Cross Sections for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ Reaction*

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Cross sections for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction have been determined by measuring the Mg^{27} radioactivity that was produced in aluminum samples that were irradiated with bremsstrahlung of maximum energy varying from 100 to 300 MeV. The observed yields were corrected for the contributions from secondary neutron reactions. The resulting cross sections have a maximum value of 13 μb at 220 MeV and the integrated cross section to 300 MeV is 1.38 ± 0.24 MeV mb. These values are a factor of about 10 smaller than those reported previously for this reaction by Masaike *et al.* The cross sections for the $Al^{27}(\gamma,\pi^+)$ reaction are compared with those observed previously for similar reactions involving other light target nuclei and are discussed in terms of an extension of the theory outlined by Laing and Moorhouse for reactions of this type.

I. INTRODUCTION

THIS paper is part of a series of related papers in which the properties of photopion production reactions in complex nuclei which lead to specific final states are discussed. Previously, we have studied the cross-section behaviors of the $B^{11}(\gamma,\pi^-)C^{11}$, $B^{11}(\gamma,\pi^+)$-Be^{11}, and the $O^{16}(\gamma,\pi^+)N^{16}$ reactions.[1,2] A study of the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction is being reported in this paper.

The observed cross sections for the $B^{11}(\gamma,\pi^-)C^{11}$ reaction have been interpreted in terms of a theory outlined by Laing and Moorhouse[3] in which the pion photoproduction process in a complex nucleus is assumed to be a single-nucleon process. The results were consistent with the process being restricted to the surface region of the nucleus. Although detailed theoretical calculations of the expected cross sections have not been made for the other reactions that have been studied, the relative values of the observed cross sections for the $B^{11}(\gamma,\pi^-)C^{11}$, $B^{11}(\gamma,\pi^+)Be^{11}$, and the $O^{16}(\gamma,\pi^+)N^{16}$ reactions could be accounted for within the framework of the Laing and Moorhouse theory by consideration of the relative number of possible single-nucleon transitions involved (i.e., by consideration of the relative number of initial and final states in each case). We have made a study of the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction to see if these same considerations can be applied to a somewhat heavier nucleus. Although a quantitative comparison can not be made at this time because of the absence of detailed information about the level structure of Mg^{27}, it appears that the Laing and Moorhouse theory can be extended to at least qualitatively account for our observed cross sections for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction.

The $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction has been studied previously by Masaike *et al.*[4] Since they were interested in the cross-section behavior over a wide range of energies from threshold to 720 MeV, they did not do a very detailed study in the energy region that we are concerned with. Their results, however, are a factor of about 10 higher than ours. Some comments on the comparison of the two sets of results are presented in Sec. IVA.

II. EXPERIMENTAL

The yield values for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction were determined by measuring the radioactivity of the product nuclei in aluminum samples that had been irradiated with bremsstrahlung from the University of Illinois 300-MeV betatron. The measurements of the Mg^{27} radioactivity were made with a gamma-ray scintillation-spectrometer system.

The aluminum used in the irradiations was commercial-grade aluminum (99% minimum purity) supplied by the Aluminum Company of America. Sheets of aluminum having a total thickness of 64 mils were placed in the collimated bremsstrahlung beam at the exit of the primary collimator. The bremsstrahlung energy range covered was from 100 to 300 MeV with yield data usually being obtained at 20-MeV intervals. An average of 30 duplicate irradiations were made at each energy. The irradiations were 17 min in duration.

In order to evaluate the contribution to the observed Mg^{27} yields from secondary reactions, the shape of the yield curve for secondary reactions was determined by making several irradiations in which the aluminum sheets to be counted were placed outside of the collimated bremsstrahlung beam next to a 1-in. cube of aluminum that was in the bremsstrahlung beam to serve as an enhanced source of secondary neutrons. The secondary reactions that are expected to interfere with the observation of the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction are the $Al^{27}(n,p)Mg^{27}$ reaction involving the main target isotope and the $Mg^{26}(n,\gamma)Mg^{27}$ reaction involving magnesium impurities in the target material. The yield behavior for the secondary reactions was determined at 40-MeV intervals from 100 to 300 MeV. Using data taken at the more widely spaced energies to determine the energy dependence for the secondary-reaction yield is

