-pole parameters can be adjusted to give the known mass and width of the ρ. If elastic unitarity is assumed, the answer is no. Calculations with inelastic unitarity are also made, and the observed ρ can be reproduced if the inelastic amplitude provides an important contribution. These conclusions are reached without a detailed identification between known exchanged systems and the interaction pole parameters.

I. INTRODUCTION

In the past few years, bootstrap calculations have played a significant role in attempts to gain a dynamical understanding of elementary particles.[1] Such calculations usually proceed via partial-wave dispersion relations and the N/D technique. If one is making a low-energy calculation, one generally adopts the view that only nearby singularities need be kept in a first approximation. Thus the left-hand cut is often approximated by contributions coming from the exchange of a few low-mass particles in the crossed channel. Such contributions comprise the long-range part of the force. The calculation can then be made either by assuming elastic unitarity or by including in addition one or more inelastic channels.[2]

Let us suppose that one wishes to use such a calculation to determine the mass and width of a resonance. Even admitting the nearby-singularity hypothesis with regard to the left-hand-cut contributions, one is still faced with the decision of whether to incorporate inelastic channels into the problem. This last question forces one to assume a basic physical mechanism for the resonance, the correctness of which is to some extent tested by a comparison of the calculated mass and width values with the experimental numbers.

What we present here is not a bootstrap calculation but a discussion of whether simple nearby singularities can reproduce the masses and widths of observed resonances.

The method we employ is the following: (1) We replace the interaction cuts by one or two nearby poles in accord with the nearby singularity hypothesis. No explicit correspondence is required between these interaction poles and the known exchange forces except for sign (i.e., attractive or repulsive) since we are merely testing a physical mechanism, not doing an actual bootstrap calculation. (2) The N/D equations are easily solved exactly in this model and one may either assume elastic unitarity or keep also a representative inelastic channel. (3) We then inquire whether for some values of the parameters which characterize the nearby interaction poles, the experimentally observed mass and width of the resonance be reproduced.

We develop this method by considering explicitly the case of the ρ-meson resonance, first assuming elastic unitarity. Here we find that no assignment of the parameters of the nearby interaction pole terms is capable of reproducing the observed mass and width of the ρ. By "nearby," we mean that the displacement from threshold of such singularities be within several orders of magnitude of the displacement of the resonance energy from threshold.

We then introduce a second, "inelastic" channel into the π-π problem with a variable threshold. Calling the hypothetical second channel "$m\bar{m}$," we test a variety of physical mechanisms which might produce the ρ from nearby singularities: (1) nonzero forces (left-hand cuts) only in $\pi\pi \to m\bar{m}$; (2) nonzero forces only in $\pi\pi \to \pi\pi$ and $\pi\pi \to m\bar{m}$; (3) nonzero forces only in $\pi\pi \to m\bar{m}$ and $m\bar{m} \to m\bar{m}$.

That mechanism (3) will easily succeed in giving the ρ with a sufficiently small width is well-known because it is possible to have an $m\bar{m}$ bound state that couples weakly to $\pi\pi$ giving an arbitrarily small width. We find, however, that mechanism (3) appears to be the only simple way of reproducing the ρ within our framework.

These results suggest that the ρ is strongly influenced by inelastic states. Thus, instead of the usual picture where one imagines the exchange of a ρ largely being responsible for the ρ and, hence, bootstrapping itself, one is led to contemplate a model for the ρ in which forces resulting from inelastic channels may be at least as important as the ρ exchange force. Further, there is no obvious reason why these results do not apply to all of the vector mesons.

The importance of the inelastic channels in determining the properties of the ρ was discussed several years ago by Blankenbecler[3] and more recently by

* Supported in part by the U. S. Air Force Office of Scientific Research and Development Command, and in part by the National Science Foundation Grant, GP-5172.

† Present address: Department of Physics, University of Massachusetts, Amherst, Massachusetts.

[1] See, for example, J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965) and references contained therein.

[2] This whole viewpoint was put forth in the original work of G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960); also see Ref. 5.

