PHYSICAL REVIEW VOLUME 143, NUMBER 3 MARCH 1966

Stripping Analysis of the $Ti^{46}(d,p)Ti^{47}$ Reactions*

J. Rapaport,† A. Sperduto, and W. W. Buechner

Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology,
Cambridge, Massachusetts

(Received 15 November 1965)

The (d,p) reactions from a target of isotopically separated Ti^{46} have been studied with the MIT multiple-gap magnetic spectrograph and the MIT-ONR Van de Graaff accelerator using a 7.0-MeV deuteron beam. Forty-seven excited states in Ti^{47} were observed up to an excitation energy of 4.0 MeV, of which seventeen showed angular-distribution characteristics of a stripping process. Sixty additional energy levels were observed between 4.0 and 6.0 MeV, and angular distributions were obtained for eleven of the most intense of these levels. The ground-state Q value was measured to be 6.665 ± 0.010 MeV, and the proton angular distribution for this state did not show a characteristic stripping curve. Transition strengths and values of the orbital angular momenta of the transferred neutrons were derived from the experimental measurements and from calculations using a zero-range distorted-wave Born-approximation analysis.

I. INTRODUCTION

The present article is concerned with the $Ti^{46}(d,p)Ti^{47}$ reaction, augmenting recent reports on (d,p) reactions from the titanium isotopes.[1]

The Ti^{47} ground-state nucleus has five $1f_{7/2}$ neutrons and two $1f_{7/2}$ protons outside the $N=Z=20$ Ca^{40} core. There are only three nuclei belonging to the $(1f_{7/2}{}^5)$ neutron configuration that can be excited by means of (d,p) reactions. The target nuclei are Ca^{44}, Sc^{45}, and Ti^{46}. The $Ca^{44}(d,p)Ca^{45}$ reaction has been reported by Cobb and Guthe,[2] and the $Sc^{45}(d,p)Sc^{46}$ reaction will be reported in a forthcoming paper.[3] In this paper the $Ti^{46}(d,p)Ti^{47}$ angular distributions are reported, and the nuclear-level structure for Ti^{47} is compared with that for Ca^{45}.

The angular distribution for the ground-state transition to Ti^{47} is of particular interest. The spin of Ti^{47} in its ground state is reported to be $\frac{5}{2}$ with odd parity.[4] The expected spin and parity, according to the simple shell-model predictions,[5] should be $\frac{7}{2}^-$.

The anomalous spin of Ti^{47} and that of the Mn^{55} ground state, where the $(f_{7/2}{}^{-3})$ configuration is in the proton shell, are two among several other odd-A nuclei where the ground-state spin J is one less than that of the unfilled level, or $J=j-1$. Attempts to account for this unusual coupling have been made, both from the point of view of nucleon-nucleon forces of intermediate range (shell model),[6] and from calculations based on quadrupole interaction (unified model).[7]

The single-particle shell-model state with $J=\frac{7}{2}$ has been reported[8] at 160 keV above the ground state of Ti^{47}. In the present experiment, distinctive features of the angular distributions of these two states are compared, and an analogy has been made with the corresponding states in Ca^{45}.[9] Pieper[10] has measured the energy levels in Ti^{47} up to 4.0-MeV excitation energy by using deuterons accelerated in a cyclotron and has observed the protons from the $Ti^{46}(d,p)Ti^{47}$ reaction. This work was later improved, and angular distributions of a few levels were obtained, but still with poor resolution.[11] More recently, Rietjens *et al.*,[12] using a 7.8-MeV deuteron beam also analyzed the proton angular distributions of the Ti^{47} levels, using a detector with improved resolution. Hansen[13] reports the energy levels observed in Ti^{47} up to 2.5-MeV excitation energy both from the (d,p) and (p,p') reactions. Zaika *et al.*,[14] using a 13.6-MeV deuteron beam from a cyclotron also reported some angular distributions for the $Ti^{46}(d,p)Ti^{47}$ reaction. Some of the levels in Ti^{47} have also been obtained by measuring the gamma rays following neutron capture in Ti^{46}.[15] The first excited state in Ti^{47} at 160 keV has been observed from Coulomb-excitation experiments.[16]

---

[*] This work has been supported in part through funds provided by the U. S. Atomic Energy Commission under AEC Contract AT(30-1)-2098. A portion of these results was presented at the Washington meeting of the American Physical Society in April 1965.

[†] This work comprised part of an investigation of the $N=25$ nuclei, partially fulfilling requirements for the Ph.D. degree at MIT, 1963.

[1] P. D. Barnes, C. K. Bockelman, Ole Hansen, and A. Sperduto, Phys. Rev. **136**, B438 (1964); **138**, B597 (1965); **140**, B42 (1965).

[2] W. R. Cobb and D. B. Guthe, Phys. Rev. **107**, 181 (1957).

[3] J. Rapaport, A. Sperduto, and W. W. Buechner, Phys. Rev. (to be published).

[4] C. D. Jeffries, Phys. Rev. **92**, 1262 (1953).

[5] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955).

[6] D. Kurath, Phys. Rev. **91**, 1430 (1953); B. H. Flowers, Phil. Mag. (London) **45**, 329 (1954).

[7] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).

[8] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D. C.).

[9] T. A. Belote, W. E. Dorenbusch, Ole Hansen, and J. Rapaport, Nucl. Phys. **73**, 321 (1965).

[10] G. F. Pieper, Phys. Rev. **88**, 1299 (1952).

[11] L. L. Lee, Jr., and W. Rall, Phys. Rev. **99**, 1384 (1955).

[12] L. H. Th. Rietjens, O. M. Bilaniuk, and M. H. Macfarlane, Phys. Rev. **120**, 527 (1960).

[13] Ole Hansen, Nucl. Phys. **28**, 140 (1961).

[14] N. J. Zaika and O. F. Nemets, Izv. Akad. Nauk. SSSR **24**, 868 (1960) [English transl.: Bull. Acad. Sci. USSR, Phys. Ser. **24**, 868 (1960)].

[15] R. M. Sinclair, Phys. Rev. **107**, 1306 (1957); J. A. Knowles, G. Manning, G. A. Bartholomew, and P. J. Campion, *ibid.* **114**, 1065 (1959).

[16] D. A. Bromley, J. A. Kuehner, and E. Almqvist, Phys. Rev. **115**, 586 (1959); G. M. Temmer and R. N. Heydenburg, *ibid.* **104**, 967 (1956).

All levels reported in Refs. 8 through 16 are in general agreement with those observed in the present experiment. Particular comments on the results of angular distributions will be discussed later in Secs. III and IV.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The 7.0-MeV deuteron beam was obtained from the MIT–ONR electrostatic generator.[17] The beam was analyzed with a 90-deg deflection magnet, and the proton groups from the (d,p) reactions were analyzed with the MIT multiple-gap spectrograph.[18] This instrument has focal properties and broad-range features similar to those of the single-gap magnetic spectrograph.[19] However, the multiple-gap spectrograph consists of 25 gaps that permit simultaneous analysis of charged particles from the target. These gaps are located every 7.5 deg from zero through 172.5 deg with respect to the direction of the incident beam.

