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Ionization Produced by Energetic Germanium Atoms within a Germanium Lattice*

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The energy dependence of the ionization produced in germanium by energetic germanium atoms was measured. Germanium solid-state detectors served simultaneously as crystalline Ge sample, neutron target, and ionization detector. The spectrum of ionization produced by prompt Ge recoil atoms energized by monoenergetic neutron bombardment was observed in a pulse-height analyzer, and the edge of the spectrum was identified with the ionization produced by Ge recoil atoms having the calculated maximum recoil energy. The electron-hole pair production of Ge recoils was measured in this manner from 21.4 to 997 keV, using monoenergetic neutrons from 400 keV to 18.6 MeV. In this energy range, the ratio of the ionization produced by a Ge recoil relative to that of an electron of the same energy increased from ~0.15 to ~0.7. At very low Ge recoil energies, most of the energy goes into atomic processes. At high recoil energies, where electronic processes become important, the ionization produced by a Ge recoil appears to approach that for an electron of the same energy. The corresponding partition of energy between electronic processes and atomic processes for an energetic Ge atom in a Ge lattice agrees favorably with predictions of the theory of Lindhard *et al.* These data together with the earlier results of ionization produced by energetic Si atoms within a Si lattice agree with the A and Z dependence of the partition of energy predicted by Lindhard.

I. INTRODUCTION

THIS paper is the second paper in a systematic study of the partition of energy of heavy energetic particles in solids. The first paper,[1] hereinafter referred to as I, presented a study of the ionization produced by energetic silicon atoms in a silicon lattice and yielded the partition of energy between atomic and electronic processes as a function of the energy of the silicon atoms. The present work is a similar study for germanium. A preliminary account of this work has been previously published.[2] More recently,[3] closely related measurements have been made for Ge over a much more limited range of energies.

Not only is a knowledge of the partition of energy of heavy energetic particles of great interest in itself, but it is necessary for a full understanding of neutron-induced effects in solids, especially for incident-neutron energies above ~1 keV. In this energy range, neutron scattering and charged-particle production can impart a very large amount of energy to recoil atoms and result in the production of a significant amount of ionization in the solid. The energy that goes into ionization is exexpected to be unavailable for the production of displacements.

In a previous review article, Seitz and Koehler[4] discussed the particion of the energy of a recoil atom into electronic and atomic processes on the basis of a semi-quantitative model having a postulated threshold energy below which ionization would no longer occur. In their recent theoretical treatment, Lindhard, Nielsen, Scharff, and Thompsen[5] gave general integral equations governing additive effects of general physical properties caused by an energetic atom slowing down in a solid. The effects are additive in the sense that in each slowing-down event all particles set in motion contribute additively to the particular physical property in question. The properties depend not only on the energy of the recoil atom but on its A and Z as well. In particular, Lindhard *et al.* treat the partition of energy of heavy recoil atoms slowing down in solids over an energy range from ~1 keV to ~100 MeV. The previous results for silicon in I agreed with the Lindhard theory to a very good approximation. As a further check of this theory, it is of interest to do measurements for an energetic atom and host lattice having an A and Z significantly different from that of silicon. The present work was therefore undertaken to study the ionization produced by energetic germanium atoms within a germanium lattice over as wide an energy range as possible and to deduce the corresponding partition of the energy of the germanium atoms into atomic and electronic processes as a function of energy.

In the first part of the paper, the experimental method is described, followed by a presentation of the data. The experimental results are then discussed in terms of the works of Seitz and Koehler, and Lindhard *et al.* Finally,

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* This work was supported by the U. S. Atomic Energy Commission.

[1] A. R. Sattler, Phys. Rev. 138, 1815 (1965).

[2] A. R. Sattler and J. M. Palms, Bull. Am. Phys. Soc. **10**, 719 (1965).

[3] C. Chasman, K. W. Jones, and R. A. Ristinen, Phys. Rev. Letters **15**, 245 (1965).

[4] F. Seitz and J. S. Koehler, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1956), Vol. 2, pp. 307-402.