[3] R. Blankenbecler, Phys. Rev. **125**, 755 (1962).

Fulco, Shaw, and Wong[1] and by Chew.[4] The present discussion differs from the others in that no correspondence is required between the interaction poles used and the actual left-hand-cut singularities. Thus our results may be more general than the others. Further, the method presented may be of use in estimating the possible importance of inelastic states in producing resonances other than the ρ.

II. π-π PROBLEM WITH ELASTIC UNITARITY

We begin our analysis of the ρ meson with the elastic scattering of pions, and consider the scattering amplitude with the threshold factors explicitly displayed:

$$T_l(\nu)=\nu^l N_l(\nu)/D_l(\nu)=\nu^l M_l(\nu)\,, \tag{1}$$

with $\nu=\frac{1}{4}s-1$, where s is the total center-of-mass energy squared, measured in (pion mass)$^2=\mu^2=1$. The scattering amplitude T_l satisfies unitarity when we define the D function as usual, with a subtraction

$$D_l(\nu;\nu_0)=1-\frac{\nu+\nu_0}{\pi}\int_0^\infty d\nu'\left(\frac{\nu'}{\nu'+1}\right)^{1/2}\times\frac{(\nu')^l N_l(\nu';\nu_0)}{(\nu'-\nu)(\nu'+\nu_0)}, \tag{2}$$

and $N_l(\nu)$ has singularities only for $\nu<0$. The function $N_l(\nu)$ is obtained from

$$N_l(\nu;\nu_0)=\frac{1}{\pi}\int_L d\nu'\frac{\operatorname{Im}N_l(\nu';\nu_0)}{\nu'-\nu}=\frac{1}{\pi}\int_L d\nu'\frac{\alpha_l(\nu')D_l(\nu';\nu_0)}{\nu'-\nu}, \tag{3}$$

where $\alpha_l(\nu)=\operatorname{Im}M_l(\nu)$, $\nu<0$, which leads to an integral equation which may or may not be well-defined depending on the asymptotic properties of $\alpha_l(\nu)$.

Most bootstrap calculations assume that the behavior of the amplitude in the low-energy physical region is determined approximately by unitarity and only the nearby left-hand singularities. This assumption is implemented by means of Eqs. (1), (2), and (3) and where $\alpha_l(\nu)$ is given by a singularity arising from the exchange of some particle(s) in the crossed channels. In the case of the π-π problem one often includes only the ρ meson exchange. In a self-consistent problem one does not specify the mass or coupling constant of the exchanged particle, but determines them by demanding that, for the ρ, $\operatorname{Re}D_1(\nu_\rho)=0$ and that the input and output coupling $(g_{\pi\pi}{}^\rho)^2$ agree. If, in fact, one finds a self-consistent, bootstrap solution, a variety of questions arise concerning the approximations made, uniqueness of the solutions, etc. Before asking such detailed questions, we propose as a first step examining the validity of the mechanism of elastic unitarity in producing the ρ. To do this we consider a scattering amplitude which satisfies unitarity on the right and has a simple pole on the left as its only left-hand singularity. This pole is taken as approximating the effects of the *nearby* singularities, and thus must be reasonably close to the physical region. The precise meaning of this will be made clear later. We do not look for a self-consistent solution here. We demand that the amplitude exhibit a resonance at the experimental position and further demand that the resonance have the correct width. The essential question then concerns the possibility of choosing the interaction pole parameters in such a way as to accomplish these demands. In fact, the only parameter which is responsible for the satisfaction of these demands is the position of the imput left-hand pole—its residue plays no important role. This happens to be a special property of a single pole model, but, as we shall show, the inclusion of more poles does not change in an essential way the results of the one-pole model.