A Ti^{46} target was prepared in the Copenhagen isotope separator by the retardation method to more than 99% relative purity. The backing was a carbon foil approximately 50 μg/cm^2 thick (see Refs. 1, 20, and 21).

The 50-μ Eastman NTA photographic plates, which were used with the multiple-gap spectrograph to detect the protons, were covered with a layer of aluminum sufficiently thick to stop the elastically and inelastically scattered deuterons. After they were exposed to the deuteron-induced reaction products, the plates were developed and scanned with a microscope. The number of tracks in one-half millimeter strips were counted and plotted as a function of distance along the plates. Figure 1 shows part of a typical spectrum of the protons recorded at an angle of 37.5 deg with respect to the incident beam. The proton energy range corresponds to excitation energies in Ti^{47} from the ground state up to 4.0 MeV. Figure 2 shows an extension of this spectrum to the lower end of the focal surface of the multiple-gap spectrograph. Up to an excitation energy of 6.0 MeV, 107 proton groups corresponding to levels in Ti^{47} were identified. The measured resolution $E/\Delta E$ over this whole range was approximately 1000 or better, which corresponds to a half-width of $\leqslant 12$ keV for the proton groups.

The area of the carbon backing onto which the Ti^{46} material was deposited measured approximately 2 mm $\times$1 mm. To normalize the data from the bombardment of the enriched target, a natural titanium target (7.93% Ti^{46}) was prepared by evaporating natural titanium metal onto a 1-in. diam carbon backing supported by a thin layer of Formvar. Its thickness was determined from measurements of elastic scattering using 3.0-MeV deuterons and assuming Rutherford scattering. The measured value was 53 μg/cm^2. This target was then bombarded with 7-MeV deuterons, and the normalization to the enriched target was made

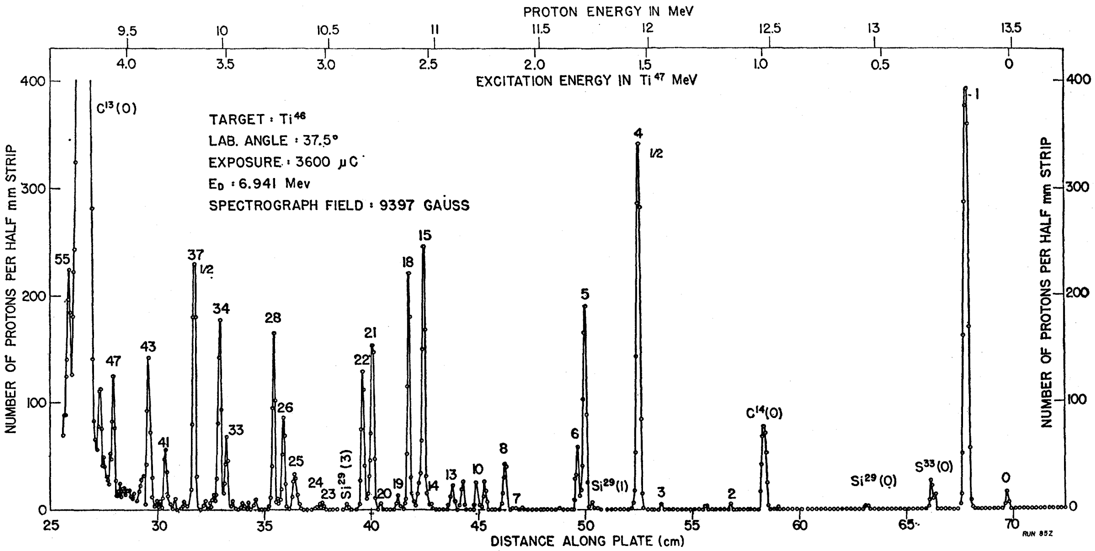

FIG. 1. Measured proton spectrum at laboratory angle 37.5°. The number of proton tracks in a 0.5-mm strip across the exposed zone is plotted against position along the photographic emulsion. Proton groups identified by their kinematic shift as arising from levels in Ti^{47} are labeled with the numbers used to identify these states in Table I. Prominent contaminant groups arising from carbon, silicon, and sulfur are also identified. The proton energy range corresponds to excitation energies in Ti^{47} from the ground state up to 4 MeV.

---

[17] W. W. Buechner, A. Sperduto, C. P. Browne, and C. K. Bockelman, Phys. Rev. **91**, 1502 (1953).

[18] H. A. Enge and W. W. Buechner, Rev. Sci. Instr. **34**, 155 (1963).

[19] C. P. Browne and W. W. Buechner, Rev. Sci. Instr. **27**, 899 (1956).

[20] G. Sidenius and O. Skilbreid, *Electromagnetic Separation of Radioactive Isotopes* (Springer-Verlag, Vienna, 1961), pp. 234, 243.

[21] J. H. Bjerregaard, P. F. Dahl, Ole Hansen, and G. Sidenius, Nucl. Phys. **51**, 641 (1964).

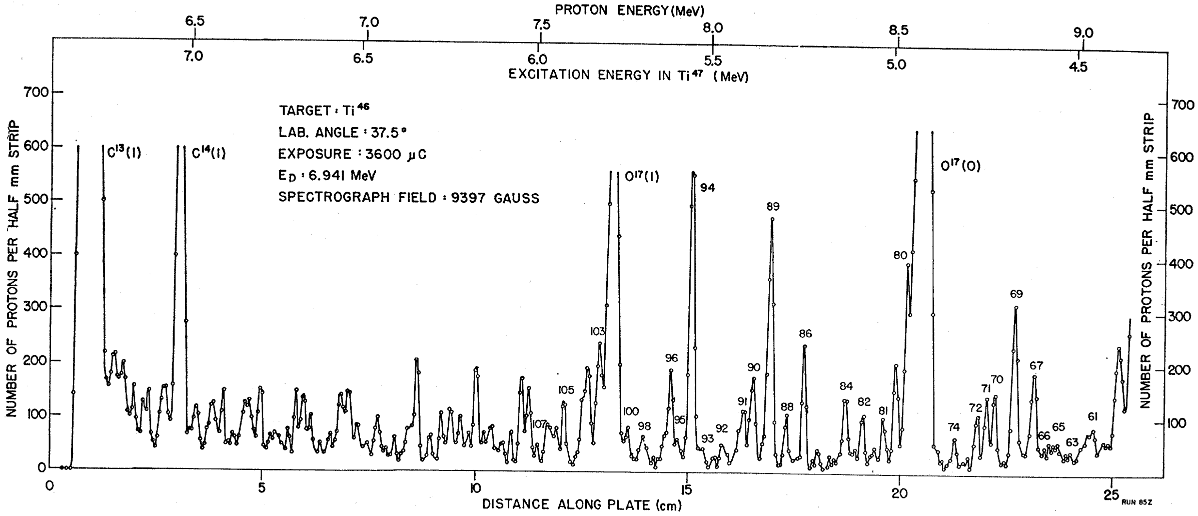

FIG. 2. Extension of the measured proton spectrum to the lower end of the focal surface of the spectrograph. Levels in Ti^{47} up to 6.0-MeV excitation energy are identified.

from measurements of the cross section of some of the more intense groups identified with the $Ti^{46}(d,p)Ti^{47}$ reactions.