[5] J. Lindhard, V. Nielsen, M. Scharff, and P. V. Thompsen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 33, No. 10 (1963).

the present results for Ge together with the previous results for Si are compared with the A and Z dependence of these effects predicted by Lindhard *et al.*

II. EXPERIMENTAL

A. General Method

Germanium atoms were energized within the depleted region of lithium-drifted germanium solid-state detectors by means of elastic and inelastic collisions with monoenergetic neutrons. The germanium solid-state detectors served simultaneously as crystalline germanium samples, neutron targets, and ionization detectors. The ionization produced by prompt Ge recoil atoms resulting from monoenergetic neutron bombardment of germanium solid-state detectors was observed in a pulse height analyzer. In a scattering event, the amount of energy transferred to a recoil atom depends on the scattering angle and is a maximum for backscattered neutrons. The maximum recoil energy from the Ge(n,n)Ge reaction was calculated from classical scattering theory. For a given incident neutron energy, the pulse height spectrum produced by the Ge recoil atoms was obtained experimentally. Such a spectrum has a high-energy cutoff which is associated with the calculated maximum Ge recoil energy. The pulse height of this cutoff was then compared with the pulse height of an electron having the same energy as the calculated maximum Ge recoil energy. In a similar manner for each incident-neutron energy, the ratio of the pulse height of a Ge recoil to that of an electron of the same energy was obtained for each Ge recoil energy.

The experimental method was generally the same as that described in I. The differences in the details and interpretations are attributable to differences in the respective solid-state properties of Ge versus Si, and also in the atomic and nuclear properties. The principal differences in the electronic solid state properties are in the band gap, density of states, and carrier mobilities. The energy to produce an electron-hole pair is a function of the band gap and densities of states in the conduction and valence bands. Since less energy is necessary for a particle, e.g., an electron, to produce an electron-hole pair in Ge, $\sim$2.9 eV/pair,[6] than in Si, $\sim$3.6 eV/pair,[7] a given amount of energy going into ionization in Ge will produce more charge pairs than in Si. The lattice drift mobilities in Ge are larger than for Si[8] and result in lower critical field strengths for holes and electrons in Ge than in Si and lower charge collection times. Thus minimum recombination losses and trapping should be obtainable at somewhat lower field strengths in Ge than in Si.

There are many atomic and nuclear properties also affecting this experiment. The energy transfer to the Ge recoil is less than that for Si due to the higher value of A. There is larger mass spread in the stable isotopes of Ge than Si, and each Ge stable isotope contributes an appreciable fraction of the total. There is a mean spread in the maximum energy transfer from the neutron to the recoil atoms of 8.1% for Ge, and a corresponding spread of only 0.5% for Si. Due to the higher value of Z, the photon efficiency of Ge is much higher than for Si (e.g., the photoefficiency $\propto Z^5$). However, charged-particle emission in Ge is suppressed over a larger range of incident-neutron energies due to the larger Coulomb barrier. The close spacing of nuclear energy levels in all the Ge isotopes[9] relative to Si will further complicate the spectrum of ionization produced by heavy recoil atoms by means of recoil atoms from the (n,n') reaction. The total and differential neutron cross sections will depend to some extent on the nucleon configuration as it varies from isotope to isotope. In summary, this experiment was somewhat more difficult to interpret than I because of the isotopic mass spread and the close spacing of the nuclear energy levels in Ge.

B. Lithium-Drifted Germanium Solid-State Detectors as Crystalline Germanium Samples

Lithium-drifted germanium detectors with depletion depths of 30 and 300 μ were used in this investigation. Small depletion depths were selected to minimize the photon efficiency of the detectors relative to the recoil efficiency. The range of Ge recoils is $\leq 1\ \mu$; therefore edge effects are small.

To insure that the electric field was strong enough to collect virtually all the charge resulting from heavy recoil particles, the pulse height of the cutoff in the ionization spectrum of Ge recoils characteristic of a particular incident-neutron energy was measured as a function of detector bias and remained constant for fields $\approx 10^3$ V/cm. Therefore, field strengths $\approx 2\times10^3$ V/cm were used. Since the Ge(n,n)Ge reactions occur throughout the sensitive volume of the detector, it is desirable to have uniformly high collection efficiency for all Ge recoils within this entire volume. Lithium-drift detectors should in principle have fairly uniform fields. Detectors having lower noise can be obtained with Ge; however, the maximum energy transfer to a Ge atom for a given monoenergetic neutron bombardment is less than that to a Si atom, and the ionization produced by a Ge recoil is less than that produced by a Si recoil of the same energy. Thus the over-all effect of noise on spectra of ionization produced by Ge recoils appeared to be about the same as that in I.

