

Ground-State (p,n) Reactions in Mirror Nuclei and the Quasielastic Model of (p,n) Reactions*

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The $Al^{27}(p,n)Si^{27}$, $Si^{29}(p,n)P^{29}$, and $P^{31}(p,n)S^{31}$ ground-state reactions have been measured using polyethylene "long counters." Absolute cross sections and angular distributions were obtained from 0° to 160° at 10° intervals. The measured angular distributions were compared with the predictions of an optical-model calculation, using an optical potential suggested by Lane, which is a function of the isotopic spins of the incident proton and target nucleus. Since the ground-state (p,n) reactions were measured below the threshold for the excited state, the proton incident energies were between 5.8 and 7.6 MeV. Because of this low proton energy, the optical-model calculations include both Coulomb effects of distortion of the proton wave and the Coulomb energy difference between the incoming proton and the outgoing neutron. Fair agreement between the theoretical and experimental results was obtained for Si^{29} and P^{31} using a surface interaction for the imaginary and isobaric parts of the optical potential. Furthermore, the angular distributions from these two nuclei were quite similar in structure, in agreement with the twin-reaction hypothesis of Bloom, Glendenning, and Moszkowski. The neutron angular distributions from Al^{27} are quite different in structure from those obtained from Si^{29} and P^{31} , and attempts to fit them with the optical-model calculation were not successful. A calculation based on distorted-wave single-particle excitation with charge-exchange was also carried out. The results of this calculation agree with those of the optical model for Si^{29} and P^{31} . For the Al^{27} reaction the results of the DWBA calculation are different from those of the optical model, since in the former calculation momentum transfers greater than zero are included; however, neither calculation fitted the Al^{27} data.

INTRODUCTION

SOME years ago Austern *et al.*¹ suggested that (p,n) reactions could be the result of a direct interaction mechanism in a number of special cases. For example, in light nuclei the (p,n) reaction measured in regions away from pronounced resonances will proceed predominantly through a direct interaction. This will be especially true for nuclei containing one or few nucleons outside closed shells.

Sawicki,^{2,3} following Austern's mathematical approach, calculated the angular distribution and polarization for $Si^{29}(p,n)P^{29}$ at 6 and 6.5 MeV. Later, Glendenning and Bloom⁴⁻⁶ argued that the ground-state (p,n) reaction on mirror nuclei proceeded mainly through a direct interaction mechanism between the incoming proton and the neutron lying beyond the double closed neutron and proton subshell (i.e., Si^{29} can be considered as a Si^{28} core plus a neutron in the $2S_{1/2}$ shell), where the predominant part of the interaction is the charge-exchange process in which the charge of the incident proton is transferred to the bound neutron.

Lane and Soper,⁷ in 1962, pointed out that a direct

(p,n) reaction will excite preferentially that state in the final nucleus which is the isobaric analog of the target state, because least rearrangement is involved. He called this selection mechanism "isobaric state selection" (I.S.S.). Another preferential excitation is one in which the (p,n) reaction excites a final configuration that is the same as the initial one, except for the replacement of a neutron by a proton. This second selection mechanism is called "isobaric configuration selection" (I.C.S.).

The isobaric state excitation dominates in the (p,n) reaction and can be considered as a special case of elastic scattering with charge exchange. The initial and final states are quite similar and the isobaric spin t of the incident proton is flipped in the interaction, hence the name "quasielastic" (p,n) reactions. Lane⁸ suggested that one add to the optical potential already used in the analysis of elastic scattering of nucleons, a term that will account for this charge exchange. This term, called the isobaric spin potential, is proportional to $t \cdot T$ where T is the isobaric spin of the target nucleus.

This new optical potential has been used with fair success⁹⁻¹² in distorted-wave Born-approximation and optical-model-type calculations to fit the experimental angular distributions of neutrons from quasielastic (p,n) reactions.

In the present work the initial and final nuclei in each of the (p,n) reactions studied are mirror nuclei. Hence

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¹ N. Austern, S. T. Butler, and H. McManus, *Phys. Rev.* **92**, 350 (1953).

² J. Sawicki, *Phys. Rev.* **104**, 1441 (1956).

³ J. Sawicki and Z. Szymanski, *Acta Phys. Polon.* **17**, 21 (1958).

⁴ S. D. Bloom, N. K. Glendenning, and S. A. Moszkowski, *Phys. Rev. Letters* **3**, 98 (1959).

⁵ R. D. Albert, S. D. Bloom, and N. K. Glendenning, *Phys. Rev.* **122**, 862 (1961).

⁶ C. Wong, J. D. Anderson, S. D. Bloom, J. W. McClure, and B. D. Walker, *Phys. Rev.* **123**, 598 (1961).

⁷ A. M. Lane and J. M. Soper, *Nucl. Phys.* **37**, 506 (1962).

⁸ A. M. Lane, *Nucl. Phys.* **35**, 676 (1962).

⁹ R. M. Drisko, R. H. Bassel, and G. R. Satchler, *Phys. Letters* **2**, 318 (1962).

¹⁰ L. F. Hansen and M. L. Stelts, *Phys. Rev.* **132**, B1123 (1963).

¹¹ J. D. Anderson, C. Wong, J. W. McClure, and B. D. Walker, *Phys. Rev.* **136**, B118 (1964).

