Dielectric Constant of a Semiconductor in an External Electric Field*

K. S. VISWANATHAN† AND JOSEPH CALLAWAY
Department of Physics, University of California, Riverside, California
(Received 8 September 1965)

The effect of a constant external electric field on the transverse dielectric constant of a semiconductor is calculated. The field produces a sharp decrease in the dielectric constant close to the threshold for an interband transition. Above the edge, the behavior is oscillatory. Numerical results have been obtained for gallium arsenide.

I. INTRODUCTION

THE effect of a constant electric field on the optical absorption of a semiconductor has been studied fairly extensively, both theoretically[1–6] and experimentally.[7–14] Since the optical absorption is described by the imaginary part of the dielectric constant, and causality implies a connection between the real and imaginary parts of this function through Kramers-Kronig relations,[15] it is obvious that an electric field must have some effect on the real part of this function, and thus on the index of refraction.[16] Such effects have been observed through measurements of the effect of an electric field on the reflectivity of a semiconductor.[17–20] We report here a direct calculation of the change in the dielectric constant of a semiconductor in an external electric field.

From a physical point of view, the effect may be described as follows: In the absence of a field, the optical absorption of an ideal semiconductor is zero below the band gap E_g, and rises rapidly above it, being proportional to $(\hbar\omega-E_g)^{1/2}$, where $\hbar\omega$ is the photon energy. The real part of the dielectric constant, κ_e, also has a discontinuous derivative at the edge. It is proportional to the function

$$(E_g/\hbar\omega)^2[2-(1+\hbar\omega/E_g)^{1/2}-(1-\hbar\omega/E_g)^{1/2}] \quad (1)$$

for $\hbar\omega<E_g$, and to the function

$$(E_g/\hbar\omega)^2[2-(1+\hbar\omega/E_g)^{1/2}] \quad (2)$$

above the gap, $\hbar\omega>E_g$. This function has a cusp when $\hbar\omega=E_g$; it is shown in Fig. 1. A more complete expression is given in Eq. (32) below.

When an electric field is applied, the absorption constant does not go to zero when $\hbar\omega=E_g$; instead it has an exponential tail into the gap. It becomes a smooth function, and so does the real part of the dielectric constant which is the Hilbert transform of the imaginary part. Removal of the cusp of Fig. 1 produces a sharp change in the reflectivity, which is observed. The general nature of these arguments indicates that similar changes are to be expected at any sort of interband edge, as has been discussed by Phillips and Seraphin.[21]

II. CALCULATION

Our work is based on an expression for the frequency-dependent transverse dielectric constant κ_e of a solid which is given below.[22] The electronic states of the solid are represented by Slater determinants of one-particle Bloch wave functions. These functions are characterized by a band index (l or n) and a wave vector $\mathbf{k}$ (crystal momentum representation or CMR). The result, expressed in mks units, is

$$\kappa_e=1+\frac{e^2}{m\omega^2\epsilon_0}\sum_{\mathbf{k},l}N_l(\mathbf{k})\times\left[1-\frac{2}{3\hbar m}\sum_{n\neq l}\frac{\omega_{nl}(\mathbf{k})|\mathbf{p}_{nl}(\mathbf{k})|^2}{\omega_{nl}(\mathbf{k})^2-\omega^2}\right]. \quad (3)$$

The quantities which appear in Eq. (3) are defined as

* Supported by the U. S. Air Force Office of Scientific Research.
† Present address: Department of Physics, Simon Fraser University, Burnaby 2, British Columbia, Canada.

