deviation of τ from the mean (the width of the histogram of Fig. 6) also had a minimum at the "best" value.

The same procedure was followed for a subset of 25 runs, the results of which are shown in Fig. 7. This subset was selected by eliminating some runs for which the dependence of τ on the lower cutoff did not appear self-consistent. This subset gave a "best" lifetime value of $\tau = 26.41$ nsec.

As a different approach, a second subset of 21 runs, eliminating runs with obvious ripples and those with few total events, was assembled into a composite run by adding the data, taking into account the calibrations. This composite run was analyzed in the same way as an actual data run, giving a "best" value of $\tau = 26.41$ nsec. For comparison, the average of the "best" values of the same 21 runs gave an average of 26.38 nsec.

It is apparent that the value of τ is somewhat dependent on the treatment of the lower cutoff, so that the statistical error of 0.02 nsec on the mean value is not realistic. We choose $\tau_\pi = 26.40$ nsec as the central value of the various data combinations and assign an uncertainty of 0.08 nsec. This reflects our sensitivity to the systematic effects tested. Also, within these limits the lifetime is insensitive to the choice of lower cutoff over a range of 3 mean lives.

This lifetime value is in disagreement with both the previously accepted value[2] of 25.51 ± 0.26 nsec and with the recently reported value of 26.01 ± 0.02 nsec of Eckhause *et al*[4]. The latter is the more serious discrepancy in view of the large amounts of data involved, and the small errors. We are unable to reconcile the discrepancy in view of the fact that only one of our 37 runs could be considered compatible with their value. The major differences in technique were as follows: (a) We required the e^+ pulse; they (Eckhause *et al*) did not. (b) Our π and μ pulse came from the same counter; theirs were from different counters. (c) We used a pulse overlap time-to-pulse-height converter with about 0.9 nsec bins; they used a 100-Mc/sec digital time analyzer with 10-nsec bins. (d) Our background is about 100 times lower than theirs.

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Remarks on the Saturation of Equal-Time Commutators and Physical Sum Rules*

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Identities relating continuations of physical amplitudes to equal-time commutators are considered. These identities, combined with fairly innocuous and justifiable approximations on the physical amplitudes, are shown to yield useful results. Some of these results look as if they emerged from a higher symmetry.

THE equal-time commutators of unrenormalized current operators have recently been the object of intensive investigation. Much of the discussion has been focused on the following aspect. Consider a theory which is invariant under a Lie group that is generated by charge operators constructed from a set of currents. Then the invariance requires that the expectation value of the equal-time commutator of two charge operators in a physical state belonging to some irreducible representation be saturated by a single intermediate state of the same irreducible representation. Conversely, if it is assumed that the expectation value of such a commutator is saturated by a single intermediate state, then results characteristic of the symmetry are obtained.[1–3] This latter circumstance has led to the interesting speculation that the saturation of equal-time charge commutators by judiciously selected intermediate states may be taken as a kind of dynamical mechanism for the induction of approximate symmetries.[2] Unfortunately, the phrase "dynamical mechanism" is difficult to define precisely in this context. Equal-time commutators *per se* are objects devoid of any special dynamical significance. In particular, the mass of an intermediate state has no apparent bearing on its importance in the sum over states. Thus, the choice of an intermediate state cannot be predicated on any simple dynamical principle of the type underlying, say, the Goldberger-Treiman formulas.

The purpose of the present paper is twofold. We first develop a field-theoretical identity that relates equal-

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[1] B. Lee, Phys. Rev. Letters **14**, 676 (1965).
[2] R. Dashen and M. Gell-Mann, Phys. Letters **17**, 142, 145 (1965) and other references cited therein.
[3] S. Fubini and G. Furlan, Physics **1**, 299 (1965).

time current commutators to the off-mass-shell analytic continuation of physical scattering amplitudes. Here our work is identical in spirit to and largely motivated by that of Fubini, Furlan, and Rossetti,[4] but it differs in detail and suffers from no ambiguities. We then investigate the possibility of extracting useful information from this identity by approximating the physical scattering amplitude. Our approximation can be justified to some extent on dynamical grounds, for it is related to the familiar technique of pole approximation.