¹² G. R. Satchler, R. M. Drisko, and R. H. Bassel, *Phys. Rev.* **136**, B637 (1964).

the ground state of the final nucleus is the isobaric analog of the target ground state. In view of this the angular distributions for the ground-state neutrons were analyzed as a case of quasielastic (p, n) reactions, using an exact optical-model-type calculation with Lane's potential. In this calculation,¹³ the Coulomb potential and the Coulomb energy difference between the target and final nuclei are included. These effects cannot be neglected because of the low energy of the incident protons.

The $\text{Al}^{27}(p, n)\text{Si}^{27}$, $\text{Si}^{29}(p, n)\text{P}^{29}$, and $\text{P}^{31}(p, n)\text{S}^{31}$ ground-state reactions were measured from 6.03 to 6.81 MeV, from 6.56 to 7.39 MeV, and from 6.61 to 7.46 MeV, respectively. Optical-model calculations (OMC) for Si^{29} and P^{31} reproduced the experimental results with fair success; however, the calculations for Al^{27} did not give any agreement with the measured neutron angular distributions. Furthermore, the angular distributions from Si^{29} and P^{31} have very similar structures but are quite different in shape from the Al^{27} angular distributions. The reason for the different shapes could be due to the different ground-state configurations of these nuclei. The ground states of Si^{29} and P^{31} are both $2S_{1/2}$, while Al^{27} has a $d_{5/2}$ ground state. For $j = \frac{1}{2}$ only $l = 0$ is allowed so there is no momentum transfer to the nucleus, while for $j = \frac{5}{2}$, $l = 0, 2$ and 4 can occur. This mixture of l values could be responsible for the different shapes of the angular distributions.

Since the OMC does not include momentum transfer ($l = 0$), it is not an exact calculation for the $\text{Al}^{27}(p, n)\text{Si}^{27}$ reaction. Therefore, comparisons of theory and experiment were also made using the direct reaction calculation program (DRC) of Gibbs *et al.*,¹⁴ in which the differential cross sections are obtained on the basis of the distorted-wave Born approximation (DWBA). The calculations were carried out¹⁵ assuming the interaction to be a single-particle excitation with charge exchange via a Yukawa interaction. The DRC and the OMC are in fair agreement for the Si^{29} and $\text{P}^{31}(p, n)$ reactions, but neither calculation reproduced the Al^{27} angular distributions.

EXPERIMENTAL METHOD

The protons of energy between 6.0 and 7.5 MeV were obtained from the Livermore 90-in. variable energy cyclotron. The description of the experimental setup has been given elsewhere.¹⁶ The observed neutrons from the ground-state (p, n) reactions were detected using eight polyethylene "long counters." The angular distributions were measured from 0° to 160° at 10° steps. The angular spread of each counter was 2.5° .

The Al targets were self-supporting foils of 2.73

¹³ E. H. Schwarzc, Bull. Am. Phys. Soc. **10**, 25 (1965).

¹⁴ W. R. Gibbs, V. A. Madsen, J. A. Miller, W. Tobocman, E. C. Cox, and L. Mowry, NASA TND-2170, 1964 (unpublished).

¹⁵ Victor A. Madsen (to be published).

¹⁶ L. F. Hansen and M. L. Stelts, Phys. Rev. **136**, B1000 (1964).

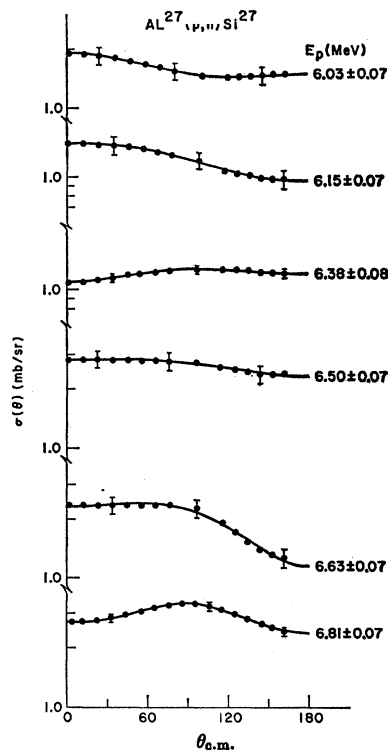


FIG. 1. Angular distributions for the ground-state neutrons from the reaction $\text{Al}^{27}(p, n)\text{Si}^{27}$ as a function of the incident proton energy.

mg/cm² and 3.24 mg/cm² thickness. The Si^{29} target was made at Oak Ridge using 70.6% enriched Si^{29} isotope evaporated on a Ta backing. The target thickness was 0.975 mg/cm². The P^{31} targets were obtained by depositing a suspension of red phosphorous in alcohol on a Au backing and letting the alcohol evaporate. The target thicknesses were 2.3, 3.4, and 3.9 mg/cm². The background neutrons were measured from blanks of thicknesses and materials identical to the ones used in the backings. Alternate measurements of neutrons from the target and blanks were made at each angle.