1 W. Franz, Z. Naturforsch. **13**, 484 (1958).
2 L. V. Keldysh, Zh. Eksperim. i Teor. Fiz. **34**, 1138 (1958) [English transl.: Soviet Phys.—JETP **7**, 788 (1958)].
3 J. Callaway, Phys. Rev. **130**, 549 (1963); **134**, A998 (1964).
4 K. Tharmalingam, Phys. Rev. **130**, 2204 (1963).
5 C. M. Penchina, Phys. Rev. **138**, A924 (1965).
6 M. Chester and L. Fritsche, Phys. Rev. **139**, A518 (1965).
7 L. V. Keldysh, V. S. Vavilov, and K. I. Britsyn, in *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Czechoslovakian Academy of Sciences, Prague, 1961), p. 824.
8 V. S. Vavilov and K. I. Britsyn, Fiz. Tverd. Tela **2**, 1936 (1960) [English transl.: Soviet Phys.—Solid State **2**, 1746 (1961)].
9 K. W. Boer, H. J. Hansche, and V. Kummel, Z. Physik **155**, 170 (1959).
10 R. Williams, Phys. Rev. **117**, 1487 (1960); **126**, 442 (1962).
11 T. S. Moss, J. Appl. Phys. **32**, 193 (1964).
12 A. Frova and P. Handler, Appl. Phys. Letters **5**, 11 (1964); *Proceedings of the International Conference on Semiconductor Physics, 1964* (Dunod Cie., Paris, 1964), p. 157.
13 M. Chester and P. H. Wendland, Phys. Rev. Letters **13**, 193 (1964).
14 L. M. Lambert, Phys. Rev. **138**, A1569 (1965).
15 J. S. Toll, Phys. Rev. **104**, 1760 (1956).
16 B. O. Seraphin and N. Bottka, Phys. Rev. **139**, A560 (1964).
17 B. O. Seraphin, in *Proceedings of the International Conference on the Physics of Semiconductors, Paris, 1964* (Dunod Cie., Paris, 1964), p. 165.
18 B. O. Seraphin, R. B. Hess, and N. Bottka, J. Appl. Phys. **36**, 2242 (1965).
19 B. O. Seraphin and R. B. Hess, Phys. Rev. Letters **14**, 138 (1965).
20 B. O. Seraphin and N. Bottka, Phys. Rev. Letters **15**, 104 (1965).
21 J. C. Phillips and B. O. Seraphin, Phys. Rev. Letters **15**, 104 (1965).
22 J. Callaway, *Energy Band Theory* (Academic Press Inc., New York, 1964), p. 296.

follows: The (circular) frequency of the incident radiation is ω. $N_l(\mathbf{k})$ is the density of occupied states in band l. Including the usual factor of 2 for spin, we get

$$N_l(\mathbf{k})=2 \quad \text{if} \quad |l\mathbf{k}\rangle \quad \text{is occupied},$$
$$N_l(\mathbf{k})=0 \quad \text{if} \quad |l\mathbf{k}\rangle \quad \text{is unoccupied}. \tag{4}$$

The quantity $\omega_{nl}(\mathbf{k})$ is given by

$$\hbar\omega_{nl}(\mathbf{k})=E_n(\mathbf{k})-E_l(\mathbf{k}), \tag{5}$$

and the elements $\mathbf{p}_{ln}$ are related to the usual momentum matrix elements by

$$\mathbf{p}_{ln}(\mathbf{k})\delta(\mathbf{k}-\mathbf{k}')=\langle l\mathbf{k}|\mathbf{p}|n\mathbf{k}'\rangle. \tag{6}$$

The sum over n in Eq. (3) includes all values of the band index (except for $n=l$), regardless of whether the band is occupied or not.

The expression we have given above for the dielectric constant does not involve the steady electric field. When a field is present, we use a different set of basis functions, as will be discussed in detail below, but in a formal sense Eq. (3) is essentially unchanged.

The dielectric function as given by Eq. (3) appears to vary as ω^{-2} for small ω. This dependence actually occurs for metals, but not in semiconductors and insulators. In order to transform Eq. (3) into a more convenient form, we use the identity

$$\frac{\omega_{nl}}{\omega_{nl}^2-\omega^2}=\frac{1}{\omega_{nl}}+\frac{\omega^2}{\omega_{nl}(\omega_{nl}^2-\omega^2)}, \tag{7}$$

and the "f" sum rule[23]:

$$\frac{2}{3\hbar m}\sum_{n\neq l}\frac{|\mathbf{p}_{ln}(\mathbf{k})|^2}{\omega_{ln}(\mathbf{k})}=1-\frac{m}{3\hbar^2}\nabla_{\mathbf{k}}^2E_l(\mathbf{k}). \tag{8}$$

