Surface-Impedance Oscillations in a Weak Magnetic Field*

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We have made a detailed experimental study of the microwave surface resistance of high-purity metal single crystals in the regime of weak magnetic field. Samples of Sn, In, and Al show oscillatory variations of surface resistance that are periodic in reciprocal field. For a given fundamental resistance maximum at field H_0, successive peaks occur at $\frac{1}{3} H_0$, $\frac{1}{5} H_0$, $\frac{1}{7} H_0$, etc., i.e., all the odd submultiples of the field of the first resistance maximum. The oscillations are observed typically in fields of just a few Oe to about 100 Oe at a frequency of 35 Gc/sec. We have studied the anisotropy of the effect with orientation of magnetic field in a given sample plane, as well as the dependence of the effect on the sample plane in which a given direction is observed. The amplitude of the effect is found to be a function of the direction of the rf current. The effect is dependent on the experimental frequency. In the range of 28 to 70 Gc/sec, we find that a given resistance maximum moves to increased fields as $\omega_{rf}^{3/2}$. Signals are readily observed also with the magnetic field tipped out of the sample plane. We suggest that the observed oscillations in the microwave absorption arise from variations with magnetic field of the time that the electron spends interacting with the electric fields in the surface region. This consideration leads us to suggest that the period of the oscillations is related to the values of radius of curvature and velocity of the electron at a single point on the Fermi surface.

I. INTRODUCTION

THE existence of complicated and detailed structure in the microwave surface impedance of metal single crystals in a weak magnetic field has been recognized by Khaikin.[1] Prior to this work, Kip *et al.*[2] in their observation of Azbel-Kaner cyclotron resonance in Sn have reported some unusual low-field structure in the surface resistance of this metal. Likewise in their report on cyclotron resonance in *W*, Fawcett and Walsh[3] note structure in the low-field region of their data. More recently, Koch and Kip[4] have reported on preliminary studies of this low-field effect, and suggested a possible explanation for the origin of the structure of the surface impedance in the region of weak magnetic fields.

This work is in large part an extension of the preliminary study. With considerable increase in the resolution of experimental data, we have been able to identify the surface impedance variations in weak magnetic fields as an oscillatory effect. The results of the present work represent an effort to provide a well documented and extensive, if not exhaustive, experimental study of this low-field effect.

To motivate the reader for the compendium of experimental results presented in Sec. III, we briefly outline the mechanism that we believe can account qualitatively for the observed oscillations. The range of magnetic field 1–100 Oe is the range where electrons make a single complete traversal of the skin region. The orbit radius is so large, that the electron is exposed to the rf electric field in the skin layer for a time comparable to the rf period during the traversal. We expect a maximum in the energy absorbed by the electron from the microwave field when the time of exposure is $\frac{1}{2}$ of an rf cycle. If the electron spends a longer time in the skin it will experience a reversal of the electric field and consequently a decrease in the energy acquired, until this time of exposure becomes a complete rf period and the net energy gained is zero. Successive maxima in the energy absorbed should result when the time of exposure is $\frac{3}{2}$, $\frac{5}{2}$, etc., of the rf period. The time spent in the skin depends on the radius of curvature of the electron orbit and consequently on the magnetic field. As we sweep the magnetic field, we expect that the microwave absorption should show some oscillatory variation.

II. EXPERIMENTAL ASPECTS

The surface impedance oscillations that we observe appear as a low-field byproduct of the cyclotron-resonance experiments. As such, the experimental arrangement as well as sample preparation techniques used in this work are essentially those developed for the observation of cyclotron resonance.

We use standard microwave reflection spectrometers operating in frequency ranges 28–38, 50–60, and 68–72 Gc/sec. The klystron frequency is stabilized to the resonant frequency of the experimental cavity. The sample forms one wall of this cavity. The magnetic field is modulated at a low audio frequency and the resulting coherent microwave absorption signal is phase detected. The output of the phase detector is the derivative with respect to magnetic field of this microwave absorption at the sample surface. This is proportional to the derivative of the real part of the surface impedance. We record dR/dH, as a function of applied magnetic field.