Figures 3 and 4 show the experimental angular distributions for 24 of the groups analyzed. Where possible, a smooth curve joining the experimental points has been drawn.

Up to 4.0-MeV excitation energy, 47 angular distributions were analyzed, of which only 17 showed a typical stripping pattern; two were given $l_n=3$ assignments, ten, $l_n=1$, three, $l_n=0$, and two were tentatively assigned $l_n=(2)$ or (3). In the spectrum region corresponding to higher excitation in Ti^{47}, the C^{13} ground-state contaminant group contributed considerably to the background and obscured a large portion of the spectra. However, the energy levels and angular distributions of the proton groups with high cross section up to 6-MeV excitation were determined by analyzing the data obtained with one-tenth of the main exposure. These were observed at the first six gaps between 7.5 and 45.0 degs, and also at 90 degs. The excitation energies for all the levels were obtained from the groups observed in the forward-quadrant 90-deg gap, where the proton-resonance fluxmeter probe is located. Known groups from excited states of S^{33} and Si^{29} observed from (d,p) reactions on S^{32} and Si^{28} impurities were used to determine the incident energy and provided as well a check on the calibration of this particular gap.

The Q value for the ground-state $Ti^{46}(d,p)Ti^{47}$ reaction was determined by using the $S^{32}(d,p)S^{33}$ and $Si^{28}(d,p)Si^{29}$ ground-state known values.[22,23] A value of 6.665 ± 0.010 MeV was calculated for the $Ti^{46}(d,p)Ti^{47}$ ground-state reaction. This is an average of values determined at three different angles and is based on this Laboratory's Po^{210} alpha-particle energy standard of 5.3042 ± 0.0015 MeV. It is in excellent agreement with the value of 6.666 ± 0.012 MeV reported by Hansen.[13] Accurate Q-value measurements using the MIT single-gap instrument have been made of reaction groups from deuteron bombardment of a natural titanium target.[24] Although the ground-state transition was not observed in these experiments, Q-value determinations of several more prominent groups identified with the $Ti^{46}(d,p)Ti^{47}$ reactions were found to be consistent with the ground-state Q value deduced here.

Table I gives the Q values and excitation energies for all of the levels found in Ti^{47} up to an excitation energy of 6.0 MeV. The position of levels reported by Hansen[13] are in good agreement with our measurements reported in Table I, except for level No. 7, which was not observed by Hansen; it appears but very weakly and only at forward angles in the present work. The level at 0.55 MeV, reported by Rietjens *et al.*,[12] was not observed. No other levels in Ti^{47} above 2.5-MeV excitation have been previously measured with sufficient resolution to permit direct identification with any specific level listed in Table I above 2.5 MeV.

Figure 5 shows graphically the observed energy levels in Ti^{47} and the results of the stripping analysis for all the excited states that do show a stripping pattern. The statistically weighted spectroscopic factors, hereafter called the Ti^{47} strength functions, are also depicted according to their l_n values in the different columns.

---

[22] P. M. Endt and C. H. Paris, Phys. Rev. **110**, 89 (1958).

[23] D. M. Van Patter and W. W. Buechner, Phys. Rev. **87**, 51 (1952).

[24] A. Sperduto and W. W. Buechner, in *Proceedings of the Second International Conference on Nuclidic Masses, Vienna, 1963* edited by Walter H. Johnson, Jr. (Springer-Verlag, Vienna, 1964), p. 289.

