requires that

$$\tfrac{1}{2}eg\alpha=2\pi n_1\,,\quad \tfrac{1}{2}eg\beta=2\pi n_2\,,$$
$$eg/4\pi=n=n_1+n_2$$

and

$$n\alpha=n_1\,,\quad n\beta=n_2\,.$$

Notice, however, that if α, or β, is specified as an irreducible fraction, the integer n must be divisible by the denominator of that fraction. Thus, in order to use symmetrical potentials $\alpha=\beta=\frac{1}{2}$, the integer n must be even.

Conversations with Bruno Zumino were helpful in stimulating this closer examination of the theory.]

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Positive-Pion Production Asymmetry with Polarized Bremsstrahlung Near Second Resonance*

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The azimuthal asymmetry $\Sigma=(\sigma_\perp-\sigma_\parallel)/(\sigma_\perp+\sigma_\parallel)$ in π^+ photoproduction by linearly polarized bremsstrahlung was measured at photon energies from 475 to 750 MeV at 90° and 135° in the center-of-mass system. The experimental results show that even in this energy region, π^+ are produced predominantly in the plane of the magnetic vector.

I. INTRODUCTION

PHOTOPRODUCTION of pions from nucleons has been studied for over a decade and much information concerning the pion-nucleon system has been gathered in the first resonance region.[1–3] In the second resonance region, however, there is still much uncertainty. The usual experimental information consisted of an angular distribution expressed by a series expansion in $\cos\theta$, θ being the angle between the outgoing-pion and the incident-photon directions in the center-of-mass system. By examining the coefficients in the expansion, one can try to infer the total angular momentum J of the final state. For a given value of J, there are, however, two values of l with different parity. A method of identifying the parity of the state would be to measure the asymmetry in the photoproduction with linearly polarized photons, thus discriminating between electric and magnetic transitions.[4–7]

In the region of the second resonance, many partial-wave amplitudes contribute simultaneously to the process and the analysis is correspondingly involved and less reliable, making experiments sensitive to the interference terms of great interest. The differential-cross-section measurements allow no separation of the real and the imaginary parts of interference terms. However, using unpolarized photons, a measurement of the recoiling-nucleon polarization gives information on the imaginary part of the interference terms and, using linearly polarized photons, a measurement of production asymmetry is sensitive to the behavior of the real part of the interference terms. The recoiling-proton polarization in π^0 photoproduction has, of course, already been extensively studied,[8] but no corresponding study has been carried out in π^+ case. In the case of π^+ photoproduction, where the resonance behavior is more evident $(I=\frac{1}{2})$, and the angular distribution is already very complex, a different experimental approach may be of interest. Asymmetry measurements with polarized photons can, it is hoped, give an additional constraint and help in evaluating the relative importance of the different multipole contributions.

The present experiment was to study this region by measuring the asymmetry in the production of positive pions with respect to the polarization plane of the incident photons. It was an extension to a higher energy region of similar work done in this laboratory[9,10] at the first resonance region. At these energies, contamination due to pion pair production is unavoidable, if reasonable polarization of the photon beam is to be achieved. However, this background cannot alter any inference that we may draw from the results of this experiment, as explained later.

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[1] An extensive bibliography of experimental data on π^+ photoproduction is given in the paper by G. Höhler and W. Schmidt, Ann. Phys. (N. Y.) **28**, 34 (1964).
[2] An extensive bibliography of experimental data on π^0 data can be found in a paper by Ph. Salin, Nuovo Cimento **28**, 1294 (1963).
[3] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963).
[4] G. T. Hoff, Phys. Rev. **122**, 665 (1961).
[5] M. J. Moravcsik, Phys. Rev. **125**, 1088 (1962).
[6] H. A. Rashid, Nuovo Cimento **33**, 965 (1965).
[7] P. Stichel, Z. Physik **180**, 170 (1964).
[8] D. Lundquist, J. A. Allaby, and D. M. Ritson, International Symposium on Electron and Photon Interactions at High Energies, Hamburg, 1965 (unpublished).
[9] R. C. Smith and R. F. Mozley, Phys. Rev. **130**, 2421 (1963).
[10] R. E. Taylor and R. F. Mozley, Phys. Rev. **117**, 835 (1959).

In Sec. II, the experimental equipment and technique, especially with respect to self-consistency checks are described. Section III contains an account of data reduction and treatment of background processes. The results are discussed in Sec. IV. General conclusions are drawn in Sec. V.

II. EXPERIMENTAL PROCEDURE

In the following subsections, we present a brief description of the equipment used in this experiment. Detailed descriptions of the major apparatus can be found elsewhere, as will be indicated. Data collection and reduction including background corrections are also treated in separate sections.