THEORY

Exact Optical-Model Calculation

A quasielastic (p, n) reaction results when the incident proton changes into a neutron, and the final nucleus is the corresponding isobaric analog of the target ground-state nucleus. Lane⁸ has shown that this reaction can be accounted for by introducing an optical potential of the form

$$V = V_0(r) + (\mathbf{t} \cdot \mathbf{T})V_1(r)/A, \quad (1)$$

where V_0 is the well-known optical potential, and \mathbf{t} and \mathbf{T} are the isotopic spin of the incident particle and target nucleus A , respectively. V_1 is the isospin potential.

The dependence of the differential (p, n) cross section on the form of the potential V_1 has been investigated by

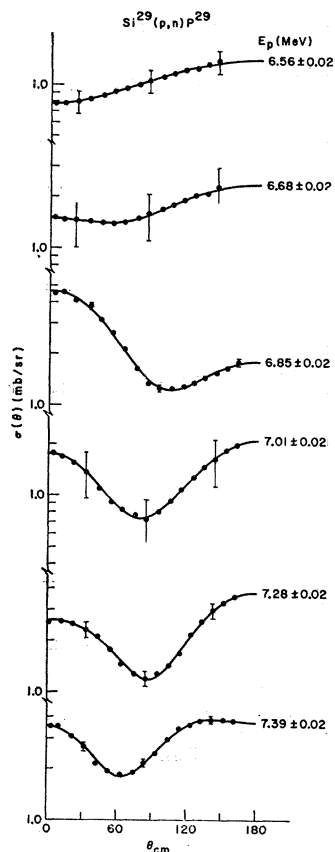


FIG. 2. Angular distributions for the ground-state neutrons from the reaction $\text{Si}^{29}(p,n)\text{P}^{29}$ as a function of the incident proton energy.

several authors.^{9,12,17,18} They have shown that the (p,n) cross sections are very sensitive to the radial form of V_1 and to the strength of the potential V_1 .

Fair agreement with experimental results has been reported¹⁰⁻¹² using the potential suggested by Lane in a distorted-wave Born-approximation calculation and in a direct optical-model-type calculation.

In all these previous calculations, approximations had been made in solving the two coupled equations, which result when the potential given by (1) is introduced in the Schrödinger equation for incident protons of energy E .

These coupled equations are (see Appendix in Ref. 8)

$$\begin{aligned} \left(\mathcal{T} + V_0 - E + V_c - \frac{1}{2A} T_0 V_1 \right) g_p \\ + \frac{1}{2A} (2T_0)^{1/2} V_1 g_n = 0, \\ \left(\mathcal{T} + V_0 - E + \Delta_c + \frac{1}{2A} (T_0 - 1) V_1 \right) g_n \\ + \frac{1}{2A} (2T_0)^{1/2} V_1 g_p = 0, \end{aligned} \quad (2)$$

¹⁷ T. Terasawa and G. R. Satchler, Phys. Letters 7, 265 (1963).

¹⁸ P. E. Hodgson and J. R. Rook, Nucl. Phys. 37, 632 (1962).

where \mathcal{T} is the kinetic energy, V_c is the Coulomb potential, T_0 is the isotopic spin of the target nucleus, Δ_c is the proton Coulomb energy, and g_p and g_n are space wave functions with asymptotic forms,

$$\begin{aligned} g_p &\sim e^{ik_p z} + f_{pp}(\theta) e^{ik_p r}/r, \\ g_n &\sim f_{pn}(\theta) e^{ik_n r}/r, \end{aligned} \quad (3)$$

where k_p and k_n are the proton and neutron wave numbers and $f_{pp}(\theta)$ and $f_{pn}(\theta)$ are the scattering amplitudes.

The coupled equations given by (2) include spin-orbit terms, the Coulomb potential, and the Coulomb energy difference between the target nucleus and the residual nucleus. The spin-orbit terms are included in the optical potential which is given by: $V_0(r) = -Vf(r) - iWg(r) + (\hbar/m\tau c)^2 V_{so} [df(r)/dr] \cdot \sigma$.

The cross section for the (p,n) process is given in this case by

$$\sigma(p,n)(\theta) = (k_n/k_p) |f_{pn}(\theta)|^2. \quad (4)$$

Recently, Schwarcz¹³ has worked out an exact solution for these coupled equations which has been coded for the 7094 computer ("LOKI 2A") for a usual type optical-model calculation.

The results of these calculations were applied here to calculate the (p,n) cross sections for the ground-state neutrons from Al^{27} , Si^{29} , and P^{31} . Because of the low energies of the incident protons, it was necessary that

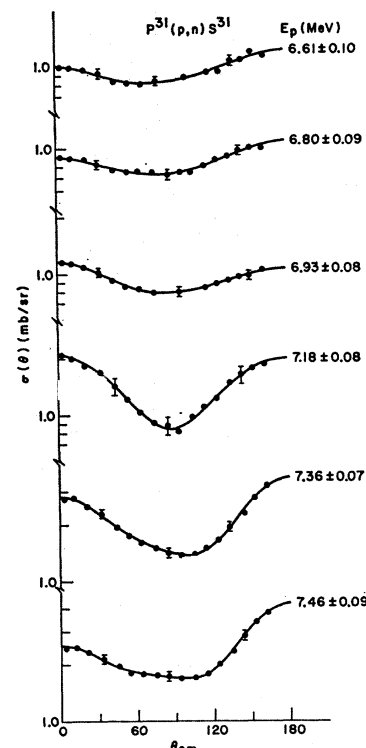


FIG. 3. Angular distributions for the ground-state neutrons from the reaction $\text{P}^{31}(p,n)\text{S}^{31}$ as a function of the incident proton energy.

