# Theoretical Investigation of the Nuclear Properties of $Pr^{143}$ in the Unified Model*

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Experimental data on the low-energy nuclear properties of $Pr^{143}$ are analyzed by using the intermediate-coupling approach in the unified nuclear model. For this purpose it is assumed that the last odd proton, having available the $1g_{7/2}$ and $2d_{5/2}$ states, is coupled to the quadrupole vibrations of the surface of the even-even core of the nucleus. The resulting Hamiltonian of the coupled system is diagonalized including all states with up to three phonons in the space of the basis eigenvectors of the uncoupled system. With reasonable values for the coupling strength and for the "effective" spacing in energy between $g_{7/2}$ and $d_{5/2}$ states, the calculated energy levels for the low-lying levels of $Pr^{143}$ and their spins and parities are in fairly good agreement with the experimental data. Further, the wave functions obtained from our model are used to calculate the electric and magnetic transition probabilities, and these results are also found to be in good agreement with the experimentally observed values. Finally, discussions are given of the validity of the approximations used in the present analysis and also of the circumstances in which more extensive experimental investigations are needed.

## I. INTRODUCTION

As is well known, the low energy properties of the odd-$A$ nuclei in the regions $A<150$ and $190<A<222$, excluding the nuclei immediately adjacent to the closed shells and in the regions of the light nuclei, are very complex and in most cases are not well understood. Recently interest has been shown in understanding the properties of the odd-$A$ nuclei whose even-even neighboring nuclei exhibit a vibrational spectrum in the abovementioned regions. During the last few years several investigations on the low-lying levels in $Pr^{143}$ populated by the $\beta$ decay of 33-h $Ce^{143}$ have been reported.[1–8] In particular, the properties of the 57-keV level, the first excited state of $Pr^{143}$, have been studied in greater detail. The most recent extensive investigations are due to Gopinathan *et al.*[1] Although there are some inconsistencies in the spin assignments of the level scheme by the various investigators, their results are more or less in reasonable agreement.

The aim of the present work, based on our preliminary report,[9] is an attempt to understand the main features of the above experiments. For this purpose we intend to apply the theory of the intermediate-coupling scheme of the collective model briefly outlined by Bohr and Mottelson[10] and later elaborated by one of us[11] to $Pr^{143}$. This intermediate-coupling approach in the past has been examined by various authors[12–20] who have met with considerable success in accounting for many nuclear properties. In our present investigation of the $Pr^{143}$ nucleus with 59 protons and 84 neutrons we may consider it as a nearly spherical nucleus with its even-even $Ce^{142}$ core susceptible to quadrupole vibrations. Since the excited states of $Ce^{142}$ show vibrational character, the properties of the low-lying energy levels in $Pr^{143}$ may be interpreted by assuming that the odd proton, having available both the $1g_{7/2}$ and $2d_{5/2}$ states, is neither weakly nor strongly coupled to the collective surface vibrations of the even-even $Ce^{142}$ core. Given the above picture, the intermediate coupling approach in the unified nuclear model should be applicable to $Pr^{143}$.

Three kinds of parameters enter into our calculations: (1) $\hbar\omega$, phonon energy; (2) $\xi$, the dimensionless coupling strength between the odd proton and the core vibrations; and (3) $\epsilon$, the "effective" spacing of the $d_{5/2}$ ang $g_{7/2}$ levels. The first of these parameters, $\hbar\omega$, is taken from the spectrum of the neighboring even-even $Ce^{142}$ nucleus which forms the core of $Pr^{143}$. The other two parameters remaining, namely, $\xi$ and $\epsilon$, are taken as adjustable, and they are varied within reasonable limits to obtain the best fit to the experimental data.

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In Sec. II we give an outline of the mathematical model and of the basic formulas needed in our calculations. Section III contains the results of our analysis and their comparison with the available experimental data. Finally, a brief discussion of our results and also of the circumstances in which more extensive experimental investigations are needed is given in Sec. IV.

## II. FORMALISM

### A. Theory of Intermediate Coupling in the Unified Model

The theory of intermediate coupling in the unified nuclear model is given in Refs. 10 and 11 so that the details are omitted here. However, for clarity, in the following paragraphs, we present a brief description of the model and the general formula for the evaluation of the matrix elements for the surface particle interaction used in our calculations.