A. Polarized Photon Beam

A bremsstrahlung beam was produced by the well-collimated electron beam from the Stanford Mark III electron linear accelerator. A very thin (0.003-in.) aluminum foil was used as the radiator. The bremsstrahlung was collimated to the order of mc^2/E_0 so that only a limited portion of the bremsstrahlung radiation cone was accepted (E_0 is the electron beam energy and m is the mass of the electron). The off-axis bremsstrahlung beam is known to be polarized with respect to the emission plane of the photon. The technique of achieving such a polarized beam and the method of calculating the amount of polarization $[P=(N_\perp - N_\parallel)/(N_\perp + N_\parallel)]$ has been described elsewhere.[9] P defines the amount of linear polarization of the photon beam; $N_\perp$ ($N_\parallel$) refers to the number of photons whose electric vector is perpendicular (parallel) to the emission plane as defined by the propagation vector and the incident electron momentum vector. The photon beam was monitored by an ionization chamber and the electron beam by a secondary emission monitor (SEM). Normalization with respect to the integrated current of the ionization chamber was used in empty target and reversed magnetic field runs. Normalization with respect to the SEM integrated current was used for no radiator runs. The general experimental setup was the same as in Ref. 11.

B. The Production and Detection of Pions

The general layout of the detection apparatus is shown in Fig. 1. The liquid-hydrogen target cell had a cylindrical wall of thin stainless steel and was large enough so that the photon beam passing through the axis of the cylinder did not hit the wall. The end windows of the cell were also shielded by the entrance slit of the spectrometer. A dummy cell could be introduced into the beam to simulate an empty target. The pions produced in the horizontal plane were momentum-analyzed by a magnetic spectrometer[9] and detected by plastic scintillation counters, operating in coincidence.

[11] D. J. Drickey and R. F. Mozley, Phys. Rev. 136, B543 (1964).

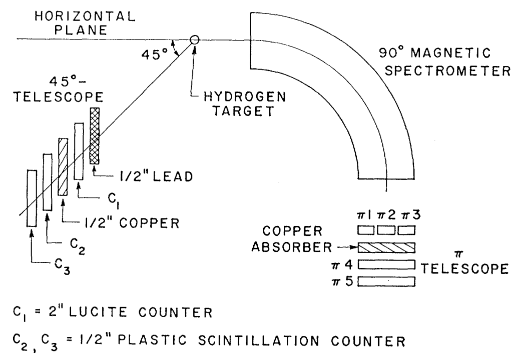

FIG. 1. General layout of the experimental apparatus.

Three momentum defining counters $\pi_{i(i=1,2,3)}$, each consisting of 5 in.×2 in.×0.25 in. of plastic scintillator, were placed in the approximate focal plane of the spectrometer, so that data on three mean photon energies were obtained at the same time. Below the momentum defining counters, copper absorbers were placed in front of two backing counters ($\pi4$ and $\pi5$) to stop protons. The plastic scintillation counters were viewed by 6810A phototubes and each of the anode pulses could be analyzed by a 256-channel pulse-height analyzer. The electronic block diagram is shown in Fig. 2. The relevant paths for only one of the momentum channels are shown in detail. The two backing counters, denoted by $\pi4$ and $\pi5$, were first put into a 10-nsec coincidence circuit, C45, whose outputs were in turn put into individual coincidence with $\pi1$, $\pi2$, and $\pi3$ momentum defining counters. Thus, a threefold coincidence was required for pion identification. The general background from the experimental area was thus reduced to a negligible amount. The counting rate due to electron contamination was found to be less than 1% of the toal

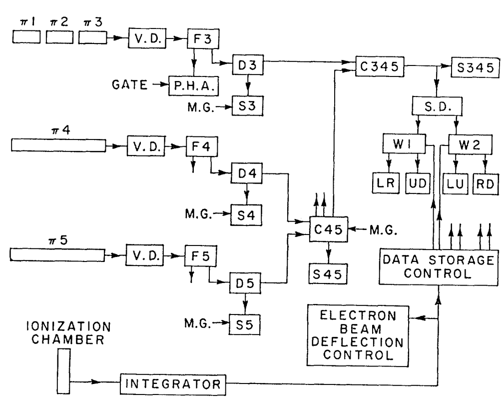

FIG. 2. Electronic block diagram: V. D. denotes a 50-Ω variable delay box, F, D, S, and C denote fanouts, discriminators, scalers, and coincidence circuits. M. G. stands for accelerator beam gate. S. D. is a scaler driver, whose two parallel outputs are routed through the switches W1 and W2 into scalers. LR denotes a scaler which stores the pulses from C345 when the electron beam is deflected to the left and right. Similar meanings hold for the other scalers.

counting rate and to be comparable with filled or empty-target rate.

