Application of Nuclear Coherence Properties to Elementary-Particle Reactions*

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We consider the use of nuclei as targets in low-momentum-transfer elementary-particle reactions. At low momentum transfer the nucleus acts coherently and may be used to enhance or suppress various exchange mechanisms for the elementary particles. We use the impulse approximation and optical-model wave-functions representing the absorption of the particles passing through the nucleus. For production reactions in which π exchange on hydrogen is dominant, coherent nuclear reactions may be used to eliminate π exchange and enhance ω-φ-type exchanges, thus allowing study of the behavior of these exchanges. Calculations are given for production on He^4, giving a crude estimate of 35 μb for $\pi+He^4\rightarrow\rho+He^4$. We also consider the general properties of coherent production reactions, including the A dependence of coherent reactions, the suppression of incoherent mechanisms at low momentum transfer, and the competition with production in the Coulomb field. We also consider "semicoherent" reactions in which the nucleus is excited to special states as a means of isolating particular components of the two-body scattering matrix.

NUCLEI offer some interesting possibilities as targets in elementary particle reactions since, in addition to the angular-momentum selection rules which have been suggested for coherent reactions,[1,2] it should be possible to use the properties of the aggregation of nucleons to enhance or suppress various basic reaction mechanisms. For instance, in a coherent reaction in which a nuclear target, quantum numbers $J^P=0^+$, $T=0$, remains unexcited, one-π exchange is forbidden although the corresponding process on hydrogen may be dominated by one-π exchange.

On the other hand, the exchange of an isoscalar particle whose nuclear vertex does not involve spin flip (e.g., vector meson such as ω or φ with mainly "charge" type coupling or a scalar meson) will not be prohibited and in fact, as we shall see, will be enhanced.

Or to take a slightly more subtle case, the restriction to a nuclear transition (J^P,T), $(0^+,0)\rightarrow(1^-,1)$, (transition to a giant dipole state) eliminates ω-φ exchange by isospin conservation, and π exchange by spin-parity arguments, and allows ρ exchange (these statements may be verified by thinking of the nucleus-nucleus-meson vertex).

The problem, of course, in using nuclear coherence properties is in experimentally selecting the final nuclear state, which we will generally want to be the ground state or some definite excited state.

If we attempt, as in nuclear physics, to identify the final nuclear state by observing the energy loss of the scattered particles, then this involves measuring energies to at least MeV accuracy with particles in the multi-BeV range, a formidable task.

Some other possibilities exist, however. One is to use the helium nucleus as a target, requiring an intact He^4 in the final state, the nonexistence of excited states of He^4 stable against particle emission thus assuring us that the nucleus remained in the $(0+,0)$ ground state. Another is to limit the inelasticity of the collision with the nucleus so that the coherent reactions will dominate although the nucleus is not definitely constrained to remain in the ground state. This might be achieved by using as a target nuclei the nuclei of a counter which rejects too violent excitations of the nucleus,[2] or by limiting the momentum-energy transfer to the nucleus as found by direct measurements on the scattered particles.

Below we attempt crude estimates of the resolution necessary to suppress the incoherent contributions to a production reaction vis-a-vis the coherent part. For the case where we wish to select a transition to a given excited state of the nucleus, we must hope for some distinct decay mode, perhaps in coincidence with the scattered particle, to separate the reaction from background.

With these points in mind, we attempt to roughly estimate the rates for some particular coherent production reactions and to consider some aspects of coherent reactions in general, including a brief discussion of "semicoherent" reactions in which the nucleus is left in special excited states.

We first discuss the kinematics and the method of calculation, i.e., the impulse approximation with eikonal wave functions to take account of absorption. Then we consider the behavior of various coherent reactions and the example of the reactions $\pi\rightarrow\rho$, $\gamma\rightarrow\pi^0$ on He^4, and the question of the supression of incoherent mechanisms contributing to a production process; we also consider briefly the Coulomb field production which will generally occur with coherent production on the nucleus proper. Finally, we consider some possible semicoherent reactions, involving coupling to special modes of excitation of the nucleus.

KINEMATICS

For a particle b scattered to a particle b' on a nucleus at rest, we have momenta $\mathbf{b},\mathbf{b}'$, of magnitude b and b',

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[1] S. M. Berman and S. D. Drell, Phys. Rev. Letters 11, 220 (1963).
[2] A. S. Goldhaber and M. Goldhaber, in *Preludes in Theoretical Physics*, edited by de Shalit, A. H. Feschbach, and L. Van Hove (North-Holland Publishing Company, Amsterdam, 1966).

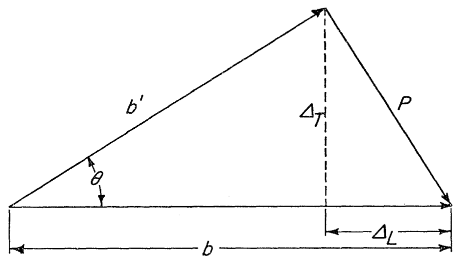

FIG. 1. The configuration of momenta in the lab in a typical low momentum transfer collision. b and b' are the incident and final scattered particles, P the recoiling nucleus, θ the scattering angle, and Δ_T and Δ_L the transverse and longitudinal momentum transfers.

energies $\omega(b)$, $\omega(b')$ in the laboratory, masses $m(b)$, $m(b')$. For the nucleus we have mass $M(A)$, equivalent uniform sphere radius $R\simeq A^{1/3}r_0$ and a recoil momentum $\mathbf{P}$. Energy conservation states $\omega(b)=\omega(b')+P^2/2M(A)$, but since we will always work at low momentum transfer, $P\sim 1/R$, $(\hbar=c=1)$ the recoil kinetic energy is negligible in the energy balance and $\omega(b)=\omega(b')$. Then $b\cong b'$ with a difference

$$b-b'=[m^2(b')-m^2(b)]/2b\equiv\Delta_0,$$

which is the minimum momentum transfer in the reaction. If the scattering angle for b' is θ, which will always be small, we have a transverse momentum transfer, $-\Delta_T=b'\sin\theta\cong b\theta$, and a longitudinal momentum transfer, which will normally be essentially a constant for a given reaction, $\Delta_L=b-b\cos\theta\simeq\Delta_0$. Momentum conservation means $\mathbf{P}=\mathbf{b}-\mathbf{b}'\equiv\mathbf{\Delta}$. Figure 1 shows the relationship of the vectors for a typical case of low momentum transfer. We have, under these conditions for the usual invariant four-momentum transfer $t=(b_\mu-b_\mu')^2\cong\mathbf{\Delta}^2=\Delta_T^2+\Delta_L^2=\mathbf{P}^2$. It should be remembered that, if the nucleus can be excited, then events are kinematically characterized not only by $\mathbf{\Delta}$, but also by E^*, the excitation energy of the nucleus, in which case the minimum momentum transfer becomes

$$\Delta_0=[m^2(b')-m^2(b)]/2b+E^*.$$

These kinematics are of course meant to apply when b and b' are highly relativistic, so that the momenta are much greater than the masses involved.