Then we get

$$\kappa_e=1-\frac{e^2}{3m\omega^2\epsilon_0}\sum_{l\mathbf{k}}N_l(\mathbf{k})\left[\frac{m}{\hbar^2}\nabla^2E_l(\mathbf{k})-\frac{2\omega^2}{\hbar m}\sum_{n\neq l}\frac{|\mathbf{p}_{ln}(\mathbf{k})|^2}{\omega_{nl}(\mathbf{k})(\omega_{nl}^2-\omega^2)}\right]. \tag{9}$$

The term in Eq. (9) involving ∇^2E_l gives rise to the usual plasma contribution in the case of a metal; however for a semiconductor in which all bands l which contain any electrons at $T=0°\text{K}$ are also full, we can write

$$\sum_{\mathbf{k}}N_l(\mathbf{k})\nabla^2E_l(\mathbf{k})=\frac{2}{(2\pi)^3}\int d^3k\nabla^2E_l(\mathbf{k})=\frac{2}{(2\pi)^3}\int\nabla E_l\cdot d\mathbf{S}=0. \tag{10}$$

[23] E. N. Adams, Phys. Rev. **85**, 41 (1952).

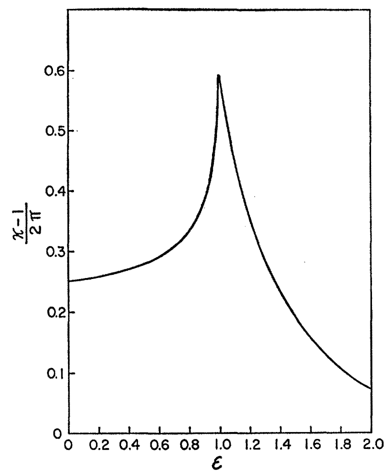

FIG. 1. Zero-field dielectric constant. The contribution to the dielectric constant from a pair of simple parabolic bands with no external electric field is shown as a function of $\mathcal{E}=\hbar\omega/E_g$, where ω is the circular frequency of the incident light, and E_g is the minimum band gap. The quantity B in Eq. (32) has been set equal to unity.

In the last step in Eq. (10), we have used Gauss' theorem to obtain an integral over a surface of constant energy in band l which surrounds all the occupied states. For a full band, however, there is no such surface, so that the sum gives zero. Thus, for a semiconductor the expression for the dielectric constant simplifies to

$$\kappa_e=1+\frac{2e^2}{3m^2\hbar\epsilon_0}\sum_{l\mathbf{k}}N_l(\mathbf{k})\sum_{n\neq l}\frac{|\mathbf{p}_{ln}(\mathbf{k})|^2}{\omega_{nl}(\omega_{nl}^2-\omega^2)}. \tag{11}$$

We are now ready to consider the effect of a steady electric field, $F=eE$. We suppose the field to be along the "x" axis, and also that this axis coincides with some reciprocal lattice vector. Instead of using Bloch functions, we use the functions introduced by Kane[24] as a basis. These functions are eigenfunctions in the presence of an electric field if tunneling is neglected, and have been used in calculations of tunneling[24,25] and of optical absorption.[3] They are characterized by discrete quantum numbers, ν, n (where ν designates a Wannier level,[26] n is still the band index), and by the wave vector $\mathbf{k}_\perp$ which refers to the components of the usual crystal momentum in directions perpendicular to the electric field. It then can be shown by a straightforward calculation that Eq. (11) remains valid for a semiconductor in which all the states characterized by different values of $\mathbf{k}_\perp$ for fixed values of ν and n are either completely full or completely empty (with tunneling neglected) provided that the quantities $N_l(\mathbf{k})$, $\omega_{ln}(\mathbf{k})$ which appear in Eq. (11) as defined in the CMR are replaced by the analogous objects com-

[24] E. O. Kane, J. Phys. Chem. Solids **12**, 181 (1959).
[25] P. N. Argyres, Phys. Rev. **126**, 1386 (1962).
[26] Reference 22, p. 281.

puted on the basis of Kane functions. Thus we have

$$\kappa_e=1+\frac{2e^2}{3\hbar m^2\epsilon_0}\sum_{\nu l\mathbf{k}_\perp}N_{\nu l}(\mathbf{k}_\perp)\sum_{\nu' n\neq\nu l}\frac{|\mathbf{p}_{\nu l,\nu' n}(\mathbf{k}_\perp)|^2}{\omega_{n\nu' l\nu}(\omega_{n\nu',l\nu}{}^2-\omega^2)}. \quad (12)$$