C. 45° Telescope

A monitoring counter telescope counted fast charged particles from the hydrogen target. The counter telescope consisted of one Lucite Čerenkov counter and two plastic scintillation counters. A sheet of lead absorber, $\frac{1}{2}$ in. thick, was placed in front of the Čerenkov counter to reduce the low-energy particles. A copper absorber, $\frac{1}{2}$ in. thick, was interposed between the Čerenkov counter and the plastic scintillation counters. The telescope was placed at about 40° in the laboratory system and pointed at 45° to the horizontal plane toward the target as shown in Fig. 1. The excitation curve for the telescope is given in Fig. 3. The curve shows that the telescope counted mainly pions produced by photons of energies above 300 MeV. The pions produced at first resonance have a large azimuthal asymmetry with respect to photon polarization. The function of this telescope was therefore, to monitor instrumental asymmetry of production of particles whose production plane was at 45° to the horizontal. The photon beam itself was polarized along and perpendicular to the horizontal plane, so that a null measurement by this telescope would be a convincing proof of our beam handling procedure. More details are given in Sec. E.

D. Data Collection

The electron beam was focused into a spot about $\frac{1}{4}$ in. in size on a screen placed in vacuum at the end of the last accelerator section. The screen was then replaced by a radiator and the electron beam centered about the axis of two coaxial collimators, one 100 in. and the other 400 in. from the radiator.[11] The electron beam was then deflected horizontally or vertically by an angle 1.25 mc^2/E_0. For example, when the electron beam was horizontally left as shown in Fig. 4, the right portion of the bremsstrahlung cone was allowed to pass through the collimator. By cyclically deflecting the electron beam about the collimator axis, a cyclical variation of the linear polarization of the beam with respect to the horizontal plane is obtained.

Let us denote by $\sigma_\perp$ and $\sigma_\parallel$ the cross sections for the production of pions whose production planes are perpendicular and parallel to the photon electric vector, respectively. When the electron beam was deflected left or right the number of pions, N', emitted in the horizontal plane was proportional to $N_\perp\sigma_\perp+N_\parallel\sigma_\parallel$. When the electron beam was deflected up or down, the number of pions emitted, N, was proportional to $N_\perp\sigma_\parallel+N_\parallel\sigma_\perp$. The measured asymmetry R_m is defined by the following equation:

$$R_m=\frac{N'-N}{N'+N}=\frac{N_\perp-N_\parallel}{N_\perp+N_\parallel}\,\frac{\sigma_\perp-\sigma_\parallel}{\sigma_\perp+\sigma_\parallel}=P\Sigma\,,$$

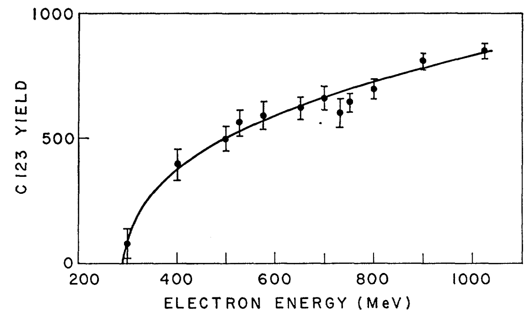

FIG. 3. Excitation curve for 45° telescope: C123 is the triple coincidence rate for the telescope for a fixed amount of collected charge in the ionization chamber. The line is drawn in to guide the eye.

where P is the polarization of the photon beam (a function of the energies of the electron and the interacting photon) and Σ is the asymmetry function containing the physics of the reaction.

The photon beam was monitored by an ionization chamber placed downstream from the hydrogen target. When the integrated charge reached a certain fixed value the integrator output actuated an electron beam deflection control and a data storage control. In this way, the electron beam was cyclically deflected to the left, down, right, and up about the collimator axis and at the same time, the data storage control caused the corresponding coincidence counts to be stored in accordance with the beam position. Through the switch W1, the data for left and right deflections of the electron beam were stored in one register and the data for up and down deflections of the electron beam in another. These numbers were denoted by N' and N, respectively. Through the switch W2, the data for left and up deflections of the electron beam were stored in one register and the data for right and down deflections in

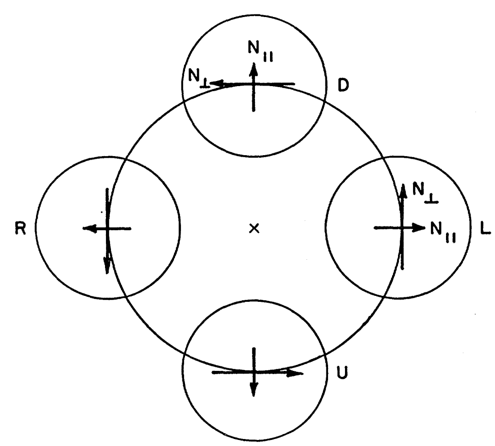

FIG. 4. Schematic diagram of electron beam deflections: the electron beam was first centered about the collimator axis X. On deflecting the electron beam to the left, the right portion of the bremsstrahlung cone passed through the collimator. The photon beam as well as the electron beam is pictured as incident into the plane of the figure.