We now take $l=1$ and choose

$$\alpha_1(\nu')=-\lambda\pi\delta(\nu'+\nu_p)\,. \tag{4}$$

From Eq. (3) this gives

$$N_1(\nu;\nu_0)=[\lambda D_1(-\nu_p;\nu_0)]/(\nu+\nu_p)\,, \tag{5}$$

and Eq. (2) yields

$$D_1(\nu;\nu_0)=1-\frac{\lambda D_1(-\nu_p;\nu_0)(\nu+\nu_0)}{\pi}\int_0^\infty d\nu'\left(\frac{\nu'}{\nu'+1}\right)^{1/2}\times\frac{\nu'}{(\nu'-\nu)(\nu'+\nu_p)(\nu'+\nu_0)}; \tag{6}$$

Eq. (6) gives $D_1(-\nu_p)$ as

$$D_1(-\nu_p;\nu_0)=\left[1-\frac{\lambda(\nu_p-\nu_0)}{\pi}\int_0^\infty d\nu'\left(\frac{\nu'}{\nu'+1}\right)^{1/2}\times\frac{\nu'}{(\nu'+\nu_p)^2(\nu'+\nu_0)}\right]^{-1}. \tag{7}$$

It would appear at this point that we have three parameters: λ, ν_p, and ν_0; however, T_l is independent of the subtraction point. This is easy to see. Let us consider a new subtraction point, $\nu=-\nu_0'$; $\nu_0'>0$, and obtain the new functions $N_l(\nu;\nu_0')$. One easily sees that

$$D_l(\nu;\nu_0')=[D_l(\nu;\nu_0)]/[D_l(-\nu_0';\nu_0)]. \tag{8}$$

Further from Eqs. (3) and (8) one has

$$N_l(\nu;\nu_0')=\frac{1}{\pi}\int_L d\nu'\frac{\alpha_l(\nu')D_l(\nu';\nu_0')}{\nu'-\nu}=\frac{1}{\pi}\int_L\frac{d\nu'}{\nu'-\nu}\times\alpha_l(\nu')\frac{D_l(\nu';\nu_0)}{D_l(-\nu_0;\nu_0)}=\frac{N_l(\nu;\nu_0)}{D_l(-\nu_0';\nu_0)}. \tag{9}$$

---
[4] G. F. Chew, Phys. Rev. **140**, B1427 (1965).

Thus

$$[N_l(\nu;\nu_0)]/[D_l(\nu;\nu_0)] = [N_l(\nu;\nu_0')]/[D_l(\nu;\nu_0')]. \quad (10)$$

It is convenient to choose the subtraction point at the position of the pole, i.e., $\nu_0=\nu_p$, and we shall do so. Dropping all reference to the subtraction point, we have

$$N_1(\nu)=\lambda/(\nu+\nu_p), \quad (11)$$

$$D_1(\nu)=1-\frac{\lambda(\nu+\nu_p)}{\pi}\int_0^\infty d\nu'\left(\frac{\nu'}{\nu'+1}\right)^{1/2} \times\frac{\nu'}{(\nu'-\nu)(\nu'+\nu_p)^2}, \quad (12)$$

since $D_1(-\nu_p)=1$. We will write the expression for $D_1(\nu)$ as

$$D_1(\nu)=1-\lambda J(\nu;\nu_p), \quad (13)$$

where $J(\nu;\nu_p)$ is given analytically by

$$\pi J(\nu;\nu_p)=\frac{\nu}{\nu+\nu_p}\left\{\left(\frac{\nu}{\nu+1}\right)^{1/2}\left[\ln\frac{[(1+\nu)/\nu]^{1/2}-1}{[(1+\nu)/\nu]^{1/2}+1}+i\pi\right] -\frac{1}{(1-1/\nu_p)^{1/2}}\ln\frac{1-(1-1/\nu_p)^{1/2}}{1+(1-1/\nu_p)^{1/2}}\right\}+\frac{\nu_p}{\nu_p-1} \times\left\{\frac{1}{2(\nu_p(\nu_p-1))^{1/2}}\ln\frac{1-(1-1/\nu_p)^{1/2}}{1+(1-1/\nu_p)^{1/2}}+1\right\}, \quad (14)$$

for $\nu>0$.

