TABLE III. The second virial coefficient compared with the computer results of Kilpatrick, Keller, Hammel, and Metropolis and with experiment.

T(°K)	*B* This work	Kilpatrick *et al.*	Experiment
2	−182.84	−177.39	−193.3[a]
5	− 59.41	− 59.14	− 62.2[b]
10	− 21.30	− 21.34	− 23.4[b]
20	− 2.49	− 2.53	− 4.04[b]
30	3.68	3.57	2.42[b]
40	6.59	6.49	6.57(40.09°K)[c]
50	8.26	8.16	8.06(50.09°K)[c]
75	10.28	10.14[e]	10.70(75.01°K)[c]
100	11.08	11.02[e]	11.85(100.02°K)[c]
273.18	11.65	11.59[e]	11.77[d]

[a] Obtained by linear extrapolation of the data in W. E. Keller, Phys. Rev. **97**, 1 (1955).
[b] David White, Thor Rubin, Paul Camky, and H. L. Johnson, J. Phys. Chem. **64**, 1607 (1960).
[c] W. H. Keesom, *Helium* (Elsevier, Amsterdam 1942).
[d] W. G. Schneider and J. A. H. Duffie, J. Chem. Phys. **17**, 751 (1949).
[e] Obtained by us from the high-temperature expansion (Ref. 14).

no dependence on density. Henshaw's measurements were taken at $T=2.2$°K, density$=0.146$ g/cm^3 and $T=5.04$°K, density$=0.095$ g/cm^3.

The second virial coefficient, Eq. (1.20), obtained from our calculations is compared with results by Kilpatrick, Keller, Hammel and Metropolis[13] in Table III. The results marked by an *e* were obtained by us using the high-temperature expansion.[14] Experimental results are displayed in column 4.

ACKNOWLEDGMENTS

We wish to thank Dr. Sigurd Larsen for communicating his results to us and for an interesting conversation. Some of this work was done while one of us (L. D. F.) had a Fellowship from the John Simon Guggenheim Memorial Foundation and was located at the Max Planck Institut für Physik und Astrophysik in Munich. He wishes to thank the foundation for its support and the Institut for its hospitality.

[13] John E. Kilpatrick, William E. Keller, Edward F. Hammel, and Nicholas Metropolis, Phys. Rev. **94**, 1103 (1954).
[14] J. O. Hirschfelder, R. B. Bird, C. F. Curtiss, *Molecular Theory of Gases and Liquids* (John Wiley & Sons, Inc., New York, 1954), p. 1119.

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Method for the Determination of Atomic-Resonance Line-Oscillator Strengths from Widths of Optically Thick Emission Lines in *T*-Tube Plasmas*

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Using the neutral-helium resonance line as an example, a method is described for the measurement of the product of oscillator strength, Stark width, and ground-state density. Directly measured is the width of the line (as emitted by an essentially homogeneous but optically thick layer), which is proportional to the square root of the above product. A T tube is filled with a known helium-hydrogen mixture, and the temperature is calculated from the measured intensity ratios of helium and hydrogen lines. Measured widths of (optically thin) visible lines yield electron densities, which then give the theoretical Stark width of the ultraviolet line. These densities are also used in Saha equations to calculate the ground-state density from the measured temperature and the mixing ratio. This leaves the oscillator strength as the only unknown, whose accuracy is limited in the helium case mostly by the error in the ground-state density to about 25%, when the plasma is sufficiently dense and long-lived for local thermal equilibrium to hold.

I. INTRODUCTION

THE atomic resonance lines of many elements lie in the vacuum-ultraviolet spectral region, i.e., below 2000 Å. Several physical and technical difficulties appear in this region in addition to those encountered in the measurement of f values of visible spectral lines, and this situation is mirrored by the scarcity of data concerning experimental vacuum-ultraviolet oscillator strengths. Apart from the early life-time measurement performed by Slack[1] for the upper level of Lyman-α, five further experiments can be quoted: Prag *et al.*[2] measured the f values of the N I and O I multiplets near 1200 and 1300 Å, respectively, by means of resonance absorption in afterglows. They determined the abundances of atomic nitrogen and oxygen by an NO titration technique and found $\sum gf=0.39\pm0.12$ and 0.30 ± 0.08, which is in good agreement with recent

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† Some of the material in this article is part of a Ph.D. thesis submitted by R. Lincke in partial fulfillment of the requirements for the Degree of Doctor of Philosophy at the University of Maryland. See also R. Lincke, U. S. Air Force Cambridge Research Laboratory Report No. AFCRL-64-960, 1964 (unpublished).

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[2] A. B. Prag, C. E. Fairchild, and K. C. Clark, Phys. Rev. **137**, A1358 (1965).

calculations.[3] Seven parameters characterizing the emission and absorption profiles of the light source, its reversing layer, and the absorption cell had to be obtained by fitting theoretical curves to the measured curves of growth. Similar measurements were reported by Morse and Kaufman,[4] who obtained $\sum gf=0.3\pm0.1$ for the oxygen resonance triplet and lower limits of 0.2 and 0.6 for the nitrogen triplet and the Lyman-α doublet.