Let $J_\nu(x)$ be a current operator and $j(x)$ any Heisenberg operator. Gauss's theorem implies the trivial identity

$$\int d^4x\, \partial^\nu\{e^{iq'x}\langle p'|i[J_\nu(x),\, j(0)]\theta(x_0)|p\rangle\}=0. \quad (1)$$

The states of momentum p' and p will, for the sake of definiteness, be taken as single baryon states. On evaluating the divergence we obtain

$$\begin{aligned}\int d^3x\, e^{-i\mathbf{q}'\cdot\mathbf{x}}\langle p'|i[J_0(\mathbf{x},0),\, j(0)]|p\rangle \\ =q'^\nu\int d^4x\, e^{iq'x}\langle p'|[J_\nu(x),\, j(0)]\theta(x_0)|p\rangle \\ -c\int d^4x\, e^{iq'x}\langle p'|i[P(x),\, j(0)]\theta(x_0)|p\rangle\,, \quad (2)\end{aligned}$$

where

$$\partial^\nu J_\nu(x)=cP(x)\,. \quad (3)$$

We take $j(x)$ to be a current density and denote by $\phi(x)$ the field generated by this current,

$$(\Box+m_\phi^2)\phi(x)=j(x)\,. \quad (4)$$

Then, by use of standard reduction techniques, Eq. (2) may be cast into the form

$$\begin{aligned}\int d^3x\, e^{-i\mathbf{q}'\cdot\mathbf{x}}\langle p'|i[J_0(\mathbf{x},0),\, j(0)]|p\rangle \\ =q'^\nu\int d^4x\, e^{iq'x}\langle p'|[J_\nu(x),\, j(0)]\theta(x_0)|p\rangle \\ -cT/(\mu^2-q'^2)\,, \quad (5)\end{aligned}$$

where μ is the mass of the P particle and T is the amplitude for ϕ+baryon $\rightarrow P$+baryon scattering.

We defer a full investigation of Eq. (5) to a later publication. In the present paper we consider only the limit in which q' vanishes

$$\begin{aligned}\langle p'|i[Q,\, j(0)]|p\rangle \\ =\lim_{q'\rightarrow 0} q'^\nu\int d^4x\, e^{iq'x}\langle p'|[J_\nu(x),\, j(0)]\theta(x_0)|p\rangle \\ -(c/\mu^2)\lim_{q'\rightarrow 0} T\,, \quad (6)\end{aligned}$$

where

$$Q=\int d^3x J_0(\mathbf{x},0)\,. \quad (7)$$

The first term on the right-hand side of Eq. (6) vanishes unless an intermediate state contributes which is degenerate in mass with either of the baryon states. If this is the case, this contribution combines with a corresponding term in the Born-approximation part of the amplitude T to give a well-defined and unambiguous limit, although the limit of the separate terms is ill defined. It is in this respect that our method is superior to that of Fubini, Furlan, and Rossetti.[4]

Equation (6) forms the basis for the discussion of the remainder of this note. We identify $J_\nu(x)$ with one of the 8 components of the axial current density which transform as the generators of $SU(3)$ or with an $SU(3)$ singlet axial current. We may then assume a generalized version of the partially conserved-axial-current hypothesis (PCAC) and take $P(x)$ to be the field operator of the corresponding member of the pseudoscalar octet with c a uniform constant for all members of the octet, or the field operator for an $SU(3)$ singlet state (η'). If $j(x)$ is identified with a nonet of polar or axial-vector currents, the equal-time commutator appearing on the left-hand side of Eq. (6) can be computed by assuming that these currents are composed of bilinear combinations of Fermi fields that satisfy canonical commutation relations. If $j(x)$ is taken to be the source of the pseudoscalar-meson octet or singlet, we obtain a generalization of a relation of Adler.[5] In this case it can be shown that the relevant commutator vanishes at the unphysical value of the momentum transfer $(p'-p)^2=\mu^2$. This is adequate for our purposes. (These statements are proved in the Appendix.)

On choosing $j(x)$ to be the pseudoscalar current we obtain a constraint on meson-baryon scattering. In order to get useful information from this constraint, we make the dynamical assumption that low-energy meson-baryon scattering is dominated by the exchange of a few systems with specific transformation properties under $SU(3)$. More precisely, we assume that systems of unit baryonic charge exchanged in the s and u channels transform only as an octet and decuplet, and systems of zero baryonic charge exchanged in the t channel transform only as singlets and octets.