* This work was supported by the U. S. Office of Naval Research under contract No. Nonr 1834 (05).
[1] P. Dyal and J. P. Hummel, Phys. Rev. **127**, 2217 (1962).
[2] R. A. Meyer, W. B. Walters, and J. P. Hummel, Phys. Rev. **138**, B1421 (1965).
[3] E. W. Laing and R. G. Moorhouse, Proc. Phys. Soc. (London) **70**, 629 (1957).
[4] A. Masaike, Y. Yoshimura, Y. Murata, A. Kusumegi, and K. Takamatsu, J. Phys. Soc. Japan **18**, 1692 (1963); A. Masaike, *ibid.* **19**, 427 (1964).

adequate since the energy dependence of the secondary-reaction yield is relatively slow and smooth.

Several additional experiments were conducted at 140 and 260 MeV to further study the properties of the interfering secondary reactions. In these experiments the Mg^{27} yield was determined for different locations of the target material, different thicknesses of the target material, and different sizes of the beam collimator. These experiments are useful in determining the source of the neutrons that are responsible for the secondary reactions, i.e., whether they are generated in the aluminum target material or elsewhere.

In all of these experiments the bremsstrahlung beam was monitored with a calibrated thick-walled copper ionization chamber. The ionization produced in the chamber was measured with a vibrating-reed electrometer. The electrometer circuit contained an appropriate *RC* circuit to correct the monitor readings for the decay of the Mg^{27} product nuclei during the irradiations and eliminate the effects due to fluctuations in the bremsstrahlung beam intensity.

The gamma-ray spectra from the irradiated samples were obtained with a scintillation spectrometer. The scintillator was a 3-in.×3-in. NaI(Tl) crystal, and the spectra were recorded by either a 512-channel or a 1024-channel pulse-height analyzer. The samples were placed directly on the top of the crystal housing. Ordinarily, each sample was counted for a 17-min period starting 2.5 min after the end of the irradiation. On most occasions the spectra from several samples irradiated in succession with bremsstrahlung of the same energy were summed in the analyzer in order to reduce the statistical fluctuations in the observed spectra and improve the accuracy of the analysis of the spectra. The resulting gamma-ray spectrum observed from ten samples that were irradiated with 220-MeV bremsstrahlung is shown

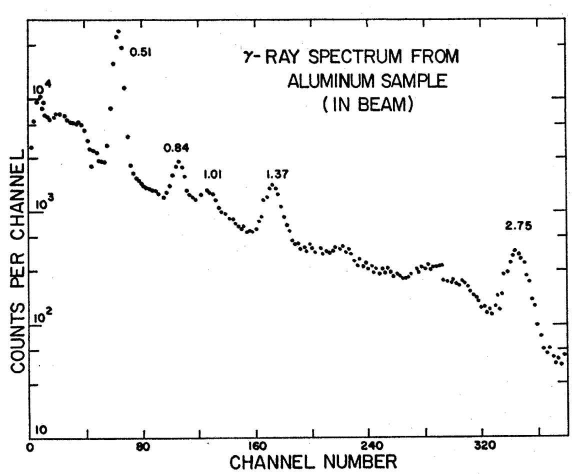

FIG. 1. Gamma-ray spectrum accumulated for ten irradiations in which the 64-mil-thick aluminum samples were in the bremsstrahlung beam. The bremsstrahlung energy was 220 MeV. The energies (in MeV) are indicated above the various photopeaks.

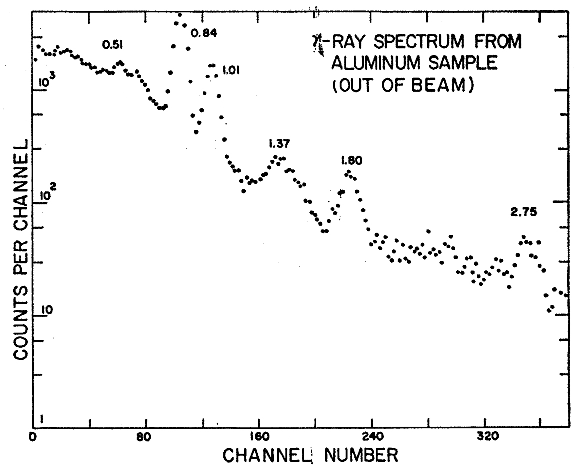

FIG. 2. A gamma-ray spectrum from samples produced in one set of irradiations in the series used to determine the yield curve for secondary reactions. This spectrum is from nine aluminum samples that were outside of the bremsstrahlung beam next to a 1-in. cube of aluminum that was in the beam. The bremsstrahlung energy was 220 MeV. The energies (in MeV) are indicated next to the various photopeaks.