The matrix elements and energy denominators may be obtained from Ref. 3. We have

$$\hbar\omega_{n\nu',l\nu}=W_{\nu',n}(\mathbf{k}_\perp)-W_{\nu l}(\mathbf{k}_\perp)=\frac{2\pi F(\nu'-\nu)}{\kappa}+\frac{1}{\kappa}\int_{-\kappa/2}^{\kappa/2}[E_n(\mathbf{k})-E_l(\mathbf{k})]dk_x, \quad (13)$$

$$\mathbf{p}_{\nu l,\nu' n}(\mathbf{k}_\perp)=\int_{-\kappa/2}^{\kappa/2}A_{\nu,l}{}^*(\mathbf{k})\mathbf{p}_{ln}(\mathbf{k})A_{\nu',n}(\mathbf{k})dk_x, \quad (14)$$

with

$$A_{\nu,l}(\mathbf{k})=\kappa^{-1/2}\exp\left\{\frac{i}{F}\int_0^{k_x}[W_{\nu l}(\mathbf{k}_\perp)-E_l(k_\perp,k_x')]dk_x'\right\}. \quad (15)$$

In Eqs. (13)–(15), κ is the length of the Brillouin zone in the "k_x" direction.

The expressions for the dielectric constant involve all bands. For this reason, Eqs. (11) and (12) are too complicated to be computed completely. Our interest here is principally the change in the dielectric constant produced by an external field. The arguments of the Introduction, which are supported by the calculation which follows, indicate that this change is largest at a frequency which corresponds to the onset of some inter-band transition (or more generally at some Van Hove singularity in the joint density of states). We will obtain the most important features of the effects in which we are interested if we use a band model appropriate to the region near a minimum gap.

More specifically, we will determine the contribution to the dielectric constant from a pair of bands which are closest at $k=0$, where they are separated by a band gap E_g, and are described by an effective mass approximation. Let

$$E_1=E_v=-\hbar^2k^2/2m_v, \quad (16a)$$

and

$$E_2=E_c=E_g+\hbar^2k^2/2m_c, \quad (16b)$$

$$\mu^{-1}=m_v^{-1}+m_c^{-1}. \quad (16c)$$

We will suppose that the CMR interband matrix element $\mathbf{p}_{cv}=\mathbf{p}_{21}$ is independent of $\mathbf{k}$, corresponding to an allowed transition. Then from Ref. 3 we have

$$\mathbf{p}_{\nu 2,\nu' 1}=(2\pi\beta^{1/3}/\kappa)\mathbf{p}_{21}(0)\mathrm{Ai}(-z), \quad (17)$$

in which Ai(z) is an Airy function defined by

$$\mathrm{Ai}(z)=\frac{1}{\pi}\int_0^\infty\cos(sz+\tfrac{1}{3}s^3)ds, \quad (18)$$

and

$$\beta=2\mu F/\hbar^2,$$
$$z=\beta^{1/3}[2\pi(\nu-\nu')/\kappa+\hbar^2\kappa^2/24\mu F]. \quad (19)$$

These expressions are substituted into Eq. (12). The quantities inside the summation are functions of $\nu'-\nu$ only. Consequently, the sum over ν and ν' can be performed using the Poisson summation formula as is done in the calculations of the absorption constant.[3] The result is

$$\kappa_e=1+\frac{8\pi}{3}\frac{e^2\beta^{2/3}|\mathbf{p}_{21}(0)|^2}{m^2\hbar\epsilon_0\kappa}\times\sum_{j=-\infty}^{\infty}\sum_{\mathbf{k}_\perp}\int_{-\infty}^{\infty}dy\frac{\mathrm{Ai}^2(-z_y)e^{2\pi ijy}}{\omega_{21}(y)[\omega_{21}{}^2-\omega^2]}, \quad (20)$$

in which we now have

$$z_y=\beta^{1/3}[2\pi y/\kappa+\hbar^2\kappa^2/24\mu F],$$
$$\hbar\omega_{21}(y)=\frac{2\pi Fy}{\kappa}+E_g+\frac{\hbar^2\mathbf{k}_\perp{}^2}{2\mu}+\frac{\hbar^2\kappa^2}{24\mu}. \quad (21)$$