While the above ansatz is somewhat *ad hoc*, it should be stressed that it is no more so than some of the assumptions that have gone into recent rederivations of some $SU(6)$ results. Indeed it is a more reasonable ansatz in the sense that one is imposing a well-defined condition on physical scattering amplitudes which has the virtue of being realizable in simple dynamical models such as the pole approximation with low-lying states.

We have investigated this constraint using routine manipulations with Clebsch-Gordan coefficients and

[4] S. Fubini, G. Furlan, and C. Rossetti, Trieste reports, 1965 (unpublished).

[5] S. Adler, Phys. Rev. 137, B1002 (1965).

also, as an algebraic check, using tensor methods (see Appendix). We find that a consistent solution is possible only if

$$(D/F)_{\text{meson-baryon coupling}}=3/2\,, \quad -1\,, \quad \text{or} \quad -3\,. \qquad (8)$$

If the ϕ field is distinct from P, we find

$$(D/F)_{\phi B\bar{B}}=(D/F)_{PB\bar{B}}\,. \qquad (9)$$

The first solution in Eq. (8) agrees with the standard predictions of $SU(6)$, $SU(6,6)$, and, more importantly, with experiment.[6] The second solution is manifestly unphysical, for it gives a vanishing pion-nucleon coupling. It corresponds to invariance under a $W(3)$ group.[7] We defer discussion of the third solution until the end of this note.

If the previous ansatz is to be understood in terms of a pole approximation, we require, in addition to the well-established baryon octet and decuplet, a low-lying nonet of mesons with even spin and positive parity which are normal under charge conjugation. There appears to be reasonable evidence for a 2^+ nonet[8] and some, albeit considerably less convincing, evidence for a 0^+ nonet.[9] The existence of either or both is sufficient for our purpose.

Since the spin of the exchanged systems is essentially irrelevant in our model, its justification in terms of an ordinary pole approximation may be replaced by one involving the exchange of Regge trajectories of prescribed signature.

A similar calculation can be carried out for the case in which the Heisenberg operator $j(0)$ is identified with the electromagnetic current density $\mathcal{J}_\mu(0)$. Here the relevant amplitude T refers to photoproduction processes. A straightforward application of Eq. (6), together with a dispersion analysis of the photoproduction amplitude, yields the sum rules for the isoscalar and isovector anomalous magnetic moments obtained by Fubini, Furlan, and Rossetti.[4] Using the same kind of pole approximation as that described above, we find that the (D/F) ratio for the magnetic moments is the same as the (D/F) ratio for the pseudoscalar coupling. Although the baryon-octet contribution does yield the anomalous magnetic moments, the dispersion analysis determines the amplitude only up to subtraction constants and does not tell us whether we should use the full moments or the anomalous parts. In any static-model calculation of low-energy photoproduction, the amplitude is proportional to the full magnetic moments.[10] If one uses the full moments, the first solution for the $(D/F)_{PB\bar{B}}$ ratio yields the well-known $SU(6)$ result

$$\mu(p)/\mu(n)=-\tfrac{3}{2}\,. \qquad (10)$$

[6] See, for example, the relevant discussion in M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965).

[7] A. Pais, Phys. Rev. Letters **12**, 632 (1964).

[8] S. Glashow and R. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[9] See, for example, L. M. Brown, Phys. Rev. Letters **14**, 836 (1965).

[10] R. Dashen, Phys. Letters **11**, 89 (1964).

The third solution in Eq. (8) gives the unacceptable result $\mu(p)=0$.

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Notes added in manuscript.

1. Since this paper was written, we have become aware of an important paper by S. Coleman [Phys. Letters **19**, 144 (1965)] in which the scheme of Dashen and Gell-Mann (Ref. 2) is shown to be self-contradictory. The scheme proposed in the present paper does not suffer from any of the troubes underlined by Coleman.

2. The apparent ambiguity in the work of Fubini *et al.* (Ref. 4) has bothered other authors as well. (See, e.g., S. Okubo, [University of Rochester report (unpublished)]. The very simple procedure used above, which effectively resolves the point, has been noticed independently by J. D. Bjorken and W. I. Weisberger (private communication).

