results of this experiment indicate the persistence of rather sharp resonances through the giant-resonance region.

Recently, Izumo[34,35] has presented a theory of "partial equilibrium" for nuclear reactions in which a few (3–7) nucleons share excitation energy. The model is an attempt to explain the existence of "intermediate resonances" or clumps of oscillator strength ranging in width from 100 to 400 keV. It may be that the resonances observed in this experiment below 14 MeV are examples of such intermediate resonances.

One prediction of the Izumo theory is that the resonance structure of different nuclei in which the same number of nucleons share in the excitation should look the same (although displaced slightly in energy). The theory of Danos *et al.*, on the other hand, suggests that the photon absorption spectrum of odd-A nuclei should be somewhat different than even-A nuclei, even though the presence of the odd nucleon does not make a dramatic difference.

It seems clear that the relative merits of the various collective oscillation theories for heavy nuclei can only be tested by a succession of high-resolution experiments on a variety of such nuclei in the near future. The results of this experiment suggest that such experiments are possible and will be carried out.

[34] K. Izumo, Nucl. Phys. **62**, 673 (1965).
[35] K. Izumo, Prog. Theoret. Phys. (Kyoto) **26**, 807 (1961).

PHYSICAL REVIEW VOLUME 143, NUMBER 3 MARCH 1966

Number-Conserving Approximation for the Theory of the Pairing Interaction in Nuclei

GIU DO DANG
Service de Physique Théorique, Orsay, Seine et Oise, France
AND
ABRAHAM KLEIN
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania
(Received 25 October 1965)

The pairing interaction is studied by means of the equations of motion. An approximation is derived which adheres strictly to particle-number conservation, but is otherwise quite close to the formalism of the Bardeen-Cooper-Schrieffer (BCS) theory. It yields ground-state energies and single-particle properties considerably improved over those of the latter theory, as evidenced by calculations for (i) the model of Pawlikowski and Rybarska, (ii) the model of the Ni isotopes solved exactly by Kerman, Lawson, and MacFarlane. Though it does not yet surpass in accuracy several alternative improvements of the BCS theory, it is capable of systematic further development, including the treatment of more realistic potentials.

I. INTRODUCTION

THE present systematic theory of medium and heavy nuclei[1,2] is based upon ideas and methods borrowed from the theory of superconductivity[3] (the BCS theory). During the past few years there have been numerous efforts, detailed below, to improve on the basic approximation of this theory—the treatment of the properties of a given nucleus as the average of the properties of an ensemble of muclei. If we restrict our attention momentarily to the pairing interaction, as has been the case in almost all work of this sort, there now exist exactly soluble models, both of spherical[4] and of deformed[5] nuclei against which to measure the accuracy of the BCS theory and its proposed extensions.

One new approach has been related to Lipkin's[6] idea of allowing for the "curvature" of the separation energy as a function of particle number, i.e., of replacing the operator used in the BCS theory,

$$\mathcal{K}=H-\lambda A\,, \tag{1.1}$$

where H is the Hamiltonian, λ the separation energy or chemical potential, and A the number of particles, by a more general operator

$$\mathcal{K}=H-\lambda f(A)\,. \tag{1.2}$$

[1] A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958); S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **31**, No. 11 (1959).
[2] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 9 (1960), referred to as KS.
[3] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957); N. N. Bogoliubov, Zh. Eksperim. i Teor. Fiz. **34**, 58 (1958) [English transl.: Soviet Phys.—JETP **7**, 41 (1958)]; J. G. Valatin, Nuovo Cimento **7**, 843 (1958). These equivalent formulations will be referred to collectively as the BCS theory.
[4] A. K. Kerman, R. D. Lawson, and M. H. MacFarlane, Phys. Rev. **124**, 162 (1961), referred to as KLM.
[5] A. Pawlikowski and W. Rybarska, Zh. Eksperim. i Teor. Fiz. **43**, 543 (1962) [English transl.: Soviet Phys.—JETP **16**, 388 (1963)].
[6] H. J. Lipkin, Ann. Phys. (N. Y.) **9**, 272 (1960).

A version of this idea has been studied by Nilsson[7] and an actual scheme of calculation developed and carried through by Nogami[8] to the extent of including terms quadratic in A in $f(A)$.