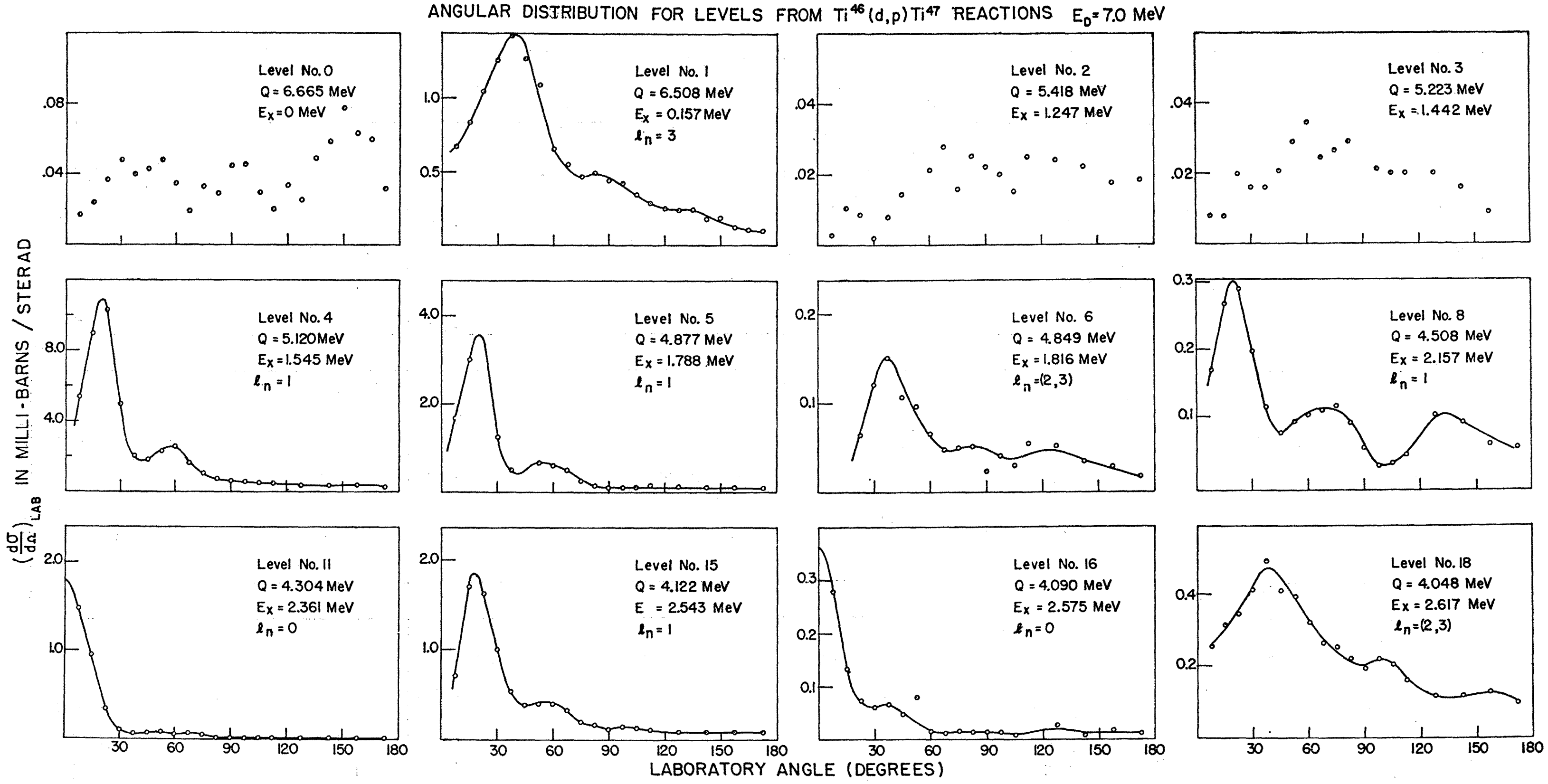

FIG. 3. Angular distributions of proton groups from Ti^{46}(d,p)Ti^{47}. A smooth curve joining the experimental points has been drawn. The absolute differential cross sections for all the observed levels up to 2.0-MeV and the stripping levels up to 2.7-MeV energy in Ti^{47} are indicated.

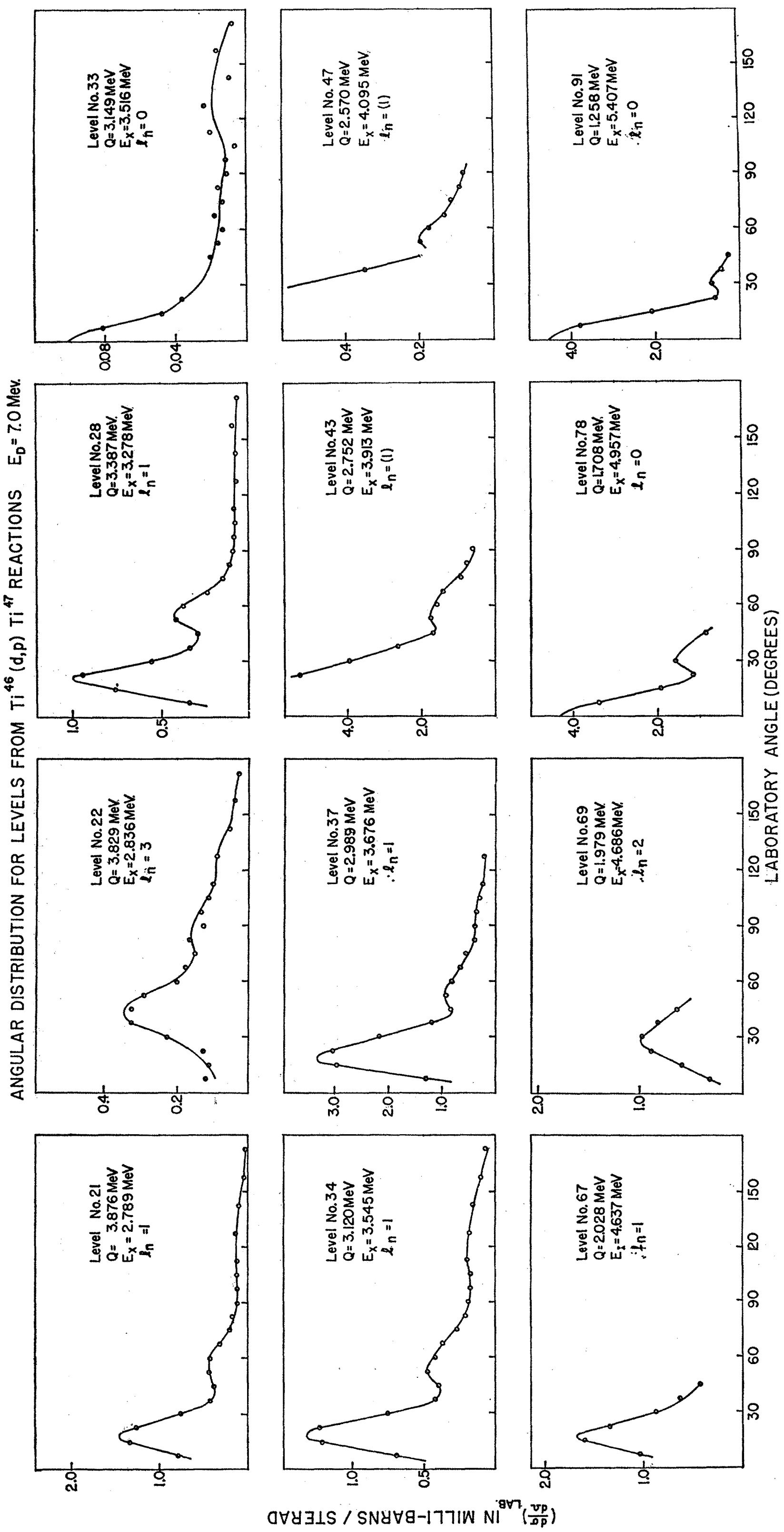

FIG. 4. Angular distributions of proton groups from $Ti^{46}(d,p)Ti^{47}$ for states that show a stripping pattern from 2.7-MeV up to 5.4-MeV excitation energy in Ti^{47}.

TABLE I. Summary of results for the levels of Ti^{47} formed through the $Ti^{46}(d,p)Ti^{47}$ reaction. $E_d=7.0$ MeV.