IMPULSE APPROXIMATION

We use the single inelastic-scattering impulse approximation,[3] according to which we neglect the effect of the binding of a nucleon on its scattering amplitude and take the scattered wave as the sum of the waves from each nucleon. If we neglect absorption in the nucleus for the moment, this gives essentially

$$\int \varphi_f^*(\mathbf{r}_1\cdots\mathbf{r}_A)\Big(\sum_{i=1}^{A} e^{i\Delta\cdot\mathbf{r}_i}T_i(\mathbf{\Delta})\Big)\varphi_0(\mathbf{r}_1\cdots\mathbf{r}_A)d\mathbf{r}_1\cdots d\mathbf{r}_A \tag{1}$$

as the matrix element for a nuclear transition $0\rightarrow f$, with the incoming particle scattered from $b\rightarrow b'$. Here r_i is the relative coordinate of the ith nucleon in the nucleus, φ_f and φ_0 are the nuclear wave functions (containing implicity spin and isospin coordinates), and T is the scattering matrix for the corresponding transition $b\rightarrow b'$ on a free nucleon at rest with three-momentum transfer $\mathbf{\Delta}=\mathbf{b}'-\mathbf{b}$, and contains operators on the nuclear coordinates:

$$T_i=a+\mathbf{b}\cdot\boldsymbol{\sigma}_i+c\tau_i+\mathbf{d}\cdot\boldsymbol{\sigma}\tau_i. \tag{2}$$

For an extensive discussion of the derivation and validity of the impulse approximation see Ref. 3. When the momentum transfer Δ is small enough, the exponential in Eq. (1) will vary slowly over the region of integration and we can have substantial matrix elements to localized states of the nuclear system, the nucleus acts in some sense coherently. This leads to the well known criterion for coherent reactions $\Delta R\sim 1$, which in turn, through the equation for Δ_0 sets an approximate lower bound on the incident momentum for a given (mass)^2 change to the scattered system. For orientation consider a 5-BeV π or K meson on a nucleus of $R=3r_0$ (Al) producing a ρ or K^* meson, which gives $\Delta_0R\sim 1.2$. On helium, which is especially tightly bound $(R\sim 1.3r_0)$, we have for these conditions $\Delta_0R\sim 0.5$. If we note that the uniform sphere form factor squared (see below) is $1-\frac{1}{5}\Delta^2R^2$ for small Δ, then these numbers indicate that we are in the coherent region at these energies.

To now see the relation between coherence and a particular reaction mechanism, consider Eq. (2) inserted into Eq. (1). If in (1) T_i operates on all the nucleons alike [the a term in Eq. (2)], and the final state is the same as the ground state, then we have full coherence with a matrix element

$$\begin{aligned} M(\Delta)&=AT(\Delta)\langle\varphi_0|e^{i\Delta\cdot\mathbf{r}}|\varphi_0\rangle\\ &=AT(\Delta)F(\Delta)\\ \frac{d\sigma}{d\Delta^2}&\simeq\frac{d\sigma}{d\Delta^2}\bigg|_{\text{free}}A^2|F(\Delta)|^2, \end{aligned} \tag{3}$$

where $F(\Delta)$ is the matter-distribution form factor, which we may take to be the same as the charge-distribution form factor used in the analysis of electron scattering. Such an operator arises, for instance, from the nonmagnetic coupling of an exchanged isoscalar vector meson, and so we may expect ω- or φ-like exchange to be a coherent effect. In general, an isoscalar $P=(-1)^J$ exchange need not be coupled to the spin-isospin and so can be coherent.

On the other hand consider a case like ρ^0 exchange, where we have the c term in Eq. (2). Then in Eq. (1) we have $M=\langle\varphi_0|\sum\tau_ie^{i\Delta\cdot\mathbf{r}_i}|\varphi_0\rangle$, which is zero on a nucleus with $N=Z$ and a small number in general for light nuclei where $N-Z$ is small, since the neutron and

[3] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), Chap. 11.

proton amplitudes tend to cancel. Thus, here, coherence suppresses ρ^0 exchange. In general, therefore, ρ^0 exchange cannot be responsible for coherent effects on light nuclei. On heavier nuclei where the neutron excess becomes considerable, the excess neutrons can, of course, contribute coherently as $(N-Z)^2$. This observation could be used to test for ρ^0 exchange: see if the effect goes away in coherent production on light nuclei and then comes back again on heavy nuclei. For the exchange of an isoscalar 0^- meson (η^0), we have the b term in Eq. (2) and thus spin-up nucleons will tend to cancel spin-down nucleons, and we cannot expect a coherent effect. For π^0 exchange $T_i \sim \tau_{3i}\boldsymbol{\sigma}_i\cdot\boldsymbol{\Delta}$ and we reach a similar conclusion. Hence, if we analyze a reaction according to exchange mechanisms, coherent effects are to be ascribed only to exchanges of particles like ω, φ, and f^0, among the presently well established mesons.

Presumably one of the ways to identify a coherent process is by its dependence on A, the mass number. This dependence is usually assumed to be A^2, as indicated by Eq. (3). Some care, however, should be used in applying this on three counts. First, because, as A increases, R also increases, the diffraction peak correspondingly narrows. Now at high energy this peak covers a very small angular range and what we may often observe is simply the total coherent rate.

Taking Eq. (3), and assuming that $T(\Delta)$ does not vary significantly over the small range of Δ in the coherent peak, we get, since F depends only on ΔR,

$$\sigma_{\text{coh}} \propto A^2 \int d\Delta^2 |F(\Delta)|^2 = \frac{A^2}{R^2} \int_{(\Delta_0 R)^2}^{\infty} |F(\Delta R)|^2 d(\Delta R)^2 .$$

This goes $\sim A^{4/3}$ for $\Delta_0 R \to 0$, and rather more slowly if Δ_0 is not negligible since $\Delta_0 R$ increases with A, which is our second point. Hence the effects of increasing the mass number of the target may not be as dramatic as might have been anticipated. This effect will be even more pronounced if we consider a case like $\pi \to \rho$, where the only possible matrix element must be[1,2] $\propto \mathbf{b}\times\mathbf{b}'\cdot\boldsymbol{\varepsilon}^\rho$ (since coherence means the only vector available from the nucleus is $\mathbf{P}=\mathbf{b}-\mathbf{b}'$), and so $d\sigma \to 0$ as $\theta \to 0$. [This holds in general for any b' whose $(-1)^J P$ is opposite to that of $\mathbf{b}$.] If we take a coherent mechanism (e.g., ω exchange), the reaction is still coherent in the sense that Eq. (3) is applicable, but the vanishing in the forward direction means we cannot take full advantage of the coherent peak. In this case, where $d\sigma \sim \theta^2$, we have for Δ_0 negligible

$$\sigma_{\text{coh}} \propto \int_0^{\infty} \Delta^2 |F(\Delta R)|^2 d\Delta^2 = \frac{A^2}{R^4} \int_0^{\infty} (\Delta R)^2 |F(\Delta R)|^2 d(\Delta R)^2 ,$$

which $\sim A^{2/3}$ and can be expected to vary more slowly for nonnegligible Δ_0. The third point concerns absorption.