in Fig. 1. The photopeaks that are present have energies of 0.51, 0.84, 1.01, 1.37, and 2.75 MeV. The 0.51-MeV photopeak is due to the annihilation radiation associated with positron emitters. These are most likely produced by high-yield (γ,n) reactions on impurities present in the aluminum samples. The 0.84- and 1.01-MeV photopeaks are due to Mg^{27} (9.5-min half-life).[5] The 1.37- and the 2.75-MeV photopeaks are due to Na^{24} (15-h half-life) that is produced in the $Al^{27}(\gamma,2pn)Na^{24}$ reaction. The assignments of the origins of the observed gamma rays are based on both their energies and their half-lives. The half-lives of the Na^{24} radiations were confirmed by following their decay over a period of three days. Because of the smallness of the Mg^{27} activity produced in these experiments, it was not possible to obtain accurate decay curves for the Mg^{27} photopeaks in the usual manner. The half-life of the Mg^{27} radiations was determined in a series of experiments in which the spectrum obtained during the first 9.5 min was recorded in one half of the analyzer memory and the spectrum for the next 9.5 min was recorded in the other half of the memory. Two sets of these half-life experiments were made, one using samples that were irradiated with 140-MeV bremsstrahlung and the other using samples that were irradiated with 260-MeV bremsstrahlung. In both cases the relative intensities of the 0.84- and the 1.01-MeV photopeaks in the first and second counts were consistent with a 9.5-min half-life.

A spectrum obtained from samples that were outside of the bremsstrahlung beam is shown in Fig. 2. The data

[5] Unless noted otherwise, all decay scheme information was taken from *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington, D. C., 1959), NRC 59-6-45.

shown there are from nine irradiations at a betatron energy of 220 MeV. One notes there that the most intense photopeaks are those for the 0.84- and the 1.01-MeV gamma rays from Mg^{27}. The large reduction in the intensity of the 0.51-MeV photopeak from that observed in the in-beam experiments in an indication that the samples were clear of the bremsstrahlung beam in the experiments used to evaluate the effects of secondary reactions. It is also apparent that the photopeaks due to Na^{24} (1.37 and 2.75 MeV) are also greatly reduced in intensity indicating that most of the Na^{24} nuclei produced in the in-beam experiments were produced in photonuclear reactions rather than secondary reactions. Also observed in the spectrum shown in Fig. 2 is a photopeak of low intensity at 1.8 MeV. This peak has a short half-life and is thought to be mainly due to Al^{28} (2.3-min half-life) which was produced by $Al^{27}(n,\gamma)$ reactions.

The yield-curve data for the production of Mg^{27} were obtained by using the intensities of the 0.84-MeV photopeak in the various spectra. The contribution to that region of the spectrum from events associated with the higher energy gamma rays from Na^{24} was determined accurately by observing the spectra from the samples after the Mg^{27} had decayed. The absolute-yield values were obtained from the intensities of the 0.84-MeV photopeak by correcting for the counting efficiency of the system. The geometry, total crystal efficiency, and peak-to-total ratio values were taken from the summary by Heath.[6] Corrections were also made for the absorption losses, summing losses, and the abundance of the 0.84-MeV gamma ray. The gamma-ray abundance was taken from the decay scheme reported by Ciuffolotti and Demichelis.[7] Their decay scheme was also used as the basis for making the correction for losses due to coincidence summing events.

III. RESULTS

The observed Mg^{27} yields are shown in Fig. 3 where the yields per monitor unit[8] are plotted as a function of the bremsstrahlung maximum energy. The existence of a large contribution from secondary reactions in the in-beam experiments (data points in Fig. 3) is evident from the presence of Mg^{27} at energies below the threshold for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction (142 MeV). The results of the experiments in which the Mg^{27} yield was measured as a function of the target thickness show that a large fraction of this secondary-reaction yield (about 75%) is due to neutrons that are generated outside of the aluminum targets. The main sources of these neutrons appear to be the interior of the betatron and the primary collimator. In order to remove the contribution from the secondary reactions, the observed yield curve for the production of Mg^{27} in aluminum targets placed outside of the bremsstrahlung beam was normalized to the yields from the in-beam bremsstrahlung irradiations in the energy region below 142 MeV. (The normalized secondary-reaction yield values are shown by the solid line in Fig. 3.) The yield values for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction are then obtained by subtracting the secondary-yield values from the observed in-beam yield values.