Boldt[5] measured C I oscillator strengths in the range of 1100 to 1800 Å. He used a wall-stabilized arc burning in argon with controlled amounts of carbon dioxide added in the central region of the arc. Observing the radiation end-on through a bore in the cathode, he thus avoided reabsorption in colder boundary layers around the cathode and the spectrograph entrance slit. The f values were then obtained from curve of growth analyses of the emission lines, using the central plateaus of saturated lines as intensity calibration standards. Since it is necessary to use argon as a transparent buffer gas in the cathode region, Boldt's method is presently limited to wavelengths larger than $\sim$790 Å (corresponding to the ionization limit of argon), and because of the rather low temperature probably also to neutral atoms.

Another method has been introduced by Kuhn and Vaughan.[6] They measured the natural and resonance widths of a visible helium line, whose upper level was sharp, while the lower level was at the same time the upper level of the resonance line at 584 Å. Both widths were therefore proportional to the oscillator strength of the resonance line. Their interferometric measurement in a liquid-helium-cooled microwave discharge yielded two f values which were, however, both substantially larger than the theoretical value[7–10] (0.38 and 0.34 from the natural and resonance widths, respectively, versus 0.276). This might be due to uncertainties in the expression used for the resonance width, and to neglecting the interdependence of Doppler and resonance broadening.[11]

The fourth method as proposed by Bashkin *et al.*[12] and Kay[13] is a modern version of Wien's experiment and has recently been applied to the resonance lines of lithium-like ions.[14] A monoenergetic ion beam is passed through a thin foil so that the emergent beam contains a mixture of various excitation and charge states, all ions or atoms having essentially the same velocity. When cascading is negligible, a measurement of the decaying intensities as function of distance from the foil, i.e., of the natural lifetimes, would then furnish oscillator strengths for cases where the decay takes place practically only through a single transition. This is of course the case for resonance lines proper, but the cascading corrections may be difficult to assess.

The above review of previous vacuum-ultraviolet oscillator-strength determinations indicates that further studies of suitable methods are of great interest. It was therefore decided to attempt the measurement of an oscillator strength for which a meaningful comparison with theoretical predictions was possible. The resonance line of neutral helium at 584 Å was chosen, since various theoretical values were available for its oscillator strength,[7–10] and because of the considerable attention that this value has received lately. Since intensity calibration methods were as much an objective as the experimental verification of the theoretical oscillator strength, we have attempted to study this line in emission. An electromagnetic T tube seemed to be a promising source for producing the ultraviolet spectrum because of the following advantages when compared with other sources: it is simple to operate, quite flexible with respect to obtainable plasma conditions, and it produces a rather homogeneous plasma so that no unfolding of the observational data is necessary when the plasma is viewed perpendicular to the tube axis. A disadvantage is the short duration of the plasma. This requires the use of fast photoelectric detection systems and may cause difficulties in the establishment of distributions of excitation and ionization corresponding to a succession of local thermal equilibrium (LTE) states.

II. METHOD OF MEASUREMENT

Since the absolute intensity of the hydrogen bremsstrahlung and recombination continuum can be calculated quite precisely when electron density and temperature are known, it would seem possible to use the absolute intensity of the Lyman continuum as a calibration standard. A fairly obvious approach would thus consist of producing a hydrogen plasma of suitable temperature and density containing a controlled trace of helium. The intensity of the helium resonance line then would follow from a comparison of the optically thin line with the underlying continuum. In order to deduce its oscillator strength, the population of the upper level of this line would have to be known. To avoid self-absorption, only a minute trace of helium could be used and none of the Stark-broadened visible helium lines would be observable above the underlying continuum. Helium-level populations would thus have to be inferred from the mixing ratio and a measurement of the temperature and electron density, applying the

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[8] A. Dalgarno and A. L. Stewart, Proc. Phys. Soc. (London) **76**, 49 (1960).
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[13] L. Kay, Proc. Phys. Soc. (London) **85**, 163 (1965).
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proper Boltzmann factors and making suitable corrections for the degree of ionization. At low temperatures, practically all the helium atoms are in the neutral ground state, and the proposed procedure is therefore permissible only when collisional processes dominate the relevant rate equations, including those for the ground state population. As long as the resonance line is optically thin, this is assured[15] only for an electron density larger than 2×10^{18} cm^{-3} at an electron temperature of 2 eV. However, at such a high electron density and relatively low temperature the hydrogen continuum is optically thick for reasonable geometrical depths, and the helium resonance line would not be seen above the (black-body) continuum.