TABLE I. Summary of instrumental checks: $\langle K\rangle_{av}$ is the average central energy of the photons. R_c is a measure of the centering of the photon beam as described in the text. r is the asymmetry value measured by the 45° telescope. The errors indicated were due to statistics only. The bracketed ratios were taken when the electron beam was deflected at 45° to the horizontal plane.

$\langle\theta\rangle_{c.m.}$ °	E_0 (MeV)	$\langle K\rangle_{av}$ (MeV)	R_m	R_c	r
90°	800	500	0.099±0.010	0.006±0.010	0.001±0.016
	1025	660	0.071±0.013	0.003±0.013	0.004±0.019
	1025	660	[−0.009±0.013]	[0.020±0.013]	[0.040±0.020]
135°	1025	550	0.043±0.013	0.006±0.013	−0.009±0.002
	1025	720	0.021±0.014	0.011±0.014	0.001±0.006

another. The ratio of these numbers R_c gave a check of the centering of the electron beam about the collimator axis. The numbers stored in these registers were checked against those in register S345, to guard against electronic-scaler bias and transcription error. A complete cycle of the deflections of the electron beam was generally made in a few minutes. Many cycles were made for each run and the numbers were added up for different runs with the same electron-beam settings.

E. Consistency Checks

In an experiment such as this one where one measures the ratio of numbers, many difficulties relating to absolute-cross-section measurements can be sidestepped. For instance, the absolute solid angle of the magnetic spectrometer, the target thickness, the various efficiencies of the counters, etc., need not be accurately known. There are, however, several reassuring cross-checks to make. One of these is a simultaneous collection of the data through switch W2 to check any electronic bias and electron beam centering. At the end of a run, the data would show an asymmetry for data collected through W1 and not those routed through W2. During 50% of the runs, the asymmetry of fast charged particles, mainly pions, emitted at 45° to the horizontal plane was also measured. For any deflection of the electron beam, the counting rate in the 45° telescope would be proportional to

$$\tfrac{1}{2}(N_\perp\sigma_\perp+N_{\parallel}\sigma_{\parallel}+N_\perp\sigma_{\parallel}+N_{\parallel}\sigma_\perp),$$

so that a measure of the asymmetry as the electron beam was cycled would be an indication of the systematic error in the experimental arrangement. The false asymmetry values r as measured by this telescope during various runs are given in Table I, column 6. These values were always consistent with zero. We assume, therefore, that the asymmetry observed in the pion counting rate in the magnetic spectrometer was due to photon polarization and not to spurious effects.

The above checks were made simultaneously with data collection. In one instance, however, the electron beam was cyclically deflected at 45° to the horizontal (see Fig. 4). In this case, the 45° telescope counting rate showed an asymmetry but not the pion telescope counting rate inside the spectrometer. The various ratios obtained for this run are given in the bracketed line in Table I. In the same table, the asymmetry, R_m, due to the polarized beam is given and compared with the ratios obtained for an unpolarized beam as denoted by R_c and r.

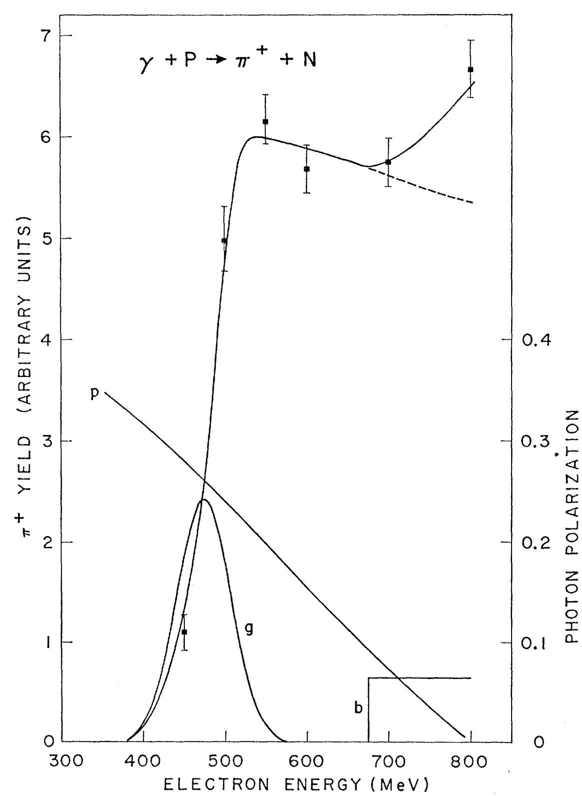

FIG. 5. π^+ excitation curve for $K=500$ MeV, $\theta=90°$. The curve g is the estimated gaussian acceptance of the magnetic spectrometer, b is a step approximating the double pion contribution as explained in the text. P is the calculated polarization of the photon beam whose end point energy is 800 MeV.