A second approach has involved an elaboration of Bayman's method[9] in which one employs a number-conserving wave function so constructed that the BCS theory can be derived from it by an argument of steepest descents; by extension of the same argument, the effects of particle-number fluctuation can be studied.[10–12] A more straightforward technique, first validated by the studies of Kerman *et al.*,[4] has been to project particle-conserving wave functions from the original BCS function and to use the former for the evaluation of all observables.[13,14] Finally, we may mention some rather more elaborate efforts to construct wave functions for ground and exicted states,[15,16] including a method for obtaining exact solutions.[17]

Without in any way gainsaying the importance of the above work, none of it refers to a "realistic" nuclear interaction or even to the pairing-plus-quadrupole model of KS. Though a brave start has been made[18] toward treating number conservation properly in the latter model, it is clear that much remains to be done.

With this note we suggest yet another method for the study of this class of problems, involving use of the equations of motion and spectral decomposition of operator products. This method has been favored previously by the authors.[19,20] Its ultimate full justification for the present application must await the further work in progress showing it to have the power to yield practical results for a realistic interaction. This will be the case because of the way in which the approximations can be controlled. Here by way of introduction we restrict ourselves to the pairing interaction and to an approximation, which though conserving the number of particles, is otherwise the simplest possible and quite close to the BCS method. The results are seen to be a large improvement over those of BCS. Comparing them with the calculations of Nogami and Zucker,[8] for example, we obtain slightly poorer ground-state energies, but *better single-particle properties*. The direction for further improvement of the theory is indicated and will be carried through in succeeding communications.

II. EQUATIONS OF MOTION WITH NUMBER CONSERVATION

We restrict our study to the usual pairing Hamiltonian

$$H=\sum_\alpha h_\alpha a_\alpha^\dagger a_\alpha-\tfrac{1}{4}G(\sum_\alpha s_\alpha a_\alpha^\dagger a_{\bar\alpha}^\dagger)(\sum_\beta s_\beta a_{\bar\beta}a_\beta)\,, \quad (2.1)$$

from which follows the equations of motion

$$[a_\alpha,H]=h_\alpha a_\alpha-\tfrac{1}{2}Gs_\alpha a_{\bar\alpha}^\dagger(\sum_\beta s_\beta a_{\bar\beta}a_\beta)\,, \quad (2.2)$$

$$[a_\alpha^\dagger,H]=-h_\alpha a_\alpha^\dagger+\tfrac{1}{2}G(\sum_\beta s_\beta a_\beta^\dagger a_{\bar\beta}^\dagger)s_\alpha a_{\bar\alpha}\,. \quad (2.3)$$

Here s_α is a phase factor with the property

$$s_{\bar\alpha}=-s_\alpha\,, \quad (2.4)$$

where $\bar\alpha$ and α refer to time-reversed, single-particle states.

Let us take the matrix element of (2.2) between the ground state $|0(A)\rangle$ of a system with A particles (A even) and a suitably chosen state $|\bar\mu(A-1)\rangle$ of the neighboring $(A-1)$-particle system. The definitions

$$\psi_\mu(\alpha)=\langle\bar\mu(A-1)|a_\alpha|0(A)\rangle\,, \quad (2.5)$$

$$e_\mu(A)=W_0(A)-W_{\bar\mu}(A-1)\,, \quad (2.6)$$

where W refers to the total energy of a state, enables us to write

$$e_\mu(A)\psi_\mu(\alpha)=h_\alpha\psi_\mu(\alpha)-\tfrac{1}{2}Gs_\alpha\sum_\beta\langle\bar\mu(A-1)|a_{\bar\alpha}^\dagger a_{\bar\beta}a_\beta|0(A)\rangle\,. \quad (2.7)$$

For the evaluation of the last matrix element in (2.7), we utilize a sum over states

$$\begin{aligned}&\langle\bar\mu(A-1)|a_{\bar\alpha}^\dagger a_{\bar\beta}a_\beta|0(A)\rangle\\&=\sum_I\langle\bar\mu(A-1)|a_{\bar\alpha}^\dagger|I(A-2)\rangle\langle I(A-2)|a_{\bar\beta}a_\beta|0(A)\rangle\\&\cong\langle\bar\mu(A-1)|a_{\bar\alpha}^\dagger|0(A-2)\rangle\langle0(A-2)|a_{\bar\beta}a_\beta|0(A)\rangle\\&\equiv\phi_{\bar\mu}^*(\bar\alpha)\sigma_\beta\,,\end{aligned} \quad (2.8)$$