C. Apparatus and Procedure

A charge-sensitive preamplifier was used for charge collection and coupled through a main amplifier to a

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[6] G. Dearnaley and D. C. Northrop, *Semiconductor Counters for Nuclear Radiations* (John Wiley & Sons, Inc., New York, 1963), p. 19.

[7] C. Bussolati, A. Fiorentini, and G. Fabri, Phys. Rev. **136**, 1756 (1964).

[8] J. N. Shive, *Semiconductor Devices* (C. D. Van Nostrand Company, Inc., New York, 1959), p. 473.

[9] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C., 1958).

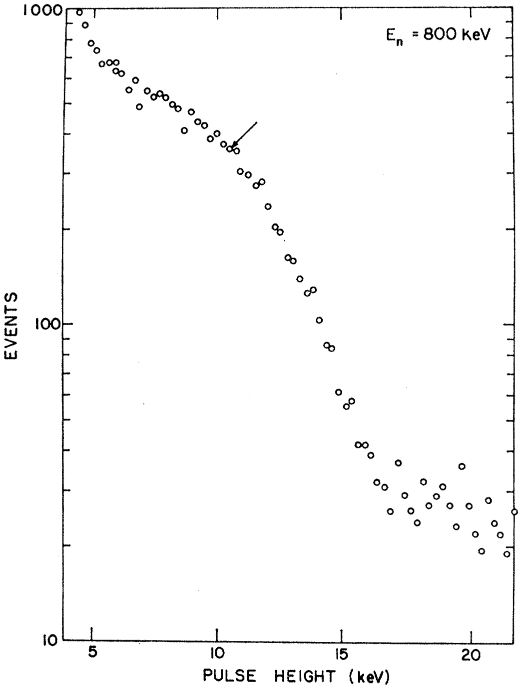

FIG. 1. Pulse-height spectrum produced by Ge recoil atoms, 800-keV neutrons incident on a Ge solid-state detector. The arrow denotes the interpretation of the end point of the ionization spectrum produced by Ge recoils.

pulse-height analyzer. The detectors were cooled to liquid-nitrogen temperature.

Low-energy gamma rays and an external pulser were used for energy calibration. Low-energy photons[9] were used which included Co^{57}, 137 and 123 keV; Am^{241}, 49.9 keV. Low-energy x rays from Cs^{137}, Bi^{207}, and Hg^{203} were used as secondary calibration sources. (The calibrated photon energy is converted to photoelectrons since virtually all of the characteristic x rays are immediately reabsorbed in this process.) Resolutions of ≈ 3 keV were obtained on the photopeaks.

Most irradiations were performed at the P-9 vertical Van de Graaff accelerator at Los Alamos Scientific Laboratory. Irradiations near 14-MeV incident-neutron energy were performed at a 180-keV neutron generator at Sandia Laboratory. The following reactions were used to obtain monoenergetic neutrons: $T(p,n)He^3$ range 400 keV-6.5 MeV; $T(d,n)He^4 > 14.1$ MeV. Estimates of neutron energy spreads are: between 300 keV–1 MeV, $\lesssim 30$ keV; 1.5–6.5 MeV, $\lesssim 40$ keV; 14.1 MeV, 100 keV; >14.1 MeV, 50 keV.

III. RESULTS AND INTERPRETATION

A. Pulse-Height Spectra from Neutron-Induced Reactions in Ge

The ionization produced by Ge recoil atoms has its origin in the $Ge(n,n)Ge$ and the $Ge(n,n')Ge$ reactions. Ideally, for a given neutron energy, the spectrum of ionization produced by Ge recoil atoms should have a high-energy cutoff corresponding to the maximum Ge recoil energy for elastically backscattered neutrons. Typical pulse height spectra are shown in Figs. 1, 2, and 3. The average maximum recoil energy $E_{Ge}(\max)$ is given from classical scattering theory for an incident neutron of energy, E_n, by

$$E_{Ge}(\max)=\frac{4\bar{M}_{Ge}m_n}{(\bar{M}_{Ge}+m_n)^2}E_n\,,$$

where $\bar{M}_{Ge}$ is the weighted average mass of all stable Ge isotopes, 76.6 mass units; and m_n is the mass of the neutron. The upper edge of the ionization spectrum produced by Ge recoils then corresponds to the calculated maximum recoil energy, $E_{Ge}(\max)$. The pulse height associated with $E_{Ge}(\max)$ was compared to that of an electron of the same energy. Detector noise, the energy spread of the neutron beam, the mass spread of the stable germanium isotopes, neutron inelastic scattering, and to some extent the statistical nature of the ionization process prevent the high-energy cutoff in a given spectrum produced by Ge recoils from being sharp. Competing reactions, resulting in charged-particle and photon production, can also cause difficulty in interpretation of the ionization spectra produced by recoils.