The total Hamiltonian for the system of core plus extra nucleon consists of three parts:

$$H=H_s+H_p+H_{\rm int}\,, \tag{1}$$

where

(i) $H_s$ is the Hamiltonian associated with the quadrupole vibrations of the surface of the core. It is equivalent to that of a system of harmonic oscillators, and is given by

$$H_s=\sum_\mu \{\tfrac{1}{2}B|\dot{\alpha}_\mu|^2+\tfrac{1}{2}C|\alpha_\mu|^2\}\,, \tag{2}$$

where $B$ is the mass parameter, $C$ is nuclear deformability, and $\alpha_\mu$ are coordinates which describe the deformation of the core, and appear as the expansion parameters of the surface of the core defined by

$$R(\theta,\phi)=R_0[1+\sum_\mu \alpha_\mu Y_{2\mu}(\theta,\phi)]\,. \tag{3}$$

In the present treatment it is convenient to express $H_s$ in the quantized form

$$H_s=\frac{\hbar\omega}{2}\sum_\mu(b_\mu b_\mu{}^\dagger+b_\mu{}^\dagger b_\mu)\,, \tag{4}$$

where $\omega$ is the frequency of the surface oscillations and is given by

$$\omega=(C/B)^{1/2}\,, \tag{5}$$

$b_\mu$ and $b_\mu{}^\dagger$ are the annihilation and creation operators, respectively, for phonons of spin 2 with $z$ component $\mu$. These operators satisfy the usual boson commutation rules,

$$[b_\mu,b_\nu{}^\dagger]=\delta_{\mu,\nu}\,. \tag{6}$$

In the above representation $H_s$ is diagonal and has the eigenvalues

$$E_s=\hbar\omega[N+\tfrac{5}{2}]\,, \tag{7}$$

where $N=0,\,1,\,2,\,\cdots$

(ii) $H_p$ is the Hamiltonian for the odd nucleon in an effective average potential. In our case it is assumed to have two values; one corresponding to $g_{7/2}$ and the other to $d_{5/2}$ single-particle states.

(iii) $H_{\rm int}$ is the Hamiltonian for the particle-surface interaction, and is given by

$$H_{\rm int}=-k(r)\sum_\mu (\hbar\omega/2C)^{1/2}\{b_\mu+(-)^\mu b_{-\mu}{}^\dagger\}Y_{2\mu}(\theta,\phi)\,, \tag{8}$$

where $Y_{2\mu}(\theta,\phi)$ is the normalized spherical harmonic of the angular coordinates of the particle and $k(r)$ determines the coupling strength.

In order to study the predictions of the coupled system given by Eq. (1), we choose the eigenvectors of the uncoupled system as the basis for our space. These basis eigenvectors are denoted by $|\alpha j;NR;IM\rangle$ and they satisfy the following equation:

$$(H_s+H_p)|\alpha j;NR;IM\rangle =[\hbar\omega(N+\tfrac{5}{2})+E_j]|\alpha j;NR;IM\rangle\,. \tag{9}$$

Here $j$ is the single-particle angular momentum and $\alpha$ represents the radial quantum numbers of the particle such as $n$ and $l$; $N$ is the number of phonons of surface oscillations, each having an angular momentum of two units; $R$ is the total angular momentum of the surface; $\mathbf{I}=\mathbf{j}+\mathbf{R}$ is the total angular momentum of the system, $M$ its $z$ component; and $E_j$ is the energy of the single particle in the quantum state of angular momentum $j$.

The matrix elements of $H_{\rm int}$ is easily evaluated by means of Racah's algebra and the result is

$$\begin{aligned}\langle\alpha j;NR;IM|H_{\rm int}|\alpha' j';N'R';IM\rangle &=(-)^{R+I+1-j}k(\hbar\omega/2C)^{1/2}[(2j+1)(2R'+1)]^{1/2}\\ &\quad\times W(j'jR'R;2I)\langle lsj\|Y_2\|l'sj'\rangle\\ &\quad\times\langle NR\|b\|N'R'\rangle,\quad\text{for}\quad N'\rangle N\,,\end{aligned} \tag{10}$$

where $W(j'jR'R;2I)$ is the Racah coefficient and $k=\langle nl|k(r)|n'l'\rangle$. $\langle lsj\|Y_2\|l'sj'\rangle$ and $\langle NR\|b\|N'R'\rangle$ are the reduced matrix elements defined as in Ref. 11. Selection rules are: $\Delta N=1$, $\Delta R\leq 2$, $\Delta j\leq 2$ and $\Delta l=0$ or 2. The values of reduced matrix elements $\langle lsj\|Y_2\|l'sj'\rangle$ are easily evaluated by means of Racah's algebra, and those of $\langle NR\|b\|N'R'\rangle$ are given in Ref. 11 up to three phonons.