At substantially higher temperatures, practically all of the admixed helium will be in the ground state of the ion He^+. The population of the upper level of the resonance line can then be obtained by connecting this level to the ionic ground state via the relevant Saha equation. The electron density necessary for LTE relations to be applicable to the relative populations of excited atomic states and the ionic ground state is quite small[15] (3×10^{16} cm^{-3} at $kT=2.5$ eV), and the previous difficulty does not arise. Because of the ever-present danger of reabsorption in cooler boundary layers, it is, however, highly undesirable to observe the neutral resonance line when most of the helium is ionized, since a slight decrease of temperature in the boundary will increase the relative abundance of neutral helium there by a significant factor. The oscillator strength of the He I 584-Å line therefore cannot be measured reliably in emission as long as this line is optically thin.

In case the resonance line is optically thick, the above-mentioned validity criterion for the existence of LTE between all atomic levels can be relaxed considerably,[15] because the dominating radiative decay term in the rate equations is largely cancelled by the contribution from photon reabsorption. Under this condition, an electron density of about 2×10^{17} cm^{-3} is sufficient to guarantee complete LTE. This density is easily obtainable in an electromagnetic T tube and small enough so that the background continuum does not cause any difficulties.

When the upper and lower states of the resonance line are connected by LTE relations, the source function for this line is equal to the Planck function. The specific intensity emitted from the homogeneous T-tube plasma of diameter l perpendicular to the tube axis then follows from the equation of radiative transfer as

$$I_\lambda'(l)=B_\lambda(T)\{1-\exp(-K_\lambda l)\}, \qquad (1)$$

where $B_\lambda(T)$ denotes the black-body intensity and K_λ is the true absorption coefficient. (Induced emission is completely negligible at 584 Å for a temperature of 2 eV.) Denoting the density of atoms in the lower state of the line by N_0 and the absorption oscillator strength by f, the absorption coefficient is obtained from

$$\int K_\nu d\nu=\frac{c}{\lambda^2}\int K_\lambda d\lambda=\frac{\pi e^2}{mc}N_0 lf, \qquad (2)$$

and the normalized line profile $S(\lambda)$. From recent Stark-broadening calculations[16] it is known that the profile of the line He I 584 Å is well represented by a dispersion profile of (half) half-width w. This half-width, furthermore, is proportional to the electron density. With $w=DN_e$, the absorption coefficient is then

$$K_\lambda=(e^2/mc^2)fN_0 l(\lambda^2/DN_e)\times\{1+[(\lambda-\lambda_0)/DN_e]^2\}^{-1}. \qquad (3)$$

The oscillator strength can be expressed in terms of the (half) half-width Δ of the saturated profile $I_\lambda'(l)$ using Eqs. (1) and (3). When $I_\lambda'(e)$ is assumed to be much broader than $S(\lambda)$, then

$$f=\frac{\ln 2}{Dl}\frac{mc^2}{e^2}\left(\frac{\Delta}{\lambda}\right)^2\frac{1}{N_0N_e}. \qquad (4a)$$

Numerically (with D taken from the tabulation in Ref. 16),

$$f_{\mathrm{He\,I}\;584}=2.15\Delta^2 10^{34}/N_eN_0. \qquad (4b)$$

Here Δ is measured in angstroms, and the value of D used corresponds to a temperature of 2×10^4 °K.

As mentioned before, boundary absorption (self-reversal) poses serious problems when the resonance line is studied in emission, especially when the line is optically thick. It is thus necessary to resolve the emission profile, since its shape will indicate the presence or absence of self-reversal. In addition, one will obtain the central black-body saturation level in this way, which can serve as an intensity-calibration standard. With an instrumental bandwidth of 1.4 Å (see below), the line profile should thus be several angstroms wide. Inserting an approximate $f=0.3$ and $\Delta=4$ Å into Eq. (4b), this is seen to require $N_eN_0\approx10^{36}$ cm^{-6}. Obviously, such a value can be obtained only with helium as a major plasma constituent.

III. DESCRIPTION OF APPARATUS AND EXPERIMENTAL PROCEDURE

The T tube apparatus employed in our measurements was similar (although much smaller) to the one used by Elton and Griem[17] to measure Lyman-α and β Stark profiles: Tungsten electrodes were sealed with O rings into a Pyrex "T" of 16-mm inner diameter. The observation section (12 cm away from the electrodes)

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[16] H. R. Griem, M. Baranger, A. C. Kolb, and G. Oertel, Phys. Rev. **125**, 177 (1962). See also the tabulation of line-broadening parameters in H. R. Griem, *Plasma Spectroscopy* (McGraw-Hill Book Company, Inc., New York, 1964).