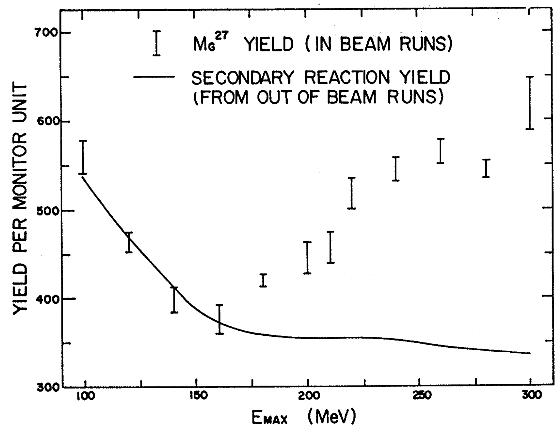

FIG. 3. Yield data (in arbitrary units) for the production of Mg^{27} in aluminum targets. The data points give the yields observed for the targets that were placed in the bremsstrahlung beam. The solid line shows the trend of the yields that were observed in the secondary reaction studies in which the samples that were counted were outside of the bremsstrahlung beam next to a 1-in. cube of aluminum that was in the beam. The out-of-beam yields were normalized to the in-beam yields below 142 MeV. (Note the suppressed zero on the ordinate scale.)

The resulting $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction yield values were then used to calculate cross sections at various photon energies by the Penfold and Leiss method.[9] The resulting cross sections for 30-MeV-wide energy intervals are shown in Fig. 4. The uncertainties shown there are those due only to the random fluctuations in the yield data. We estimate that the uncertainties in the factors which were used to calculate the absolute yield values from the counting and monitor data will contribute an additional 15% to the uncertainties in the absolute-cross-section values.

The integrated cross section from threshold to 300 MeV is 1.38±0.24 MeV mb. The uncertainty quoted here includes the uncertainties in the factors used to obtain the absolute-yield values from the raw data.

IV. DISCUSSION

A. Comparison with the Results of Masaike *et al.*

Before discussing the significance of the observed cross sections shown in Fig. 4, we want to comment on the comparison of our results with those observed pre-

[6] R. L. Heath, Atomic Energy Commission Research and Development Report No. IDO-16408, 1957 (unpublished).
[7] L. Ciuffolotti and F. Demichelis, Nucl. Phys. 39, 252 (1962).
[8] The response of the beam monitor is energy-dependent. At 120 MeV one monitor unit corresponds to 4.27×10^7 ergs of energy in the bremsstrahlung beam. At 280 MeV the response is 5.62×10^7 ergs per monitor unit. The response varies approximately linearly with the bremsstrahlung maximum energy over this energy range.

[9] A. S. Penfold and J. E. Leiss, Phys. Rev. **114**, 1332 (1959).

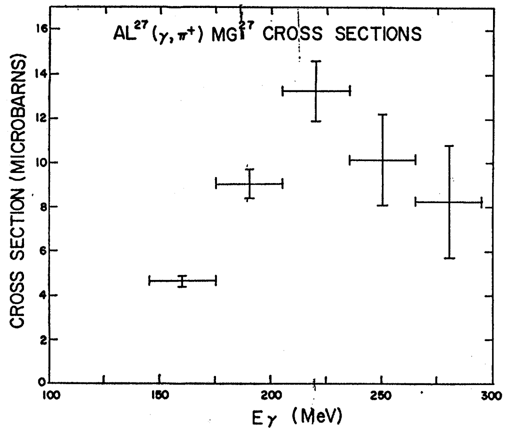

FIG. 4. Cross sections (in units of microbarns) for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction.

viously by Masaike *et al.*[4] They measured the yields of Mg^{27} in aluminum samples that were irradiated with bremsstrahlung varying in energy from 148 to 720 MeV at approximately 50-MeV intervals. There are two features of their results which are quite different from ours. The first one is that their total Mg^{27}-yield values are much larger than ours. For example, at 300 MeV their yield value is about a factor of 4 times our yield. The second difference is that they estimate that the contribution to the observed Mg^{27} yields from secondary reactions is negligible at all energies. Since a large fraction of our observed yield was due to secondary reactions, this means that there is about a factor of 10 difference between their values and our values for the (γ,π^+) reaction yields and cross sections.