The eigenvalues and the expansion coefficients of the eigenvectors are obtained by diagonalizing the total Hamiltonian whose diagonal and off-diagonal matrix elements are given by Eqs. (9) and (10), respectively, in the space of the above defined basis eigenvectors. The eigenvectors of the total Hamiltonian are then a linear combination of the basis eigenvectors and can be expressed as

$$|E;IM\rangle=\sum_{jNR}\langle j;NR;I|E\rangle|\alpha j;NR;IM\rangle\,, \tag{11}$$

where $\langle j;NR;I|E\rangle$ are the expansion coefficients, $E$ the eigenvalues, and the eigenvectors $|E;IM\rangle$ satisfy the following equation:

$$H|E;IM\rangle=E|E;IM\rangle\,. \tag{12}$$

### B. Electromagnetic Transitions

The diagonalizations of the total Hamiltonian described in the preceding subsection yield the wave functions that describe the nuclear energy levels of our model. These wave functions will now be used to calculate the electric and magnetic transitions. Only by such calculations and comparing these results with the experimental data, we will be able to test the validity of our assumed model.

The transition probability, $T(\lambda)$, for the emission of a photon of multipolarity $\lambda$ and frequency $\omega$ is given by[21]

$$T(\lambda)=\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{\omega}{c}\right)^{2\lambda+1}B(\lambda), \qquad (13)$$

where $B(\lambda)$ is the reduced transition probability. $B(\lambda)$ can be expressed in terms of the matrix elements of the multipole operator $M(\lambda,\mu)$ of order $\lambda$, $\mu$ between the initial state $|IM\rangle$ and the final state $|I'M'\rangle$, as

$$B(\lambda)=\frac{1}{(2I+1)}\sum_{MM'\mu}|\langle I'M'|M(\lambda,\mu)|IM\rangle|^2. \qquad (14)$$

For the coupled system consisting of a single particle and the quadrupole oscillations of the surface of the even-even core, the electric quadrupole operator[10] is

$$M_e(2,\mu)=[e_p-(Ze/A^2)]r^2Y_{2\mu}(\theta,\phi)+(3/4\pi)ZeR_0^2\alpha_{2\mu}^*, \qquad (15)$$

and the $\mu$ component of the magnetic dipole operator[10] expressed in the spherical tensor representation is

$$M_m(1,\mu)=(e\hbar/2Mc)(3/4\pi)^{1/2}[(g_l l_\mu+g_s s_\mu)+g_c R_\mu], \qquad (16)$$

where $e_p$, $g_l$, and $g_s$ refer to the charge, orbital, and spin $g$ factors of the particle; $g_c=Z/A$ is the $g$ factor of the core; $R_0$ is the nuclear radius, $Z$ the number of protons and $A$ the atomic number. It is worthwhile to note that the last term in each of the above expressions refers to the moment generated by the collective motion of the nucleons.

Using techniques similar to the ones used in calculating the matrix elements of $H_{\text{int}}$, we can easily evaluate the reduced transition probabilities between an initial state of spin $I$ to a final state of spin $I'$ by inserting the wave functions given by Eq. (11) into Eqs. (15) and (16). The result for the electric quadrupole transitions is

$$\underset{I\to I'}{B(E2)}=(2I'+1)\Big|\sum_{\substack{jNR\\ j'N'R'}}\langle j;NR;I|E\rangle\langle j';N'R';I'|E'\rangle \times\{\chi_1(-)^{I'+j-R}(2j'+1)^{1/2}\langle j'\|Y_2\|j\rangle W(jj'II';2R)\delta_{NN'}\delta_{RR'} +\chi_2(-)^{I-j}W(RR'II';2j)[(-)^{R'}(2R'+1)^{1/2}\langle N'R'\|b^+\|NR\rangle+(-)^R(2R+1)^{1/2}\langle N'R'\|b\|NR\rangle]\delta_{jj'}\delta_{ll'}\}\Big|^2, \qquad (17)$$

where

$$\chi_1=[e_p-(Ze/A^2)]\langle r^2\rangle,\quad \chi_2=(3/4\pi)(\hbar\omega/2C)^{1/2}ZeR_0^2,$$

and the reduced matrix elements $\langle j'\|Y_2\|j\rangle$, etc., are again defined as in Ref. 11. The corresponding expressions for the magnetic dipole transitions are

$$\underset{I\to I'}{B(M1)}=(3/4\pi)(e\hbar/2Mc)^2(2I'+1)|(-)^{I'-1/2}B_p(M1)+(-)^I g_c B_c(M1)|^2, \qquad (18)$$

where $B_p(M1)$ refers to the contribution of the particle and is

$$B_p(M1)=\sum_{jj'NR}\langle j;NR;I|E\rangle\langle j';NR;I'|E'\rangle(-)^{R+l} \times[(2j+1)(2j'+1)]^{1/2}W(jj'II';1R)[g_l(-)^{j+j'}[l(l+1)(2l+1)]^{1/2} \times W(jj'll;1\tfrac{1}{2})+g_s(\tfrac{3}{2})^{1/2}W(jj'\tfrac{1}{2}\tfrac{1}{2};1l)],\quad \text{for}\quad l=l',$$

and $B_c(M1)$ that of the surface vibrations of the core and is

$$B_c(M1)=\sum_{jNR}\langle j;NR;I|E\rangle\langle j;NR;I'|E'\rangle(-)^{R+j} \times[R(R+1)(2R+1)]^{1/2}W(RR'II';1j),\quad \text{for}\quad l=l'.$$

We have now obtained the basic formulas needed for the present work. In the following section we present the results of our calculations and their comparison with the experimental data.