ABSORPTION EFFECTS IN THE NUCLEUS

Finally, we must consider the screening of nucleons due to absorption. Qualitatively, we have for elementary particles passing through nuclei at high energy a situation of semitransparency. For an incoming π, say, with a total cross section σ on nucleons of 20–30 mb we have a mean free path[3,4] in the nucleus $\lambda = (\rho\sigma)^{-1}$ (ρ = density of nucleons) of $(3\text{–}4)r_0$, while the nucleus has radius $R = A^{1/3} r_0$. Thus while we will have quantitative effects even on He, our general picture of coherence is still correct.

As for the A dependence, the incident wave is attenuated as it passes through the nucleus so we may expect that something less than the full volume of the nucleus is effective in Eq. (3) and that the dependence on A is further reduced.

To take into account the absorption in passing through the nucleus, we modify the simple Eq. (3) with the effect of the phenomenological nuclear potential on the incoming and scattered particle, so that Eq. (1) becomes[3]

$$\int \varphi_f^*(\mathbf{r}_1\cdots\mathbf{r}_A)\Big(\sum_{i=1}^{A} \psi_{b'}^{-*}(\mathbf{r}_i)\psi_{b}^{+}(\mathbf{r}_i)T_i(\Delta)\Big) \times \varphi_0(\mathbf{r}_1\cdots\mathbf{r}_A)d\mathbf{r}_1\cdots d\mathbf{r}_A , \quad (4)$$

where ψ^+ and ψ^- are the optical model wave functions for the outgoing particle of momentum $\mathbf{b}'$ and incoming particle of momentum $\mathbf{b}$. We use eikonal approximation wave functions[3,4]

$$\psi_b^+(x) = \exp\left(i\mathbf{b}\cdot\mathbf{x} - \int_{-\infty}^{z} \frac{1}{2\lambda} dz'\right)$$
$$\psi_{b'}^-(x) = \exp\left(i\mathbf{b}'\cdot\mathbf{x} - \int_{z}^{\infty} \frac{1}{2\lambda} dz'\right), \quad (5)$$

where λ is the mean free path which for simplicity (and from ignorance) we take to be the same for b and b', and the paths of integration are along the directions of b and b', which we take to be parallel, since we always deal with small angles.

We may feel some apprehension about the neglect of the real part of the optical potential in Eq. (5), since the real part of the forward scattering amplitude on nucleons, which gives the real part of the optical potential for elementary particles has been found to be non-negligible (~20% of the imaginary part) and this can cause some bending of rays and an alteration of our momentum transfer dependence. However, in terms of the index of refraction of the nuclear material $n-1 = (2\pi/b^2)\rho f(0)$ [ρ = density of nucleons, $f(0)$ = forward scattering amplitude on nucleons], Snell's

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[4] R. J. Glauber, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959), Vol. 1. We neglect a Pauli-principle effect on λ discussed here since it gives a modification $\lesssim 10\%$, comparable to other uncertainties going into λ.

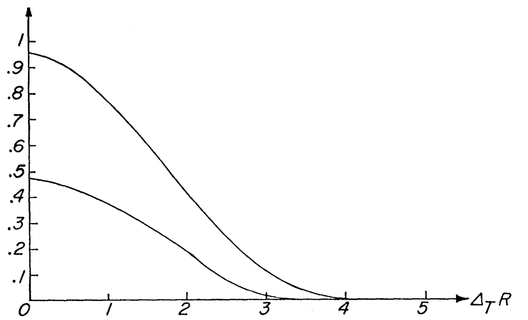

FIG. 2. The form factor squared $F^2(\Delta_T,\Delta_0,\lambda)$ for $\Delta_0 R=0.5$ and no absorption (upper curve) and with $\Delta_0 R=0.5$ and $R/2\lambda=0.24$ (lower curve).

law gives, for a ray with angle of incidence to the nuclear surface φ, a deviation θ from a straight path $b\theta \cong (2\pi/b)\rho\, \mathrm{Re}f(0)\tan\varphi$. Thus it turns out, we have a momentum transfer spread for, say, a pion characterized by

$$\frac{2\pi}{b}\rho\, \mathrm{Re}f(0) \cong \frac{2\pi}{b}\rho(0.2)\, \mathrm{Im}f(0) = (0.2)\frac{1}{2\lambda} \sim 5\ \mathrm{MeV},$$

which we may safely neglect. Furthermore, in the case that the particles have spin, we also assume the optical potential to be spin-independent.

Now if we consider the model of a coherent reaction on a $(0+,0)$ nucleus treated as a uniform sphere, we get Eq. (3) except that the form factor $F(\Delta)$ becomes

$$F(\Delta_T,\Delta_0,\lambda)=\frac{1}{\frac{4}{3}\pi R^3}\int_{\mathrm{sphere}} e^{i\Delta\cdot\mathbf{x}-D/2\lambda}d^3x$$
$$=\frac{3}{R^3}\int_0^R \rho d\rho J_0(\Delta_T\rho)\frac{\sin[\Delta_0(R^2-\rho^2)^{1/2}]}{\Delta_0} \times\exp[-(R^2-\rho^2)^{\frac{1}{2}}/\lambda]. \quad (6)$$

J_0 is the Bessel function, D is the distance through the sphere at a given impact parameter ρ, $D=2(R^2-\rho^2)^{1/2}$, where we have taken cylindrical coordinates with z as the beam direction. For the interesting special case in which the incoming particle is not strongly interacting (e.g., photon) it is more reasonable to use a plane wave for ψ^+ instead of Eq. (5), in which case we use $D=$ distance to travel out $=(R^2-\rho^2)^{1/2}-z$, giving just Eq. (6), but with Δ_0 replaced by $\Delta_0-i/2\lambda$, where λ is the mean free path for just the outgoing particle, and $1/\lambda$ in the exponential replaced by $1/2\lambda$.

To see the effect of absorption on the "form factor" $F(\Delta_T,\Delta_0,\lambda)$, we show in Fig. 2, F^2 versus $\Delta_T R \cong bR\theta$ for a case of low Δ_0, $\Delta_0 R=0.5$ and small absorption $R/2\lambda=0.24$ as compared to a curve with no absorption. The curve with absorption tends to be simply a constant multiple of the other at first and thus appears somewhat flattened. In Fig. 3 we have a case of substantial absorption $R/2\lambda=1$, low Δ_0, $\Delta_0 R=0.5$ and show the form factor squared curve I, as compared with the situation just mentioned where we have absorption only for the outgoing particle, curve II. Although there are considerable changes of scale among these curves, the shape does not appear to alter significantly.