APPENDIX

In this Appendix we shall give some of the details of the calculation that yields the values of the D/F ratio displayed in Eq. (8). To this end, we write Eq. (8) in the slighly altered form

$$\langle \mathbf{p}'|i[Q^i,\phi^j(0)]|\mathbf{p}\rangle$$

$$=\lim_{q'\to 0} q'^{\nu}\int d^4x\, e^{iq'\cdot x}\langle \mathbf{p}'|[J_\nu{}^i(x),\phi^j(0)]\theta(x_0)|\mathbf{p}\rangle$$

$$-\frac{c}{\mu^2}\frac{1}{\mu^2-(p'-p)^2}\lim_{q'\to 0}T^{ij}. \qquad (\text{A1})$$

$$i,j=0,1,2,\cdots 8\,.$$

Here Q^i is a nonet of axial charges corresponding to the axial current $J_\nu{}^i$ and ϕ^j the nonet of pseudoscalar fields. We use the generalized PCAC hypothesis

$$\partial^\nu J_\nu{}^i=c\phi^i \qquad (\text{A2})$$

so that T^{ij} is the off-mass-shell ϕ^j+baryon octet $\to \phi^i$ $+$baryon-octet scattering amplitude. We shall suppress the baryon charge and spin indices.

Our first task is to explicitly demonstrate the cancellation of the ill-defined terms in the $q'\to 0$ limit. On introducing a complete set of intermediate states in the first term on the right-hand side of (A1) and performing the space-time integral, we find that the contribution of each intermediate state vanishes in the $q'\to 0$ limit save for the mass-degenerate baryon-octet states. With an implicit sum over intermediate spin and charge indices we thus obtain

$$q'^{\nu}\int d^4x\, e^{iq'\cdot x}\langle \mathbf{p}'|[J_\nu{}^i(x),\phi^j(0)]\theta(x_0)|\mathbf{p}\rangle$$
$$=iq'^{\nu}\Bigg[\frac{1}{2E(\mathbf{p}'+\mathbf{q}')}\frac{\langle \mathbf{p}'|J_\nu{}^i(0)|\mathbf{p}'+\mathbf{q}'\rangle\langle \mathbf{p}'+\mathbf{q}'|\phi^j(0)|\mathbf{p}\rangle}{q_0'+E(\mathbf{p}')-E(\mathbf{p}'+\mathbf{q}')}$$
$$-\frac{1}{2E(\mathbf{p}-\mathbf{q}')}\frac{\langle \mathbf{p}'|\phi^j(0)|\mathbf{p}-\mathbf{q}'\rangle\langle \mathbf{p}-\mathbf{q}'|J_\nu{}^i(0)|\mathbf{p}\rangle}{q_0'-E(\mathbf{p})+E(\mathbf{p}-\mathbf{q}')}\Bigg]+O(q_\lambda)\,. \quad \text{(A3)}$$

We write

$$\langle \mathbf{p}'|\phi^j(0)|\mathbf{p}\rangle=[\mu^2-k^2]^{-1}\bar{u}(p')\gamma_5K^j(k^2)u(p)\,, \quad \text{(A4)}$$

in which the form factor $K^j(k^2)$ is a matrix in charge space and $k=p'-p$. Then, in accordance with the PCAC condition (A2), we have

$$\langle \mathbf{p}|J_\nu{}^i(0)|\mathbf{p}\rangle=-i(c/2M\mu^2)\bar{u}(p)\gamma_\nu\gamma_5K^i(0)u(p)\,. \quad \text{(A5)}$$

In the $q'\to 0$ limit the denominators occurring in Eq. (A3) become simply $2p_\mu'q'^\mu$ and $2p_\mu q'^\mu$. Accordingly, the right-hand side of this equation may be written as

$$(c/2M\mu^2)[\mu^2-k^2]^{-1}\bar{u}(p')\times\Bigg[\gamma\cdot q'\gamma_5K^i(0)\left(\frac{M+\gamma\cdot p'}{2p'\cdot q'}\right)\gamma_5K^j(k^2)$$
$$-\gamma_5K^j(k^2)\left(\frac{M+\gamma\cdot p}{2p\cdot q'}\right)\gamma\cdot q'\gamma_5K^i(0)\Bigg]u(p)+O(q_\lambda)$$
$$=(c/2M\mu^2)[\mu^2-k^2]^{-1}\bar{u}(p')\times\Bigg[\left(1-\frac{M\gamma\cdot q'}{p'\cdot q'}\right)K^i(0)K^j(k^2)$$
$$+\left(1-\frac{M\gamma\cdot q'}{p\cdot q'}\right)K^j(k^2)K^i(0)\Bigg]u(p)+O(q_\lambda)\,. \quad \text{(A6)}$$