[17] R. C. Elton and H. R. Griem, Phys. Rev. **135**, A1550 (1964); see also R. C. Elton, U. S. Naval Research Laboratory Report No. NRL 5967, 1963 (unpublished).

consisted of an aluminum block with the 50-μ monochromator entrance slit mounted flush with the inside of the tube. A shock reflector was positioned 2 mm downstream from this slit, and a quartz window permitted observation and plasma diagnosis in the visible. Since no window materials are available for the vacuum-ultraviolet range of interest, the entrance slit had to serve as plasma boundary. With the gas pressure necessary for proper operation of the T tube (4.5 Torr of a mixture consisting of 95% He and 5% H_2) the gas flow through the entrance slit was quite large, and the pumping system of the 0.5-m Seya-Namioka vacuum monochromator was therefore designed for 600 l/sec at 10^{-4} Torr. The electrical-power requirements of such a small T tube are very modest, and one capacitor of 0.5 μF charged to 35 kV was sufficient at half-cycle times of 1 μsec. To avoid discharges between the electrodes and the grounded vacuum spectrograph or the pumping system, the capacitor had to be insulated against ground.

The above-mentioned admixture of 5% of hydrogen proved to be necessary for proper operation of the T tube at this relatively high initial pressure, since in pure helium the signal rise time was extremely long (probably because of ionization-relaxation effects)—about 5 μsec, a time comparable to the total transit time for the 12-cm-long tube—and the signal shape was irreproducible. With hydrogen added, however, typical signals associated with the reflected shock wave had a rise time to maximum intensity of about 0.5 μsec, and the reproducibility was excellent ($\pm 10\%$), provided the capacitor voltage and tube pressure were controlled within $\pm 2\%$ (see Fig. 1). Because of this reproducibility the vacuum ultraviolet and visible spectra could be scanned by a shot-to-shot procedure. For the vacuum uv scans, the bandwidth of the monochromator was 1.4 Å (with approximately a Gaussian profile), and successive discharges were observed with wavelength separations as small as 0.5 Å. Oscilloscope traces were evaluated at the maximum amplitude of the He I 3889 Å signal, which was displayed on the second channel of a dual-beam oscilloscope and always served as reference standard. The latter signal and also the visible spectra necessary for the plasma diagonosis were obtained from a high-speed (large-aperture) grating spectrograph equipped with a photoelectric scanning attachment.

In our first experiments, a grating coated with magnesium fluoride was used in the vacuum uv monochromator, and the radiation was detected with the usual sodium-salicylate coated photomultiplier. The reflectivity of such a grating is excellent in the visible and vacuum uv down to approximately 1200 Å but drops rapidly toward shorter wavelengths.[18] Because of the transmissivity of the fluorescent layer for visible light, the response of the detection system extended to the long-wavelength cutoff of the photomultiplier,

[18] W. R. Hunter, Opt. Acta 9, 255 (1962).

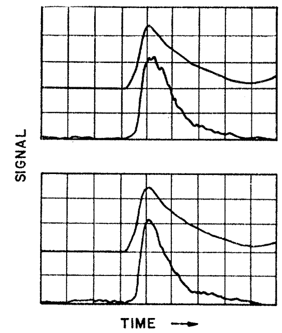

FIG. 1. Reproducibility of T-tube signals on two successive shots. Upper traces: Center of He I 3889 Å. Lower traces: 2 Å from center of He I 584 Å. Time scale: 1 μsec per division with 1-μsec delay.

i.e., approximately to 6000 Å. The results of this unfavorable combination were severe difficulties with scattered light of longer wavelengths, i.e. it proved impossible to detect the helium resonance line from the T tube plasma. The amount of scattered radiation near 1000 Å was considerably reduced by using a photoemission-scintillator detector.[19] In the 500 to 600 Å region, however, an aluminum coated ($\sim$1000 Å thick) sodium salicylate detector was successfully employed.[20,21] Further improvement was obtained by installing a platinum-coated grating.

IV. PLASMA DIAGNOSIS

Before calculating the oscillator strength from the measured width of the emission profile according to Eq. (4b), the electron density and density of helium atoms in the ground state must be determined. When complete LTE prevails in the plasma and when the temperature is known, then one can calculate the ground-state density from the electron density via the Saha equations

$$N_e N_{\mathrm{He}^+}/N_{\mathrm{He}^0} = S_{\mathrm{He}}(T,N_e)\,, \qquad (5a)$$

and

$$N_e N_{\mathrm{H}^+}/N_{\mathrm{H}^0} = S_{\mathrm{H}}(T,N_e)\,, \qquad (5b)$$

the quasineutrality condition

$$N_e = N_{\mathrm{He}^+} + N_{\mathrm{H}^+}\,, \qquad (6)$$

and the mixing ratio $M = N_{\mathrm{H}} : N_{\mathrm{He}}$. The electron density ($N_e$) enters the Saha functions S_{He} and S_{H} through the reduction of the ionization potentials.[22] With N_e as a parameter and T as free variable, these four equations can be solved quite easily; the results are shown in Fig. 2 for $N_e = 2.5$, 5, and 7.5×10^{17} cm^{-3}. Since the complete Saha equations were used, these curves are reliable only when complete LTE exists down to the ground states in hydrogen and helium. According to the validity criteria given in Ref. 15, this is indeed the case

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[21] R. Lincke and G. Palumbo, Appl. Optics **4**, 1677 (1965).