To see how a differential cross section would be affected by the rise of $\Delta_0 R$ and the increase of absorption with A, we show in Fig. 4 F^2 at $\theta=0$ for $\Delta_0 \simeq 45$ MeV versus A for various degrees of absorption, σ being the free-nucleon cross section which goes into λ. If Δ_0 and absorption are negligible $F^2=1$ at $\theta=0$ for all A, so we see that even in a differential cross section we will generally have marked departure from A^2 behavior for relatively modest values of Δ_0 and σ. We can also use Eq. (6) to estimate the behavior of the total coherent cross section for the case where $T(0^0)$ nonzero, in the presence of absorption by squaring and using

$$\int_0^\infty d\Delta_T\Delta_T J_0(\Delta_T\rho)J_0(\Delta_T\rho')=\frac{1}{\rho}\delta(\rho-\rho')$$

to get

$$\sigma_{\mathrm{coh}}\cong\int_0^\infty d\Delta_T{}^2\frac{d\sigma}{d\Delta_T{}^2}|F(\Delta_T,\Delta_0,\lambda)|^2$$
$$\cong\frac{d\sigma(0)}{d\Delta^2}\bigg|_{\mathrm{free}} \times A^2 2\int_0^R \rho d\rho \left\{\frac{3}{R^3}\frac{\sin[\Delta_0(R^2-\rho^2)^{1/2}]}{\Delta_0}e^{-(R^2-\rho^2)^{\frac{1}{2}}}\right\}.$$

Note that since the integral in the above equation is practically energy independent (except for Δ_0) the energy dependence is just given by $(d\sigma/d\Delta^2)|_{\mathrm{free}}$. In Fig. 5 we show the variation of σ_{coh} with A for different amounts of absorption. If we consider coherent reactions vanishing as θ^2 in forward direction, then as mentioned above, we may expect to lose roughly another power of $\frac{2}{3}$ in the A dependence; thus we can have coherent reactions of this type whose total cross

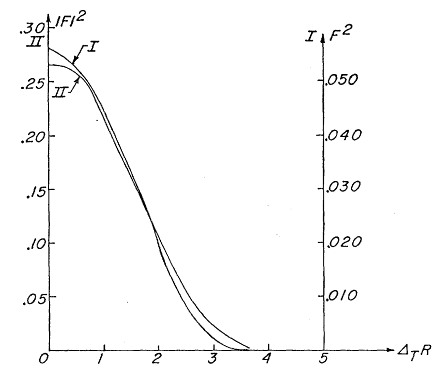

FIG. 3. The form factor squared for $\Delta_0 R=0.5$, $R/2\lambda=1$ (curve I), and the corresponding form factor modulus squared with absorption only for the outgoing particle with $\Delta_0 R=0.5$, $R/2\lambda=1$ (curve II). Use the right-hand scale for curve I.

section increases hardly, if at all, with A. These calculations are of course in some degree model-dependent and would change somewhat with the use of more sophisticated nuclear models or with a variation in the nuclear radius parameter r_0, which we have taken as roughly 1.3 F. For instance a 10% increase in r_0 corresponds to a 20% increase in the σ and a 10% decrease in the Δ_0 labeling the curves in Figs. 4 and 5.

INCOHERENT CONTRIBUTIONS

Now let us imagine that we want to study a coherent reaction on a complex nucleus, and that while the experimental resolution is not good enough to guarantee that the nucleus remains in the ground state, some limit can be placed on the maximum momentum transfer $\mathbf{\Delta}$ and energy E^* transferred to the nuclear system. Now if E^* and Δ are kept relatively small, we expect to still have some suppression of incoherent mechanisms. We will have transitions to relatively localized states of the final system, and the tendency of proton and neutron amplitudes for isovector matrix elements or of spin up and spin down amplitudes for $\boldsymbol{\sigma}$ matrix elements to coherently cancel, will be retained until we reach large momentum transfers where all the nucleons act incoherently.

To get an estimate of the region of the suppression effect, consider the particles scattered into some angle, summed over the excited states of the nuclear system. Using Eq. (1)

$$\frac{d\sigma}{d\Omega}\sim\sum_f |\langle f|\sum_i e^{i\Delta\cdot\mathbf{r}_i}T_i(\Delta)|0\rangle|^2\rho(\omega(b')) . \quad (7)$$

Now in the matrix element, Δ_0, which varies with E^* is actually a function of f, but we replace it with some average value within the region of E^* allowed. Now the sum in Eq. (7) stops at a certain value of E^*, but we can get an overestimate by carrying the sum over all f,

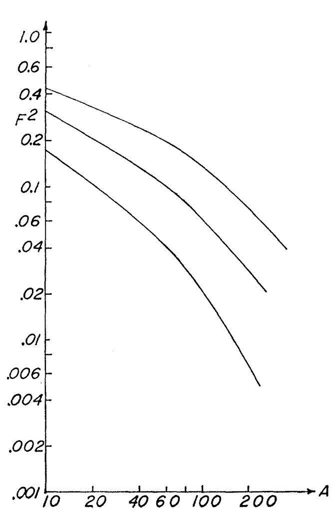

FIG. 4. F^2 at $\theta=0$ versus A for $\Delta_0=45$ MeV and various values of absorption corresponding to free nucleon cross sections $\sigma=18$, 27, and 40 mb from the top down).

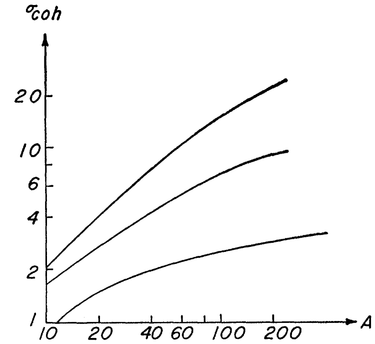

FIG. 5. The total coherent cross section

$$\sigma_{\text{coh}}\sim A^2\int F^2(\Delta_T\Delta_0\lambda)d\Delta_T^2,$$

appropriate to a differential cross section not vanishing at $\theta=0$, for $\Delta_0=45$ MeV and varying degrees of absorption (arbitrary normalization). $\sigma=18$, 27, and 40 mb (from the top down).

neglecting the variation of the density of states $\rho(\omega(b'))$ over the actual allowed E^*, and use closure, to get

$$\frac{d\sigma}{d\Omega}\lesssim 0|\langle \sum_i T_i^\dagger(\Delta)T_i(\Delta) + \sum_{i\neq j} e^{i\Delta\cdot(\mathbf{r}_i-\mathbf{r}_j)}T_i^\dagger(\Delta)T_j(\Delta)|0\rangle ,$$

$$\frac{d\sigma}{d\Omega}\lesssim\frac{d\sigma}{d\Omega}\bigg|_{\text{free}}\langle 0|A+\sum_{i\neq j} e^{i\Delta\cdot(\mathbf{r}_i-\mathbf{r}_j)}\theta_i^\dagger\theta_j|0\rangle , \quad (8)$$

where the θ_j are the σ and τ operators on the nucleons. For a crude estimate we neglect correlations and using the average of the θ_i for a (0,0) nucleus[3] to get for the partial cross section from a particular nuclear operator (the different nuclear operators add up incoherently since they lead to different final states)

$$\frac{d\sigma}{d\Omega}\bigg|_{\theta_i}\lesssim\frac{d\sigma}{d\Omega}\bigg|_{\text{free},\theta_i}A(1-|F(\Delta)|^2) . \quad (9)$$

We have neglected absorption, but its effect can be presumably estimated simply by multiplying Eq. (9) by the attenuation factor (Ref. 3, p. 825) $(1/\text{vol}\times\int_{\text{sphere}} e^{-D/\lambda}dv)$. In (9) we evaluate $\Delta^2=(b\theta)^2+\Delta_0^2$ at $\Delta_0=[m^2(b')-m^2(b)]/2b+E^*$, where E^* is on the order or our energy-loss resolution. This may then be used, along with estimates of the partial cross sections for various mechanisms, to decide what resolution is needed to suppress a spin, isospin coupled mechanism sufficiently as compared to the coherent part, Eq. (6).