## III. COMPARISON OF CALCULATED AND EXPERIMENTAL RESULTS

The basic formulas obtained in the preceding section will now be used to calculate the nuclear properties of the low-lying levels in $Pr^{143}$. Furthermore, these results will be compared with the available experimental data. For this purpose we suppose that the odd proton,

[21] S. A. Moszkowski, *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), p. 373; J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 595.

having available both the $g_{7/2}$ and $d_{5/2}$ states, is coupled to the quadrupole type collective surface vibrations of its even-even $Ce^{142}$ core. We limit ourselves to the following configurations:

$$(1g_{7/2};NR) \quad \text{and} \quad (2d_{5/2};NR),$$

where $N$ is the number of phonons ($\leq 3$) and $R$ is the core angular momentum. The above configurations give rise to many possible values of the total angular momentum of the nucleus $\mathbf{I}=\mathbf{j}+\mathbf{R}$. We study only the levels with spins $\frac{1}{2}$ to 11/2 inclusive. States with higher spins are not studied; for should they occur experimentally, they would probably be due to particle excitations, and, moreover, might not occur at low energy. Since in the present investigation we are mainly interested in the low energy nuclear properties, we include in our calculations only states up to three phonons. We shall see later that the effects of the higher phonon states are negligible.

As has been pointed out in the introduction, three kinds of parameters enter into our formulation: $\xi$, the dimensionless coupling strength between the odd proton and the core vibrations; $\epsilon$, the effective spacing in energy between the $g_{7/2}$ and $d_{5/2}$ states; and $\hbar\omega$, the quantum energy associated with the quadrupole oscillations of the core. In order to obtain the numerical values for energy spectra and their properties from the theory developed in the last section, it is necessary to fix these parameters.

The first of these parameters is regarded as adjustable, and so is the "effective" single-particle spacing $\epsilon=(d_{5/2}-g_{7/2})$, since it cannot be deduced from the experimental information and to deduce it from theory is beyond the purpose of the present work. The last parameter $\hbar\omega=630$ keV, the phonon energy, is fixed from the experimental value of the frequency of the quadrupole vibrations of the surface of the neighboring even-even nucleus which in our case is $Ce^{142}$. The dimensionless coupling parameter is conveniently taken to be

$$\xi=k(5/2\pi\hbar\omega C)^{1/2}.$$

### A. Energy Levels

The Hamiltonian given by Eq. (1) is diagonalized for each $I$, $\frac{1}{2}\leq I\leq 11/2$, for various values of the "effective" energy spacing $\epsilon$ of the $d_{5/2}$ and $g_{7/2}$ levels, and of the dimensionless coupling parameter $\xi$. As a result of diagonalizing the Hamiltonian matrices for a given $I$ and $\epsilon$ we obtain the energy eigenvalues and expansion coefficients of the eigenvectors of our system as a function of $\xi$. As an example, the matrices diagonalized for $I=\frac{7}{2}$ is $17\times 17$. In our model the eigenvectors of the coupled system have the form given by Eq. (11) where the sum extends over $j$ corresponding to the $g_{7/2}$ and $d_{5/2}$ single particle states; over the core oscillator states $N=0$, 1, 2, 3; and over all allowed values for $R$.

Figure 1 shows the energy eigenvalues corresponding to $\epsilon=0.1\hbar\omega$ as a function of the dimensionless coupling parameter $\xi$. Notice that for large $\xi$ the spectrum exhibits almost multiplet structure for many levels. Values of $\epsilon$ and $\xi$ have been chosen to give the best fit to the experimental energy levels and their properties.