The sum over $\mathbf{k}_\perp$ can be converted to an integral in the usual way:

$$\sum_{\mathbf{k}_\perp}\rightarrow\frac{1}{(2\pi)^2}\int d^2k_\perp.$$

The integral over $k_\perp$ can then be performed. Since the integrand decreases as $k_\perp{}^{-5}$ for large $k_\perp$, the upper limit on the $k_\perp$ integration may be made infinite without serious error. From the derivation of the expression for the dielectric constant in Ref. 22, we see that the integral is to be interpreted as a Cauchy principal value. Also, the sum over j and the integral over y can be transformed with the aid of the relation[27]

$$\sum_{j=-\infty}^{\infty}e^{2\pi ijy}=\sum_{n=-\infty}^{\infty}\delta(y-n). \quad (22)$$

After a straightforward calculation, we obtain

$$\kappa_e=1+\frac{2\mu e^2\beta^{2/3}|\mathbf{p}_{21}(0)|^2}{3\kappa\hbar^2m^2\epsilon_0\omega^2}\times\sum_{n=-\infty}^{\infty}\mathrm{Ai}^2(-z_n)\ln\frac{a_n{}^2}{|\omega+a_n||\omega-a_n|}, \quad (23)$$

[27] G. Goertzel and N. Tralli, *Some Mathematical Methods of Physics* (McGraw-Hill Book Company, Inc., New York, 1960), p. 123.

in which z_n is obtained from Eq. (21) by replacing y by n, and

$$a_n = \hbar^{-1}(E_g + 2\pi nF/\kappa + \hbar^2\kappa^2/24\mu). \tag{24}$$

Equation (23) is our fundamental expression for the dielectric constant. It is worth noting that this result can also be obtained from the expressions of Ref. 3 for the absorption constant with the use of a Kramers-Kronig relation in the form

$$\kappa_e = 1 + \frac{2}{\pi}\frac{c}{\omega}\int_0^\infty \frac{\bar{n}\alpha(\omega')}{\omega'^2 - \omega^2} d\omega', \tag{25}$$

in which α is the absorption constant and $\bar{n}$ is the index of refraction.[28] A similar procedure was employed by Seraphin and Bottka.[16]

Equation (23) contains a sum over terms arising from the discrete Wannier levels. In the low-field limit (and also because of the smearing of these levels by collisions), it is legitimate to replace the sum over n by an integral. Then let

$$\sum_n f(z_n) = \sum_n f[\beta^{1/3}(2\pi n/\kappa + \hbar^2\kappa^2/24\mu F)] \rightarrow$$

$$\int_{-\infty}^{\infty} dz f[\beta^{1/3}(2\pi z/\kappa + \hbar^2\kappa^2/24\mu F)] = \frac{\kappa}{2\pi}\int_{-\infty}^{\infty} f(\beta^{1/3}t)dt.$$

Thus,

$$\kappa_e = 1 + \frac{\mu e^2\beta^{2/3}|\mathbf{p}_{21}(0)|^2}{3\pi m^2\hbar^2\epsilon_0\omega^2}\int_{-\infty}^{\infty} dt\, \mathrm{Ai}^2(-t\beta^{1/3}) \times \ln[a^2(t)/(|\omega - a(t)||\omega + a(t)|)]. \tag{26}$$

In order to obtain a more convenient expression, we introduce the variable $s = Ft/E_g$, and define dimensionless parameters

$$\mathcal{E} = \hbar\omega/E_g,$$
$$\gamma = E_g\beta^{1/3}/F, \tag{27}$$
$$B = \frac{e^2|\mathbf{p}_{21}(0)|^2}{12\pi^2 m^2\epsilon_0\hbar}\left(\frac{2\mu}{E_g}\right)^{3/2}.$$