Level	Q^{a} (MeV)	$E_x{}^{b}$ (MeV)	$(d\sigma/d\Omega)_{max}$ (mb/sr) (lab)	θ_{max} (deg) (lab)	l_n	$(2J+1)S$
0	6.665	0	0.04			
1	6.508	0.157	1.47	37	3	5.1
2	5.418	1.247	0.02			
3	5.223	1.442	0.03			
4	5.120	1.545	10.8	18	1	2.36
5	4.877	1.788	3.4	18	1	0.71
6	4.849	1.816	0.16	37	(2) (3)	0.22 0.44
(7)	(4.563)	(2.102)	$\leqslant 0.008$			
8	4.508	2.157	0.3	20	1	0.06
9	4.413	2.252	0.03			
10	4.373	2.292	0.04			
11	4.304	2.361	1.8	0	0	0.06
12	4.263	2.402	0.02			
13	4.247	2.418	0.02			
14	4.148	2.517	0.03			
15	4.122	2.543	1.84	17	1	0.32
16	4.090	2.575	0.36	0	0	0.09
17	4.069	2.596	0.02			
18	4.048	2.617	0.46	37	(2) (3)	0.53 1.28
19	3.996	2.669	0.03			
20	3.911	2.754	0.01			
21	3.876	2.789	1.44	17	1	0.25
22	3.829	2.836	0.34	40	3	0.89
23	3.633	3.032	0.01			
24	3.611	3.054	0.01			
25	3.492	3.173	0.08		d	
26	3.441	3.224	0.07		d	
27	3.410	3.255	0.03			
28	3.387	3.278	1.0	20	1	0.02
29	3.296	3.369	0.03			
30	3.272	3.393	0.02			
31	3.236	3.429	0.02			
32	3.183	3.482	0.02			
33	3.149	3.516	0.1	0	0	0.003
34	3.120	3.545	1.36	18	1	0.20
35	3.086	3.579	0.04			
36	3.046	3.619	0.03			
37	2.989	3.676	3.2	18	1	0.48
38	2.964	3.701	0.04			
39	2.941	3.724	0.04			
40	2.889	3.776	0.03			
41	2.842	3.823	0.18^{c}			
42	2.827	3.838	0.03^{c}			
43	2.752	3.913	6.0^{c}		1	0.82
44	2.704	3.961	0.05^{c}			
45	2.647	4.018	0.02^{c}			
46	2.625	4.040	0.04^{c}			
47	2.570	4.095	0.7^{c}		(1)	0.1
48	2.551	4.112				
49	2.533	4.132				
50	2.465	4.200				
51	2.448	4.217				
52	2.422	4.243				
53	2.401	4.264				
54	2.384	4.281				
55	2.362	4.303				
56	2.329	4.336				
57	2.306	4.359				
58	2.285	4.380				
59	2.274	4.391				
60	2.199	4.466				
61	2.173	4.492				
62	2.147	4.518				
63	2.124	4.541				
64	2.112	4.553				
65	2.077	4.588				
66	2.060	4.605				
67	2.028	4.637	1.7	18	1	0.21
68	1.995	4.670				
69	1.979	4.686	1.0	30	2	0.30
70	1.922	4.743				
71	1.872	4.793				
72	1.854	4.811				
73	1.836	4.829				
74	1.818	4.847				
75	1.789	4.876				
76	1.767	4.898				
77	1.741	4.924	0.9	18	1	0.10
78	1.708	4.957	4.5	0	0	0.09
79	1.683	4.982				
80	1.652	5.013	1.1	18	1	0.13
81	1.622	5.043				
82	1.595	5.070				
83	1.563	5.102				
84	1.540	5.125				
85	1.517	5.148				
86	1.470	5.195				
87	1.400	5.265	1.5	0	0	0.06
88	1.364	5.301				
89	1.352	5.313	0.65	24	1 2	0.07 0.15
90	1.310	5.355	3.4	17	1	0.37
91	1.258	5.407	4.7	0	0	0.09
92	1.232	5.433				
93	1.187	5.478				
94	1.125	5.540				
95	1.085	5.580	2.9	17	1	0.32
96	1.050	5.615				
97	1.030	5.635				
98	0.995	5.670				
99	0.963	5.702				
100	0.910	5.755				
101	0.891	5.774				
102	0.855	5.810	3.3	18	1	0.33
103	0.829	5.836				
104	0.793	5.872				
105	0.728	5.937				
106	0.689	5.976				
107	0.641	6.024				

[a] The estimated uncertainty is 10 keV for level Nos. 0, 1, and 2; 12 keV for level Nos. 3 through 14; 16 keV for all other levels.
[b] The estimated uncertainty is 8 keV for level Nos. 1 and 2; 12 keV for level Nos. 3 through 14; 16 keV for all other levels.
[c] The C^{13} ground-state contaminant obscured level Nos. 41 through 47.
[d] Obscured by contaminant; uncertain l_n values.

III. RESULTS

A. Stripping Analysis

A distorted-wave code (distorted-wave Born approximation, DWBA), originated by Bassel *et al.*,[25] was used to fit the experimental proton angular distributions. The optical-model parameters, set B4 used in the previous Ti papers,[1] were used to predict the differential cross section $\sigma(\theta)$ for the (d,p) reactions. This, set, shown in Table II, was chosen because it corresponds to an average of fits for 6.0 MeV elastically scattered deuterons from isotopes of Ti^{48}, Ti^{49}, and Ti^{50}.

---

[25] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL 3240 (unpublished); and G. R. Satchler, Nucl. Phys. 55, 1 (1964).

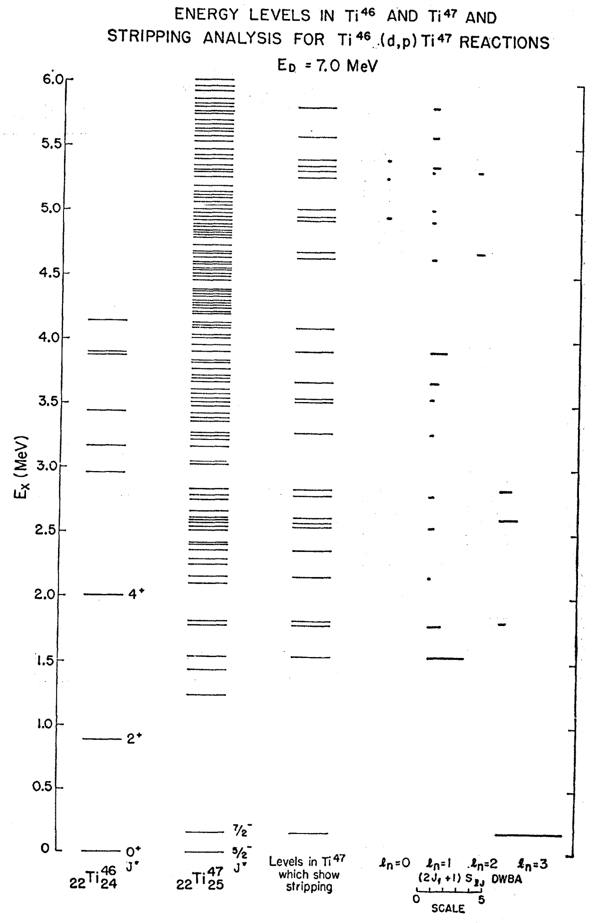

FIG. 5. Energy levels in Ti^{46} and Ti^{47} and stripping analysis for the $\mathrm{Ti}^{46}(d,p)\mathrm{Ti}^{47}$ reactions. The transition strengths $(2J_f+1)S$ are indicated in columns according to the l_n value.

Although the present experiment was performed with 7.0-MeV deuterons and no elastic data were obtained, set B4 was used without change. It is assumed that possible deviations in the calculated (d,p) cross section from use of the 6.0-MeV parameters would not be large and would fall within the limits of the experimental errors.[26] This assumption is partially supported from recent results observed in this Laboratory from the $\mathrm{Cr}^{52}(d,p)\mathrm{Cr}^{53}$ analysis,[27] where the set B4 parameters were compared with others extracted from elastic scattering of 7.5-MeV deuterons from Cr^{52}. The deviations in the predicted (d,p) cross sections were less than 10%. The parameters for the emitted protons were extrapolated from fits to data at higher bombarding energies,[1] and the captured neutron in the final state was described as moving in a Saxon well. The calculations were zero-range calculations with no radial cutoff (LCO=0).