III. BACKGROUND SUBTRACTION

In order to have appreciable polarization in the photon beam, it is necessary to work considerably below the end point of the bremsstrahlung spectrum. The polarization in the bremsstrahlung beam is a sensitive function of the photon energy for a given end-point energy; the lower the photon energy, the higher the polarization. At the end point of the bremsstrahlung spectrum, the polarization is zero. We select with the spectrometer singly photoproduced π^+ corresponding to a photon energy which is at least 30–40% lower than the end-point energy. It is, therefore, no longer possible to effect a kinematic separation of single- and double-pion production. The electron-beam energy was, however, chosen to exclude the reaction $\gamma+p\rightarrow\pi^-+N^*$. According to recent experimental evidence,[12] most of the pairs are photoproduced through the channel $\gamma+p\rightarrow\pi^-+N^{*++}$ at these energies; therefore, it is not unreasonable to expect double-pion background to be small in our case. The counting rate with the current reversed in the magnetic spectrometer gave us an indication of this background. This counting rate as compared to positive pion counting rate ranged from 16.5% for the lower photon energy point to less than 1% for the higher energy points.

The size of total background due to double-pion production by photons of higher energy in the bremsstrahlung spectrum was determined from an analysis of the excitation curve. An excitation or yield curve is the counting rate for a fixed angle and momentum settings of the spectrometer, as the electron beam energy is increased from threshold to some maximum energy. In general, the electron energy was selected to be 30–40% higher than the chosen photon energy in single-pion production. The shape of the yield curve depended on the resolution of the spectrometer, the bremsstrahlung shape and the response of the ionization monitor.

Each point on the excitation curve was normalized to a fixed integrated charge in the ionization chamber. The response of the ionization chamber to the bremsstrahlung spectrum of end-point energies between 400–1000 MeV was directly determined in a separate calibration run. The response of the chamber was found to depend logarithmically on the end-point energy and to extrapolate to a cutoff energy of about 55 MeV. This energy response was then used together with Schiff's[13] bremsstrahlung formula to calculate the shape of the excitation curve. A Gaussian photon acceptance for the single-π production was assumed, and a step function for double-pion production starting at 200 MeV above the central of the Gaussian was used. Within the limit of our accuracy the assumption of a step function for double-pion production is almost indistinguishable from the assumption of phase-space production with constant matrix elements. The value of this step relative to the normalized Gaussian was obtained by a χ^2 fit to the excitation curve. An example of such a fit is shown in Fig. 5. This particular point had a large background from double-pion production. The percentage counting rates δ due to this background as calculated above are shown in Table II.

[12] J. V. Allaby, H. L. Lynch, and D. M. Ritson, International Symposium on Electron and Photon Interactions at High Energies, Hamburg, 1965 (unpublished), and CEA data at same conference.

[13] H. W. Koch and J. W. Motz, Rev. Mod. Phys. 31, 920 (1959).

TABLE II. Summary of results. $\langle P\rangle$ gives the average calculated polarization of the photon beam. Σ is the measured asymmetry function, $(\sigma_\perp-\sigma_\parallel)/(\sigma_\perp+\sigma_\parallel)$, after correction for double pion production background δ, as described in the text. $\Delta\Sigma_S$ is the maximum possible systematic error introduced in Σ by the presence of double pion background. $\Delta\Sigma$ gives the error in Σ, taking into account only counting statistics and the uncertainty in the determination of δ.

E_0 (MeV)	K_0 (MeV)	$\langle P\rangle$	δ	Σ	$\Delta\Sigma$	$\Delta\Sigma_S$
			$\theta=90°$			
800	475	0.262	0.248	0.526	0.108	0.038
	500	0.241	0.280	0.585	0.128	0.036
	535	0.212	0.133	0.417	0.106	0.014
1000	545	0.261	0.335	0.523	0.103	0.071
	570	0.247	0.305	0.381	0.073	0.062
	600	0.230	0.275	0.416	0.080	0.054
1000	620	0.219	0.291	0.430	0.103	0.047
	660	0.195	0.238	0.412	0.095	0.034
	705	0.167	0.110	0.486	0.097	0.015
1025	620	0.225	0.326	0.431	0.138	0.061
	660	0.202	0.279	0.582	0.155	0.049
	705	0.177	0.141	0.317	0.144	0.023
1000	700	0.170	0.045	0.411	0.068	0.007
	735	0.149	0.029	0.408	0.071	0.003
	765	0.132	0.065	0.380	0.098	0.005
			$\theta=135°$			
1025	510	0.282	0.376	0.249	0.126	0.087
	550	0.262	0.408	0.150	0.129	0.085
	595	0.238	0.274	0.262	0.139	0.057
1025	655	0.205	0.167	0.194	0.148	0.032
	725	0.165	0.137	0.186	0.158	0.020
	780	0.133	0.042	0.016	0.196	0.008

The measured asymmetry R_m, given in Table III, has to be corrected for this background. The effects of this background on the data are twofold. Firstly, it tends to

TABLE III. Measured values of asymmetry $R_m=(N'-N)/(N'+N)$. Here E_0 is the electron beam energy and K_0 is the central photon energy for each of the three momentum intervals accepted by the spectrometer. $\theta_{\rm lab}$ is the pion-production angle with the incident photon direction in the laboratory system. The errors indicated were due to statistics only.