The scattering amplitude T^{ij} is well-defined as q' vanishes except for its baryon pole term $T_{\text{pole}}{}^{ij}$. We write

$$T^{ij}=T_{\text{pole}}{}^{ij}+\left(\frac{1}{2M}\right)\bar{u}(p')\bar{A}^{ij}u(p)\,. \quad \text{(A7)}$$

This pole term,

$$T_{\text{pole}}{}^{ij}=\bar{u}(p')\Bigg[\gamma_5K^i(q'^2)\frac{1}{M-\gamma\cdot(p'+q')}\gamma_5K^j(q^2)$$
$$+\gamma_5K^j(q^2)\frac{1}{M-\gamma\cdot(p-q')}\gamma_5K^i(q'^2)\Bigg]u(p)$$
$$=-\bar{u}(p')\Bigg[K^i(0)K^j(k^2)\frac{\gamma\cdot q'}{2p'\cdot q'}$$
$$+K^j(k^2)K^i(0)\frac{\gamma\cdot q'}{2p\cdot q'}\Bigg]u(p)+O(q_\lambda)\,, \quad \text{(A8)}$$

precisely cancels the terms in (A6) which are ambiguous in the $q'\to 0$ limit. Thus the limit in (A1) is well defined, and we may write this relation as

$$\bar{u}(p')[K^i(0)K^j(k^2)+K^j(k^2)K^i(0)]u(p)$$
$$=\bar{u}(p')\bar{A}^{ij}(q'=0)u(p)$$
$$+(2M\mu^2/c)[\mu^2-k^2]\langle \mathbf{p}'|i[Q^i,\phi^j(0)]|\mathbf{p}\rangle\,. \quad \text{(A9)}$$

The evaluation of the equal-time commutator of the axial charge with the pseudoscalar field can be performed only with the aid of some specific model. We shall avoid this evaluation by analytically continuing (A9) to the unphysical point $k^2=\mu^2$ where this term does not contribute. We shall also neglect the variation of the form factor $K^i(k^2)$ and the amplitude A^{ij} in the region $0\leqslant k^2\leqslant\mu^2$. Accordingly, we find

$$K^i(0)K^j(0)+K^j(0)K^i(0)=\bar{A}^{ij}(q'=0)\,. \quad \text{(A10)}$$

This relation forms the basis of our calculation of the D/F ratio. We assume that the amplitude $\bar{A}^{ij}$ is dominated by the exchange of systems in the s and u channels that transform as a decuplet and by the exchange of systems in the t channel that transform as octets and singlets. It is only necessary to specify these $SU(3)$ transformation properties; the spin of these exchanged systems does not enter into the calculation.

The tensorial method will be used to illustrate the calculation. Although this method is perhaps somewhat clumsy, it has the virtue of being self-contained. The baryon octet-pseudoscalar octet coupling may be written as

$$\alpha\psi^\dagger{}_b{}^a\psi_c{}^b\phi_a{}^c+\beta\psi^\dagger{}_b{}^a\psi_a{}^c\phi_c{}^b \quad \text{(A11)}$$
$$a,b,c \text{ etc.}=1,2,3\,.$$

Here *lower* indices transform under the defining representation of $SU(3)$; upper indices transform as the complex conjugate of this representation. The conventional D/F ratio is then given by

$$D/F=(\alpha+\beta)/(\alpha-\beta)\,. \quad \text{(A12)}$$

We must also list the various couplings to the pseudoscalar octet that are assumed to dominate the amplitude A. In the s and u channels there is a decuplet Ψ_{abc} coupling

$$\Psi_{abc}\psi^\dagger{}_d{}^a\phi_e{}^b\epsilon^{cde}+\text{Hermitian adjoint}. \quad \text{(A13)}$$