The absence of an important contribution from secondary reactions to the Mg^{27} yields observed by Masaike *et al.* is very different from our observations. In view of the similarity of the experimental set-ups involved, it is quite difficult to understand why there should be such a big difference in the effects of secondary reactions. Their conclusion about the unimportance of secondary reactions is based on two observations. The first is that the Mg^{27} yield in samples placed outside of the collimated bremsstrahlung beam was less than 3% of the yield observed in samples that were in the beam. The second is that they observed a very small yield of Mg^{27} at 148 MeV. The first observation is similar to results that we have obtained for samples that were outside of the bremsstrahlung beam. In our case this reflects the fact that the neutron contamination of the bremsstrahlung beam which is causing most of our below-threshold yield is collimated to a large extent and is not evidence for a small contribution from neutron-induced reactions. The second point about the very small yield obtained at 148 MeV is quite different from our observations. In our experiments at the lower energies we observed an appreciable yield due to interfering reactions. We found that the yield of the interfering reactions varied with the location of the target material and the collimator size, but we were not able to find any irradiation conditions for which the below-threshold yield could be reduced to less than half of the yield shown in Fig. 3. Furthermore, since much of the yield is due to neutrons that are generated outside of the aluminum target, it can not be eliminated by extrapolating the yield to zero target thickness (which Masaike *et al.* did). We thus find it difficult to reconcile the difference in the importance of neutron-induced reactions in the two sets of results. One might attempt to reconcile this difference by assuming that Masaike's result at 148 MeV is in error and is much too small. Justification for this may be found in the fact that 148 MeV is near the lower limit of operation of the University of Tokyo synchrotron and the stability of operation at that energy is poor.[10] However, in order to obtain a net yield curve for the $Al^{27}(\gamma,\pi^+)$ reaction that agrees with ours to 300 MeV, it would also be necessary to assume that there were large errors in Masaike's results at 200 and 250 MeV. It appears that there is more to the difference between the two sets of results than just the difference in the behavior of the below-threshold yields.

It is possible that some of the difference between the two sets of results might be due to errors in the calculation of the absolute yield values from the counting and monitor data. A partial check against this possibility can be made by comparing the absolute yields for the production of Na^{24} in the aluminum targets. Our values for the Na^{24} yield are generally within 25% of those reported by Masaike *et al.* This represents fairly satisfactory agreement and indicates than an assumption that large errors were made in the calculation of the absolute yield values would be unwarranted. It thus appears that we are unable to account for the differences between our results and those of Masaike *et al.*

B. Discussion of the Results

The cross sections for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction that are shown in Fig. 4 have the same energy dependence as that observed previously by Hughes and March[11] and by us[1] for the $B^{11}(\gamma,\pi^-)C^{11}$ reaction. They also have the same energy dependence as that observed previously by us for the $B^{11}(\gamma,\pi^+)Be^{11}$ and the $O^{16}(\gamma,\pi^+)N^{16}$ reactions.[1,2] In all these cases the cross section has a maximum value near 210 MeV and then decreases somewhat at the higher energies.

These reactions involving the emission of only a pion are compared in more detail in Table I where the integrated cross sections from threshold to 300 MeV are tabulated along with the values of integrated cross sections relative to that for the $B^{11}(\gamma,\pi^-)C^{11}$ reaction. Also given there are the relative values that one would expect if the cross sections for these reactions varied in the same way that the π^+ or π^- production cross sections

[10] A. Masaike (private communication).
[11] I. S. Hughes and P. V. March, Proc. Phys. Soc. (London) 72, 259 (1958).

TABLE I. Integrated cross sections for several (γ,π^+) and (γ,π^-) reactions.