REACTIONS ON HELIUM

As mentioned above, the helium nucleus is a particularly simple case for studying coherent production because it has no excited states stable againstparticle emission. Also, its small radius makes it possible to take up relatively large momentum transfers without breakup. Furthermore, there may not be a great loss in

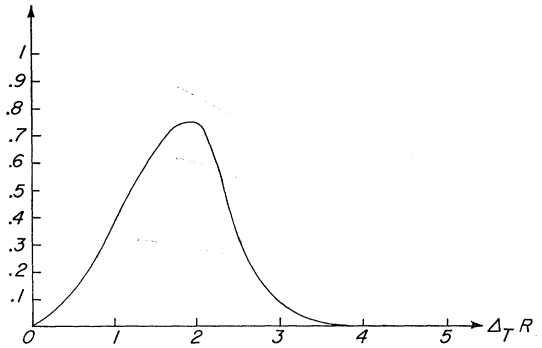

FIG. 6. The angular distribution for a coherent reaction on He going as θ^2 in forward direction (e.g., $\pi+He^4 \to He^4+\rho$, $\gamma+He^4 \to He^4+\pi^0$). The shape of the curve is essentially energy-independent (in the BeV range).

the total rate if we use a light nucleus, as the calculations above have indicated.

Let us consider the reaction $\pi^-+He^4 \to He^4+\rho^-$, as a means of examing the ω-φ exchange contribution to ρ production, without the π exchange found predominant on a free nucleon. First of all, on general grounds[1,2] a characteristic of coherent production is that the ρ decay in its own rest frame is given by a matrix element $\propto (\mathbf{q}\times\mathbf{P})\cdot\mathbf{q}'$ ($\mathbf{q}'$=momentum of decay π, $\mathbf{q}$=that of incident π^-, $\mathbf{P}$=that of He nucleus) i.e., a distribution isotropic around the $\mathbf{q}\times\mathbf{P}$ axis and $\cos^2\theta d\Omega$ with respect to it. Now the cross section given by an isoscalar vector meson exchange (where in this model the vector meson may be a linear combination of ω and φ or worse) is (we append a "form factor" f for absorption effects, Regge behavior, and the like)

$$\left.\frac{d\sigma}{d\Delta^2}\right|_{\text{free}}=\pi\frac{f_{vNN}^2}{4\pi}\frac{(f_{v\rho\pi}/m)^2}{4\pi}\frac{K^2\sin^2\theta}{(\Delta^2-m_v^2)^2}f^2(\Delta^2,K)\,, \quad (10)$$

inserting in (9), we get the cross section on He (small θ)

$$\frac{d\sigma}{d\Delta^2}\cong\pi\frac{f_{vNN}^2}{4\pi}\frac{(f_{v\rho\pi}/m)^2}{4\pi}\frac{K^2\sin^2\theta}{(\Delta^2-m_v^2)^2}f^2(\Delta^2,K)\,. \quad (11)$$

Figure 6 shows a sketch of this angular distribution where we use the absorption parameter $R/2\lambda=0.24$ and $\Delta_0\cong m_\rho^2/2K\simeq 60$ MeV, which in the case of He is well within the coherent region.

To try to estimate the size of cross section we can use the comment of Hagopian and Selove in their compilation of Penn-Saclay data (unpublished) on ρ^- production at 2.75–3 BeV that the enhancement in their $d\sigma/d\Delta^2$ plot at a $\Delta^2=50\,\mu^2$, above the obvious π exchange region, may correspond to vector exchange. This enhancement appears to correspond to about

$$\frac{d\sigma}{d\Delta^2}\approx(1\text{ mb})\frac{f^2(\Delta^2,K)}{f^2(50\mu^2,K)}\times\frac{(K\theta)^2}{(\Delta^2+m_V^2)^2}\,.$$

Theoretical estimates of this can vary widely because of the uncertainties in the coupling constants, but in the ω-exchange model (since the φ couplings are probably weaker)[5] this corresponds to

$$\pi\frac{f_{\omega NN}^2}{4\pi}\frac{f_{\omega\rho\pi}^2/m^2}{4\pi}\frac{1}{f^2(50\mu^2,3\text{ BeV})}\sim 1\text{ mb}$$

which is well within the range of various speculations.[5,6] An experiment of this type should help to clarify this situation and allow study of the energy behavior. If we now use this to gauge the total cross section to be expected we get

$$\frac{f^2(0,K)}{f^2(50\mu^2,3\text{ BeV})}\times 35\,\mu\text{b}\,,$$

where absorption in the nucleus has caused a reduction of about 50%.

Now it appears experimentally[7] that cross sections corresponding to vector exchange are dropping with energy, perhaps like the π-exchange cross sections; $f^2(0,K)$ must be decreasing to compensate the increase of the simple vector exchange mechanism. On the other hand, the enhancement in coming to $\Delta=0$ from 50 μ^2 may be an order of magnitude or more, so that 35 μb seems a reasonably conservative estimate at a few BeV.

A source of enhancement may be found in indication[8] from experiments on deuterium that there are a surprising number of deuterons left intact in relatively high momentum-transfer collisions which may show that we should expect a similar effect on He. This would mean that the high Δ tails of our distribution are much too small, so that cross sections varying as Δ_T^2 may be increased considerably. Also, on helium, the use of the more complicated and realistic Gaussian distribution for the nucleons may give a somewhat different value for the screening effect, but our calculation should be adequate, considering the other uncertainties in the problem. All that has been said about $\pi\to\rho$ applies equally well to $K\to K^*$ at the same momentum (since $m_\rho^2-m_\pi^2=m_{K^*}^2-m_K^2$,$\Delta_0$ is unchanged) with appropriate adjustment in λ.

An obvious analog to the case just discussed is photoproduction ($\gamma\to\pi^0$, η) on nuclei to separate out any putative $T=0$, $P=(-1)^J$ exchange. Because of the low mass of the π^0, small values of $\Delta_0=m_\pi^2/2K$ (K=photon energy) are reached at low energies (in fact, well below a BeV, where π-n resonances are important and our simple semiclassical treatment of the wave function becomes questionable, thus we remain in the BeV region). For η, however, we must have $K=3$ BeV for $\Delta_0=50$ MeV. Because of the low Δ_0 for π^0 production,

[5] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).
[6] S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964).
[7] Derrick *et al.*, presented at the Athens Conference, June 1965, by T. Fields, in *Resonant Particles*, edited by B. A. Munir (Ohio University, Athens, Ohio, 1965).
[8] Butterworth *et al.*, Phys. Rev. Letters **15**, 500 (1965).

the estimate Eq. (9) indicates that it would be possible to suppress non-ω-like exchanges even on complex nuclei if sufficient energy resolution were possible for the incoming photons o if violent excitations of the nucleus could be anti-d out.