An octet $\Phi_b{}^a$ and singlet χ are exchanged in the t channel. The coupling of this octet to the initial and final pseudoscalar wave functions ϕ and ϕ' is generally

$$\lambda\phi'{}_b{}^a\phi_c{}^b\Phi_a{}^c+\kappa\phi'{}_b{}^a\phi_a{}^c\Phi_c{}^b. \quad \text{(A14a)}$$

However, crossing symmetry of the amplitude A for zero four-momentum mesons demands that $\lambda=\kappa$, or that this coupling be pure D type. This pure D coupling leads to the assignment of the quantum numbers for this system that was noted in the text. The octet $\Phi_b{}^a$

baryon coupling is

$$\alpha'\psi^{\dagger}{}_{b}{}^{a}\psi_{c}{}^{b}\Phi_{a}{}^{c}+\beta'\psi^{\dagger}{}_{b}{}^{a}\psi_{a}{}^{c}\Phi_{c}{}^{b}, \quad \text{(A14b)}$$

and the remaining singlet couplings are

$$\phi'{}_{b}{}^{a}\phi_{a}{}^{b}\chi, \quad \psi^{\dagger}{}_{b}{}^{a}\psi_{a}{}^{b}\chi. \quad \text{(A15)}$$

These coupling terms operate in conjunction with the octet projection tensor

$$\langle\Phi_{b}{}^{a}\Phi^{\dagger}{}_{d}{}^{c}\rangle=\mathcal{P}_{bd}{}^{ac}=\delta_{d}{}^{a}\delta_{b}{}^{c}-\tfrac{1}{3}\delta_{b}{}^{a}\delta_{d}{}^{c} \quad \text{(A16)}$$

and the decuplet projection tensor

$$\langle\Psi_{abc}\Psi^{\dagger a'b'c'}\rangle=\mathcal{P}_{abc}{}^{a'b'c'}=\tfrac{1}{3}[\delta_{a}{}^{a'}\delta_{b}{}^{b'}\delta_{c}{}^{c'}+\cdots], \quad \text{(A17)}$$

where the omitted terms refer to the other 5 permutations of the indices *abc*. In terms of this notation, the constraint (A10) for the case of the pseudoscalar octet becomes (within traceless wave functions)

$$\begin{aligned}&\alpha^{2}\delta_{d'}{}^{a'}\mathcal{P}_{b'd}{}^{c'a}\delta_{b}{}^{c}+\alpha\beta\delta_{d'}{}^{a'}\mathcal{P}_{b'b}{}^{c'c}\delta_{d}{}^{a}+\beta\alpha\delta_{b'}{}^{c'}\mathcal{P}_{d'd}{}^{a'a}\delta_{b}{}^{c}\\&\quad+\beta^{2}\delta_{b'}{}^{c'}\mathcal{P}_{d'b}{}^{a'c}\delta_{d}{}^{a}+(a\leftrightarrow a',\, b\leftrightarrow b')\\&\quad=\mathcal{A}_{10}[\epsilon^{ec'a'}\mathcal{P}_{d'b'e}{}^{cae'}\epsilon_{e'db}+(a\leftrightarrow a',\, b\leftrightarrow b')\\&\qquad+\mathcal{A}_{8}[\alpha'\delta_{b'}{}^{a}\mathcal{P}_{bd'}{}^{a'c}\delta_{d}{}^{c'}+\beta'\delta_{b'}{}^{a}\mathcal{P}_{bd}{}^{a'c'}\delta_{d'}{}^{c}\\&\qquad+(a\leftrightarrow a',\, b\leftrightarrow b')]+\mathcal{A}_{1}[\delta_{b}{}^{a'}\delta_{b'}{}^{a}\delta_{d}{}^{c'}\delta_{d'}{}^{c}]. \quad \text{(A18)}\end{aligned}$$

Here the index pairs (a,b) and (c,d) refer to the initial meson and baryon, respectively; the corresponding final-state pairs are primed.