With these substitutions Eq. (26) becomes

$$\kappa_e = 1 + \frac{2\pi B\gamma^{1/2}}{\mathcal{E}^2} \times \int_{-\infty}^{\infty} ds\, \mathrm{Ai}^2(-\gamma s)\ln\frac{(1+s)^2}{|\mathcal{E}-1-s||\mathcal{E}+1+s|}. \tag{28}$$

The quantity B can be obtained from the optical absorption in the absence of a steady field. The absorption constant α in this case can be written as

$$\alpha = \alpha_0(\hbar\omega - E_g)^{1/2}, \tag{29}$$

with

$$\alpha_0 = 2\pi E_g^{3/2}B/(\bar{n}\hbar\omega c).$$

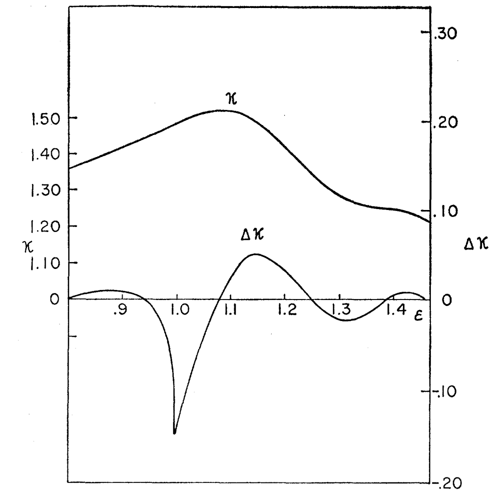

FIG. 2. Dielectric constant in the presence of an electric field. The upper curve (left-hand scale) shows the contribution of a pair of simple parabolic bands to the dielectric constant in the presence of an electric field as a function of $\mathcal{E} = hw/E_g$. The lower curve (right-hand scale) shows the difference between curve 1 and the zero-field dielectric constant for the same pair of bands. The curves have been calculated with parameters appropriate to the light-hole conduction-band transition in GaAs and a field strength of $F = 10^6$ eV/cm.

Our numerical computations, which are discussed below, are based on Eq. (28). Before considering these in detail, it is desirable to see how Eq. (28) behaves as the electric field becomes very small. In such a situation, γ is large, and we can introduce asymptotic expansions for the Airy functions. Let x be a large positive number. Then

$$\mathrm{Ai}(-x) = (\pi^{1/2}x^{1/4})^{-1}\sin(\tfrac{2}{3}x^{3/2} + \tfrac{1}{4}\pi),$$
$$\mathrm{Ai}(x) = (2\pi^{1/2}x^{1/4})^{-1}\exp(-\tfrac{2}{3}x^{3/2}). \tag{30}$$

From this we see that for very large γ, only the portion of the integrand coming from positive values of s survives. We have

$$\kappa_e(F=0) = 1 + \frac{B}{\mathcal{E}^2}\int_0^\infty \frac{ds}{s^{1/2}}\ln\frac{(1+s)^2}{|\mathcal{E}-1-s||\mathcal{E}+1+s|}. \tag{31}$$

The integral can be performed in a straightforward manner. We obtain

$$\kappa_e = 1 + (2\pi B/\mathcal{E}^2)[2 - (1+\mathcal{E})^{1/2} - (1-\mathcal{E})^{1/2}], \quad (\mathcal{E} < 1), \tag{32}$$
$$\kappa_e = 1 + (2\pi B/\mathcal{E}^2)[2 - (1+\mathcal{E})^{1/2}], \quad (\mathcal{E} > 1).$$

The properties of this function were discussed in the Introduction. The change in the dielectric constant

[28] In performing the calculation leading from Eq. (25) to Eq. (23), one should note that the constant K in Ref. 3 is too large by a factor of 4, and that we have replaced $|\boldsymbol{\varepsilon}\cdot\mathbf{p}_{21}|^2$ by its average $\frac{1}{3}|\mathbf{p}_{21}|^2$.