In any case in an experiment with intact α particles we have a clear-cut situation with ρ exchange forbidden and the cross section in an ω-exchange model is ($K\theta$ small)

$$\frac{d\sigma}{d\Delta^2}=(\text{const})\frac{(K\theta)^2}{m_\omega^4}f^2(\Delta^2=0,K)\times 16|F(\Delta_T,\Delta_0,\lambda)|^2, \quad (12)$$

where F now only has absorption for the outgoing π or η. Talman *et al.*[9] have observed vector-meson-exchange effects in $\gamma+p\rightarrow p+\pi^0$ at 1140 MeV with a cross section corresponding to $d\sigma/d\Delta^2\cong(19\ \mu\text{b})(K\theta)^2/m_\omega^4$. If we attribute all of this to ω-φ exchange, we get for the total coherent helium cross section of $\cong 3.0\ \mu\text{b}\times[f^2(0,1140)/f^2(0,K)]$. Here the absorption on the outgoing π^0 has caused a reduction in the cross section of $\sim 30\%$. The angular distribution should be the same as in Fig. 6.

We do not discuss any specific cases of coherent reactions nonzero at $\theta=0$, such as $\pi\rightarrow A'$, $\gamma\rightarrow\rho^0$ since these reactions can proceed without exchange of quantum numbers. In this case the reaction can go by a dissociation mechanism and a different kind of theory would seem to be required.[9a]

COULOMB PRODUCTION

We should briefly mention Coulomb production here, both because it may overlap with strong coherent production and because it offers interesting possibilities for studying photon-induced processes which are not otherwise accessible. Discussing again processes $\sim\theta^2$, Coulomb production is characterized by a very sharp forward peak due to the zero mass of the photon giving a Coulomb denominator in the cross section $[\Delta_0^2+(K\theta)^2]^{-2}$ which leads to a peaking characterized by a momentum transfer region $K\theta\sim\Delta_0$ as opposed to the processes on the nucleus characterized by $K\theta\sim 1/R$. Since with present energies and resolutions we are often dealing with the case $\Delta_0\sim 1/R$, these two regions are not so very different.

The Coulomb denominator, as the energy increases and Δ_0 decreases leads to an increasing differential cross section while the corresponding nuclear part is essentially constant, aside from the energy dependencies of the non-nuclear factors in the matrix element; so that for a given process the differential Coulomb cross section can apparently always become bigger at a high enough energy.

[9] Talman *et al.*, Phys. Rev. Letters **9**, 177 (1962).

[9a] See R. T. Deck, Phys. Rev. Letters **13**, 169 (1964); U. Maar and T. A. O'Halloran, Phys. Letters **15**, 281 (1965); M. Ross and L. Stodolsky (to be published).

The reaction $\gamma\rightarrow\pi^0$ in the Coulomb field was originally suggested by Primakoff as a means of measuring the π^0 life time and has been studied experimentally on Pb.[10] It appears from this data that the Coulomb and strong production are comparable in the region of the coherent peak in heavy nuclei and thus there are interference effects. As we go to smaller R, the nuclear production peak moves out and the ratio to the Coulomb matrix element in the vicinity of the nuclear peak ($\Delta_T R\approx 2$) goes as $(Z/A)[\Delta_0^2+(2/R)^2]^{-1}\sim Z/AR^2$ (for $\frac{1}{4}\Delta_0\ll R^{-1}$). If the two matrix elements are roughly equal at the nuclear peak or Pb, then we have a 3% interference term on He, where we have taken another factor of $\frac{1}{2}$ for the reduction of the nuclear absorption.

For $\pi^\pm\rightarrow\rho^\pm$, $K^\pm\rightarrow K^{*\pm}$ reactions, the Coulomb production[6] is of interest as a way of studying $\pi\gamma\rho$ and $K\gamma K^*$ interactions, since in principle all other relevant quantities are known.[10a]

If we neglect absorption[11] (since presumably most of the production takes place outside the nucleus), but take account of the nuclear charge form factor we get the analog to Eq. (11)

$$\frac{d\sigma}{d\Delta^2}=\pi Z^2\frac{e^2}{4\pi}\frac{(f_{\gamma\pi\rho}/m)^2}{4\pi}\frac{(K\theta)^2}{[\Delta_0^2+(K\theta)^2]^2}F_Q^2(\Delta)\,. \quad (13)$$

This differential cross section has a peak at $K\theta=\Delta_0$, which grows with energy as K^4, and therefore should, as pointed out by Berman and Drell, become greater than the nuclear production at high energy (if the coherent nuclear production, as seems likely, is decreasing with energy).[12]

[10] Tollestrup *et al.*, in *Proceedings of the Tenth International Conference on High Energy Physics at Rochester, 1960*, edited by E. C. G. Sudarshan *et al.* (Interscience Publishers, Inc., New York, 1961).

[10a] Note: In a recent experiment R. Huson *et al.*, Phys. Letters **20**, 91 (1966) with low statistics have attempted but were unable to identify this mode of ρ production.

[11] We may attempt to get an idea of the effect of absorption on a production process in a Coulomb field by taking the extreme model of a totally absorbing nucleus using wave functions ψ^+ and ψ^- with shadows fore and aft of the nucleus, respectively. Then performing the integral over the Coulomb field, we find that

$$F_Q(\Delta_0^2+\Delta_T^2)\rightarrow J_0(\Delta_T R)(\Delta_0 R)K_1(\Delta_0 R)-(R\Delta_T)J_1(\Delta_T R)K_0(\Delta_0 R)\,,$$

where the K's and J's are Bessel functions. This gives a diffraction structure for relatively small $\Delta_T R$ if $\Delta_0 R$ is small; for $\Delta_T R<1$, then J_0 term dominates and $(\Delta_0 R)K_0(\Delta_0 R)<1$ shows the effect of removing the shadows from the interaction region. The truth is presumably somewhere between this and F_Q when we have semi-transparency involving both the effects of absorption and the charge distribution (and for very small Δ_0 and large Z, the variation of the Coulomb phase over the large spatial extent of the interaction region).

[12] If the nuclear production is not decreasing relative to the Coulomb then the solid angle in which we have clear Coulomb production is limited by $\Delta_T\lesssim R$ in which case the total Coulomb production in this interval is

$$\int_0^{1/R}\frac{d\sigma_{\text{coul}}}{d\Delta_T^2}d\Delta_T^2\sim\ln\left(1+\frac{1}{\Delta_0^2R^2}\right)-\frac{1}{1+\Delta_0^2R^2},$$

which is varying relatively slowly with energy (factor of 2.6 in going from 10–20 BeV on Al). With $\Gamma(\rho\rightarrow\pi+\gamma)=0.15$ MeV we get for this integrated cross section 8 μb on Al at 10 BeV.

TABLE I. Nuclear operators, with corresponding simple exchange mechanisms, and transitions excited by the given operator.