The low fields required for the experiment are produced by Helmholtz coils. The field is determined by measuring the current supplied to the coils and cali-

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1 M. S. Khaikin, Zh. Eksperim. i Teor. Fiz. **39**, 212 (1960) [English transl.: Soviet Phys.—JETP **39**, 152 (1961)].

2 A. F. Kip, D. N. Langenberg, B. Rosenblum, and G. Wagoner, Phys. Rev. **108**, 494 (1957).

3 E. Fawcett and W. M. Walsh, Jr., Phys. Rev. Letters **8**, 476 (1962).

4 J. F. Koch and A. F. Kip, *Proceedings of the Ninth International Conference on Low-Temperature Physics, Columbus, Ohio*, edited by J. G. Daunt, D. V. Edwards, F. J. Milford, and M. Yaqub (Plenum Press, Inc., New York, 1964).

brated against the reading of a rotating coil probe. The probe, in turn, had been calibrated with an EPR signal at higher fields. Since the sweep circuit for the current to the coils was not strictly linear in time, we mark field values at frequent intervals and interpolate linearly between each pair of markers. By averaging magnetic-field readings between successive increasing and decreasing field runs, we eliminate the effects of phase-detector time constant in the display of the signal. The scales of magnetic field used in the figures are approximate in that they have been drawn linearly and have not been corrected for phase-detector time constant.

We use cylindrical cavities operating in the TE_{111} mode. The sample forms the end wall of this cavity. The currents in the TE_{111} mode are very nearly linear. In much of the work we use such a cavity coupled to the side of the waveguide in an arrangement that allows three-dimensional search for signals.[5] This geometry permits the choice between two mutually perpendicular current modes, which are respectively parallel and perpendicular to the applied magnetic field when the field is in the surface of the sample. A second arrangement that we have used couples the cavity at the end of the waveguide, with the two perpendicular current modes fixed relative to the sample surface independent of the direction of the magnetic field.

For some of the frequency scaling work we use multiple half-wavelength cavities resonating at two or more different frequencies in the desired range. With this arrangement we avoid possible errors in relocating a certain direction in a sample after remounting to a new cavity. This allows us to directly and accurately compare the data at different frequencies.

We briefly outline the preparation of the samples in this work. The Sn crystals were prepared from a special spectro grade Sn, supplied by Vulcan Detinning. The material has a resistivity ratio in excess of 40 000. Single-crystal boules are grown in vacuum in a Pyrex mold, subsequently oriented, spark cut, and electropolished. Some others of these Sn crystals were grown directly against optically flat quartz plates. The Al crystals were obtained from 69 grade Cominco Al with a resistivity ratio of about 5000. The material supplied contained large sections of single crystals. These were spark cut and electropolished. The In samples were grown in high-purity graphite crucibles and likewise spark cut and electropolished. The starting material was 69 grade Cominco In. The Cu crystal in which we have searched for the low field effect is the same used in previous cyclotron resonance studies.[6,7]

The samples are oriented with x-ray Laue diffraction photographs prior to cutting. The final surfaces obtained proved generally to be within about 2° of the desired symmetry plane. The orientation of symmetry axes in such planes could be determined from the corresponding symmetries of the signals that were observed, with an accuracy of approximately 2°.

The low-field experiments are done in the liquid-He range of temperatures.

III. EXPERIMENTAL RESULTS

1. The Low-Field Effect

In the range of magnetic field 0–100 Oe and at frequencies between 28 and 70 Gc sec, we have observed detailed structure in the microwave absorption of single crystals of Sn, Al, and In. This range of fields is generally well below the magnetic field necessary to observe cyclotron resonance in the samples. That is, $\omega_c\tau$ is less than one and electrons are unable to complete cyclotron orbits. The observed structure appears as a low-field byproduct of the cyclotron resonance experiment.