Our analysis proceeds as follows: We choose a value for ν_p and determine λ by the demand

$$1/\lambda=\mathrm{Re}J(\nu_\rho;\nu_p), \quad (15)$$

where ν_ρ is the experimental mass of the ρ meson. We can always find a λ satisfying Eq. (15). The width is given as

$$\Gamma(\nu_p)=\frac{\nu_\rho/(\nu_\rho+\nu_p)}{\mathrm{Re}J'(\nu_\rho;\nu_p)}. \quad (16)$$

We emphasize that with only one left-hand pole, $\Gamma(\nu_p)$ is independent of λ and depends only on ν_p and ν_ρ. Since ν_ρ is fixed by experiment, as in Γ, Eq. (16) determines ν_p. The experimental value of Γ is approximately $1\mu^2$, and we demand that ν_p yield this value for the width. A plot of Eq. (16) is given in Fig. 1 which shows that even for values of $\nu_p=41(s=-160)$, the width is still 30 times too large. (In fact to obtain the experimental value of $1\mu^2$ one must choose $\nu_p\simeq10^7$–$10^8\mu^2$.) If this calculation is accepted seriously, then one immediately wonders how the initial bootstrap calculations gave anything approaching reasonable values [$\nu_\rho\simeq0.6$ (350 MeV); $\Gamma\simeq2$–$3\mu^2$].[5] We have repeated the calculation given in

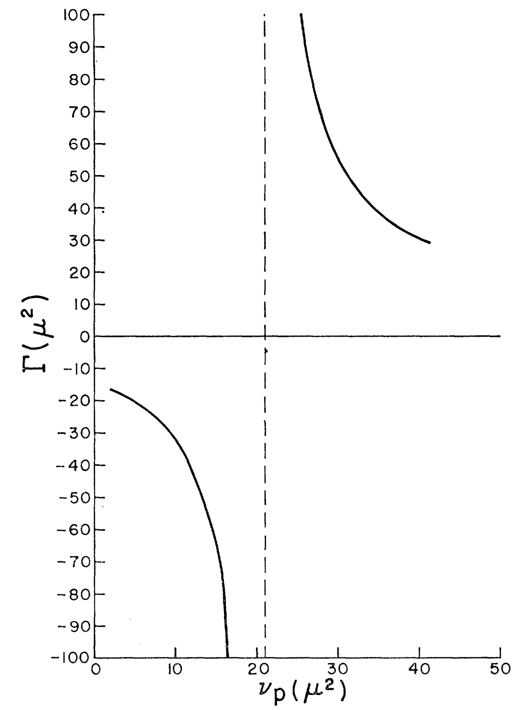

FIG. 1. The ρ width Γ plotted against the pole position ν_p. The experimental value is $\Gamma\simeq1\mu^2$.

Fig. 1 using the mass value $\nu_\rho=0.6$. In this case we find that we may obtain the bootstrap value for the width for a rather reasonable value of ν_p, viz. $\nu_p\simeq3$. (For a ρ with a mass of 350 MeV, the branch point for its exchange would occur at $\nu=-1.6$.) It is important to emphasize here that we do not conclude from these results that a bootstrap solution *must* exist at $m_\rho\simeq350$, but that it *may* exist consistent with the assumption of nearby singularities.

One may object to the above analysis on the basis that the results may depend in an essential way on the use of only one interaction pole. To investigate this objection we have studied explicitly models with two interaction poles. If both poles are attractive (positive residues), the situation is quantitatively the same: It is impossible to make the width of the ρ sufficiently small if the magnitude of the distance of the two poles with respect to threshold is within several orders of magnitude of the resonance energy.

The situation changes in an important way if the interaction pole nearest the physical region has a negative residue corresponding to a long-range repulsion. In this case it appears that one can succeed in making the width of the ρ small. This result is easily understood intuitively because the long-range repulsion aids the centrifugal barrier in "trapping" the particle, producing a long lifetime or narrow width for the resonance.