Reaction	Observed integrated cross section to 300 MeV (MeV mb)	Observed integrated cross section relative to $B^{11}(\gamma,\pi^-)C^{11}$ reaction	Expected relative integrated cross section based on π^+ and π^- yields[a]
$B^{11}(\gamma,\pi^+)Be^{11}$	0.63 ± 0.13[b]	0.16 ± 0.04	0.56
$B^{11}(\gamma,\pi^-)C^{11}$	3.95 ± 0.43[b]	1.00	1.00
$O^{16}(\gamma,\pi^+)N^{16}$	0.96 ± 0.16[c]	0.24 ± 0.05	0.97
$Al^{27}(\gamma,\pi^+)Mg^{27}$	1.38 ± 0.24[d]	0.35 ± 0.07	1.28

[a] Based on the π^+ and π^- yield measurements of Littauer and Walker (Ref. 12).
[b] Reference 1.
[c] Reference 2.
[d] This work.

are thought to vary. These values are based on the $A^{2/3}$ dependence of the total charged-pion production yields and the systematics for the variation of the π^- to π^+ ratio observed by Littauer and Walker.[12] The comparison of the values given in Table I indicates that the observed (γ,π^+) and (γ,π^-) cross sections do not follow the behavior of the total π^+ or total π^- production cross sections very closely. These reactions are apparently sensitive to other properties of the nuclei involved besides the total pion-production cross sections. To see what properties might be important, we turn to the theoretical work of Laing and Moorhouse[3] and some previous results.[1,2]

The cross sections observed for the $B^{11}(\gamma,\pi^-)C^{11}$ reaction were found to be consistent with those predicted by Laing and Moorhouse.[3] In making their predictions, they considered that the pion photoproduction process in a complex nucleus was a single-nucleon process. They then calculated transition probabilities for the various possible single-nucleon transitions using a shell-model description of the initial and final states involved. The observed cross-section values were in agreement with their calculated values that were based on a limitation of the production process to the surface region of the target nucleus.

Although detailed cross-section calculations using the Laing and Moorhouse approach had not been made for the other reactions that we studied previously, the smallness of the observed cross sections for the $B^{11}(\gamma,\pi^+)Be^{11}$ and the $O^{16}(\gamma,\pi^+)N^{16}$ reactions could be accounted for in terms of one of the features of the results of the Laing and Moorhouse calculations for the $B^{11}(\gamma,\pi^-)C^{11}$ reaction. That was that the results for the $B^{11}(\gamma,\pi^-)C^{11}$ reaction calculations depended more on the number of possible transitions involved than on the specific details of the initial and final states involved. In the reactions in which Be^{11} and N^{16} are produced, the numbers of final states stable to nucleon emission are much smaller than in the C^{11} case. Using the numbers of states involved, it was possible to account for the observed cross-section values for the production of Be^{11} and N^{16}.

Turning our attention to the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction, we note that there are many more final states stable against particle emission in Mg^{27} than in the lighter nuclei that were just discussed (34 known levels in Mg^{27} as compared to two in Be^{11}, eight in C^{11}, and four in N^{16}).[13,14] On the basis of the total number of particle-stable final states available, we would expect that the cross sections for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction would be much larger than the cross sections observed for the other reactions involving the lighter nuclei. Using this viewpoint the cross sections observed by us for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction are unexpectedly small.

A possible explanation for the small cross sections observed by us for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction can be found by looking at the Mg^{27} level structure in more detail. Although a thorough analysis of all of the particle stable levels in Mg^{27} has not yet been made because of the absence of complete spin and parity assignments, it appears quite likely that many of the observed levels are rotational states and that the number of intrinsic single-particle states involved is quite small. That this is true for the low-lying levels in Mg^{27} has been demonstrated by Glover and Weigold[15] who have assigned most of the levels up to an excitation energy of 3.8 MeV to three rotational bands.[16] A similar analysis of the higher states in Mg^{27} has not been attempted, but we can anticipate the results of such an analysis by noting the results that have been obtained for Mg^{25}. Sheline and Harlan[17] have analyzed the known level structure in Mg^{25} in terms of the collective model. They find that five rotational bands can account for over 90% of the observed levels through 6.2 MeV and about 80% of the levels through 6.8 MeV. Thus, in Mg^{27} where the number of levels seems to be comparable to the number observed in Mg^{25}, we might expect that the states stable against particle emission could be accounted for by considering the collective excitations associated with only a relatively small number of different single-particle configurations. This number of different intrinsic states would probably be larger than five but possibly not larger than about ten. If the pion photoproduction process is governed by the number of intrinsic single-particle states available to the product

12 R. M. Littauer and D. Walker, Phys. Rev. 86, 838 (1952).

13 S. Hinds, H. Marchant, and R. Middleton, Proc. Phys. Soc. (London) 78, 473 (1961).

14 T. Lauritsen and F. Ajzenberg-Selove, Supplement to *Nuclear Data Sheets* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington, D. C., 1962), NRC 61-5, 6-103, 119, 223.