where

$$\phi_{\bar\mu}^*(\bar\alpha)=\langle\bar\mu(A-1)|a_{\bar\alpha}^\dagger|0(A-2)\rangle\,, \quad (2.9)$$

$$\sigma_\beta=\langle0(A-2)|a_{\bar\beta}a_\beta|0(A)\rangle\,. \quad (2.10)$$

Deferring temporarily a discussion of the approximation made in (2.8), we now find for (2.7) the form

$$e_\mu\psi_\mu(\alpha)=h_\alpha\psi_\mu(\alpha)-\Delta s_\alpha\phi_{\bar\mu}^*(\bar\alpha)\,, \quad (2.11)$$

$$\Delta=\tfrac{1}{2}G\sum_\beta s_\beta\sigma_\beta\,. \quad (2.12)$$

A second equation for the pair of amplitudes $\psi_\mu(\alpha)$,

[7] S. G. Nilsson, Nucl. Phys. **55**, 97 (1964).
[8] Y. Nogami, Phys. Rev. **134**, B313 (1964); Y. Nogami and I. J. Zucker, Nucl. Phys. **60**, 203 (1964).
[9] B. F. Bayman, Nucl. Phys. **15**, 33 (1960).
[10] I. N. Mikhailov, Zh. Eksperim. i Teor. Fiz. **45**, 1102 (1962) [English transl.: Soviet Phys.—JETP **18**, 761 (1964)].
[11] J. Bang, J. Krumlinde, and S. G. Nilsson, Phys. Letters **15**, 55 (1965).
[12] K. Dietrich, H. J. Mang, and J. Pradal, Phys. Rev. **135**, B22 (1964).
[13] M. Rho and J. O. Rasmussen, Phys. Rev. **135**, B1295 (1964).
[14] A. Lande, Ann. Phys. (N.Y.) **31**, 525 (1965).
[15] R. R. Chasman, Phys. Rev. **132**, 343 (1963); **134**, B279 (1964); **138**, B326 (1965).
[16] S. Wahlborn, Bull. Am. Phys. Soc. **8**, 626 (1963).
[17] R. W. Richardson and N. Sherman, Nucl. Phys. **52**, 225, 253 (1964); R. W. Richardson, J. Math. Phys. **6**, 1034 (1965); Phys. Letters **14**, 325 (1965).
[18] I. Unna and J. Weneser, Phys. Rev. **137**, B1455 (1965).
[19] Giu Do Dang and A. Klein, Phys. Rev. **133**, B257 (1964).
[20] A. Kerman and A. Klein, Phys. Rev. **132**, 1326 (1963).

$\phi_{\bar{\mu}}^*(\alpha)$, using basically the same approximation as in (2.8), can be derived from (2.3). It is

$$[e_\mu - 2\lambda(A)]\phi_{\bar{\mu}}^*(\bar{\alpha}) = -[h_{\bar{\alpha}} - G]\phi_{\bar{\mu}}^*(\bar{\alpha}) + \Delta s_{\bar{\alpha}}\psi_\mu(\alpha), \quad (2.13)$$

where

$$2\lambda(A) = W_0(A) - W_0(A-2), \quad (2.14)$$

and the extra term involving G on the right-hand side of (2.13) resulted from a commutation of operators necessary to obtain an equation for the same pair of amplitudes as occur in (2.11).

We emphasize that the "equations of motion" (2.11) and (2.13) are exact insofar as conservation of number of particles is concerned. They are approximate in the neglect of all excited intermediate states of seniority zero in the sum (2.8) and analogous sums. To show that one can retain some of these states exactly or all of them approximately and still have a viable method of calculation will be the burden of future papers, where we shall also show that similar techniques can be applied to more realistic interactions.[21]