The long-range part of the force in any problem is given in terms of the exchange of particles of low mass. This is just the part of the force which is generally

[5] F. Zachariasen, Phys. Rev. Letters 7, 112, 268(E) (1961).

considered as the best understood. In the case of the ρ meson, no known particles that can be exchanged will produce a long-range repulsion.

On the basis of the above analysis it would appear quite doubtful that a bootstrap calculation performed in the usual manner neglecting inelastic states will produce the experimental position and width of the ρ meson. There is, in fact, little reason to believe that these conclusions will change for any of the vector mesons, since they are all at reasonably large energies (compared with the threshold energy). On the other hand, we have seen that the results change markedly for lower energy resonances (for the ρ, the results are much more reasonable at 350 MeV). Thus one might conjecture that the reciprocal bootstrap[6] (involving the N and N^*) in fact contains the essential force structure for that problem. The success of the static model here also indicates that this is a reasonable conjecture.

If we accept the above results, what forces are then responsible for the ρ (and the vector mesons in general)? We shall continue to assume the nearby-singularity hypothesis for the left-hand-cut contributions. However, in the next section, we shall consider the effect of including two-body inelastic channels in the problem.

III. INELASTIC CHANNELS

Having established that one cannot produce the ρ meson with elastic unitarity and nearby interaction poles, we now investigate various mechanisms arising from the incorporation of inelastic channels into the problem. Let us begin with a definition of terms. We suppose that there is a single "elastic" channel which is the only (or at least primary) decay mode of some resonance, (e.g., $\pi\pi$ and the ρ) and which is described by an "elastic" amplitude (e.g., $\pi\pi\to\pi\pi$). In addition, there can be a number of "inelastic" channels, (e.g., $K\bar{K}$, $\pi\omega$, etc.) which are described by "inelastic" amplitudes, (e.g., $K\bar{K}\to K\bar{K}$, etc.). These inelastic amplitudes are coupled by unitarity to the elastic amplitude by "production" amplitudes (e.g., $\pi\pi\to K\bar{K}$, etc.). Although this is a somewhat artificial language, it is convenient.

We shall find in the problem of the ρ that two mechanisms involving additional channels are of interest, viz., a strong contribution from one or more production amplitudes with the inelastic amplitudes playing a nonessential role, or a strong contribution from one or more inelastic amplitudes in which the production amplitudes only serve to connect the elastic amplitude to the inelastic amplitudes through unitarity. To keep the analysis simple, we shall consider only two channels with equal masses in each channel, μ and m. The two-channel problem is easily handled in terms of matrices and we write for the S matrix

$$\mathbf{S}=\mathbf{I}+2i\varrho^{1/2}\mathbf{T}\varrho^{1/2}=\mathbf{I}+2i\varrho_l^{1/2}\mathbf{M}\varrho_l^{1/2}, \tag{17}$$

[6] G. F. Chew, Phys. Rev. Letters 9, 233 (1962).

where

$$\varrho_l=\frac{2}{s^{1/2}}\begin{pmatrix}\nu_1^{l+1/2} & 0\\ 0 & \nu_2^{l+1/2}\end{pmatrix}, \tag{18}$$

and $\nu_1=\frac{1}{4}s-\mu^2$ and $\nu_2=\frac{1}{4}s-m^2$. Again we approximate the left-hand singularities in all the amplitudes by simple poles. Further, in terms of ν_1 we place all poles at the same position ν_p and write for $\mathbf{M}$,

$$\mathbf{M}(\nu_1)=\frac{1}{\nu_1+\nu_p}\boldsymbol{\lambda}+\frac{1}{\pi}\int_0^\infty\frac{d\nu'}{\nu'-\nu_1}\,\mathrm{Im}\mathbf{M}(\nu'), \tag{19}$$

where $\boldsymbol{\lambda}$ is a 2×2 matrix. The unitarity constraint is given as

$$\mathrm{Im}\mathbf{M}^{-1}=-\varrho_l. \tag{20}$$

This is satisfied if we define $\mathbf{MD}=\mathbf{N}$ with

$$\mathbf{N}(\nu_1)=(\nu_1+\nu_p)^{-1}\boldsymbol{\lambda},\quad \mathbf{D}(\nu_1)=\mathbf{I}-\mathfrak{J}\boldsymbol{\lambda}. \tag{21}$$