E_0 (MeV)	$\theta_{\rm lab}°$	K_0 (MeV)	R_m
800	68.0°	475	0.110±0.018
		500	0.110±0.017
		535	0.078±0.019
1000	66.8°	545	0.102±0.010
		570	0.072±0.009
		600	0.075±0.010
1000	64.6°	620	0.073±0.014
		660	0.065±0.013
		705	0.073±0.014
1025	64.6°	620	0.073±0.023
		660	0.092±0.022
		705	0.049±0.022
1000	62.6°	700	0.067±0.011
		735	0.059±0.010
		765	0.047±0.012
1025	115.2°	510	0.051±0.024
		550	0.028±0.023
		595	0.049±0.025
1025	110.9°	655	0.034±0.026
		725	0.027±0.023
		780	0.002±0.025

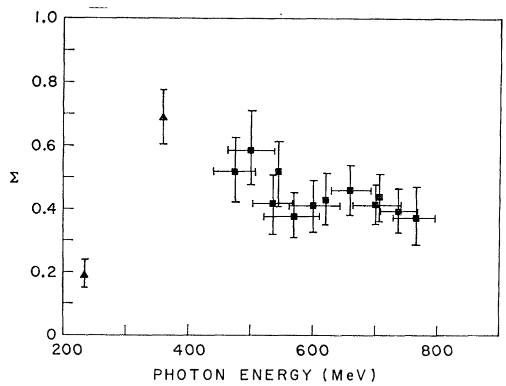

FIG. 6. The asymmetry function $\Sigma=(\sigma_\perp-\sigma_\parallel)/(\sigma_\perp+\sigma_\parallel)$ at 90° in the center-of-mass system. The error flags are explained in the text. The triangular points (▲) are taken from Ref. 9.

lower the measured asymmetry and secondly, it introduces an unknown systematic bias. The corrected asymmetry R_0 is easily shown to be given by

$$R_0\equiv\langle P\rangle\Sigma=R_m(1+\delta)-\delta\langle p\rangle_B\alpha,$$

where α is a possible asymmetry in the photoproduction of double pions from hydrogen. We assigned to it a maximum possible value of ±1. Therefore, $\delta\langle p\rangle_B/\langle P\rangle$ gives the maximum possible systematic error of Σ, where $\langle p\rangle_B$ is the effective polarization of the beam contributing to this background. $\langle p\rangle_B$ at the end point of the bremsstrahlung is generally less then 20% of $\langle P\rangle$ in the single-pion-production energy region. The error $\Delta\Sigma$ tabulated in Table II is, however, calculated from statistics and an assigned error of 0.5δ in the subtraction, and does not include this possible systematic error which is given separately in Table II. The asymmetry Σ is plotted in Figs. 6 and 7. The previously reported points[9] are also indicated by triangles. The abscissa uncertainty represents the full width at half-maximum of the Gaussian acceptance of the photon energy obtained from the excitation curve.

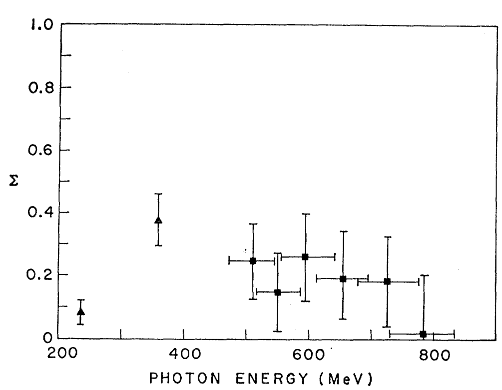

FIG. 7. The asymmetry function $\Sigma=(\sigma_\perp-\sigma_\parallel)/(\sigma_\perp+\sigma_\parallel)$ at 135° in the center-of-mass system. The error flags are explained in the text. The triangular points (▲) are taken from Ref. 9.

IV. DISCUSSION

It is now well known that between the photon energies from threshold to about 450 MeV, conventionally called the first resonance, the $M1$ amplitude completely dominates over other amplitudes. This is most convincingly demonstrated by Smith and Mozley[9] in their asymmetry measurement of pion production by polarized photons. Their measured values reached the theorectical value of $\frac{3}{5}$ for a pure $M1$ transition at 90° in the c.m. system. As the photon energy is increased, the picture becomes more complicated, and there is no single dominating amplitude.