TABLE II. Optical-model parameters used in the (d,p) analysis; set B4.

Particle	V (MeV)	W' (MeV)	r_0 (F)	a (F)	r_0' (F)	a' (F)	r_{0c} (F)
d	103	25.0	1.00	0.90	1.41	0.65	1.30
d	52	12.0	1.25	0.65	1.25	0.47	1.25

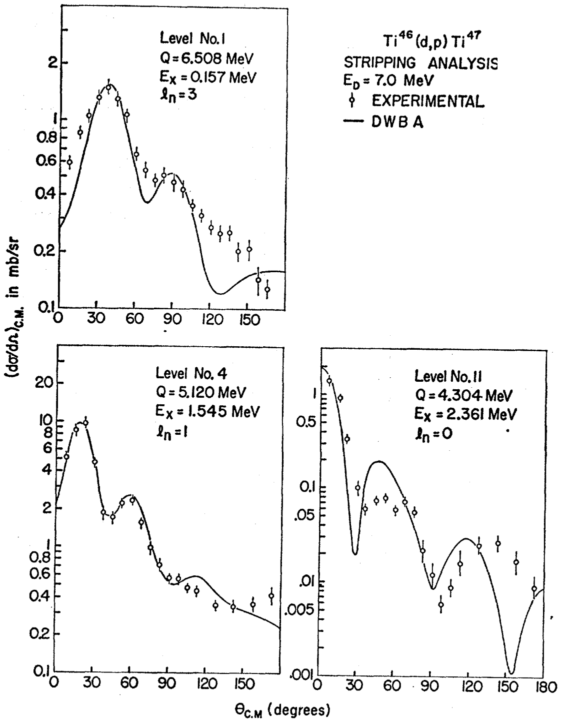

FIG. 6. Stripping analysis for states in Ti^{47} with largest cross sections for $l_n=3$, $l_n=1$, and $l_n=0$. The solid curves are derived from DWBA predictions.

Figure 6 shows the experimental data or three of the levels with the largest cross section and the calculated DWBA predictions. The orbital angular momentum l_n of the captured neutron was assigned on the basis of best match for the position of the experimental maximum with that predicted by DWBA calculations. The relationship between the experimental cross section $d\sigma/d\Omega$ and the DWBA results $\sigma(\theta)$ is given by

$$\frac{d\sigma}{d\Omega}=1.5\frac{2J_f+1}{2J_i+1}S\sigma(\theta).$$

Here, J_i and J_f represent the initial and final nuclear spins, and the numerical factor 1.5 is related to the use of the Hulthén wave function to describe the deuteron. The maximum of the differential cross sections from

---

[26] R. H. Bassel, R. M. Drisko, G. R. Satchler, L. L. Lee, Jr., J. P. Schiffer, and B. Zeidman, Phys. Rev. **136**, B960 (1964); and L. L. Lee, Jr., J. P. Schiffer, B. Zeidman, R. H. Bassel, R. M. Drisko, and G. R. Satchler, *ibid.* **136**, B971 (1964).

[27] A. Sperduto, D. L. Smith, M. N. Rao, H. A. Enge, W. W. Buechner, and H. Y. Chen, Bull. Am. Phys. Soc. **9**, 470 (1964).

the theoretical predictions was normalized to match the experimental maximum, and the normalizing factor thus obtained yielded the transition strength, $(2J_f+1)S$, for each excited state. The values of the transition strengths are indicated in column 7 of Table I.

In the following paragraphs, the analysis of the observed angular distributions will be discussed. A comparison of the experimental results with the shell-model sum rule[28] will be summarized in Table III.

B. The $l_n=3$ Groups

In Ti^{47}, the strongest $l_n=3$ group is the first excited state at $E_x=0.157$ MeV. The spin value for this state is known to be $\frac{7}{2}$, and therefore, using the transition strength value from Table I, one obtains $S=0.64$ for the spectroscopic factor. In the simple shell-model picture, a spectroscopic factor of $S=0.5$ would be expected. If consideration is given to the 15% error assigned to the measured absolute cross sections, the observed agreement is regarded as being good. Also, it has been reported that the B4 set of optical-model parameters give spectroscopic factors about 15% too high for the other titanium isotopes.[1]

The Ti^{47} ground-state spin[4] is known to be $\frac{5}{2}^-$. Since the Ti^{46} ground state has spin 0^+ and the captured neutron goes to the $1f_{7/2}$ shell, then from the jj-coupling viewpoint, the $Ti^{46}(d,p)Ti^{47}$ ground-state transition should be forbidden since it must involve a $1f_{5/2}$ nucleon. The experimental fact shows a ratio, approximately 35, for the maximum differential cross section between the state with $J=\frac{7}{2}$ and $J=\frac{5}{2}$. This is also seen in the $Ti^{48}(d,t)Ti^{47}$ reaction,[29] where, with 21.4-MeV deuterons, only the transition to the 0.157-MeV excited state was observed; the ground-state transition thereafter was much weaker. Since the shape of the angular distribution for the ground-state transition is certainly nonstripping, it could be inferred that the amount of $1f_{5/2}$ admixture in both the Ti^{46} and Ti^{48} ground-state wave functions therefore is negligible. This is quite reasonable, since the $1f_{5/2}$-$1f_{7/2}$ splitting is approximately 5.5 MeV.[30] These results are further confirmed in recent (p,d) experiments[31] on the titanium isotopes reported by Kashy and Conlon at 17.5 MeV and by Sherr *et al.* at 28 MeV.

In addition to these two levels arising from the $_\pi(1f_{7/2}{}^2)_\nu(1f_{7/2}{}^{-3})$ proton-neutron configurations, another level, No. 22, at 2.836-MeV excitation energy in Ti^{47} is observed with a characteristic $l_n=3$ stripping pattern. In the 28-MeV (p,d) pickup experiment,[31] this level is also reported with $l_n=3$, thus indicating that the state is probably associated with the $(1f_{7/2})$ configuration.

McCullen *et al.*[32] have recently made theoretical calculations on the spectroscopy of the nuclear $1f_{7/2}$ shell. Their model introduces a two-body residual interaction between pairs of nucleons in the $1f_{7/2}$ shell which is represented by the interaction matrix elements of the one-neutron, one-proton configuration. Those matrix elements are adjusted so as to fit the known Sc^{42} spectrum. For the Ti isotopes, they assume that the low-lying states can be described by a pure linear combination of $1f_{7/2}$ neutron and proton configurations. In Ti^{47}, McCullen *et al.*[32] thus predict two $l_n=3$ levels at 2.498 and 2.875 MeV. The latter is remarkably close to the observed $l_n=3$ level (No. 22), although the predicted spectroscopic factor is much lower than the value deduced in the present experiment. Levels in the region of 2.498 MeV are observed with low cross section, and they show no evidence of stripping character.