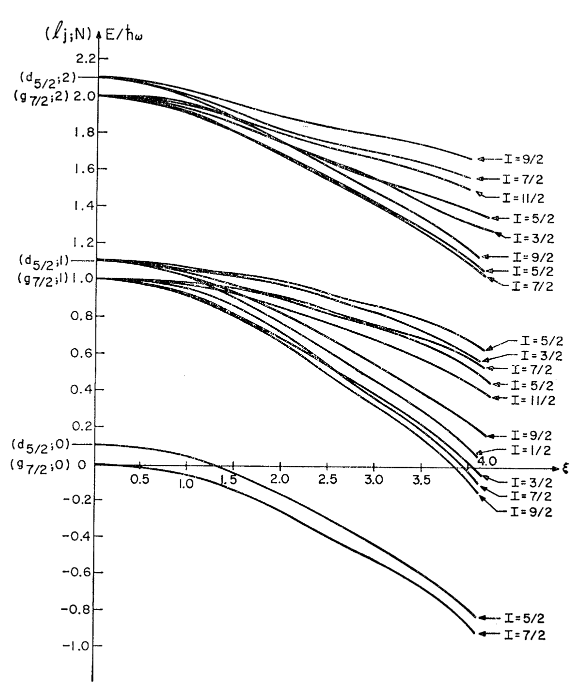


FIG. 1. The energy levels of the Hamiltonian consisting of a single proton coupled to the quadrupole oscillations of the core of the nucleus are plotted as a function of the dimensionless coupling parameter $\xi$. The odd proton has available the $d_{5/2}$ and $g_{7/2}$ states separated by $\epsilon$, where $\epsilon=0.1\hbar\omega$.

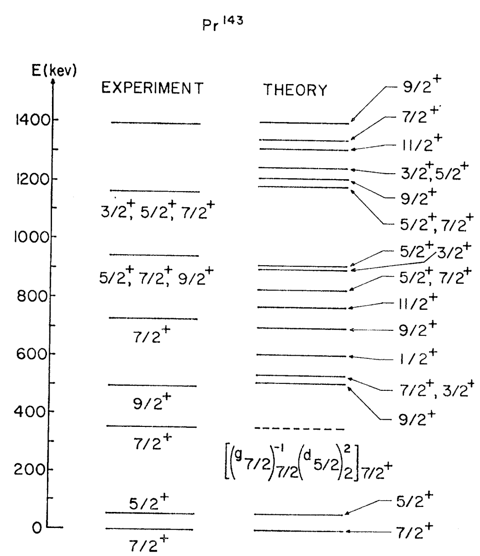


FIG. 2. Comparison of theoretical with experimental spectrum of $Pr^{143}$. The values of the parameters used to obtain the theoretical spectrum are $\xi=3.0$ and $\epsilon=0.1\hbar\omega$. The 351-keV level is discussed in the text.

TABLE I. Calculated $M1$ and $E2$ transition probabilities in $Pr^{143}$.[a]

| Initial state $I$ | Final state $I'$ | $\Delta E$ (keV) | $M1$ transitions per second | $E2$ transitions per second | Theory $E2/M1$ | Experiment |
|---|---|---|---|---|---|---|
| $\frac{5}{2}$ | $\frac{7}{2}$ | 57 | $3.25\times10^{7}$ | $3.76\times10^{5}$ | 1.15% | $E2/M1\approx0.3\%$[b,c] |
| $\frac{9}{2}$ | $\frac{7}{2}$ | 507 | $1.29\times10^{10}$ | $1.45\times10^{11}$ | 11 to 1 | The experimental data are inconclusive.[c] |
| $\frac{9}{2}$* | $\frac{9}{2}$ | 203 | $1.69\times10^{10}$ | $1.69\times10^{7}$ | 0.1% | |
| $\frac{9}{2}$ | $\frac{5}{2}$ | 450 | Forbidden | $3.37\times10^{9}$ | Pure $E2$ | |
| $\frac{9}{2}$* | $\frac{7}{2}$ | 710 | $1.04\times10^{11}$ | $2.01\times10^{10}$ | 20% | |
| $\frac{9}{2}$* | $\frac{5}{2}$ | 653 | Forbidden | $3.44\times10^{11}$ | Pure $E2$ | |
| $\frac{5}{2}$** | $\frac{5}{2}$ | 863 | $1.57\times10^{12}$ | $6.54\times10^{11}$ | 40% | $\frac{5}{2}$** $\rightarrow \frac{5}{2}$ transition is more intensive than $\frac{5}{2}$** $\rightarrow \frac{7}{2}$.[c] |
| $\frac{5}{2}$** | $\frac{7}{2}$ | 920 | $2.76\times10^{11}$ | $2.26\times10^{10}$ | 8% | |

[a] The asterisk and double asterisk represent the second and third states, respectively, of a given spin from theory (cf. Fig. 2).
[b] The 57-keV level has been extensively investigated (Ref. 1-8) and almost all results confirm the transition to be mainly $M1$ in character, consistent with our prediction.
[c] These experimental results are taken from Ref. 1.