There are considerations which facilitate interpretation of the spectrum produced by recoil atoms. The energy distribution of recoil particles is directly related to the angular distribution of the elastically (and inelastically) scattered neutrons. From symmetry, the slope of either the differential elastic or inelastic neutron-scattering cross section $d\sigma/d\theta$ is zero at $\theta = 180°$, where θ is the center-of-mass angle. Furthermore, the number of heavy recoil particles of any isotope per energy interval in the laboratory system is proportional to $\cos\theta$. It can therefore be shown that the curve of the number of heavy recoil particles of any isotope as a function of energy is, within a proportionality factor, identical to the curve of $d\sigma/d\theta$ as a function of $\cos\theta$.[10] Over small increments of energy then, a pulse-height spectrum produced by the Ge recoils will be almost proportional to the sum of $d\sigma/d\theta$ over all Ge isotopes; and ideally the slope of this spectrum should be zero just below the cutoff.

At incident-neutron energies $\lesssim 1$ MeV, the photon background is quite small relative to the ionization spectrum produced by the Ge recoils. A spectrum of ionization generated by Ge recoils from 800-keV neutrons incident on a Ge detector is shown in Fig. 1 with the arrow denoting the interpretation of the cutoff of the spectrum produced by recoils.

For incident neutron energies from ≈ 1.5 to 6.5 MeV, the background produced by photons is somewhat larger; nonetheless, the edge of the ionization spectrum

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[10] L. C. Bidenharn, E. Baumgartner, P. Huber, and H. B. Willard, in *Fast Neutron Physics*, edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1963), Part 2, pp. 1217–1316.

produced by recoils is still predominant or at least is readily discernible on the background produced by photons. The only prominent photopeaks seen were from x rays caused by the gold beam-stop target. Interpretation of the spectrum was possible when the pulse height of the recoil edge was either above or below that of these x ray lines. Figure 2 shows a pulse-height spectrum produced by 2-MeV neutrons incident on a Ge detector; the end point of the ionization spectrum produced by heavy recoils is denoted by an arrow. The x rays from the stop target appear above the ionization spectrum produced by Ge recoils. Also seen in the spectrum in Fig. 2 is a decay from the 67-keV level of Ge^{73}.[9] The 13-keV conversion-electron decay is not observed among the much more prominent spectrum of ionization produced by the Ge recoils. Most of the photons produced in the Ge detector do not appear in the ionization spectrum due to the relatively small sensitive volume of the detector.

The effect of neutron inelastic scattering in Ge and the mass spread in the Ge isotopes further prevents the cutoff of the spectrum of ionization produced by Ge recoil particles from being sharp. The first and some higher excited states of the Ge nuclei[9] are fairly close to the ground state for all isotopes. The maximum energy transferred to recoils from inelastic scattering is therefore quite close to the maximum recoil energy from elastic scattering. This makes an energy overlap of (n,n') recoils (from a lighter Ge isotope) and (n,n) recoils (from a heavier Ge isotope) possible. The effects of inelastic scattering and the overlapping on the determination of the end point should be smaller, however, than the energy deviations caused by the mass spread of the stable Ge isotopes. The mass spread of the stable Ge isotopes, neutron inelastic scattering, plus different total and differential cross sections for each Ge isotope are reflected not only in the smearing out of the cutoff in the spectrum of ionization produced by Ge recoils but also give an uncertainty in the Ge recoil kinetic energy to properly ascribe at the cutoff. These uncertainties are included in the display of results.