Levels Nos. 6 and 18 are both given probable $l_n=(2)$ or (3) assignments. Arguments for $l_n=2$ are given in the next section; here, we present evidence in support of the $l_n=3$ possibilities.

Level No. 6 at 1.816-MeV excitation energy is spaced only 26 keV from the strong $l_n=1$ state, No. 5. In the previously reported (p,d) work,[31] where the experimental resolution did not permit discrimination between the two states, the observed angular distribution was fitted with an admixture of $l_n=1$ and $l_n=2$. The (d,p) angular distributions observed in the present experiment for states Nos. 6 and 18 are shown in Fig. 7, together with the DW predictions for both $l_n=2$ and $l_n=3$ stripping.

Recently, a J dependence of the angular distributions has been reported from (d,p) reactions to $1f$ single-particle states. In this work, Alty *et al.*[33] depict the ratios of the differential cross sections of known $1f_{5/2}$ states to those of the known $1f_{7/2}$ states and observe an increase in these ratios at backward angles. The $l_n=3$ states from the present data were analyzed in a similar fashion; the ratio for the 2.836-MeV state to the 0.157-MeV state $(J=\frac{7}{2})$ is observed to be nearly constant over the whole angular range. However, each of the two other states tentatively assigned $l_n=3$ shows a ratio (relative to the 0.157-MeV state) that increases with angle up to a factor of 4 at 172.5 deg. If the $l_n=3$ assignments are correct, this pronounced difference could be indicative of a spin-dependent effect and would imply that the 1.816-MeV and 2.617-MeV excited states in Ti^{47} probably belong to the $(1f_{5/2})$ configuration, while the 2.836-MeV state in Ti^{47} belongs to the $(1f_{7/2})$ configuration. However, the $Ti^{46}(d,p)Ti^{47}$ angular-distribution data of Alty *et al.*[33], obtained with

---

[28] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

[29] J. L. Yntema, Phys. Rev. **127**, 1659 (1962).

[30] T. A. Belote, A. Sperduto, and W. W. Buechner, Phys. Rev. **139**, B80 (1965).

[31] E. Kashy and T. W. Conlon, Phys. Rev. **135**, B389 (1964); R. Sherr, B. F. Bayman, E. Rost, M. E. Rickey, and C. G. Hoot, *ibid.* **139**, B1272 (1965).

[32] J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. **134**, B515 (1964).

[33] J. L. Alty, L. L. Green, G. D. Jones, and J. F. Sharpey-Schafer, Phys. Letters **13**, 55 (1964).

9.15-MeV incident deuterons, are not in accord with our results here. On the basis of the ratio test, their data indicate, instead, a $\frac{7}{2}$ assignment to the 2.617-MeV state and a $\frac{5}{2}$ assignment to the 2.836-MeV state. The proton group corresponding to the level at 1.816 MeV is not mentioned in Ref. 33. The group very likely was not resolved and thus was obscured by the more prominent $l_n=1$ group at 1.788 MeV.

Our present $1f_{5/2}$ assignment to the 2.617-MeV state may explain why the 2.167-MeV state in Ti^{47} was not seen in the (p,d) experiments of Ref. 31, while the 2.836-MeV state, although having a lower spectroscopic factor than the 2.617-MeV state, is excited in the pickup experiment[31] with $l_n=3$ distribution.

Sherr *et al.*[31] in their 28-MeV pickup experiment report an additional $l_n=3$ state at 3.18-MeV excitation energy in Ti^{47}. This state is either level No. 25 at 3.173 MeV or level No. 26 at 3.224 MeV, or both, since the resolution of the pickup experiment was only 120 keV. Unfortunately, in the present 7.0-MeV (d,p) experiment, level Nos. 25 and 26 are only weakly excited and furthermore are obscured by contaminant peaks (S^{33}) at several angles, thereby making unambiguous l_n assignments rather difficult in these cases. Thus, from the present analysis, only the states at 0.157 MeV and 2.836 MeV (level Nos. 1 and 22) can be assumed to belong to the $1f_{7/2}$ configuration. The sum of the transition strengths, $\sum(2J_f+1)S$, is therefore 6.0, compared with the shell-model sum-rule prediction[28] of 4.0.

If the state corresponding to level No. 18 ($E_x=2.617$ MeV) alone is assigned to the $1f_{5/2}$ orbital, the observed transition strength of 1.28 is only about one-fifth of the sum-rule prediction, which would indicate that we failed to observe other $l_n=3$ distributions, most likely those above 4.0-MeV excitation. If the state at 1.816 MeV (level No. 6) is added to the $1f_{5/2}$ strength, although evidence in III C below favors $l_n=2$, the combined sum of 1.72 then corresponds to about 30% of the expected single-particle strength. It is possible that neither of these assignments is correct. In the following section, these two states are given a different interpretation in the light of other available data.

C. The $l_n=2$ Groups

Two states at 4.686- and 5.313-MeV excitation energy in Ti^{47} were assigned $l_n=2$. The $2d_{5/2}$-$1f_{7/2}$ single-particle spacing[30] is approximately 5.0 MeV. Therefore, these are most probably states belonging to the $2d_{5/2}$ configuration. Since, in this region of the Ti^{47} spectrum, only the levels that showed the largest cross sections were analyzed, it is very likely that there are several other $l_n=2$ states. The observed levels show less than 10% of the total $2d_{5/2}$ expected strengths.

In Table I, level Nos. 6 at 1.816 MeV and 18 at 2.617 MeV are each given a tentative $l_n=2$ assignment. However, as seen in Fig. 7, the $l_n=3$ theoretical calculations are in better agreement with the present data. The ratio test applied above to the corresponding angular-distribution data in an attempt to distinguish $J=\frac{7}{2}$ from $J=\frac{5}{2}$ states had led to a possible assignment of $J=\frac{5}{2}^-$ to both of these states. In the case of level No. 6, however, there is a clear conflict. Kashy and Conlon[31] observe an $l_n=2$ distribution with appreciable spectroscopic strength ($S=0.7$) for this state from the (p,d) pickup experiment. Their assignment is further strengthened by the systematics observed in the excitation of $\frac{3}{2}^+$ states in both Ti^{45} and Ti^{49}. In all three cases, the Q values, the differential cross sections, and the spectroscopic factors are very nearly the same. Hence, the present (d,p) conflicting "best fit" $l_n=3$ possibility for level No. 6 appears to be in error and suggests perhaps the need for more careful and critical analyses of both the experimental data and the DWBA description. Evidence of anomalous angular distributions in (d,p) data from this Laboratory has already been reported in the cases of the $Ca^{40}(d,p)Ca^{41}$ reaction[30] and the $Ni^{58}(d,p)Ni^{59}$ reaction.[34] Comparison of (p,d) and (d,p) spectroscopic information on specific states in such nuclei leads to the conclusion that particle excitation to hole states via the (d,p) reaction may well exhibit characteristic distributions that could be misinterpreted.