The theoretically determined energy spectrum corresponding to the parameters $\epsilon=0.1\hbar\omega$ and $\xi=3.0$ as well as the experimentally determined energy spectrum are presented in Fig. 2. The agreement of the theory with the experimental spectrum is quite good. The theory confirms most spins and their parities, which are firmly assigned. From the simple shell model one expects the ground state of $Pr^{143}$ to have spin and parity $\frac{5}{2}^+$ instead of the measured $\frac{7}{2}^+$. This indicates the importance of the residual nucleon-nucleon interactions which must be incorporated into the shell model. We assume the ground state to have spin $\frac{7}{2}^+$. The 725-keV level is listed[1] tentatively as being most probably $\frac{7}{2}^+$ while we have assigned to it a spin of $\frac{9}{2}^+$.

Our approach does not give a state near the 351-keV level. This state which cannot be explained by using the above model alone may arise as a result of a partial excitation of the core in which the $(d_{5/2})^2{}_0$ proton pair is decoupled and excited from spin state 0 to spin state 2. Furthermore, we suggest that this level can have most probable spin $\frac{7}{2}^+$ (since $\frac{7}{2}^+$ is the mean of the five spin states $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{7}{2}^+$, $\frac{9}{2}^+$, $11/2^+$, which can arise due to the coupling of the angular momenta 2 and $\frac{7}{2}$) and consequently it can be represented by configuration,

$$[(d_{5/2})^2{}_2(g_{7/2})^{-1}{}_{7/2}]_{7/2}.$$

This assumption is further strengthened by the fact that in the most recent experiment,[1] the 351-keV level has been assigned spin $\frac{7}{2}^+$ instead of $\frac{3}{2}^+$ or $\frac{7}{2}^+$ previously assigned.[2,7] Our rough estimate indicates that a weak two-body interaction is sufficient to give an energy separation of approximately 351 keV between $(d_{5/2})^2{}_0$ and $(d_{5/2})^2{}_2$ states.

### B. γ-Decay Transition Probabilities

It would now be of interest to calculate the electromagnetic transition rates by using the wave functions obtained as described above in the present section. The results of such calculations and their comparison with the experimental data will enlighten further this investigation.

The calculations for the $M1$ and $E2$ transition rates have been performed by using the general expressions given by Eqs. (17) and (18), respectively. These results and available experimental data are compared in Table I. For most cases our results are in good agreement with the experimental data. The parameters used in obtaining the theoretical results are $g_l=1$, $g_s=5.587$, $g_c=0.412$, $R_0=1.2\times10^{-13}$ $A^{1/3}$ cm, and $k=40$ MeV. For the radial matrix element in Eq. (17) we have used the approximation $\langle r^2\rangle=\frac{3}{5}R_0^2$.

The 57-keV level has been extensively investigated[1-8] and almost all results confirm that the transition from this level to the ground state is mainly $M1$ in character, consistent with our result. Quantitatively, the calculated $E2/M1$ ratio is found to be 1.15% as compared to the experimental value which is found to be mainly $M1$ with $E2$ admixture $\leq0.3\%$. In addition, our estimated half-life for this $M1$ transition is found to be $2.13\times10^{-8}$ sec, indicating that this transition is highly retarded. Experimentally this transition is also found to be highly retarded and the measured value for the half-life is $(4.17\pm0.09)\times10^{-9}$ seconds. These results are very satisfactory in view of the fact that there are certain uncertainties in the parameters used in obtaining the absolute transition probabilities. It is worthwhile to mention here that had we excluded the mixing of states, the $M1$ transition from the 57-keV level would have been forbidden because of its $\Delta l$-forbidden character, manifesting the importance of mixing of states on electromagnetic transitions.[22-25]

Furthermore, we find that the transition from the 920-keV level of spin $\frac{5}{2}$ (this level corresponds to the experimental 942-keV level; see Fig. 2) to the 57-keV level is more intense than to the ground state by a ratio of 6 to 1. These results have also been confirmed experimentally (cf. Table I).

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[23] A. Arima and H. Horie, Progr. Theoret. Phys. (Kyoto) **12**, 623 (1954).
[24] A. de-Shalit, in *Proceedings of the Rehovoth Conference on Nuclear Structure, 1957* (North-Holland Publishing Company, Amsterdam, 1958).
[25] D. C. Choudhury, Phys. Rev. **129**, 1754 (1963).

Finally, in Table I, we also give the calculated results for several $M1$ and $E2$ transition probabilities other than those discussed above. In view of the fact that sufficient experimental information is not available on these transition rates, there is a need for more extensive experimental investigations, before any valuable comment can be made on these results. In general, the experimentally determined electromagnetic transition rates which are most conclusively known are consistent with the predictions of our model, indicating that our wave functions give a fairly good description of the nuclear states.