TABLE I. Absolute (p, n_0) cross sections in Al^{27} , Si^{29} , and P^{31} .

Energy (MeV)	Al^{27}		Si^{29}		P^{31}	
	Energy (MeV)	σ_{pn} (mb)	Energy (MeV)	σ_{pn} (mb)	Energy (MeV)	σ_{pn} (mb)
6.03	19.6 ± 1.96	6.56	13.7 ± 1.37	6.61	10.2 ± 1.02	
6.15	15.4 ± 1.54	6.68	21.4 ± 2.14	6.93	11.1 ± 1.11	
6.38	15.3 ± 1.53	6.85	24.5 ± 2.45	7.18	15.4 ± 1.54	
6.50	33.3 ± 3.33	7.01	14.2 ± 1.42	7.36	22.7 ± 2.27	
6.63	25.8 ± 2.58	7.28	24.2 ± 2.42	7.46	25.8 ± 2.58	
6.81	38.2 ± 3.82	7.39	34.9 ± 3.50			

Coulomb effects be taken into account in the optical-model calculation.

RESULTS

The angular distributions for the ground-state (p, n_0) reaction in Al^{27} are shown in Fig. 1. The threshold for the reaction is 5.795 MeV and the threshold for the (p, n_1) (neutrons from the first excited state in Si^{27}) is 6.880 MeV. The continuous lines through the experimental points are the Legendre polynomial fits calculated to obtain the total cross sections. The errors indicated are absolute errors and they are a combination of the statistical errors, neutron detection efficiency errors,¹⁶ target thickness errors, and calibration errors of the current integrator. This combined error is about 10% or less, except for the measurements in Si^{29} at 7.01 MeV proton energy where the errors are 30%. The errors in the proton energy are the result of the energy spread of the proton beam due to target thickness.

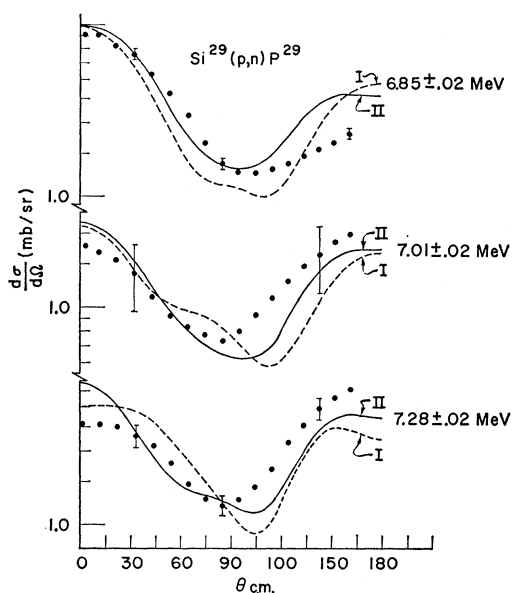


FIG. 4. Experimental and theoretical angular distributions for the ground-state neutrons from the $\text{Si}^{29}(p, n)\text{P}^{29}$ reaction. The theoretical curves were calculated with LOKI 2A. I. The optical parameters are from Perey's work with the exception of W , r_1 , and b_1 which were obtained from a search routine. II. Obtained by a rough search for all the optical parameters.

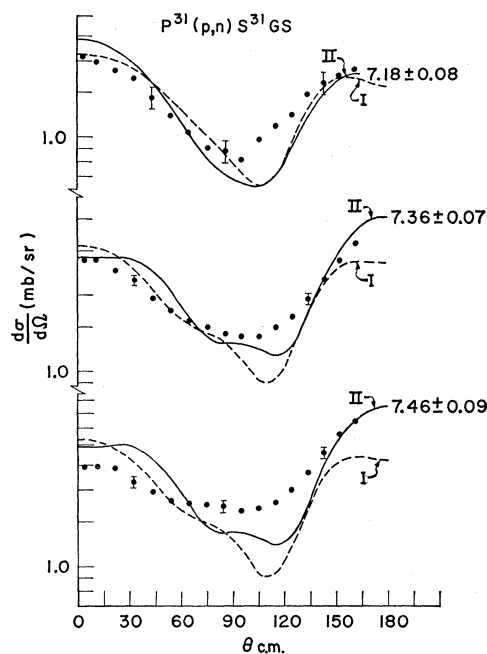


FIG. 5. Experimental and theoretical angular distributions for the ground-state neutrons from the $\text{P}^{31}(p, n)\text{S}^{31}$ reaction. The theoretical curves were calculated with LOKI 2A. I. The optical parameters are from Perey's work with the exception of W , r_1 , and b_1 which were obtained from a search routine. II. Obtained by a rough search for all the optical parameters.

The angular distributions are not symmetric about 90° . They are quite smooth and seem to indicate a resonance behavior.

Figures 2 and 3 show the ground-state angular distributions for the Si^{29} and P^{31} reactions. In Si^{29} the threshold for the (p, n_0) reaction is 5.948 MeV and for the (p, n_1) reaction, 7.292 MeV. In P^{31} the threshold for (p, n_0) is 6.493 MeV and for (p, n_1), 7.599 MeV. The angular distributions from these two nuclei change gradually with increasing proton energy and their features are quite similar, which will suggest that the "twin reaction hypothesis"³⁷ holds in this case.