There are, however, two recent attempts to fit the experimental photoproduction data from threshold to 800 MeV. Höhler and Schmidt[1] used only pole terms of the dispersion theory and the resonant magnetic dipole term. They were able to fit the data below 400 MeV rather well. At higher energies, however, they concluded that the theoretical expressions are too complex and contain too many terms for a comparison with the existing data to be meaningful. Another approach is that of the isobaric model of Gourdin and Salin.[3] Using this model, Salin[2] was able to fit the existing data up to 800 MeV surprisingly well. He also found that the first and second resonances are mainly through P and D

TABLE IV. The theoretical values of asymmetry Σ for single amplitude transitions. The notation is the same as in Chew, Goldberger, Low, and Nambu.[a] R is the matrix element for the retardation term. Note the difference in sign between electric and magnetic transitions. A similar table was given in Ref. 6 in which there was an error in M_{2+}.

Pion state	J	Photon state	Transition amplitude	$\Sigma=(\sigma_\perp-\sigma_\parallel)/(\sigma_\perp+\sigma_\parallel)$
S	$\frac{1}{2}$	$E1$	E_{0+}	0
P	$\frac{1}{2}$	$M1$	M_{1-}	0
P	$\frac{3}{2}$	$E2$	E_{1+}	$-\dfrac{\sin^2\theta}{1+\cos^2\theta}$
P	$\frac{3}{2}$	$M1$	M_{1+}	$\dfrac{3\sin^2\theta}{5-3\cos^2\theta}$
D	$\frac{3}{2}$	$E1$	E_{2-}	$-\dfrac{3\sin^2\theta}{5-3\cos^2\theta}$
D	$\frac{3}{2}$	$M2$	M_{2-}	$\dfrac{\sin^2\theta}{1+\cos^2\theta}$
D	$\frac{5}{2}$	$E3$	E_{2+}	$\dfrac{-4\sin^2\theta(6-5\sin^2\theta)}{5+6\cos^2\theta+5\cos^4\theta}$
D	$\frac{5}{2}$	$M2$	M_{2+}	$\dfrac{\sin^2\theta(6-5\sin^2\theta)}{1+6\cos^2\theta-5\cos^4\theta}$
...	...	...	R	-1

[a] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

TABLE V. The expected behavior of asymmetry Σ for transitions involving more than one amplitude.

Transition amplitude	$\Sigma=(\sigma_\perp-\sigma_{\|\|})/(\sigma_\perp+\sigma_{\|\|})$
E_{2-}, M_{1+}	$\dfrac{3\sin^2\theta(\|M_{1+}\|^2-\|E_{2-}\|^2)}{(5-3\cos^2\theta)(\|M_{1+}\|^2+\|E_{2-}\|^2)+4\,\mathrm{Re}\|M_{1+}{}^*E_{2-}\|\cos\theta}$
E_{2-}, E_{0+}	$\dfrac{3\sin^2\theta(2\,\mathrm{Re}\|E_{0+}{}^*E_{2-}\|-\|E_{2-}\|^2)}{2\|E_{0+}\|^2+\|E_{2-}\|^2(5-3\cos^2\theta)-2\,\mathrm{Re}\|E_{0+}{}^*E_{2-}\|(1-3\cos^2\theta)}$
E_{2-}, M_{1-}	$\dfrac{-3\|E_{2-}\|^2\sin^2\theta}{2\|M_{1-}\|^2+\|E_{2-}\|^2(5-3\cos^2\theta)-4\,\mathrm{Re}\|E_{2-}{}^*M_{1-}\|\cos\theta}$
M_{1+}, M_{1-}	$\dfrac{3\sin^2\theta[\|M_{1+}\|^2+2\,\mathrm{Re}\|M_{1+}{}^*M_{1-}\|]}{2\|M_{1-}\|^2+\|M_{1+}\|^2(5-3\cos^2\theta)+2(1-3\cos^2\theta)\,\mathrm{Re}\|M_{1+}{}^*M_{1-}\|}$
M_{1+}, E_{0+}	$\dfrac{3\|M_{1+}\|^2\sin^2\theta}{2\|E_{0+}\|^2+\|M_{1+}\|^2(5-3\cos^2\theta)+4\,\mathrm{Re}\|E_{0+}{}^*M_{1+}\|\cos\theta}$
E_{2-}, M_{1+}, R	$\dfrac{\sin^2\theta[3(\|M_{1+}\|^2-\|E_{2-}\|^2)-2R^2(1-\cos\theta)+2\,\mathrm{Re}(M_{1+}{}^*R)-2\,\mathrm{Re}\|E_{2-}{}^*R\|(2-3\cos\theta)]}{(5-3\cos^2\theta)(\|M_{1+}\|^2+E_{2-}\|^2)+2R^2\sin^2\theta(1-\cos\theta)+4\,\mathrm{Re}\|M_{1+}{}^*E_{2-}\|\cos\theta-2\,\mathrm{Re}\|M_{1+}{}^*R\|\sin^2\theta+2\,\mathrm{Re}\|E_{2-}{}^*R\|\sin^2\theta(2-3\cos\theta)}$

waves, respectively, but that one could not neglect the background S and P waves. Our results also tend to support this view.