### C. Nuclear Moments

Since the above results are very interesting and informative, it is of value to calculate the nuclear moments using the wave functions obtained from our model even when to our knowledge, there exist as yet no such experimental data. Our results for the electric quadrupole and magnetic dipole moments of the ground state of $Pr^{143}$ are found to be $+0.605$ b and 1.775 nm, respectively. The sign of the predicted quadrupole moment has been assigned positive, opposite to that of the calculated, because in obtaining the wave functions we coupled the particle states to the collective oscillations of the core, while the ground state configuration to first order is one-hole configuration represented by $[(d_{5/2})^2{}_0(g_{7/2})^{-1}{}_{7/2}]_{7/2}$. Therefore, we believe that our estimate for the quadrupole moment might be slightly higher than its actual value, for when the calculated result is multiplied by minus one, the $d_{5/2}$ configuration (approximately 5%) is also changed into one-hole configuration while actually it is in a particle state. However, there is a possibility that part of the hole effects of the $d_{5/2}$ state introduced into our calculation might have been compensated for by the judicious selections of $\epsilon$ and $\xi$. In addition, we have calculated the magnetic moment for the first excited 57-keV level, and our result is $\approx 4.2$ nm. Hence an experimental measurement of the nuclear moments would be very illuminating in this connection.

## IV. CONCLUSIONS AND DISCUSSIONS

It is clear from the preceding section that the intermediate coupling approach in the unified nuclear model gives a fairly good description of the low-lying levels in $Pr^{143}$. From Fig. 2 and Table I, one sees that the agreement between the theoretical results and experimental data for energy spectrum and for their spins and parities as well as for the electromagnetic transition rates, for most of the well known cases, is satisfactory. There is a disagreement between the theoretical prediction and the experimental data in connection with the 725-keV level to which we assign a spin of $\frac{9}{2}^+$ instead of $\frac{7}{2}^+$; however, this experimental result is not conclusive. There is another disagreement between the theory and experiment as pointed out in Sec. IIIA: the 351-keV level cannot be explained by only using the above model, but it is necessary to include a partial excitation of the core in order to account for this level. Thus we see that it is possible to account for most of the observed experimental facts in the framework of the present approach. It would also be interesting and informative to investigate the properties of the above nucleus in the framework of the Kisslinger and Sorensen approach.[26]

Now it would be extremely desirable to examine the validity of the use of only core oscillator states $N=0$, 1, 2, 3, and neglecting higher states in our calculations. It would be, furthermore, desirable to determine as to what extent the values of the parameters used in the calculations are reasonable.

We may estimate the validity of including states only up to three phonons in our calculations by examining the expansion coefficients of the eigenvectors given by Eq. (11) corresponding to the highest phonon state. Their square should be negligible compared to unity. This condition has been satisfied in our calculations, particularly for the lower-lying energy levels, as can be seen from Table II, in which the expansion coefficients of some of the eigenvectors obtained from our calculations have been shown. From Table II one also sees that the ground state and the first excited state are highly pure. The ground state $\frac{7}{2}^+$ has a 95% $g_{7/2}$ admixture, while the first excited state $\frac{5}{2}^+$ has a 95% $d_{5/2}$ admixture. In view of our results, it would be very interesting if these admixtures are determined experimentally from stripping reactions, or from the angular distributions of the inelastic scattering cross sections from this nucleus. We believe that the admixtures of various states present in our wave functions, some of which are given in Table II, will be in agreement with the experimental values because our wave functions give excellent results for many of the observed nuclear properties. It is very significant to mention here that we find from our calculations that the mixing of states has very little effect on the energy spectrum; nevertheless, the electromagnetic transition rates depend most crucially on mixing of states, and even as small as a few percentages of admixtures can immensely affect the transition rates.

We now turn to the discussion of the values of the dimensionless coupling parameter $\xi$ and the effective single-particle level spacing $\epsilon$. The values chosen for these parameters to obtain the best fit to the experimental data are $\xi=3.0$ and $\epsilon=0.1\hbar\omega$. This particular value of $\xi$, for $k=40$ MeV, implies the value for the nuclear deformability

$$C\simeq(140/\hbar\omega)\text{MeV},$$

and it further implies the equilibrium value of $\beta$ given

[26] L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 9 (1960); Rev. Mod. Phys. **35**, 853 (1963).

TABLE II. Expansion coefficients of some of the eigenvectors.[a]