Table I gives the absolute cross sections for the (p, n_0) reactions for these nuclei as a function of energy.

The theoretical angular distributions for the ground-state neutrons in Si^{29} and P^{31} using an optical-model calculation are shown in Figs. 4 and 5. They were calculated using the LOKI 2A¹⁸ program.

The best agreement with the measured angular distributions was obtained with surface potentials for the imaginary potential W and the isobaric potential V_1 . (The surface potential in these calculations is a Gaussian function of r .) The real potential, as is customary, is a Saxon potential.

The optical parameters for the nuclear potential were taken from Perey's¹⁹ optical-model analysis of proton

¹⁹ F. G. Perey, Phys. Rev. 131, 745 (1963).

TABLE II. Values of the optical parameters. I. This set of parameters was taken from Perey's work with the exception of W and the parameters for the isobaric potential, V_1 , r_1 , and b_1 , which were obtained using a search routine. II. Obtained from a rough search for all the parameters.

	$\text{Si}^{29}(p,n)\text{P}^{29}$				$\text{P}^{31}(p,n)\text{S}^{31}$							
	6.85 MeV		7.01 MeV		7.28 MeV		7.18 MeV		7.36 MeV		7.46 MeV	
	I	II	I	II	I	II	I	II	I	II	I	II
V (MeV)	50.77	45.77	50.68	50.68	50.54	50.54	50.53	50.53	50.43	50.43	50.28	50.28
r_0 (F)	1.25	1.25	1.25	1.31	1.25	1.31	1.25	1.26	1.25	1.29	1.25	1.29
a (F)	0.65	0.55	0.65	0.45	0.65	0.45	0.65	0.55	0.65	0.45	0.65	0.45
W (MeV)	5.061	3.00	2.471	4.00	4.186	3.00	3.561	4.50	2.975	3.00	2.946	3.00
b (F)	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
V_{so} (MeV)	7.0	6.82	7.0	6.43	7.0	6.44	7.0	7.0	7.0	7.0	7.0	7.00
V_1' (MeV)	350.3	275.0	158.2	400.0	502.9	400.0	512.3	481.0	400.7	481.0	400.6	481.0
r_1 (F)	1.373	1.25	0.792	1.31	1.244	1.31	1.213	1.25	1.241	1.29	1.228	1.29
b_1 (F)	1.204	1.08	1.537	0.58	0.759	0.68	0.744	0.88	0.973	0.98	0.962	0.98
χ^2	5.64	7.31	0.79	0.42	6.12	4.14	2.51	3.37	2.71	2.04	7.26	5.26

elastic scattering. The well depth for the real potential was calculated from the expression $V = 53.5 - 0.55E + 0.4Z/A^{1/3} + 27(N-Z)/A$. The imaginary potential was derived from $W = 3A^{1/3} \pm 1.5$ MeV and the strength of the spin-orbit potential, $V_{so} = 7$ MeV. The geometrical parameters were: $r_0 = 1.25$ F, $a = 0.65$ F, and $b = 0.98$ F. For the nuclei studied here, and for the energy of the incoming protons, Table II gives the value of the parameters taken from Perey's work, which will be called the "standard set." The strengths of isobaric potential were calculated according to the expression $V_1' = v_1 R/na$, where $v_1 \approx 100$ MeV has been obtained from the proton potential anomaly,²⁰ $R = r_0 A^{1/3}$ and $na = b$ for a Gaussian surface potential. The geometrical

parameters for the isobaric potential were taken equal to those of the nuclear potential.

The common feature of the absolute differential cross sections calculated with the set of parameters just described is that they are lower than the experimental values by a factor of almost ten. Since the magnitudes of the cross sections are roughly proportional to $V_1'^2$, larger values could have been obtained by increasing the value of V_1' ; however, larger values of V_1' did not yet give a good fit to the shape of the angular distributions. Furthermore, the already accepted value of $v_1 \approx 100$ MeV seems to be supported by previous experiments.^{11,19}

A study of the effects of variations of the different parameters on the magnitude and shape of the differential cross sections suggested that lower values for the imaginary potential than those obtained by Perey would give better fits to the data (Fig. 6). Taking the depth of the imaginary potential W at a tentative value of 3 MeV, a new fit to the experimental distribution was obtained by keeping the other nuclear optical parameters¹⁹ fixed and searching for W and the isobaric parameters r_1 and b_1 (where $V_1' = 100R/b$). The search was done by the least-squares method using Maddison's²¹ search routine.

The curves labeled I in Figs. 4 and 5 show the calculated angular distributions using the criterion just described in the selection of the optical parameters. The final values of the parameters are given in Table II as well as the values of χ^2 , which give a measurement of the accuracy of the fit.

χ^2 is defined here as

$$\chi^2 = N^{-1} \sum_{i=1}^N [\sigma_{\text{th}}(\theta_i) - \sigma_{\text{exp}}(\theta_i) / \Delta\sigma_{\text{exp}}(\theta_i)]^2,$$

where σ_{th} and σ_{exp} are the calculated and experimental values of the cross sections at θ_i , and $\Delta\sigma_{\text{exp}}$ is the experimental error in the measurement at that angle.