In Eq. (21) we have

$$\mathfrak{J}=\begin{pmatrix}\mathfrak{J}_1 & 0\\ 0 & \mathfrak{J}_2\end{pmatrix}, \tag{22}$$

where

$$\mathfrak{J}_1(\nu_1)=\frac{\nu_1+\nu_p}{\pi}\int_0^\infty\frac{\nu_1'd\nu_1'}{(\nu_1'-\nu_1)(\nu_1'+\nu_p)^2}\left(\frac{\nu_1'}{\nu_1'+\mu^2}\right)^{1/2},$$

$$\mathfrak{J}_2(\nu_1)=\frac{\nu_1+\nu_p}{\pi}\times\int_0^\infty\frac{\nu_2'd\nu_2'}{[\nu_2'-(\nu_1+\mu^2-m^2)][\nu_2'+(\nu_p+\mu^2-m^2)]^2}\times\left(\frac{\nu_2'}{\nu_2'+m^2}\right)^{1/2}. \tag{23}$$

This may be reduced to

$$\mathbf{M}(\nu_1)=(\nu_1+\nu_p)^{-1}[\boldsymbol{\lambda}^{-1}-\mathfrak{J}]^{-1}. \tag{24}$$

Now then let us consider as the first case one where only the production amplitude is important and thus we choose

$$\boldsymbol{\lambda}=\begin{pmatrix}0 & \lambda_1\\ \lambda_1 & 0\end{pmatrix}. \tag{25}$$

In this case, the elastic amplitude [always the (11) amplitude] for p waves is given as

$$T_{11}=\frac{\nu_1}{\nu_1+\nu_p}\,\frac{\lambda_1^2\mathfrak{J}_2(\nu_1)}{1-\lambda_1^2\mathfrak{J}_1(\nu_1)\mathfrak{J}_2(\nu_1)}. \tag{26}$$

It is clear from Eq. (26) that, as in the elastic situation, the width of the resonance does not depend on λ_1. On the other hand, the width will, in principle, depend on the physical threshold of the inelastic channel. In fact, Γ is rather insensitive to the position of the inelastic threshold as is shown in Fig. 2, where the width of the ρ is plotted against ν_p for two inelastic thresholds, $s_{\text{inel}}=32\mu^2$ and $s_{\text{inel}}=56\mu^2$. The results are quite similar to those given in Fig. 1. Although the situation has improved somewhat, it is clear that a nearby singularity in the production amplitude alone will not enable one to reproduce the experimental characteristics of the ρ. The two additional curves in Fig. 2 show the results of a model including a pole in both the elastic and production amplitudes. In this case one chooses for $\boldsymbol{\lambda}$

$$\boldsymbol{\lambda}=\begin{pmatrix}\lambda_0 & \lambda_1\\ \lambda_1 & 0\end{pmatrix}, \tag{27}$$

and obtains

$$T_{11}=\frac{\nu_1}{\nu_1+\nu_p}\frac{\lambda_0/\lambda_1^2+\mathfrak{I}_2(\nu_1)}{1/\lambda_1^2-\mathfrak{I}_1(\nu_1)\mathfrak{I}_2(\nu_1)-(\lambda_0/\lambda_1^2)\mathfrak{I}_1(\nu_1)}. \tag{28}$$

In this case the width of the resonance depends on ν_p and the ratio of residues $\lambda_0/\lambda_1^2=G_0$, $G_0=0$ being the case of a singularity in the production amplitude alone. As Fig. 2 shows, adding an elastic contribution worsens the situation. Again if we include a long-range repulsive force in the elastic channel, $\lambda_0<0$, then we may improve the situation. But as we remarked earlier this is just that part of the force which we believe we understand best, and there is no evidence of such a repulsive force. We conclude therefore that a model which strongly emphasizes the production amplitude is not likely to be the basic mechanism responsible for the ρ.