[22] H. R. Griem, Phys. Rev. **128**, 997 (1962).

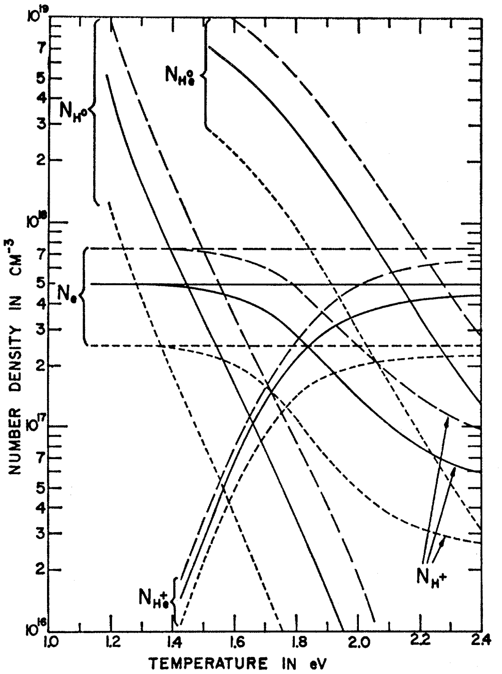


FIG. 2. Plasma composition for a mixture of 95% He and 5% H_2. ($N_e = 2.5\times10^{17}$ cm^{-3} - - - - -, 5×10^{17} cm^{-3} ——, 7.5×10^{17} cm^{-3} — — —).

for the range of temperatures and densities shown as long as the resonance lines are optically thick and time variations not too fast.

The electron density was obtained from the half-width of the He I 3889 Å line. This line was practically the only one in the visible spectrum that was not affected by overlap or self-absorption and was strong enough to be scanned accurately. After applying an empirical correction[23] of -10% to the tabulated Stark widths,[16] the experimental half-width of (15.2 ± 1.0)Å yielded an electron density of $(5.7\pm0.4)\times10^{17}$ cm^{-3}.

The plasma temperature was derived from the ratio of neutral helium and hydrogen Balmer-line intensities. Because of the large difference in the ionization potentials of hydrogen and helium, such ratios are very strong functions of temperature. For the particular case of He I 5876 Å and Hα this ratio times the mixing ratio is

$$\frac{I_{\mathrm{HeI}\,5876}}{I_{\mathrm{H}\alpha}}M=\left(\frac{6563}{5876}\right)^3\frac{g_{5876}}{g_{\mathrm{H}\alpha}}\frac{f_{5876}}{f_{\mathrm{H}\alpha}}\frac{1}{2}\exp\left(\frac{\Delta E}{kT}\right)\frac{\alpha(T,N_e)}{\beta(T,N_e)}. \quad (7)$$

In this equation, g and f are the statistical weights of the lower states and the absorption oscillator strengths, respectively, $\alpha(T,N_e)$ is the degree of ionization of helium and $\beta(T,N_e)$ that of hydrogen. With $\Delta E=\chi_{\mathrm{He}}-E_{5876}-\chi_{\mathrm{H}}+E_{\mathrm{H}\alpha}=0.01$ eV (χ and E are the ionization and excitation potentials, respectively), the exponential is practically equal to unity in the temperature range of interest, and the temperature dependence stems solely from the degrees of ionization. The above expression has been evaluated numerically for electron densities between 10^{17} and 10^{18} cm^{-3} and temperatures between 1 and 2.2 eV, and the curves are presented as Fig. 3. Again, the existence of complete LTE in hydrogen and helium had to be assumed, and the validity range is therefore the same as that for Fig. 2.

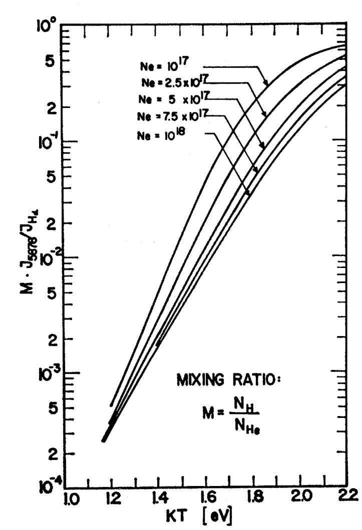
The line intensities in the preceding discussion are those of optically thin lines. In our high-density plasma, however, the strong neutral helium and hydrogen lines exhibited a considerable amount of self-absorption, and corresponding corrections had to be applied. Assuming that the central plateau of the experimental He I 5876 Å profile was the black-body level (the consistency of this assumption was verified later on by calculating the central absorption coefficient from the complete plasma analysis), the black-body intensity could be established over the whole region of the visible scan by using the theoretical wavelength dependence of this intensity and a relative calibration of the detection system. With the black-body intensity thus known, the optically thin line profiles I_λ and the total line intensities were reconstructed from the measured intensities I_λ' according to

$$I_\lambda=B_\lambda(T)\ln[B_\lambda(T)/(B_\lambda(T)-I_\lambda')], \quad (8)$$

an expression that is obtained by comparing the intensity I_λ' from Eq. (1) with the optically thin intensity, $I_\lambda=B_\lambda(T)K_\lambda l$. The good fit between these corrected profiles and the theoretical profiles of He I 5876 Å[16] and Hα[24] calculated for the measured electron density was taken as further evidence for the correctness of this procedure and the reliability of the total line intensities.