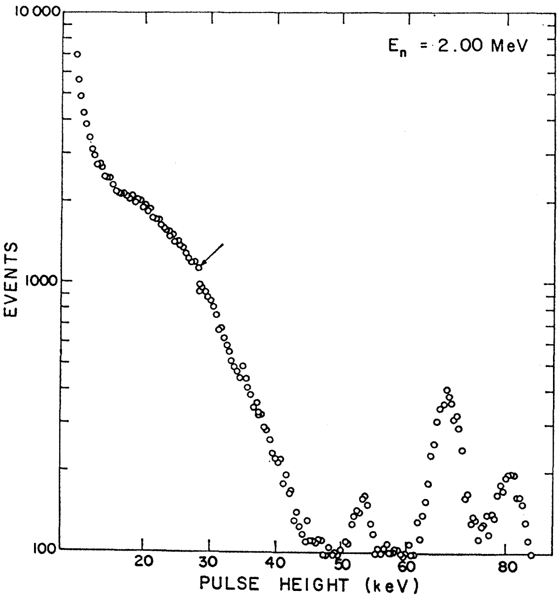

FIG. 2. Pulse-height spectrum produced in a Ge solid-state detector by 2.00-MeV neutrons. The arrow denotes the interpretation of the end point of the ionization spectrum produced by Ge recoils.

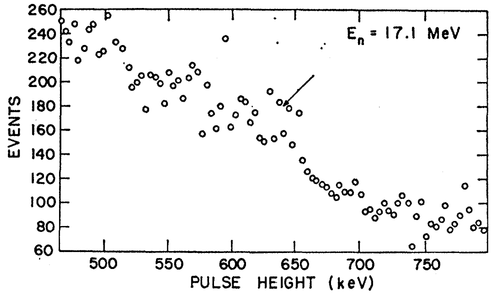

FIG. 3. Low-energy portion of pulse-height spectrum produced by 17.1-MeV neutrons incident upon a Ge solid-state detector. The arrow denotes interpretation of the end point of the ionization spectrum produced by Ge recoils.

In the higher range of incident-neutron energies >14.1 MeV, charged-particle production must be considered. Charged particles can affect the spectrum produced by recoil atoms since charged particles not stopped in the detector leave a smooth energy tail which could overlap the spectrum of ionization produced by the recoils. For germanium the Coulomb barrier is ~6.4 MeV for protons. Since the end point of the ionization spectrum produced by Ge recoils is less than 1 MeV, the possibility is remote of direct superposition of charged particle groups or a continuum (evaporation spectrum) upon the spectrum of ionization produced by Ge recoils. Although at these incident-neutron energies the effects of the smooth energy tail from charged particles and photons are not small, the edge of the spectrum of ionization caused by heavy Ge recoil atoms still appears to be discernible. Figure 3 shows such an ionization spectrum for 17.1-MeV neutrons incident on the germanium detector. The interpretation of the data at these incident-neutron energies is not complicated by the effects of noise, which is relatively small at the energy of the edge of the ionization spectrum produced by recoil atoms. Furthermore, the spectrum of ionization of recoil atoms is concentrated in a relatively small part of the total ionization spectrum because of the large difference in mass between a Ge atom and a neutron. This enhances the observation of Ge recoils against a background of very energetic charged particles and photons.

B. Energy Dependence of Ionization Produced by Germanium Atoms

The pulse-height ratio produced by a maximum-energy Ge recoil atom relative to an electron of the same