15 R. N. Glover and E. Weigold, Nucl. Phys. 24, 630 (1961).

16 A similar conclusion concerning the presence of rotational bands at low energies in Mg^{27} has been presented by B. Cujec, Phys. Rev. 136, B1305 (1964). However, many of his assignments are different from those made by Glover and Weigold in Ref. 15.

17 R. K. Sheline and R. A. Harlan, Nucl. Phys. 29, 177 (1962).

nucleon rather than by the total number of states available, we might expect to observe a relatively small cross section for the $Al^{27}(\gamma,\pi^+)Mg^{27}$ reaction. A quantitative prediction would require information about both the number of final states available for population in Mg^{27} and the number of initial protons available for the pion production process. Such a prediction can not be made until more detailed information about the nature of the level structure in Mg^{27} is available. However, it does appear that this type of extension of the Laing-and-Moorhouse theory could account for our results.

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Study of Nucleon-Nucleus Elastic Scattering in Second-Order Multiple-Scattering Theory*

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Nucleon-nucleus scattering is studied in the energy range 95–350 MeV for targets ranging from carbon to lead. The relative importance of the first and second order terms in Watson's multiple-scattering expansion of the optical potential in terms of the two-nucleon scattering matrix is investigated with the nucleon-nucleon phase parameters obtained at Yale. Effects of including the angular dependence of the two-nucleon amplitudes, as Cromer has done, are compared with those of the second-order potential, and they are found to be equally important so that both must be included for consistency. The possibility of investigating nuclear-structure parameters, especially the two-nucleon correlation lengths brought in by the second-order potential, is considered.

I. INTRODUCTION

THE treatment of inelastic processes which remove the incident nucleon from the entrance channel in nucleon-nucleus scattering by including an imaginary part in the potential representing the interaction was introduced by Ostrofsky, Breit, and Johnson.[1] The method was further investigated by Bethe,[2] and several early analyses[3] demonstrated the ability of the optical model to correlate scattering data over a range of energies and targets with relatively few parameters. Continued successes with this model have inspired extensive phenomenological analyses, with many refinements of detail receiving careful attention. The review in Hodgson's book[4] summarizes much of this work.

The assumed potential has the form

$$V(r)=(U_c+iW_c)\rho(r)+\left(\frac{\hbar}{\mu c}\right)^2(U_s+iW_s)\frac{1}{r}\frac{d\rho}{dr}(\boldsymbol{\sigma}\cdot\mathbf{L}), \quad (1)$$

where the potential strengths are adjustable parameters, and $\rho(r)$ is a dimensionless radial function that is nearly constant from $r=0$ to the nuclear radius and there falls rapidly but smoothly to very small values. It is natural to assume that $\rho(r)$ has some connection with the distribution of matter in the nucleus, so that the range R_h (defined as the distance at which $\rho(r)$ has half its maximum value) will depend on $A^{1/3}$, where A is the mass number. However, both range and surface thickness, t_s (the distance in which the radial function falls from 90% to 10% of its central value) are treated as adjustable. The electron-scattering experiments, analyzed in terms of a similar $\rho(r)$,[5] show a discrepancy when compared with the results of nucleon-nucleus scattering experiments: the extent of the nuclear-matter distribution is smaller for electron scattering than for nucleon scattering. More sophisticated phenomenological analyses by Hodgson[6] show that a satisfactory

* This research was supported by the U. S. Atomic Energy Commission and by the U. S. Army Research Office-Durham.

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[1] M. Ostrofsky, G. Breit, and D. P. Johnson, Phys. Rev. **49**, 22 (1936).

[2] H. A. Bethe, Phys. Rev. **57**, 1125 (1940).

[3] S. Fernbach, R. Serber, and T. B. Taylor, Phys. Rev. **75**, 1352 (1949); R. E. LeLevier and D. S. Saxon, Phys. Rev. **87**, 40 (1952); H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev. **96**, 448 (1954).

[4] P. E. Hodgson, *The Optical Model of Elastic Scattering* (Oxford University Press, London, 1963).

[5] R. Hofstadter, Ann. Rev. Nucl. Sci. **7**, 231 (1957).

[6] P. E. Hodgson, Phys. Rev. Letters **6**, 358 (1961).