Figure 1 shows a recorder tracing of the surface resistance derivative dR/dH as a function of magnetic field observed with H along the [001] axis of a (100) crystal of Sn. This complicated pattern of dR/dH peaks is not associated with cyclotron resonance. The last harmonics of the usual Azbel-Kaner cyclotron resonance have disappeared at about 100 Oe. This spectrum of peaks is typical of signals observed in many other orientations as well as other symmetry planes of Sn. Crystals of Al and In show similar structure in a comparable range of magnetic field (Figs. 3 and 6). In each of the metals there appears series of dR/dH peaks that rapidly diminish in amplitude as the field is decreased.

The amplitudes of the surface resistance variations that we observe are usually considerably less than typical cyclotron resonance signals and represent a change in the microwave absorption by one part in 10^3 or less. We obtain an approximate measure of amplitude by comparing the low-field signals to the superconducting transition signals in Sn and In.

A careful examination of the dR/dH traces shows that they should be analyzed in terms of series of peaks. This is particularly evident from data on the anisotropy of the signals with orientation of the magnetic field (see Figs. 3 and 4). There are observed always several peaks that show the same variation with the orientation of H and are observed over the same range of angles. In Fig. 2 we explore the relationship between the various peaks of such a series. The first part of the figure is the recorder tracing for the [100] axis in the (100) plane of Sn. In the lower half we separate out the structure belonging to the dominant series of peaks, omitting the other peaks and filling in with the necessary imagination where required. We mark both the dR/dH maxima and the dR/dH zeros corresponding to resistance maxima. The relationships between the successive peaks of a series are expressed in terms of the ratio H_0/H_n, where H_0 is the value of the field for the fundamental of the series, i.e., the highest field peak of the series. Within reasonable experimental error, we find that the resist-

[5] J. F. Koch and A. F. Kip, Phys. Rev. Letters 8, 473 (1962).
[6] A. F. Kip, D. N. Langenberg, and T. W. Moore, Phys. Rev. 124, 359 (1961).
[7] J. F. Koch, R. A. Stradling, and A. F. Kip, Phys. Rev. 133, A240 (1964).

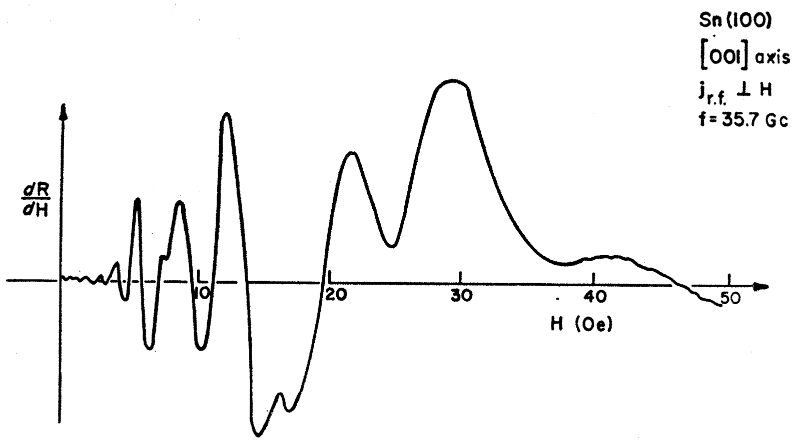

FIG. 1. Low-field effect signals in Sn(100) sample, observed with magnetic field along the [001] axis. The observed structure represents a superposition of three series of peaks.

ance maxima ratios are odd integers 1, 3, 5, 7, etc., or equivalently, that resistance maxima of a series of peaks occur at $H_0/(2n+1)$ with $n=0$, 1, 2, 3, etc. This same periodic behavior is observed for each of the series of Fig. 2 as well as for low-field effect data in Al and In. Likewise the dR/dH maxima ratios found in Fig. 2 are characteristic of the low-field effect in each of the metals studied, but there is no simple relationship between these ratios. In terms of reciprocal magnetic field $1/H$, the resistance maxima form a series $1/H_0$, $3/H_0$, $5/H_0$, etc. We see that the effect is oscillatory in $1/H$, with period $2/H_0$; i.e., twice the reciprocal of the field for the fundamental.