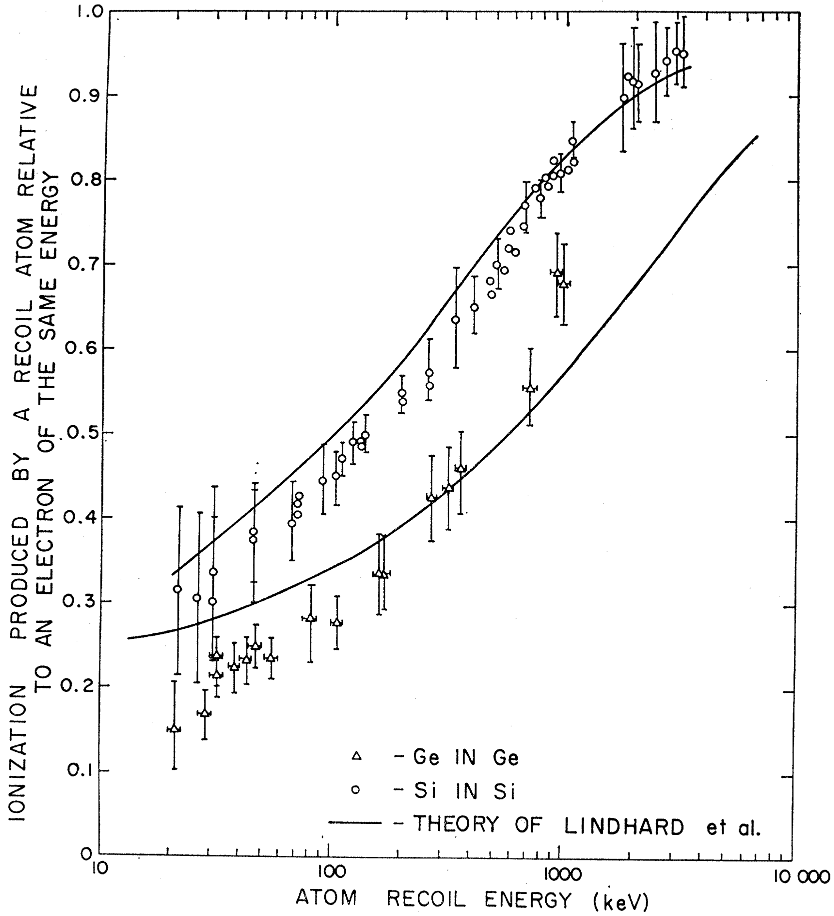

FIG. 4. Pulse height produced by a recoil atom relative to an electron of the same energy, Si in Si (top), Ge in Ge (bottom). Solid lines represent respective predictions of Lindhard *et al.*, in variables η/E from numerical solutions of the integral equations.

energy is given as a function of Ge recoil energy in Fig. 4 and Table I. This fraction varies from ≈ 0.15 for a Ge recoil of ~ 20 keV to ~ 0.7 for a Ge recoil of ~ 1 MeV. The ionization of a Ge recoil relative to an electron of the same energy increases monotonically with energy and appears to approach unity. The vertical error includes estimated systematic error in interpreting individual spectra. Random error can be estimated from the scatter of the data points and is fairly small. The horizontal error bars represent the uncertainty of the kinetic energy corresponding to the recoil edge. This uncertainty is due mainly to the spread in the maximum energy transfer to the Ge isotopes, and to a lesser extent, to the uncertainties in the relative amount of the backscattering for each Ge isotope and the effect of inelastic scattering on the recoil edge. The results for Si from I have also been included in Fig. 4 to show the A and Z dependence of these effects.

IV. DISCUSSION

Of particular interest in the Seitz and Koehler survey[4] of the behavior of solids under irradiation is the estimate of the ionization threshold. They introduce an energy parameter ϵ, where $\epsilon = (m/M)E$; E is the energy of the moving atom, m the electronic mass, and M the atomic mass. The parameter ϵ is then the energy of an electron having the same velocity as that of the atom. They assume a moving atom has an appreciable probability of exciting only those electrons in a solid for which the transition energies are $\leq \epsilon$; implying therefore that ϵ is about the maximum energy that the atom can transfer to the electron. They further assume electrons on a moving atom with a binding energy less than ϵ will be swept off by the field of atoms in a solid; those with a significantly greater energy should be retained. By setting ϵ equal to the Ge band-gap energy, $E_g = 0.67$ eV, an ionization threshold of about 90 keV is obtained on the above assumptions. Since Seitz and Koehler feel a sharp threshold is very unlikely, they estimate that electronic excitation will only be small when the value of ϵ is smaller than $E_g/4$ corresponding to a threshold of 22 keV for Ge. In this investigation, ionization has been seen at smaller values relative to the predicted threshold than for I; however, no sharp ionization threshold was seen. The data in Fig. 4 show that in the neighborhood of recoil energies corresponding to $\epsilon = E_g/4$ or even $\epsilon = E_g$, ionization is not large.

The recent theory of Lindhard *et al.*[5] is quite general since it shows how any cumulative effect may be derived for all kinds of energetic particles slowing down in a particular medium. Lindhard gives a basic integral equation which describes the behavior of some unspecified cumulative physical quantity which is arbitrarily taken to be zero before irradiation. As important examples to the solution of the basic integral equation, Lindhard gives for a particle of energy E in atomic matter the energy $\bar{\eta}(E)$ that is lost to electronic processes, and that part $\bar{\nu}(E)$ lost to atomic processes excluding internal excitation of the atoms; and $\bar{\eta}+\bar{\nu}=E$. The functions $\bar{\eta}(E)$ and $\bar{\nu}(E)$ are given in terms of A, Z, and E. Lindhard specifically calculates $\bar{\eta}(E)$, hence $\bar{\eta}(E)/E$, for energetic Ge atoms in a Ge lattice and for energetic Si atoms in a Si lattice.[5,11] A summary of the assumptions Lindhard *et al.* make in the solution of the basic integral equation for $\bar{\eta}$ and $\bar{\nu}$ in Ref. 5 is also given in I.