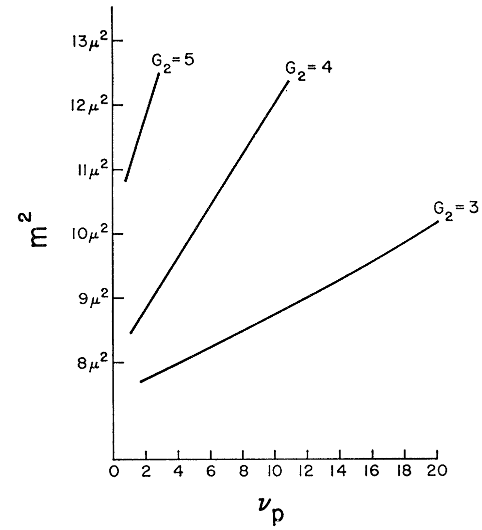

FIG. 3. A plot of the values for the mass in the inelastic channel m^2 against the pole position ν_p such that the ρ width is $1\mu^2$. G_2 determines the coupling between the inelastic and production channels (see text).

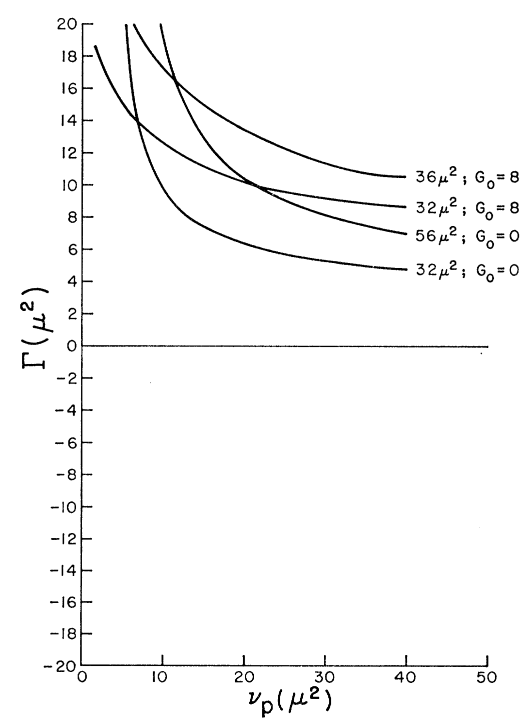

FIG. 2. The ρ width Γ plotted against the pole position ν_p in the presence of a single inelastic threshold at $s_{\text{inel}}=32\mu^2$, $36\mu^2$, or $56\mu^2$. G_0 determines the coupling between the elastic and production channels (see text).

Let us turn now to our second possibility in which we consider a strong contribution arising from the inelastic channel. We choose

$$\boldsymbol{\lambda}=\begin{pmatrix}0 & \lambda_1\\ \lambda_1 & \lambda_2\end{pmatrix}, \tag{29}$$

which yields

$$T_{11}=\frac{\nu_1}{\nu_1+\nu_p}\frac{(\lambda_1^2/\lambda_2)\mathfrak{I}_2(\nu_1)}{1/\lambda_2-(\lambda_1^2/\lambda_2)\mathfrak{I}_1(\nu_1)\mathfrak{I}_2(\nu_1)-\mathfrak{I}_2(\nu_1)}. \tag{30}$$

Of course, if $\lambda_1\to 0$, then $T_{11}\to 0$. Here the width of the resonance depends on the ratio $\lambda_2/\lambda_1^2=G_2$, on m^2, and on ν_p. Figure 3 plots m^2 against ν_p for various values of G_2 such that $\Gamma=1\mu^2$. It is no surprize that a range of values for all of the parameters allows us to produce the correct width; however, the figure does have an interesting feature. The inelastic channels of interest for the ρ meson are supposedly the π-ω and K-$\bar{K}$ channels, and although the π-ω channel cannot be characterized by a single mass, m^2, we should expect the effective mass for these two inelastic channels to be approximately between $10\mu^2$ and $12\mu^2$. Taking a reasonable value for ν_p, say $\nu_p\simeq 5$, we see from Fig. 3 that $G_2\gtrsim 4$ will produce the correct width for the ρ. However, we also demand that the denominator in Eq. (30) vanish at the experimental position of the ρ; using $G_2=4$ we may then calculate that the ratio $\lambda_2/\lambda_1=5$. Since this is an approximate measure of the relative strength of the inelastic "force" to the production "force" necessary to yield the properties of the ρ, we may argue that the inelastic processes are of great importance in an understanding of the ρ. This of course supposes that we are confined to nearby singularities. If we increase ν_p, then, as Fig. 3 shows, G_2 decreases as does λ_2/λ_1 so that the inelastic states become less important.