It is straightforward (but tedious) to work out (A18) in terms of various products of four Kronecker-delta symbols. There is one linear relation among these products for, since the indices take on only the values 1, 2 or 3, their completely antisymmetrical sum vanishes. That is,

$$\sum_{\text{perm}}\delta_{P}\delta_{a'}{}^{a}\delta_{b'}{}^{b}\delta_{c'}{}^{c}\delta_{d'}{}^{d}=0, \quad \text{(A19)}$$

where the sum extends over all 4! permutations of *abcd* and $\delta_{P}=+1(-1)$ for even (odd) permuations. This linear dependence reduces the number of such products that contribute within traceless wave functions to 8, which is, of course, precisely the number of $SU(3)$ invariants in the octet-octet scattering amplitude. Upon comparing coefficients of the independent combinations of Kronecker symbols that occur in (A18), or otherwise by making appropriate contractions, one finds the relations

$$\begin{aligned}\alpha^{2}-\tfrac{1}{3}(\alpha+\beta)^{2}&=-\tfrac{1}{6}\mathcal{A}_{10}+\mathcal{A}_{8}\alpha',\\2\alpha\beta-\tfrac{1}{3}(\alpha+\beta)^{2}&=-\tfrac{1}{6}\mathcal{A}_{10},\\\beta^{2}-\tfrac{1}{3}(\alpha+\beta)^{2}&=-(7/6)\mathcal{A}_{10}+\mathcal{A}_{8}\beta',\\\tfrac{1}{3}(\alpha+\beta)^{2}&=\tfrac{3}{2}\mathcal{A}_{10}-\tfrac{2}{3}\mathcal{A}_{8}(\alpha'+\beta')+\mathcal{A}_{1}.\end{aligned} \quad \text{(A20)}$$

We may also apply the constraint (A10) to the case where one of the pseudoscalar fields is a singlet, the other remaining an octet. In this case the only exchanged system that contributes to the amplitude $\bar{A}$ is the t channel octet. It is easily seen that the constraint then requires that the D/F ratio of the system exchanged in the t channel be identical with that of the pseudoscaler field, or that

$$\alpha'/\beta'=\alpha/\beta. \quad \text{(A21)}$$

With this additional condition, Eqs. (A20) have a solution only if

$$\alpha/\beta=0,\ \tfrac{1}{2},\quad\text{or}\quad 5. \quad \text{(A22)}$$

These values yield the D/F ratio quoted in Eq. (8) of the text.

The case in which $\partial^{\nu}J_{\nu}{}^{i}=cP^{i}$ is not proportional to the pseudoscalar field ϕ^{i}, can be treated in a similar manner. One arrives at a constraint identical in form to (A10) with only $K^{i}(0)$ replaced by a new matrix $K'^{i}(0)$,

$$K'^{i}(0)K^{j}(0)+K^{j}(0)K'^{i}(0)=\bar{A}^{ij}. \quad \text{(A23)}$$

If we take the index i to correspond to a singlet and the index j to correspond to an octet we conclude as above that

$$(D/F)_{\phi B\bar{B}}=(D/F)_{t\text{-channel exchange }B\bar{B}}. \quad \text{(A24)}$$

Then, interchanging the roles of i and j we obtain

$$(D/F)_{PB\bar{B}}=(D/F)_{t\text{-channel exchange }B\bar{B}}. \quad \text{(A25)}$$

Therefore,

$$(D/F)_{PB\bar{B}}=(D/F)_{\phi B\bar{B}} \quad \text{(A26)}$$

and Eqs. (A20) are not altered for this more general case. We thus find the same values for the (D/F) ratio as previously obtained. This was indicated in Eq. (9) of the text.

The procedures outlined above should enable the reader to work out the results on magnetic moments stated in the text. While there are obvious differences between the space-time kinematics encountered in the two situations, the $SU(3)$ analysis is essentially identical. The only nontrivial difference lies in the circumstance that the relevant equal-time commutator, in the electromagnetic case, must perforce be gleaned from some model, such as the quark model which leads to

$$[Q^{i}(x_{0}),\,\mathcal{J}_{\mu}{}^{j}(x)]=if^{ijk}J_{\mu}{}^{k}(x), \quad \text{(A27)}$$

$\mathcal{J}_{\mu}{}^{j}$ being the electromagnetic vector current. One is free, of course, to postulate that the commutator in (A27) represents a higher degree of truth than the model from which it emerged.