Recently pion-nucleon scattering-data analysis has shown a complexity in the scattering amplitudes hitherto unsuspected.[14,15] Besides the well-known D-wave amplitude, a large $J=\frac{1}{2}$, $T=\frac{1}{2}$ phase shift was noticed.[14] More recent work shows a large S-wave background as well, at about the same energy.[15] These phase shifts have a large inelasticity parameter associated with them, and they do not show up conspicuously in any experimental data. These findings therefore all tend to make any simple approach to pion-photoproduction analysis unrealistic. Fortunately, there is in progress an attempt[16] to do an analysis similar to that done for pion-nucleon scattering and our type of data would therefore be very useful in distinguishing between the different solutions.

Our experimental results are shown in Figs. 6 and 7, together with the measurement of Smith and Mozley.[9] At 90° the asymmetry decreases only slightly from that measured near the first resonance. The lower asymmetries observed at 135° are mainly due to the $\sin^2\theta$ dependence of Σ. It is obvious at once that the predominant state cannot be $D_{3/2}$, since the asymmetry would have an opposite sign. This is, of course, not surprising since the existing data on angular distributions already show a complex structure very different from $5-3\cos^2\theta$.

[14] L. D. Roper, Phys. Rev. Letters **12**, 340 (1964); P. Anvil, C. Lovelace, A. Donnachie, and A. T. Lea, Phys. Letters **12**, 76 (1964).

[15] B. H. Bransden, P. J. O'Donnel, and R. G. Moorhouse, Phys. Rev. **139**, B1566 (1965).

[16] P. Finkler, University of California Radiation Laboratory, Livermore, California (private communication).

Since there is a possibility of systematic error associated with the subtraction of the double-pion background, we wish to discuss this point in greater detail. The double-pion-background subtraction is made from a fit to the yield curve as described in the previous section. The asymmetry of the background will depend on the mechanism of the double-pion production in this energy range, and several investigations[12] have already shown that a two-body channel through Δ^+ (1238 MeV) is strongest in this energy range. A separate experiment[17] to look for the asymmetry in this channel has produced little or no evidence of departure from isotropy. Moreover, the peak end of the bremsstrahlung is chosen to exclude Δ^+ production, and the remaining background from the 3-body process presumably has no definite plane of production. The amount of background at each point is given in Table II. The maximum possible systematic error, i.e., assuming 100% asymmetry in pion-double production, is also tabulated. Therefore, even in this most unlikely case, the effect of this background is negligible.

In Table IV, we present the asymmetry Σ as expected from different amplitudes. These energy-dependent amplitudes are denoted by $M_{l\pm}$ or $E_{l\pm}$, magnetic or electric transitions leading to final states of orbital angular momentum l and total angular momentum $l\pm\frac{1}{2}$. It is clear that electric and magnetic transitions leading to the same final states have different signs for the asymmetry Σ. However, the presence of important contributions from pole terms in π^+ production makes the interpretation more uncertain.

The importance of the pole term is seen in the angular

[17] R. Morrison, HEPL, Stanford University (private communication).

distribution measurements. The general trend[2] is that of a large forward peaking, first observed just beyond the (3,3) region, followed by a striking minimum at a small angle. This minimum has been interpreted as due to the interference of a "retardation-like" term and other terms of low angular momentum. The data at small angles,[18] being completely dominated by the retardation-like term, are therefore not sensitive to the resonating state, presumably $D_{3/2}$. At 90° and backward angles, the contribution of the retardation term can be expected to be less important, so that other terms may be separated out. It may be pointed out that a retardation-like term contributes only along the electric vector of the photon, so that if it predominates over other terms, a negative azimuthal asymmetry should be observed. Unfortunately, the interference of this term with other background terms may also drastically modify this asymmetry value.

The interference effects of the retardation term with E_{2-} and M_{1+} are given in Table V. The presence of only the retardation term and the E_{2-} term (leading to $D_{3/2}$) cannot explain the sign of our data in this energy range. Indeed, both E_{2-} and R contribute negatively and if E_{2-} is assumed to be in resonance, the interference term should vanish at the resonance energy, so that the observed behavior of Σ is not consistent with the simple scheme of a D-wave resonance together with a retardation term. A $D_{3/2}$ final state may be reached through an $M2$ transition also, and a strong $M2$ transition is consistent with our data, since $M2$ transition gives the correct sign. Enhanced M_{1-} transition leading to $P_{1/2}$ state might help to explain the data because of the interference with the large M_{1+} residual amplitude at these energies.

In summary, the presence of a strong S- and P-wave background is indicated by our data. Our data also show possibly a large amount of $M2$ transition. However, the number of transition amplitudes present seems to be large, so that a systematic fitting of all relevant data on photoproduction may be necessary in order to separate out the different contributions.

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[18] R. L. Walker, in *Proceedings of the Tenth Annual International Conference on High-Energy Physics at Rochester, 1960*, edited by E. C. G. Sudarshan, J. H. Tincot, and A. C. Melissins (Interscience Publishers, Inc., New York, 1961).