An attempt was made to get better agreement between the calculated and measured (p,n) cross sections

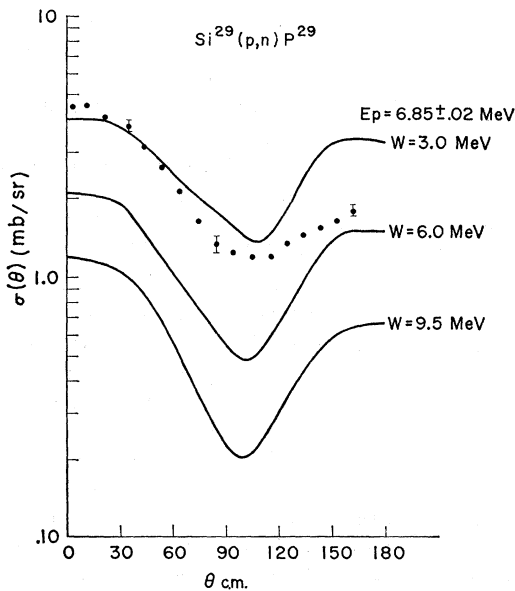


FIG. 6. Theoretical angular distributions for the neutrons as a function of the depth of the imaginary potential. The calculations were done with LOKI 2A for the optical parameters (Ref. 19) designated in the text as the "standard set."

²⁰ P. C. Sood, Nucl. Phys. **37**, 624 (1962).

²¹ R. N. Maddison, Proc. Phys. Soc. (London) **79**, 265 (1962).

by making a rough search for all the parameters of the optical model. The results obtained are given by curves labeled II in Figs. 4 and 5, and the final values of the parameters are shown in Table II with the corresponding values of χ^2 .

By comparing the measured differential (p, n) cross sections with the theoretical calculations using Set I and Set II parameters, it seems that the over-all agreement is better for the calculations done with the optical parameters given by Set II, especially with respect to the depth and location of the minima in the angular distributions. This was accomplished with a larger nuclear radius parameter [$(r_0)_{av}=1.285$ F], a smaller value of a ($a_{av}=0.48$ F), and a smaller value for the imaginary potential W ($W_{av}=3.42$ MeV).

Qualitative comments can be made to justify the variation of some of the optical-model parameters just discussed. For example the accepted value of $r_0=1.25$ F has been obtained mainly from elastic scattering data in medium weight nuclei. Because of the more loosely bound structure of the nucleons in lighter nuclei, one anticipates a larger r_0 . The value of ~ 9 MeV (i.e., $W=3A^{1/3}\pm 1.5$) for the imaginary potential depth also has been obtained from fitting elastic scattering data. Its purpose is to account for all nuclear process, other than the elastic scattering, initiated by the incident proton. The analysis of the quasielastic (p, n) cross section with the optical-model calculation effectively removes the process from the inelastic group. Hence, the imaginary potential has to account for a smaller number of nuclear processes which results in a smaller value for the well depth. However, the measured (p, n) cross sections are here only a very small fraction of the total absorption cross section, so this will not account for the much lower value of the imaginary potential used in the fittings.

Of course these remarks are only of a tentative character, since they are based on the behavior of the parameters found for the optical potential in a very narrow energy region and for only two nuclei.

For the lower proton incident energies shown in Figs. 2 and 3, the calculated differential cross sections for the Si^{29} and P^{31} (p, n) reactions did not give any reasonable agreement with experimental measurements.

$\text{Al}^{27}(p, n)\text{Si}^{27}$ Reaction: The $\text{Al}^{27}(p, n_0)\text{Si}^{27}$ angular distributions shown in Fig. 1 were not reproduced by the optical-model calculations. It was assumed that this failure of the quasielastic model was due to the fact that the ground-state configuration of $\text{Al}^{27}(d_{5/2})$ allows momentum transfer values of $l=0, 2, \text{ and } 4$. Since the optical model considers the (p, n) reaction to be an elastic scattering with charge exchange, only values of $l=0$ are included (no momentum transfer). Of course, this is sufficient for the (p, n_0) reactions in Si^{29} and P^{31} , where the ground states in both nuclei are $2S_{1/2}$.

To determine the importance of the contributions from $l=2$ and $l=4$, the (p, n_0) differential cross sections were calculated using the direct reaction calculation of

TABLE III. Optical parameters for the proton and neutron used in the DRC program to calculate the $\text{Al}^{27}(p, n)\text{Si}^{27}$ cross section.

	V (MeV)	W (MeV)	r_0 (F)	a (F)	b (F)
Proton	51.0	3	1.25	0.65	0.98
Neutron	47.8	3	1.25	0.65	0.98

Gibbs *et al.*^{14,15} In this calculation the cross section is obtained using a distorted-wave Born approximation (DWBA). The transition amplitude for the reaction $A(p, n)B$ has the form^{22,23} of a matrix element between product wave functions.

$$T_{p,n} = \langle \psi_B \psi_n \chi_n^{(-)}(\mathbf{k}_n, \mathbf{r}_n) V \psi_A \psi_p \chi_p^{(+)}(\mathbf{k}_p, \mathbf{r}_p) \rangle,$$

where $\psi_A, \psi_B, \psi_p, \psi_n$ are the internal wave functions for the noninteracting particles A, B, p , and n . The functions $\chi_p^{(+)}(\mathbf{k}_p, \mathbf{r}_p)$ and $\chi_n^{(-)}(\mathbf{k}_n, \mathbf{r}_n)$ are the distorted waves describing the relative motion of the proton and initial nucleus A and the neutron and final nucleus B , respectively. They are obtained from an optical-model analysis of the appropriate elastic-scattering data.