As assumed in I for energetic Si atoms in a Si lattice, if the number of pairs produced per unit amount of energy going into electronic processes is constant and independent of energy, and in addition if this number is the same for an electron in Ge or a Ge atom in Ge, then the pulse height ratio in Fig. 4 actually yields $\bar{\eta}(E)/E$. (The Ge detectors were calibrated by low-energy photons where it is assumed that virtually all of the energy of the resulting photo electrons and x rays, which are reabsorbed with successive ejection of additional photoelectrons, goes into electronic processes.) Identical theoretical and experimental justifications of this assumption presented in I are also applicable here. Moreover, the predictions of $\bar{\eta}/E$ versus E from a numerical solution of the basic integral equation of Lindhard *et al.* for Ge in Ge as well as Si in Si shown in Fig. 4 are in good agreement with the data. Agreement of experiment and theory has then been found for more than one A and Z.

At low recoil energies, the present results are somewhat lower than the theory of Lindhard predicts. This is in contrast to the recent results of Chasman *et al.*[3] who show pulse-height ratios somewhat larger than are given in the present investigation. The range of Ge recoils covered by Chasman *et al.* was from about 20–100 keV. Their results seem to agree more closely with the theory of Lindhard in this energy range. Due to a misinterpretation of a formula of Lindhard, however, they introduce an error $\sqrt{2}$ in the conversion of their data to compare it with the Lindhard theory.[12] They compare their Ge recoil energies of energy E with Lindhard's predictions for Ge recoils having energy $E\sqrt{2}$ instead of E. This does not affect their measured values, but it obscures their comparison with the theory of Lindhard. In addition, although excellent agreement with theory is claimed, at least their tabulated values of $\bar{\eta}/E$ below $\approx$50 keV do not increase monotonically with E as is predicted by theory, and indeed in this range their results are somewhat higher than the theory of Lindhard predicts.

TABLE I. Tabulation of some values of ionization parameters of Ge shown in Fig. 4.

Incident neutron energy (MeV)	Energy of maximum Ge recoil (keV)	Pulse height of maximum Ge recoil relative to electron of same energy	Predictions of Lindhard $\bar{\eta}/E$ (numerical estimate)
0.400	21.4	0.149±0.057	0.267
0.500	26.8	0.175±0.027	0.272
0.600	32.2	0.215±0.022	0.281
0.600	32.2	0.224±0.022	0.281
0.700	37.5	0.223±0.030	0.288
0.800	42.9	0.234±0.027	0.293
0.900	48.2	0.249±0.023	0.298
1.000	54.3	0.235±0.021	0.307
1.500	80.4	0.281±0.015	0.329
2.000	107	0.274±0.030	0.350
3.000	161	0.312±0.031	0.370
5.000	268	0.422±0.045	0.419
6.000	321	0.438±0.039	0.439
6.500	342	0.460±0.044	0.445
14.3	765	0.555±0.045	0.525
17.1	916	0.692±0.045	0.551
17.3	921	0.692±0.045	0.552
18.6	997	0.680±0.045	0.573

Chasman *et al.* obtain spectra from the Ge$^{72}(n,n'e^{-})$-Ge^{72} reactions which are a summation of conversion-electron and recoil spectra from inelastic scattering from the first excited state of Ge^{72}. The broadening in the conversion-electron linewidth is produced by the ionization made by the Ge recoils; the minimum Ge recoils produce virtually no ionization. The end points (i.e., the points corresponding to minimum and maximum Ge recoil energies) of their spectra were approximated by half-maximum points, in contrast to the knees of the spectra which are assumed in the present investigation to be associated with E_{Ge}(max). In their paper, Chasman *et al.* recognized that their method does not take into account the effect of detector resolution which at low energies is of the same order of magnitude as the ionization produced by recoils. The positions of the half-maximum points of their spectra, unlike the knees, are a function of electronic noise and detector response. Furthermore, it is not obvious that their method accounts for the difference in the response of the detector to electrons and to heavy recoils as evidenced by the difference in sharpness of the leading and trailing edge of their pulse-height distribution. Use of the half-maximum points might lead to systematically high values of ionization corresponding to E_{Ge}(max). Indeed, the interpretation of the knee of the spectra as the point corresponding to E_{Ge}(max), in addition to being suggested by symmetry considerations and by the rela-

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[11] J. Lindhard (private communication).