Operator	Simple exchange mechanisms	Transition (J^P,T)
1	Isoscalar $0^+, 1^-, 2^+\cdots$meson (ω,φ,f^0)	$(0^+,0)\rightarrow(0^+,0)$ (coherent)
τ	Isovector $0^+, 1^-, 2^+\cdots$meson (ρ,A_2)	$(0^+,T)\rightarrow(0^+,T)$ (isoanalog) $T_z\rightarrow T_z\pm1$
$\boldsymbol{\sigma}$	Isoscalar $0^-, 1^+\cdots$meson (η,X^0) mag. coupling of 1^- meson (ω)	$(0^+,0)\rightarrow(1^+,0)$ (spin fiips)
$\boldsymbol{\sigma}\tau$	Isovector $0^-, 1^+\cdots$meson (π) mag. coupling of 1^- meson (ρ)	$(0^+,0)\rightarrow(1^+,1)$ (isovector spin flip)
$\mathbf{r}\tau$	Same as τ	$(0^+,0)\rightarrow(1^-,1)$ (giant dipole)

Now if we put everything together and scale our previous estimate of the nuclear production to Al at 10 BeV we have the following picture—very schematically, since the magnitudes of the cross sections are obviously based on very rough guesses: At $K\theta=28$ MeV or $\theta=2.8$ mrad we have, taking $\Gamma(\rho\rightarrow\pi+\gamma)=0.15$ MeV, the Coulomb peak with $d\sigma/d\Omega=130$ mb/sr; at $K\theta=2/R=92$ MeV or $\theta=9.2$ mrad we have the coherent nuclear peak where the Coulomb cross section is $\frac{1}{4}$ maximum and the coherent peak corresponds to (25 mb/sr)$[f^2(0,10\text{ BeV})/f^2(50\mu^2,3\text{ BeV})]$. If nothing is done about holding the nucleus together, then we also have forward ρ's from the ordinary incoherent production on the 27 nucleons $\sim a\times27\times d\sigma/d\Omega$ where a is the absorption factor $\sim\frac{1}{3}$, we have $\frac{1}{3}\times17\times\frac{1}{3}0\sim90$ mb/sr, where we have taken some average value of the ρ production[13] near the forward direction because th Fermi motion gives momentum transfer even at 0^0.

SEMICOHERENT REACTIONS

Certain excited states of nuclei have particularly strong transitions from the ground state through a given operator, taking up most of the probability associated with applying that operator to the ground state. If we could observe the excitation of such states in coincidence with a scattering process, then we could, in principle, have a means for studying that component of the scattering matrix Eq. (2) which in Eq. (1) or (4) induces transitions to one of these states.[14]

Since some of these excitations may involve a large number of the nucleons the corresponding cross sections are enhanced as compared with a single nucleon, and we have a kind of "semicoherence," as in the "isotopic analog" transitions on nuclei with a large neutron excess $|\langle f|\sum T_i e^{i\Delta\cdot\mathbf{r}_i}|0|^2\sim N-Z$ or "giant dipole" transitions $|\langle f|\sum T_i e^{i\Delta\cdot\mathbf{r}_i}|0\rangle|^2\sim A$ (on light nuclei). In Table I, we shown some simple operators arising from $T_i e^{i\Delta\cdot\mathbf{r}_i}$ with the transition shown in a given row selecting the corresponding operator, and we give some simple exchange mechanisms corresponding to the nuclear operator. Generally, of course, a given operator does not uniquely correspond to the exchange of a given meson or class of mesons unless we have some model of the reaction or some selection rule (e.g., G parity) limiting the possible exchanges. Aside from the obvious restriction that a change in isospin necessitates an isovector exchange (higher isospin exchange is not allowed if we consider only single nucleon vertices), the point of principle interest is that 0^- exchange (π,η) is forbidden for nuclear transitions $J^P=0^+\rightarrow0^+$, 1^-, 2^+, so that we can forbid π exchange by fixing on an appropriate transition, while still permitting other than simple coherent exchanges. There is no analogous prohibition for the exchange of spin-bearing particles except for the nuclear transition $0^+\rightarrow0^\pm$, in which case the exchange particle must have $P=\pm(-1)^J$. It would be interesting if a "giant," $T=0$, 0^- state were to exist, for then η exchange would be coupled, but ω, ρ, and π type exchanges would be forbidden. Now let us examine three simple cases which have been studied in nuclear physics: isotopic analog, giant dipole, and isovector-spin flip transitions.

Isotopic multiplets are of course well known in light nuclei and the study of narrow, well defined analog states in heavier nuclei[15] has recently become a subject of intense investigation in nuclear physics. In the

[13] J. D. Jackson *et al.*, Phys. Rev. **139**, B428 (1965).

[14] A question which arises, particularly when we consider the relatively weak noncoherent transitions, is the effect of multiple scattering in which the nuclear transition may occur in steps, thus arriving at a given final state by a mechanism other than the one we are trying to isolate. For instance, in a process where π exchange is inherently bigger than say ρ exchange, but we try to isolate the latter by proper choice of the final nuclear state, we may fear that the reaction may proceed by first exchange of a π, followed by a final scattering of the outgoing particle leading to the final state we observe. First of all, since we always discuss bound or quasibound states, the intermediate state is also a bound or localized state since if the nucleus is broken up it is very unlikely that a second collision will put it back together again. Secondly, although the mean free path for scattering in the nucleus is not negligible in general, the σ, τ parts of the scattering amplitude which can cause spin flip or isospin change are small at high energy so that a second scattering involving one of these changes does have an effectively long mean free path so that we need only worry about the case when one of the steps can be through the scalar $(e^{i\Delta\cdot\mathbf{r}})$ part of the scattering. To see this consider a transition in steps $0\rightarrow i\rightarrow f$ through $T^{(1)}$ and $T^{(2)}$, respectively, where the direct transition is $\sim\langle\varphi_f|T^0e^{i(\mathbf{q}-\mathbf{P})\cdot\mathbf{r}}|\varphi_0\rangle$. The two-step process is then essentially (Ref. 3)

$$\int d^3K\frac{1}{\omega_P-\omega_K+i\epsilon}\langle\varphi_f|e^{i(\mathbf{P}-\mathbf{K})\cdot\mathbf{r}}T^{(2)}|\varphi_i\rangle$$
$$\times\langle\varphi_i|T^{(1)}e^{i(\mathbf{K}-\mathbf{q})\cdot\mathbf{r}}|\varphi_0\rangle\sim K^2\langle f|T^{(2)}|i\rangle\langle i|T^{(1)}|0\rangle$$
$$\times\int d\Omega_K\langle\varphi_f|e^{i(\mathbf{P}-\mathbf{K})\cdot\mathbf{r}}|\varphi_i\rangle\langle\varphi_i|e^{i(\mathbf{K}-\mathbf{q})\cdot\mathbf{r}}|\varphi_i\rangle.$$

Now since we have localized states the integration introduces a factor of $(KR)^{-2}$ (as can be seen with simple Gaussian forms of the wave functions), so we get essentially $\langle f|T^{(2)}|i\rangle\langle i|T^{(1)}|0\rangle/R^2$ as compared with $\langle f|T^0|0\rangle$ which may be used to gauge the multiple-scattering effect. Now for the scalar part of the scattering amplitude $\text{Im}T\sim\sigma$ (the total cross section on free nucleons) so that there may be cause for concern if a two-step process is possible via a level strongly connected by $e^{i\Delta\cdot\mathbf{r}}$ and an intrinsically stronger mechanism. On the other hand, a step requiring an isospin change, say, with high energy π's, $\text{Im}T^V\sim1$ mb, is characterized by ~1 mb$/R^2$ which is small. Finally, we remark heuristically that the single inelastic-scattering approximation has been successful in nuclear physics.