| Basic states | Expansion coefficients corresponding to states $\|E(\text{keV});I\pi\rangle$ | | | | | |
|---|---|---|---|---|---|---|
| | $\|0;\frac{7}{2}^+\rangle$ | $\|57;\frac{5}{2}^+\rangle$ | $\|507;\frac{9}{2}^+\rangle$ | $\|710;\frac{9}{2}^+\rangle$ | $\|920;\frac{5}{2}^+\rangle$ | $\|1180;\frac{5}{2}^+\rangle$ |
| $\|1g_{7/2};00\rangle$ | 0.8134 | 0 | 0 | 0 | 0 | 0 |
| $\|1g_{7/2};12\rangle$ | −0.5142 | 0.2190 | 0.7950 | −0.1731 | 0.0361 | −0.1251 |
| $\|1g_{7/2};20\rangle$ | 0.1237 | 0 | 0 | 0 | 0 | 0 |
| $\|1g_{7/2};22\rangle$ | −0.0991 | 0.0382 | 0.3304 | 0.1054 | 0.1895 | −0.4192 |
| $\|1g_{7/2};24\rangle$ | 0.1176 | −0.1029 | −0.3776 | 0.2259 | −0.1834 | −0.3285 |
| $\|1g_{7/2};30\rangle$ | 0.0175 | 0 | 0 | 0 | 0 | 0 |
| $\|1g_{7/2};32\rangle$ | −0.0495 | 0.0206 | 0.1064 | −0.0168 | 0.0839 | 0.0400 |
| $\|1g_{7/2};33\rangle$ | −0.0009 | −0.0002 | 0.0106 | −0.0644 | 0.0477 | −0.2802 |
| $\|1g_{7/2};34\rangle$ | 0.0217 | −0.0080 | −0.0921 | −0.0496 | −0.0633 | 0.2233 |
| $\|1g_{7/2};36\rangle$ | −0.0039 | 0.0144 | 0.0026 | −0.0047 | 0.0543 | 0.0874 |
| $\|2d_{5/2};00\rangle$ | 0 | 0.8089 | 0 | 0 | −0.5189 | −0.0383 |
| $\|2d_{5/2};12\rangle$ | −0.1588 | −0.4977 | 0.1517 | 0.8105 | −0.5430 | −0.1218 |
| $\|2d_{5/2};20\rangle$ | 0 | 0.1247 | 0 | 0 | 0.3836 | 0.1157 |
| $\|2d_{5/2};22\rangle$ | −0.0260 | −0.0943 | 0.1135 | −0.2524 | −0.2464 | 0.5808 |
| $\|2d_{5/2};24\rangle$ | 0.0748 | 0.1002 | −0.2191 | −0.4012 | 0.2895 | 0.2400 |
| $\|2d_{5/2};30\rangle$ | 0 | 0.0174 | 0 | 0 | 0.0807 | −0.2493 |
| $\|2d_{5/2};32\rangle$ | −0.0147 | −0.0483 | 0.0225 | 0.0925 | −0.2234 | −0.0380 |
| $\|2d_{5/2};33\rangle$ | −0.0011 | −0.0010 | −0.0244 | 0.0904 | 0.0064 | 0.2334 |
| $\|2d_{5/2};34\rangle$ | 0.0052 | 0.0092 | −0.1012 | −0.0230 | 0.0245 | −0.1425 |
| $\|2d_{5/2};36\rangle$ | −0.0106 | 0 | 0.0338 | 0.0468 | 0 | 0 |

[a] $\pi$ represents the parity of the state of spin $I$.

by Eq. (II.22) in the paper of Bohr and Mottelson,[10]

$$\beta \simeq 0.04.$$

These values are consistent with the rough knowledge of the parameters for $Pr^{143}$ nucleus. Let us now examine the value of the effective single-particle spacing $\epsilon=(d_{5/2}-g_{7/2})$ used in our case. Glendenning[15] has calculated energy levels of the positive parity-states of some tellurium and xenon isotopes by application of the pairing-correlation theory to the nuclear structure. According to his calculation, the energy separation between $\frac{5}{2}$ and $\frac{7}{2}$ spin states can be in the range of 25 to 110 keV depending on the structure of the isotopes. In our case we have used $\epsilon=63$ keV, the energy separation between the $d_{5/2}$ and $g_{7/2}$ states which seems reasonable with respect to the results obtained by Glendenning. All these considerations justify our approach.

Before any further discussions can be given, there is a need for more extensive experimental investigations on the energy spectrum because the theory predicts many more levels than experimentally observed as yet; the determination of the spins and parities, in particular for those levels whose spin assignments are still tentative; the measurement of nuclear moments, since there exist as yet no such experimental data to the best of our knowledge; and finally, measurement of the electromagnetic transition rates, since the data available on transition rates are not sufficient.

In conclusion, we believe that our present analysis has clarified many of the difficulties in understanding the observed properties of the low-lying levels in $Pr^{143}$, and hope that it will further stimulate new experiments.

Application of the same model with some appropriate modifications to several other odd-$A$ nuclei is being studied by one of us (D. C. C.) and T. F. O'Dwyer.

*Note added in proof.* After submission of our paper, K. P. Gopinathan, Phys. Rev. **139**, B1467 (1965); and R. V. Mancuso, J. P. Roalsvig, and R. G. Arns, *ibid.* **140**, B525 (1965) have reported new experimental results. This new data appears to lend further support to our conclusions.

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