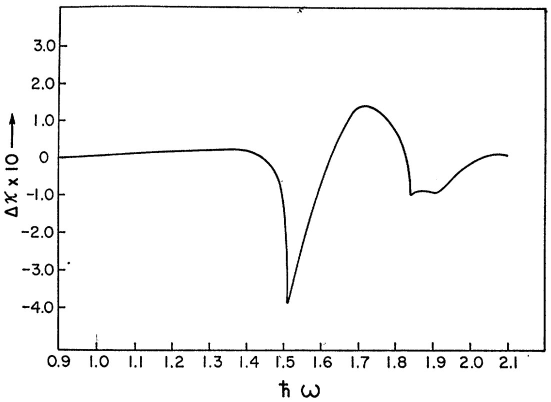

FIG. 3. Change in dielectric constant for GaAs. The change in the dielectric constant of gallium arsenide in an electric field of 10^6 eV/cm is shown as a function of photon energy in electron volts.

produced by the field can now be computed as the difference of Eqs. (28) and (32).

$$\Delta\kappa=\kappa_e(F)-\kappa_e(0). \tag{33}$$

III. RESULTS AND DISCUSSION

The dielectric constant in the presence of an electric field for a pair of parabolic bands is shown in Fig. 2 as computed from Eq. (28). In this example, we also show the change in the dielectric constant, $\Delta\kappa$ for the same pair of bands. The quantity $\Delta\kappa$ has a cusp when $\hbar\omega=E_g$ and oscillates with decreasing amplitude when $\hbar\omega>E_g$.

We have also evaluated $\Delta\kappa$ using band structure parameters appropriate to gallium arsenide. One minor complication arises from the complexity of the valence-band structure. Two bands are degenerate at $k=0$, and a valence band, detached from the first two, by spin orbit coupling lies approximately 0.33 eV below. It is, however, not difficult to include these three bands, since their effective masses are known. According to Ehrenreich,[29] the conduction band has an effective mass ratio $m_c^*=0.072$, while the three valence bands have masses $m_{v1}^*=0.68$, $m_{v2}^*=0.13$, $m_{v3}^*=0.20$. The band gap at $T=0$°K is $E_g=1.515$ eV, while from the optical absorption measurements of Sturge,[30] we have $\alpha_0=5.6\times10^4$ cm^{-1} eV$^{-1/2}$ close to the edge. The resulting change in the dielectric constant, including all three valence bands, is shown in Fig. 3.

[29] H. Ehrenreich, Phys. Rev. **120**, 1951 (1960).

[30] M. D. Sturge, Phys. Rev. **127**, 768 (1962).

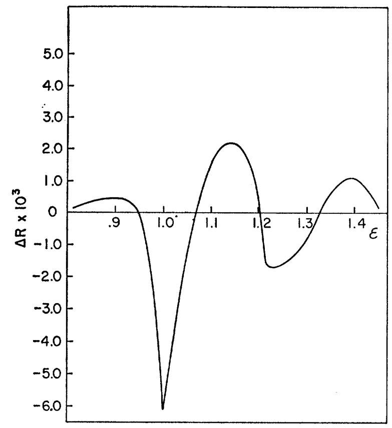

FIG. 4. Change in reflectivity of GaAs. The change in the reflectivity of GaAs in the presence of a field of 10^6 eV/cm is shown as a function of $\mathcal{E}=\hbar\omega/E_g$. The second minimum at $\mathcal{E}=1.22$ is produced by the split-off valence band.

The field-induced change in the dielectric constant has been observed through measurement of the change in the reflectivity.[16–20] The reflectivity is given by

$$R=\frac{[\bar{n}(F)-1]^2+K^2(F)}{[\bar{n}(F)+1]^2+K^2(F)}, \tag{34}$$

in which $\bar{n}(F)$ is the index of refraction and $K(F)$ is the extinction coefficient, both in the presence of the field. The index of refraction is related to dielectric constant by

$$\bar{n}(F)=[\kappa_e(F)]^{1/2} \tag{35}$$

(when the absorption is small). Close to the gap, the extinction coefficient is negligible, and the change in the reflectivity is dominated by the behavior of the dielectric constant. The change in the reflectivity ΔR for GaAs in a field of 10^6 eV/cm is shown in Fig. 4.

The change in the reflectivity of GaAs in an external field has been observed.[31] It is qualitatively in agreement with our results (Fig. 4); however quantitative comparison is not possible because of lack of knowledge of the precise electric fields existing in the material, and of the complicating effects of impurity and surface states.

[31] B. O. Seraphin (to be published).