[15] J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. **129**, 2718 (1963).

heavier nuclei these states can decay by particle emission and may be identifiable in this way, while in the lighter nuclei where the Coulomb energy shift is less, we may have decay by gamma emission. The isotopic analog to the ground state of a given target nucleus is presumed to be the same state with the value of t_z differing by one. In the region of neutron excess, the stable target nucleus is taken to have the minimum t possible: $t=-t_z=\frac{1}{2}(N-Z)$, and the analog state has a neutron changed to a proton, with an energy higher by $\sim(6/5)Ze^2/R$. Now let us consider a process involving charge exchange on a 0^+ nucleus making a transition to an anlog state, so that the relevant nuclear operator is $\tau_{\pm i}e^{i\Delta\cdot r_i}$ (e.g., $\rho^\pm$ exchange). For $\Delta=0$ we would have in Eq. (1)

$$\tfrac{1}{2}\langle f|\ \textstyle\sum_i \tau_{\pm i}|0\rangle=\langle t,\ t_z\pm 1|T_\pm|t_z,\ t\rangle\ ,$$

and thus we get

$$\frac{d\sigma}{d\Delta^2}=\frac{d\sigma}{d\Delta^2}\bigg|_{\text{free}}[t(t+1)-t_z(t_z\pm 1)]F^2(\Delta_T,\Delta_0,\lambda)\ . \quad (14)$$

For an analog transition on a nucleus with a large neutron excess, the quantity in the bracket is $2t=N-Z$, and we have a kind of semicoherent enhancement of charge exchange. We might consider an experiment which triggers on the decay of an analog state as a way of isolating the ρ exchange contribution to charged ρ photo-production.[6] Berman and Drell indicate that the 0^0 cross section may be on the order of microbarns/sr, while in the region of $A\sim 200$ we have $N-Z\sim 40$, $F^2=0.14$ at 5 BeV and 0.23 at 10 BeV, with the absorption parameters the same as for π's.

Another transition strongly excited through τ_i (ρ exchange) is to the giant dipole resonance,[16] $T=1$, $J^P=1^-$, which could presumably be identified by its nucleon decay spectrum. Essentially the matrix element needed here has been calculated[17] for carbon and oxygen and we have, taking over this result, (assuming that the "momentum" $1/2\lambda$ is neglected relative to Δ)

$$\frac{d\sigma}{d\Delta^2}\cong\frac{d\sigma}{d\Delta^2}\bigg|_{\text{free}} A\frac{(\Delta R)^2}{5}F^2(\Delta_T,\Delta_0,\lambda)\ . \quad (15)$$

Here again we have an enhancement over a single nucleon rate since essentially all the nucleons participate. By isospin symmetry this holds for transitions to all 3 members of the $T=1$ giant triplet from the $T=0$ ground state.

Finally, we consider a case, analyzed by Kawai, Terasawa, and Izumo[18] when we have coupling through the isovector spin-flip component of the scattering matrix. These authors obtain good agreement with inelastic proton scattering experiments for the excitation of the $J^P=1^+$, $T=1$, 15.1 MeV level of C^{12} (which is part of an isotriplet with B^{12} and N^{12}) using the impulse approximation, the known nucleon-nucleon scattering parameters, and taking the nuclear matrix element as $\langle f|\sum\sigma_i\tau_i|0\rangle$, which can be found from the ρ decay ft values for N^{12} and B^{12}. Now if we consider the two-body amplitude for a process on a free nucleon, where the isovector-spin flip part is $\mathbf{g}^V\cdot\boldsymbol{\sigma}\tau_3$ so that the elastic cross section on a free unpolarized nucleon would be $d\sigma/d\Delta^2|_{\text{free}}=|\mathbf{g}^V|^2$ (due to this part of the amplitude), then the cross section for excitation of the level is approximately, taking the results of Ref. 18,

$$\begin{aligned}\frac{d\sigma}{d\Delta^2}&\cong|\mathbf{g}^V|^2\sum_{mf}|\langle f|\ \textstyle\sum\sigma_i\tau_{3i}|0\rangle|^2F^2(\Delta_0,\Delta_T,\lambda)\\&\cong|\mathbf{g}^V|^2(0.6)|F(\Delta_0,\Delta_T,\lambda)|^2.\end{aligned} \quad (16)$$

[If the scattering particle is an unpolarized baryon then $|\mathbf{g}^V|^2=\frac{1}{2}\sum_{if}(\mathbf{g}^\dagger\cdot\mathbf{g})_{if}$]. This formula also applies to the $C^{12}\to B^{12}$, N^{12} transitions by isospin symmetry. Although this is a strong transition by nuclear physics standards, we have no enhancement over the free-particle cross section; however we have done away with the isoscalar and non-spin-coupled parts of the scattering matrix.

We might now consider turning the analysis around to use the excitation of the spin flip level as a way of roughly gauging the magnitude and behavior of the $\mathbf{g}^V$ term at small angles in a scattering amplitude. Thus for elastic πN or KN scattering we have $\mathbf{g}^V\cdot\boldsymbol{\sigma}=g^V\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{q}'/|\mathbf{q}\times\mathbf{q}'|$, $g\to 0$ as $\theta\to 0$. In terms of a ρ-exchange model for $\pi+p\to\pi+p$

$$\frac{d\sigma}{d\Delta^2}=\pi\frac{f_{\rho\pi\pi}{}^2}{4\pi}\frac{f_{\rho NN}{}^2}{4\pi}\left(\frac{K\theta}{m_\rho{}^2}\frac{1+\mu_V}{2M}\right)^2(0.6)F^2(\Delta_T,\Delta_0,\lambda)\ ,$$

which in the lab will have a peak at $K\theta R=2$, of the shape in Fig. 6 ($\theta=2.5°$) at 3 BeV of $f^2(0,K)$ 0.35 mb/sr. Note that as opposed to polarization measurements, which measure the interference between $\mathbf{g}\cdot\boldsymbol{\sigma}$ and a scalar term, this is proportional to $|\mathbf{g}|^2$ directly. It may also be worth noting that the 15-MeV $M1$ photon emitted as a result of such an excitation must be correlated with the normal to the scattering plane of the π as $(\mathbf{K}_\gamma\times\hat{n})^2$.

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[16] E. G. Fuller and Evans Hayward, in *Nuclear Reactions*, edited by P. M. Endt and P. B. Smith (North-Holland Publishing Company, Amsterdam, 1962), Vol. II.

[17] S. Fallieros *et al.*, Nucl. Phys. **15**, 363 (1960).

[18] M. Kawai, T. Terasawa, and K. Izumo, Progr. Theoret. Phys. (Kyoto) **27**, 404 (1962). We would like to thank Dr. G. R. Satchler for bringing this work to our attention.