Equations (2.11) and (2.13) can be simplified, first by the definition of new, more familiar amplitudes,[22]

$$\psi_\mu(\alpha) = \delta_{\mu\alpha} s_\alpha v_a(A), \quad (2.15)$$

$$\phi_\mu^*(\alpha) = \delta_{\mu\alpha} u_a(A-2). \quad (2.16)$$

Here we shall suppose that the single-particle levels have, as usual, a certain degeneracy $2\Omega_a$, i.e., $\epsilon_\alpha = \epsilon_a$, where a is the set of quantum numbers common to all the $2\Omega_a$ levels. The amplitudes $v_a(A)$ and $u_a(A-2)$ satisfy the equations

$$E_a v_a(A) = -\epsilon_a v_a(A) + \Delta(A) u_a(A-2), \quad (2.17)$$

$$E_a u_a(A-2) = \epsilon_a u_a(A-2) + \Delta(A) v_a(A), \quad (2.18)$$

where

$$E_a = -e_a + \lambda_1, \quad 2\lambda_1 = 2\lambda + G, \quad \epsilon_a = h_a - \lambda_1. \quad (2.19)$$

Notice that discounting the difference between λ and λ_1,

$$E_a = W_a(A-1) - \tfrac{1}{2}[W_0(A) + W_0(A-2)]. \quad (2.20)$$

Were we to ignore the difference between A and $(A-2)$, (2.17) and (2.18) would be the familiar equations of the BCS theory. Since the normalization condition[22]

$$v_a^2(A) + u_a^2(A) = 1 \quad (2.21)$$

connects the solution of (2.17), (2.18) with that for $A \to A+2$, however, there now arises the necessity of treating several A values simultaneously, if we insist—as we do—on retaining the difference

$$\delta_a(A) = u_a^2(A-2) - u_a^2(A) \quad (2.22)$$

as nonvanishing.

We shall therefore seek solutions to (2.17), (2.18) subject to (2.21) and to the condition

$$A = \sum_a 2\Omega_a v_a^2(A). \quad (2.23)$$

For the ground-state energy, our approximation (2.8) applied to (2.1) yields

$$W_0(A) = \langle 0(A)|H|0(A)\rangle = \sum_a 2\Omega_a h_a v_a^2(A) - [\Delta^2(A)/G]. \quad (2.24)$$

III. METHOD OF SOLUTION

From Eqs. (2.17) and (2.18) we find, for the quasiparticle energies, the standard expression

$$E_a^2 = \epsilon_a^2 + \Delta^2. \quad (3.1)$$

On the other hand, with the help of (2.21) and (2.22), (2.17) and (2.18) are equivalent to the equations

$$2u_a(A-2)v_a(A)E_a = \Delta(A)(1+\delta_a(A)), \quad (3.2)$$

$$[u_a^2(A-2) - v_a^2(A)]E_a = \epsilon_a(1+\delta_a(A)), \quad (3.3)$$

which yield in turn

$$v_a^2(A) = (1+\delta_a)\tfrac{1}{2}[1 - (\epsilon_a/E_a)], \quad (3.4)$$

$$u_a^2(A-2) = (1+\delta_a)\tfrac{1}{2}[1 + (\epsilon_a/E_a)]. \quad (3.5)$$

From Eq. (2.12) we have

$$\Delta = G\sum_a \Omega_a u_a(A-2)v_a(A), \quad (3.6)$$

which becomes, inserting (3.2),

$$1 = \tfrac{1}{2}G\sum_a \frac{\Omega_a(1+\delta_a)}{E_a}. \quad (3.7)$$

Finally, Eq. (2.23) may be rewritten

$$A = \sum_a \Omega_a[1 - (\epsilon_a/E_a)](1+\delta_a). \quad (3.8)$$

For $\delta_a = 0$, Eqs. (3.7) and (3.8) reduce to the standard equations of the BCS theory and serve to determine Δ and λ. In the present instance we describe two procedures for the determination of these quantities as a function of A.

(i) *Iterative method.* This method depends only on the

[21] In the conventional factorization of the matrix element (2.8), one extracts both a pairing term, as we have, as well as a Hartree-Fock field term. (A general technique for doing this is described in Ref. 20.) It is clear, however, that insofar as we sum over *all* relevant intermediate states in (2.8), the Hartree-Fock terms must be omitted for the pairing interaction, their inclusion in this case involving a double counting. Since our aim in future work is to carry through the more complete sum, we have chosen to omit such terms here. For a more general interaction, self-consistent-field terms will appear in the theory.