In addition to the spectral lines mentioned, the line

FIG. 3. Temperature from the ratio of He I 5876 Å and Hα.



[23] R. Lincke (to be published).

[24] H. R. Griem, A. C. Kolb and K. Y. Shen, Phys. Rev. 116, 4 (1959); see also U. S. Naval Research Laboratory Report No. NRL 5805, 1962 (unpublished).

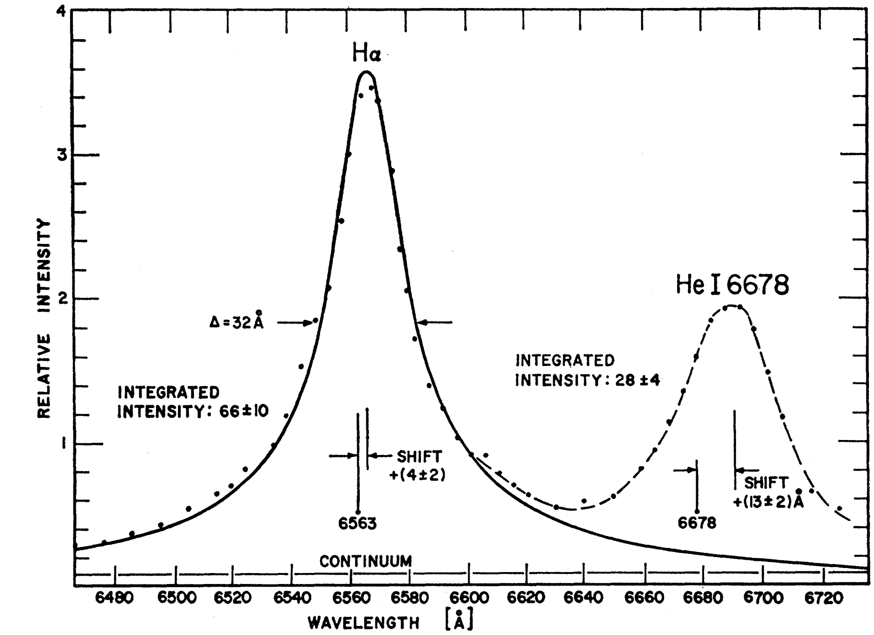


FIG. 4. Diagnosis of plasma: Hα—He I 6678 Å scan. Experimental points corrected for detector sensitivity and self-absorption. Solid line is theoretical profile for $N_e=5.7\times10^{17}$ cm^{-3}.

He I 6678 Å was measured. (Because of Debye screening of the perturbing electrons, the experimental profile was approximately 30% narrower than the one predicted.[23]) Based on the curves shown in Fig. 3 and theoretical ratios for the intensities of the helium lines, these three line intensities together with the measured electron density yielded a temperature of $kT=2.02\pm0.04$ eV. As an illustration of the plasma diagnosis, Fig. 4 shows the Hα—He I 6678 Å scan. Note also that any uncertainties from calculated f values[7] of the visible helium lines are negligible, because both upper and lower levels are nearly hydrogenic.

For our plasma conditions the partition functions of neutral helium and hydrogen are practically equal to the ground-state statistical weights, and N_0, the desired density of helium atoms in the ground state, can with good precision be taken as equal to the number density of all neutral helium atoms, i.e., for the measured values of T and N_e one finds from Fig. 1 that $N_{\mathrm{He}^0}=N_0=(1.20\pm0.25)10^{18}$ cm^{-3}.