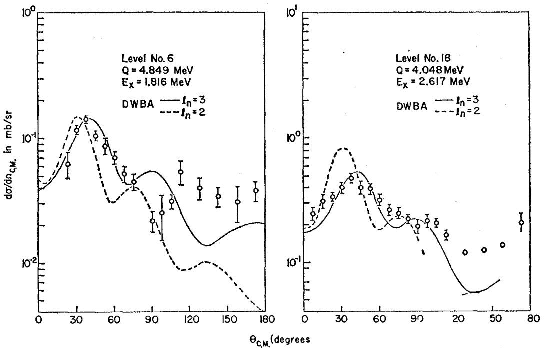

FIG. 7. Angular distributions to the 1.816- and 2.617-MeV levels in Ti^{47}. The experimental data are shown in comparison with the calculated curves, assuming 1*d* and 1*f* transfers.

In the pickup experiment,[31] a state at 2.56 MeV is reported, but no l_n assignment is given. Since this level in the (p,d) work was not completely resolved from the strong $l_n=1$ state at 2.789 MeV, it is conceivable that the 2.56-MeV level is the same excited state with $l_n=2$ (No. 18) in the present (d,p) work.

If it is assumed that the 1.816-MeV excited state is formed by coupling a $1d_{3/2}$ hole with the eight nucleons in the $1f_{7/2}$ shell with isotopic spin $T=2$, and that the 2.617-MeV state is formed by coupling a $1d_{3/2}$ hole with the eight nucleons in the $1f_{7/2}$ shell with isotopic spin $T=1$, then the latter state should not be reached by means of a pickup reaction because the Ti^{48} ground state has mainly a $T=2$ component.

---

[34] E. R. Cosman, C. H. Paris, A. Sperduto, and H. A. Enge, Phys. Rev. **142**, 673 (1966).

D. The $l_n=1$ Transitions

The $l_n=1$ states observed are due to $2p_{3/2}$ and $2p_{1/2}$ shell-model states. A sum-rule analysis of these states, taken from the data in Table I, gives

$$\sum(2J_f+1)S=6.84.$$

This permits the assumption that virtually all the $2p_{3/2}$ and $2p_{1/2}$ single-particle strengths have been observed.

The angular distributions were carefully analyzed in an attempt to find the "dip" effect reported by Lee and Schiffer.[35] This may have permitted differentiation between $p_{3/2}$ and $p_{1/2}$ states. However, probably because the bombarding energy was only 7.0 MeV, no such effect was observed (see also Ref. 1).

E. The $l_n=0$ Transitions

Titanium-46 has a ground-state spin and parity of $J^\pi=0^+$; therefore, all levels in Ti^{47} for which an $l_n=0$ assignment was made should have $J^\pi=\frac{1}{2}^+$. There are two $l_n=0$ states observed below 3.0-MeV excitation energy in Ti^{47}. Since those states are rather low-lying, they may arise from $2s$ core excitation.[36] Similar states have been reported in (d,p) experiments on the calcium isotopes.[30]

Levels with $l_n=0$ and large cross sections were found at about 5.0-MeV excitation energy in Ti^{47}. One may assume that those levels are mainly states from the $3s_{1/2}$ configuration. A sum-rule analysis, taken from Table I, gives $\sum(2J_f+1)S=0.24$, which is only one-eighth of the $3s_{1/2}$ single-particle strength. This would imply that there are several other $l_n=0$ states with lower cross sections or at higher excitation energies in Ti^{47}.

TABLE III. Sum-rule limits. The experimental data shown in the second row are $\Sigma(2J_f+1)S$ for the observed levels up to 6.0-MeV excitation energy in Ti^{47}.

Transition	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$(2p_{1/2}+2p_{3/2})$	$1f_{5/2}$	$2d_{5/2}$	$3s_{1/2}$
Theory[a]	0	0	4.0	6.0	6.0	6.0	2.0
Experiment	0.15[b]	0.22[c] 0.75[d]	6.0[e]	6.84	1.28[f] 1.72[g]	0.45[h]	0.24[i]

[a] Assuming pure $\pi(1f_{7/2}{}^2)\nu(1f_{7/2}{}^4)$ proton-neutron configuration relative to an inert Ca^{40} core (see, for example, Ref. 28).
[b] Includes level Nos. 11 and 16.
[c] Includes level No. 6.
[d] Includes level Nos. 6 and 18.
[e] Includes level Nos. 1 and 22.
[f] Includes No. 18.
[g] Includes Nos. 18 and 6.
[h] Includes Nos. 69 and 89.
[i] Includes Nos. 78, 87, and 91.

In Table III, the experimental and theoretical sum-rule limits for the observed transitions are compared.

IV. REMARKS

Of the 47 analyzed angular distributions up to 4.0-MeV excitation energy, only 17 showed a typical stripping pattern. The transitions of nonstripping character may result from compound-nucleus formation or from transitions involving excited proton configurations present in the Ti^{46} ground states or in states in which the neutron capture is accompanied by rearrangement of the outer shell. This interpretation is in line with the theoretical results of McCullen *et al.*[32] Up to 4.0 MeV, they predict 25 levels belonging to the $_\pi(1f_{7/2}{}^2)_\nu(1f_{7/2}{}^{-3})$ configuration, for which no stripping is predicted.

It is interesting to compare the energy levels of Ti^{47} with the known levels in Ca^{45}, both nuclei having 25 neutrons. First, the number of levels observed in Ca^{45} and Ti^{47} are the same up to 2.0-MeV excitation energy. This would indicate that they are due mainly to neutrons outside the closed shell. However, the number of levels in Ti^{47} between 2.0- and 3.5-MeV excitation energy is almost twice as great as the number of levels in Ca^{45} in the same region of excitation. This would mean that excited proton configurations are present in the Ti^{46} ground-state wave function.

Also, the strength of the $Ca^{44}(d,p)Ca^{45}$ ground-state transition, which was analyzed in the same way as were the data presented here, gives the same value as the $Ti^{46}(d,p)Ti^{47}$ first excited-state transition. This implies that both final states have essentially the same neutron configuration.

ACKNOWLEDGMENTS

We should like to express our thanks to Ole Hansen from the Niels Bohr Institute in Copenhagen for putting at our disposal the Ti^{46} isotope-separated target that was used in the present investigation and for his helpful and valuable comments. We are also indebted to H. Y. Chen for carrying out the DWBA calculations. The authors particularly wish to express appreciation to the MIT scanning group and especially to W. A. Tripp for his careful work. We extend special thanks to Mrs. Mary E. White for her expert assistance in the preparation of the manuscript. One of the authors (J.R.) wishes to express appreciation to the Agency for International Development, Department of State, which sponsored his stay at MIT and to thank the John Simon Guggenheim Memorial Foundation for granting him a one-year fellowship. He is also indebted to the DuPont Foundation for a grant which permitted a month's visit to MIT in January 1965.

---

[35] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. Letters **12**, 108 (1964).

[36] R. K. Bansal and J. B. French, Phys. Letters **11**, 144 (1964).