[22] We remind the reader that the choice of phases in (2.15) and (2.16) serves to satisfy the anticommutation relation $\{a_\alpha, a_\beta\} = 0$, whereas (2.21) is the expression of $\{a_\alpha, a_\beta^\dagger\} = \delta_{\alpha\beta}$. In both cases the approximation of (2.8) is involved.

local validity (as a function of A) of our approximations. Here we adjoin to Eqs. (3.7) and (3.8) the expression

$$A-2=\sum_a 2\Omega_a v_a^2(A-2)$$
$$=\sum_a 2\Omega_a\{1-\tfrac{1}{2}[1+\delta_a(A)]\times[1+(\epsilon_a/E_a(A))]\}. \quad (3.8')$$

We utilize the fact that (3.8) and (3.8′) as stated must be satisfied by the same values of $\Delta(A)$, $\lambda(A)$, $\delta_a(A)$. Suppose, however, that all we know is an nth approximant $\delta_a^{(n)}(A)$ to $\delta_a(A)$. Then the simultaneous solution of (3.7) and (3.8) will yield a set $\Delta^{(n)}(A)$, $\lambda^{(n)}(A)$, whereas the simultaneous solution of (3.7) and (3.8′) yields a different set $\Delta^{(n)'}(A)$, $\lambda^{(n)'}(A)$. From the former set, we compute $u_a{}^{(n)2}(A)$ and from the latter $u_a{}^{(n)2}(A-2)$ and define

$$\delta_a{}^{(n+1)}(A)=u_a{}^{(n)2}(A-2)-u_a{}^{(n)2}(A). \quad (3.9)$$

The iteration starts with the BCS value $\delta_a{}^{(0)}(A)=0$. From (3.8) and (3.8′) follows the sum rule

$$\sum_a \delta_a\Omega_a=1, \quad (3.10)$$

which will serve as a check on our results.

(ii) *Step-by-step method.* Suppose that $u_a{}^2(A-2)$ is known. This allows us through (3.5) to express $\delta_a(A)$ in terms of known quantities and of $\Delta(A)$ and $\lambda(A)$. This means in turn that (3.7) and (3.8) depend only on the unknowns Δ and λ and can be solved. But

$$u_a(0)=1, \quad (3.11)$$

and this is sufficient to solve (3.7) and (3.8) for $A=2$, obtain $u_a(2)$, and then proceed step by step to higher A.

Comparing the two procedures (i) and (ii), we see that they should give the same results if (and only if) the theory was exact. As we know that the theory is *not* exact (except in the degenerate case, see below) the two methods may give different results. The second procedure is inherently less accurate than the first because an error in the calculation of $u_a(A-2)$ may entail an error in the calculation of $u_a(A)$ in addition to the error in the theory for *that* A. The two kinds of errors may compensate each other, but this can hardly be foreseen (or expected).

IV. DEGENERATE USE

For this case of a single value of a, the theory of this paper is exact. This is because there are no excited states of seniority zero, and the factorization (2.8) and all similar factorizations can be made without approximation. To solve our equations, we simply realize that

$$v^2(A)=(A/2\Omega), \quad u^2(A)=1-(A/2\Omega). \quad (4.1)$$

Thus

$$\delta=(1/\Omega), \quad (4.2)$$

TABLE I. Values of the ground-state energy in the model of Ref. 5, both exactly and in various approximation schemes. Except for the results of the present work, the table duplicates one of Nogami and Zucker, Ref. 8. The energies are in units of the single-particle separations.

W_0 \ G	1.25	1.00	0.80	0.50
Exact	4.953	6.828	8.213	9.998
Projected BCS	4.966	6.850	8.249	10.095
BCS	6.510	8.082	9.215	10.562
Nogami and Zucker	4.975	6.864	8.264	10.067
Present work	5.256	7.182	8.617	10.490

and we can easily obtain the following exact results:

$$E^2=(G^2\Omega^2/4)[1+(1/\Omega)]^2, \quad (4.3)$$
$$\Delta^2=G^2\Omega^2(A/2\Omega)[1-(A/2\Omega)+(1/\Omega)], \quad (4.4)$$
$$\epsilon=\tfrac{1}{2}G\Omega[1-(A-1)/\Omega], \quad (4.5)$$
$$\lambda=h-\tfrac{1}{2}G\Omega[1-(A-2)/\Omega], \quad (4.6)$$
$$e=h-G\Omega[1-(A-2)/\Omega], \quad (4.7)$$
$$W_0(A)=-\tfrac{1}{2}G\Omega A[1-(A-2)/2\Omega]. \quad (4.8)$$

V. NUMERICAL RESULTS AND DISCUSSION

Numerical calculations were carried out for two examples: (i) The problem of Pawlikowski and Rybarska[5] where $A=6$ particles are distributed over 5 doubly degenerate ($\Omega_a=1$), equidistant levels ($h_a=1, 2, 3, 4, 5$). Exact calculations as well as various approximate results are available for $G=0.5$, 0.8, 1.00, 1.25. In Table I, we compare our values for the ground state energy obtained by iterative method (i) with previous calculations as compiled by Nogami and Zucker.[8] Though in almost every case a considerable improvement over BCS, the results are inferior to those of other methods. On the other hand, Table II illustrates that the ground-state wave functions are rather better than those of the other approximation methods in their representation of single-particle properties.