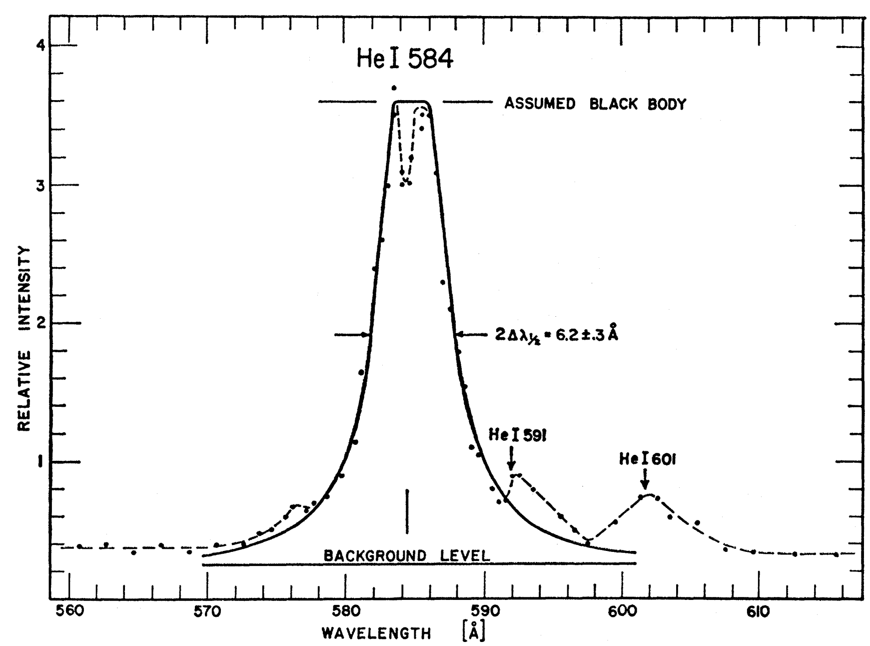


FIG. 5. Profile of self-absorbed He I resonance line. Solid line: theoretical profile $I_\lambda'(l)=B_\lambda(T)\{1-\exp[-C/(\lambda-\lambda_0)^2]\}$ with background and C adjusted for best fit.

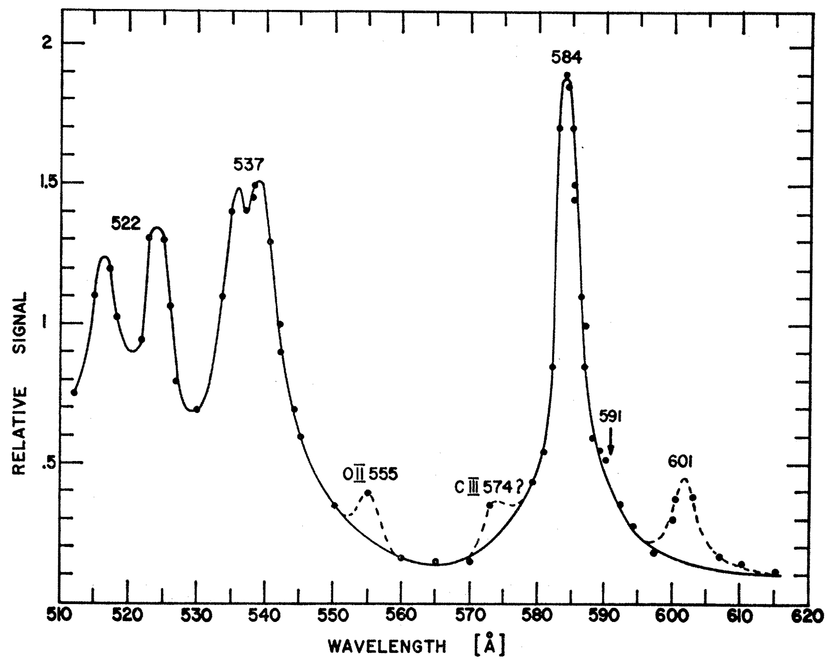


FIG. 6. First three members of the resonance series of neutral helium. (The plasma conditions differ slightly from those corresponding to Fig. 5.)

V. RESULTS

The experimental profile of the saturated resonance line of neutral helium is given in Fig. 5. The line center indicates some self-reversal, and one wing is disturbed by weaker lines, possibly by the forbidden lines He I 591 and 601 Å and oxygen impurity lines.

Because of the self-reversal, the experimental profile does not exhibit a constant saturation level. It may, however, be assumed that this black-body saturation level lies rather close to the points of maximum intensity, since the theoretical line shape from Eqs. (1) and (3), fitted at the top, the half-width, and the background, represents the experimental points quite well. Further support for this assumption can be obtained from a scan of the higher members of the resonance series as shown in Fig. 6. Calculations indicate that the lines He I 537 and 522 Å are also optically thick, and these lines should, therefore, saturate also at the black-body level. If allowance is made for a 30% decrease in detector sensitivity from 520 to 584 Å (which is consistent with an independent comparison of the aluminum coated detector with an uncoated sodium salicylate disc), then these peak intensities can indeed be connected by a theoretical black-body curve for $kT=2$ eV.

A comparison of the half-widths of the saturated lines He I 537 and 584 Å may serve as another test for the above assumption. From Eq. (4a) one finds that the ratio of these widths is independent of plasma parameters and solely determined by the ratio of the oscillator strengths and the broadening constants. Using the theoretical values of these quantities,[10,16] one calculates a ratio $\Delta_{537}/\Delta_{584}=2.15$. The experimental ratio of approximately 2.3 is thus certainly consistent with the theoretical relative oscillator strengths and Stark broadening parameters. If the black-body levels were significantly above the peak intensities, no agreement in the relative f values could be expected, because then the peak of He I 584 Å would be below its black-body level by a much larger factor than He I 537 Å.