[12] *Note added in proof.* C. Chasman, K. W. Jones, and R. A. Ristinen have subsequently corrected their earlier error in the relationship between energy and the quantity $\epsilon(E)$, Phys. Rev. Letters **15**, 684 (1965). The difference in the $\eta(E)/E$ values between the work of Chasman *et al.* and the present work is ~25% for a Ge recoil energy of 20 keV, and the difference decreases to ~15% at 100 keV.

tionship of the neutron cross section to the energy distribution of heavy recoils, was checked in I by obtaining monoenergetic recoils from a coincidence technique. Interpretation of the knee of (ungated) pulse-height spectra as the end point was found to be more consistent with the spectrum of ionization produced by monoenergetic recoils than use of the positions of the half-maximum points.

A measurement[5,13] of the ionization produced by low-energy K ions in Ge by a quite different technique (for K-ion energies of 0.5, 1, 3, and 8 keV) yielded values of $\bar{\eta}(E)/E$ about a factor of 2 lower than predicted by the theory of Lindhard. It is difficult to see how the low-energy (below 50 keV) values of $\bar{\eta}(E)/E$ for Ge in Ge can be higher than predicted by the theory of Lindhard when significantly lower $\bar{\eta}(E)/E$ values than predicted by theory are observed for the lighter K ions in Ge (over only a slightly lower energy range). One would expect the lighter ion to have a higher value of $\bar{\eta}(E)/E$. In fact, a simplified approximation derived by Lindhard *et al.*, discussed below, predicts for K ions in Ge that $\bar{\eta}(E)/E \approx 0.051E^{1/2}$, whereas for Ge in Ge this approximation yields $\bar{\eta}(E)/E \approx 0.029E^{1/2}$.

For very low recoil energies, Lindhard[5] derived a simplified analytical approximation to the more general integral equation. In this approximation, it is assumed that the atomic stopping cross section is constant with energy. This would hold at very low recoil energies where E (in keV) $\ll AZ/2$. In addition, the differential atomic scattering cross section in this simplified approximation is approximated by scattering corresponding to a power potential $\propto 1/r^2$. For $A_1 = A_2$ and $Z_1 = Z_2$, the analytical expression obtained by Lindhard[5] then reduced to $\bar{\eta}(E)/E \approx (2E/AZ)^{1/2}$. The theory also holds for $A_1 \neq A_2$ and $Z_1 \neq Z_2$, but the expression is more complicated.

The predictions obtained from the simplified expression actually give better agreement with experiment in the lower range of Ge recoils than the numerical solutions of the basic integral equation derived by Lindhard. (The simplified approximation also gives an excellent fit for K ions in Ge.[5,12]). A comparison of the two predictions with the present data are shown in Table II. The simplified approximation does not agree quite as well with the data for Si; but in the lower range of Si recoil energies that could be measured, the condition $E \ll AZ/2$ is not satisfied as well as it was for Ge. Generally, however, for recoil energies where $E \ll AZ/2$ (≤ 20 keV for Si, ≤ 200 keV for Ge), the simplified theory of Lindhard appears to give a somewhat better agreement with experiment.

TABLE II. Predictions of $\bar{\eta}(E)/E$ from results of simplified approximation of Lindhard *et al.* and those obtained numerical solutions of basic integral equation compared with values interpolated from experiment.

Recoil energy (keV)	Si			Ge		
	From expt.	Simplified theory	Numerical solution	From expt.	Simplified theory	Numerical solution
20	0.32	0.32	0.33	0.16	0.13	0.26
40	0.37	0.45	0.40	0.21	0.19	0.29
60	0.41	0.56	0.44	0.24	0.23	0.31
80	0.43	0.64	0.48	0.27	0.26	0.33
100	0.45	0.72	0.50	0.29	0.29	0.34
200	0.55	1.01	0.59	0.35	0.42	0.39

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