A better approximation to the energy requiring the study of excited states of seniority zero will be included in our next communication.

In Table III, the values of δ_a are given. It can be verified that they satisfy the sum rule (3.10). The results for λ and Δ are recorded in Table IV.

TABLE II. Values of the single-particle level occupation probabilities, $v_a{}^2$ in the model of Ref. 5. Again compare Ref. 8.

$v_a{}^2$ \ Level	1	2	3	4	5
BCS	0.892	0.810	0.649	0.418	0.229
Nogami and Zucker	0.883	0.797	0.641	0.430	0.249
Present work	0.895	0.827	0.649	0.401	0.227
Exact	0.911	0.853	0.716	0.338	0.182

TABLE III. Values of δ_a [see Eq. (2.22)] for the model of Ref. 5. These numbers may be considered as a qualitative measure of the corrections to the BCS theory.

Levels \ G	1.25	1.00	0.80	0.50
1	0.146	0.122	0.091	0.028
2	0.222	0.228	0.229	0.165
3	0.262	0.298	0.357	0.614
4	0.222	0.228	0.229	0.166
5	0.146	0.122	0.091	0.028

TABLE IV. Results for the chemical potential λ and the energy-gap parameter Δ, for the model of Ref. 5, in units of the single-particle energy difference.

G	1.25	1.00	0.80	0.50
λ	2.370	2.500	2.600	2.750
Δ	3.500	2.700	2.051	1.071

(ii) As a second example, we consider the model of the even Ni isotopes solved exactly by KLM.[4] Here A runs from 2 to 10, with the particles distributed over four levels: $h(p_{3/2})=0.00$ (MeV), $h(f_{5/2})=0.78$, $h(p_{1/2})=1.56$, $h(g_{9/2})=4.52$, and $G=0.331$. The solution was obtained by both methods and is compared with some previous results for the energy in Table V. The energies in this case are much nearer to the exact values than in the previous example, indicating the relative unimportance for this case of excited states of seniority zero. (This assertion remains to be verified.) In Table VI the values of $\delta_a(A)$ are given, again obeying the sum rule (3.10). From these values it is a short step to the occupation probabilities. Finally, in Table VII, we record the results for λ and Δ.

TABLE V. Values of the ground state energy in the model of the Ni isotopes solved exactly by KLM, Ref. 4. Except for the solutions of the present work, carried out by both methods described in the text, the table duplicates one of Nogami and Zucker, Ref. 8. Energies are measured in MeV.

W_0 \ A	2(Ni^{58})	4	6	8	10(Ni^{66})
Exact (KLM)	−1.49	−2.11	−1.75	−0.51	1.70
Kisslinger and Sorenson	−1.13	−1.51	−1.09	−0.22	2.48
Nogami and Zucker	−1.49	−2.07	−1.72	−0.44	1.85
Present work (i)	−1.48	−2.06	−1.69	−0.40	1.83
(ii)	−1.48	−2.06	−1.67	−0.38	1.82

TABLE VI. Values of δ_a for the Ni isotopes.

Levels \ A	2	4	6	8	10
$p_{3/2}$	0.319	0.247	0.167	0.113	0.083
$f_{5/2}$	0.094	0.139	0.184	0.207	0.199
$p_{1/2}$	0.041	0.055	0.081	0.126	0.229
$g_{9/2}$	0.007	0.007	0.006	0.005	0.001

TABLE VII. Results for λ and Δ for the Ni isotopes, in MeV.

A	2	4	6	8	10
λ	−0.710	−0.262	0.192	0.654	1.143
Δ	0.891	1.167	1.315	1.366	1.322

In conclusion, we may claim as the chief merit of the present work a considerable step toward a more exact solution of the pairing problem at an essentially negligible increase in complexity.