IV. DISCUSSION

We have proposed here a method by which one may determine whether a given observed resonance can possibly be reproduced by an N/D calculation that assumes the interaction cuts can be approximated by simple nearby singularities. Our method is only applicable if the actual nearby interaction singularities are simple enough to be approximated by a few poles.

In considering the ρ meson, we have concluded that elastic unitarity is not an acceptable mechanism and that inelastic channels are at least as important as the elastic channel. This conclusion was reached without a detailed consideration of the left-hand cut in terms of known exchanged systems.

The limited validity of the elastic mechanism for the vector mesons has consequences in other situations and a few remarks are in order.

Recently, there have been several attempts[7] to obtain symmetries as a consequence of a bootstrap calculation. A particular and relevant example is the symmetry between the pseudoscalar and vector mesons obtained by means of pseudoscalar-pseudoscalar scattering. Since it is just this case which we have discussed, one can argue that the validity of such a calculation is questionable. However, it is not clear that our analysis applies directly to such a program. First, a rather artificial distinction has been made here with respect to the various channels. In the symmetry bootstrap these are treated in a completely equivalent manner. In addition such calculations are always made for equal mass configurations; in this case the vector mesons are low-energy resonances. As we saw in Sec. II, reasonably narrow widths can be produced with a nearby singularity if the resonance energy is not large with respect to the elastic threshold. Thus, although the correctness of including only nearby singularities in such "symmetry bootstraps" is not assessed here, it may be that such a mechanism is consistent with the assumed properties of the octet. The continuation of the solutions to the physical masses is a separate question.

That rather narrow widths can be produced for resonances where the energy is sufficiently low also has a bearing on the π-N reciprocal bootstrap.[6] In particular, because of the possibility of narrow widths it would appear that elastic unitarity may reproduce properties of the N and N^*.

Finally let us remark briefly on the influence of the high-energy behavior. We have explicitly excluded high energy effects in assuming the dominance of nearby singularities, but it is not at all clear that this is reasonable. This question is being treated in at least two recent programs of calculations: the first is a relativistic calculation which incorporates Regge asymptotic behavior into the problem[8] (a modification of the strip approximation); the second involves work within the framework of the static model.[9] The first work is still in progress whereas in the second, calculations involving (admittedly somewhat artificial) models have shown that the high energy behavior is determined to a large extent by the properties of bootstrap solutions.[9] An examination of the two-pole model in Sec. II shows that a nearby pole together with a pole at a very large distance will produce narrow widths. If one takes such a situation seriously, then the distant pole may be interpreted as an approximation to necessary effects arising from the high-energy behavior in the crossed channels. We do not wish to emphasize this point, but simply remark that narrow widths may occur in a bootstrap calculation if some explicit high-energy behavior is included.

---

[7] For example, see R. H. Capps, Phys. Rev. Letters **10**, 312 (1963); H. M. Chan, P. C. DeCelles, and J. E. Paton, *ibid.* **11**, 521 (1963); L. F. Cook and J. E. Paton, Phys. Rev. **137**, B1267 (1965).

[8] G. F. Chew, Phys. Rev. **129**, 2363 (1963); D. C. Teplitz and V. L. Teplitz, *ibid.* **137**, B142 (1965).

[9] K. Huang and F. Low, J. Math. Phys. **6**, 795 (1965).