The experimental He I 584 Å half-width of (6.2 ± 0.3) Å was corrected for the instrumental bandwidth of 1.4 Å, which reduced the width by 6%. Inserting the corrected width and the measured values of N_e and N_0 into Eq. (4b) yields an oscillator strength of $f=0.265\pm0.07$. Several corrections might have to be applied to this value because of the rather simplified treatment of the Stark profile of the resonance line. To be consistent, one has to apply the above-mentioned empirical correction of -10% also to the tabulated value of D used in arriving at Eq. (4b). Further, in addition to electron broadening, which gives rise to the dispersion profile used in Eqs. (3), (4a) and (4b), one has to consider ion broadening. In the present case this is most conveniently done by using the asymptotic wing formulas[16]

$$j_{as}=1/\pi x^2+3\alpha/4x^{7/4}, \quad (9)$$

$$j_{as}=1/\pi x^2,$$

and

$$S(\lambda)=wj_{as}(x). \quad (10)$$

The first expression in (9) represents the wing towards which the line is shifted, x being $\Delta\lambda/w$, α is the ion-broadening parameter, and the electron impact width w is needed in (10) for the normalization of the line profile. When these equations are evaluated for our conditions, one finds that the intensity on one wing has increased by 22%. The resulting profile, however, is practically indistinguishable from a (slightly shifted) dispersion profile with both wing intensities 11% larger than in the pure electron-broadened profile.

Of the higher order contributions to the line broadening, only the quadrupole interaction in the ionic contribution need be evaluated. Considering just the quadrupole interactions as given by Unsöld,[25] one finds that the Zeeman state $M=0$ of the upper level 2^1P gives rise to a blue-shifted line profile of shape and magnitude similar to the ionic (dipole) contribution discussed above, while the states $M=\pm 1$ yield an equal but red-shifted profile. There is thus some cancellation of quadratic Stark effect and quadrupole contribution to the ionic broadening, and the remaining effect is probably well estimated by the 22% increase on one line wing or an 11% increase in a shifted dispersion profile. This ion-broadening correction practically cancels the empirical correction in the electron impact width, and the final result is $f_{\mathrm{He\,I}\ 584}=0.26\pm 0.07$.

VI. DISCUSSION

Of the various sources of error considered, namely, experimental errors in the widths of ultraviolet and visible helium lines, remaining uncertainties in calculated Stark widths, and errors in the measurement of helium-to-hydrogen line intensity ratios, the latter errors are the most important ones. Even though they are reflected in a rather small temperature error, the corresponding uncertainty in the helium ground-state density accounts for almost all of the estimated 25% error in the oscillator strength, when all errors are treated as independent of each other. This error does not include any contribution for deviations from LTE, which in this experiment might well occur from relaxation effects. Estimated ionization relaxation times[15] are 0.4 μsec for the measured conditions, while, e.g., the electron density changes by 15% in this time (see Fig. 1).

With different initial conditions, the electron density had already decayed to half its maximum value after 0.4 μsec. The maximum value was 1.0×10^{18} cm^{-3} in this case, corresponding to an estimated ionization relaxation time of 0.2 μsec. Thus LTE could not be expected to hold, and indeed the oscillator strength came out too small by almost a factor of 4. Accordingly, the ground-state density was below its instantaneous LTE value by a similar factor. Qualitatively, one can say that the ground-state density is to be averaged over about one ionization relaxation time. As it depends essentially on the square of the electron density, a reduction by the above factor then appears quite plausible. For the original experiment, a similar argument would suggest a 30% upward correction of the oscillator strength derived in the previous section. However, because some averaging is necessarily introduced by the recording apparatus, this would most likely be an over-correction. Therefore no significant change in the above value of $f=0.26\pm 0.07$ is expected, even though the upper limit might have to be extended to, say, $f=0.35$, again assuming all errors to be independent of each other.

While an f value between 0.19 and 0.35 is consistent both with the theoretical value[7–10] of 0.276 and Kuhn and Vaughan's[6] measured values which range from 0.31 to 0.41 (assuming an error of ± 0.03 for both of their values), the accuracy is unfortunately not sufficient to decide experimentally whether the latter values were significantly affected by the assumption of Doppler and resonance broadening as independent mechanisms (see Sec. I). The reliability of the theoretical value most likely greatly exceeds the accuracy attained in both experiments. Therefore the most important result of the present work is seen in the establishment of a method for the measurement of resonance line oscillator strengths which should be applicable to many atoms. This method would probably be capable of a better accuracy for heavier atoms since characteristic times for the plasma decay, which are essentially hydrodynamic in nature, will then be longer, whereas ionization relaxation times are shorter than for helium because of the smaller ionization potentials. Thus instantaneous LTE relations would be more reliable, and the ground-state densities would depend less critically on the temperature. An accuracy approaching 20% might therefore be possible for other elements, assuming that errors in Stark-broadening parameters and the temperature measurement do not increase very much.

[25] A. Unsöld, *Physik der Sternatmosphären* (Springer-Verlag, Berlin, 1955).