The potential V was taken¹⁵ to be a single-particle excitation with charge exchange via a Yukawa interaction.

$$V_{01} = \tau_0 \cdot \tau_1 (a \sigma_0 \cdot \sigma_1 + b) V(r_0, \mathbf{r}_1),$$

where τ_0, τ_1, σ_0 , and σ_1 are the isospins and spins of the

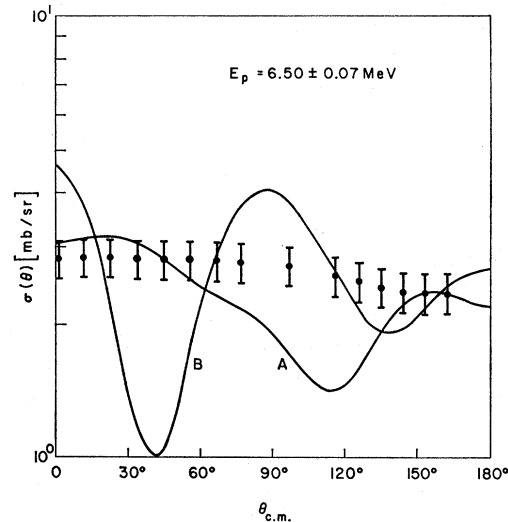


Fig. 7. Experimental and theoretical angular distributions for the ground-state neutrons from the $\text{Al}^{27}(p, n)\text{Si}^{27}$ reaction. A: Calculated with LOX1 2A with the optical parameters from Perey's work with the exception of the values of $W=2.78$ MeV, $r_1=1.13$ F, and $b_1=0.996$ F obtained from a search routine. B: Calculated with the DRC.

²² N. Austern, in *Fast Neutron Physics*, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1963) Vol. IV, p. 1113-1216.

²³ F. Bjorklund and S. Fernbach, *Phys. Rev.* **109**, 1295 (1958).

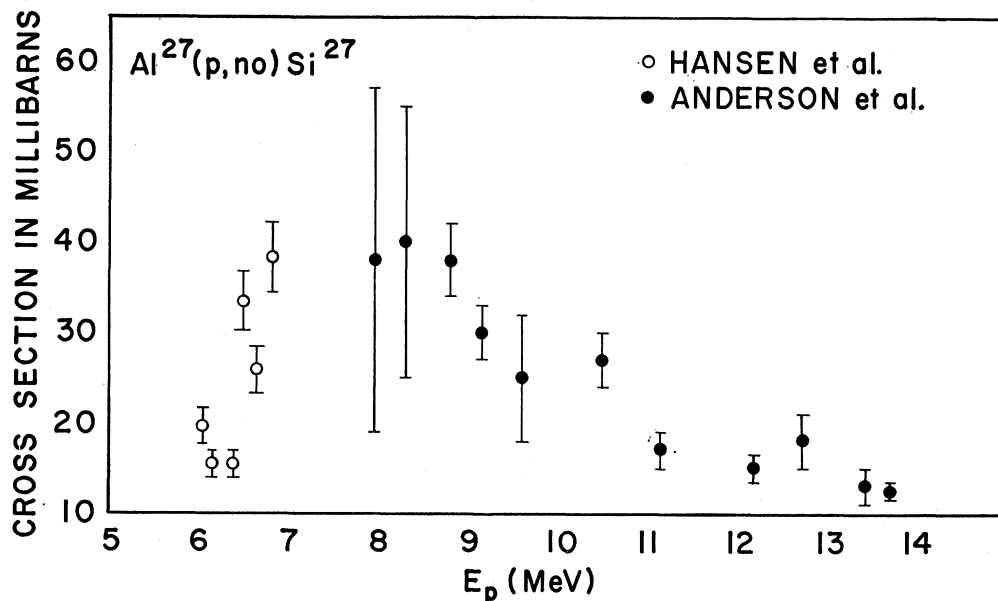


FIG. 8. Excitation function for the ground-state neutrons from the $Al^{27}(p,n)Si^{27}$ reaction.

incident proton and the bound particle 1 in the target nucleus A , taken as composite ("particle" + "core"); a and b are parameters that can take values between 0 and 1. $V(r_0, r_1)$ is the Yukawa interaction.

This picture of the initial nucleus as composed of a "core" plus a bound particle can be applied to the three nuclei studied here. Al^{27} is taken to be a Si^{28} core with a proton hole in the $d_{5/2}$ shell, Si^{29} is a Si^{28} core plus a neutron in the $2S_{1/2}$ shell, and P^{31} is a S^{32} core with a $2S_{1/2}$ proton hole.

Calculations were done with the DRC program of Gibbs.¹⁴ The optical parameters for the protons¹⁹ and neutrons²³ which were used to obtain $\chi^{(+)}$ and $\chi_n^{(-)}$ are given in Table III. The strength of the Yukawa interaction was taken equal to 30 MeV and the range equal to the Compton length of the π meson. Figure 7 shows the calculated differential cross sections for $Al^{27}(p,n)Si^{27}$ at 6.5 MeV.

Since the DRC calculation and the optical-model calculation are in rough agreement for the Si^{29} and P^{31} reactions, the failure to reproduce the Al data indicates that the contributions from $l=2$ and 4 are not significant.

The failure of both direct interaction calculations to explain the $Al^{27}(p,n_0)Si^{27}$ cross sections is possibly due to major contributions of compound nucleus effects at these energies. For example, the structure of the angular distributions in Fig. 1 seems to indicate resonance effects. Further, the excitation function of Fig. 8²⁴ appears to confirm this point.²⁴

CONCLUSIONS

In view of the low energies of the incident protons for the (p,n) reactions studied here, it is quite surprising

²⁴ The cross sections above 8 MeV were measured with time-of-flight techniques by J. Anderson *et al.*, Ref. 11.

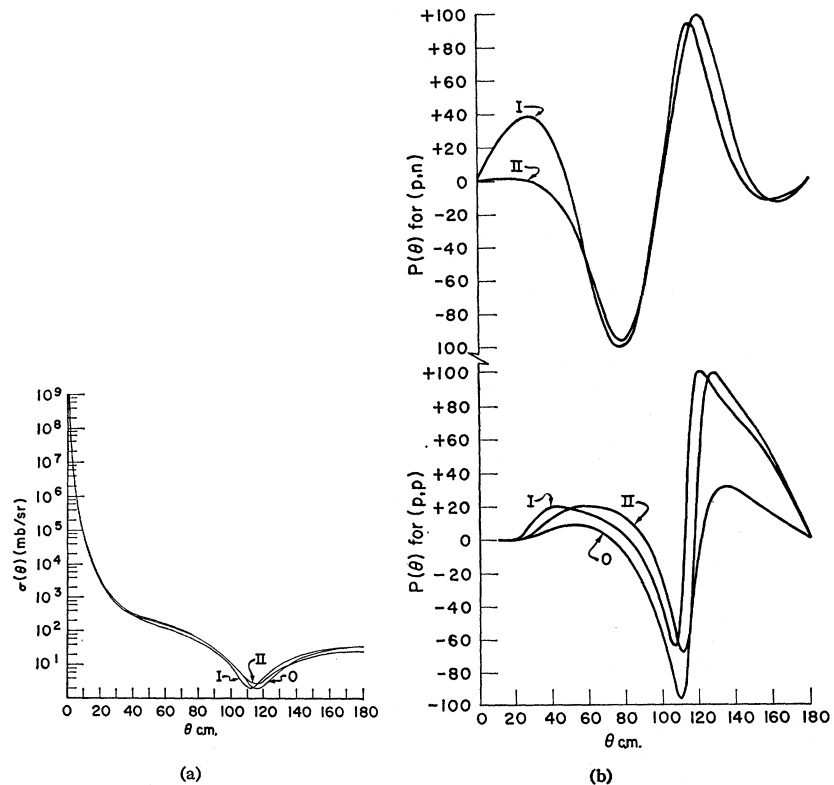
that reasonable agreement was obtained with the optical-model potential. At these energies compound nucleus contributions can be important, since the observed (p,n) channel is the only neutron channel open.

The fact that the calculations were unable to reproduce the experimental (p,n_0) differential cross sections at the very low energies for Si^{29} and P^{31} and at all energies for Al^{27} could be the result of a coherent interference between resonances and direct effects.

A surface potential for the imaginary and isobaric potentials gave better agreement with the experimental data than volume potentials, in agreement with what was found for (p,n) reactions in middle nuclei.¹¹

The over-all better agreement obtained with optical parameters different from those taken from Perey's proton scattering analysis suggests that it will be interesting to analyze simultaneously for a given nucleus the proton elastic scattering with the quasielastic (p,n) scattering using the exact calculation of Schwarcz¹³ for the optical potential. To minimize compound-nucleus effects one must make these analyses at higher energies, where more channels would be available for neutron emission. Figures 9(a) and 9(b) show the calculated differential cross section and polarization for (p,p) and (p,n) scattering for 7.46-MeV protons in P^{31} with LOKI 2A for the parameters given in Table II. The (p,p) scattering is relatively insensitive to the introduction of the isobaric potential, whereas the quasielastic (p,n) cross section and respective polarization are quite sensitive to the form of the potential, as well as to the values of the parameters used. This indicates that any serious attempt to obtain the optical parameters has to include experimental information other than the (p,p) scattering.

FIG. 9(a). Theoretical angular distributions for the elastic scattering of protons from P^{31} at 7.46 MeV calculated with LOKI 2A. O: Calculated with the optical parameters given by Perey (standard set) and no isobaric potential. I and II. Calculated with the optical parameters given in Table II. (b). Theoretical angular distributions for the polarizations of the protons from the elastic scattering and for the neutrons from the quasielastic scattering in P^{31} bombarded with 7.46-MeV protons. O: Calculated with the optical parameters given by Perey and no isobaric potential. I and II. Calculated with the optical parameters given in Table II.



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