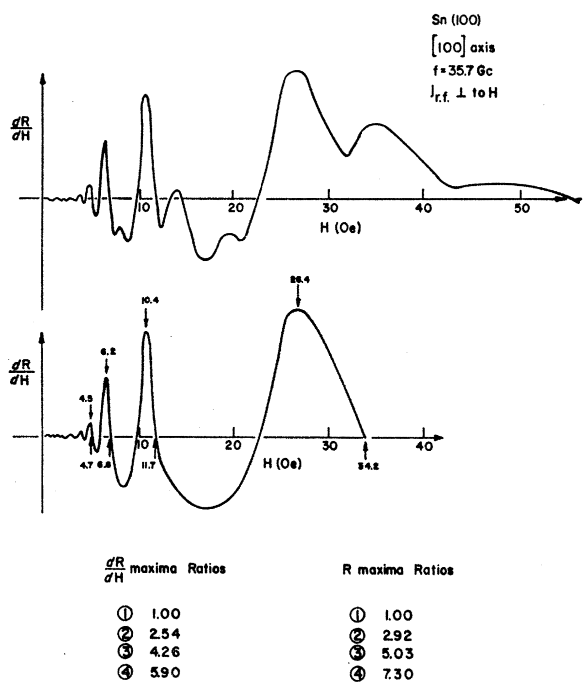

FIG. 2. The first portion is the recorder tracing for the Sn(100) sample and magnetic field along the [100] axis in the plane of the sample. The lower portion separates out the structure belonging to the dominant series of peaks of the tracing above, filling in with some imagination where necessary. The observed ratios of fields of the resistance maxima referred to the fundamental at 34.2 Oe are very nearly 1, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc. This odd submultiple periodicity is characteristic of the low-field oscillations.

The periodicity of the low-field signals has not been identified correctly in previous studies of the effect. Khaikin[1] reports that the surface impedance variations are oscillatory. From the limited number of peaks resolved in his work, he derives a relationship between the surface reactance maxima of a series. For Sn data this is $H_0/(1.6)^n$ with $n=1$, 2, etc. For the In signals it is $H_0/(2.1)^n$ with $n=1$, 2, etc. The work reported by Koch and Kip[4] failed to confirm these periodicities for peaks observed in the resistance data. The peaks studied could not reasonably be fitted to series as proposed by Khaikin, so that the conclusion, based on these data, was that the effect is not oscillatory. With the increased resolution of the spectra obtained in this study, it is apparent that peaks seen in the earlier work are generally fundamentals with successive peaks at lower fields not adequately resolved.

In addition to the metals discussed above, the oscillatory low-field effect has been observed in samples of Cd [1] and W.[3] We have also made an extensive search for the effect in a Cu crystal. In Cu there is observed no oscillatory effect in the range of magnetic field in which the signals have been found in the other metals. At slightly higher fields, about 120 Oe at 30 Gc/sec, there is found a broad dR/dH maximum that has also been observed in the cyclotron resonance work.[6,7] Comparison of this low field dR/dH maximum in Refs. 6 and 7 shows that this signal is moved to higher fields with increased frequency, as is also the case for the low-field-effect signals in Sn, In, and Al. It is not at all clear if this Cu signal is associated with the effect studied here. In any case, there is no evidence for oscillations of the surface resistance as is the case in the other metals. Also, cyclotron resonance studies in Na and K[8] have failed to show an oscillatory low-field effect.

[8] C. C. Grimes and A. F. Kip, Phys. Rev. 132, 1991 (1963).

2. Anisotropy with the Orientation of the Magnetic Field

The period $2/H_0$, as well as the amplitude of the observed low field oscillations, vary with the direction of the magnetic field relative to crystal axes. We study this anisotropy by rotating the magnetic field in small increments in the plane of the sample surface. Figure 3 shows a series of low-field effect spectra observed in the (111) surface of an In crystal. As the direction of the field is varied in the plane of this specimen, dR/dH maxima shift their position in magnetic field. In particular, here there are two groups of peaks. At the [110] axis these two can barely be resolved, but as the field is rotated, one of the series splits off and moves to increased fields. This angular variation allows us to identify which of the peaks belong to a given series.

We have made a plot of field position of dR/dH maxima as a function of the orientation of the magnetic field in the (100) plane of Sn. Data taken with rf current modes along the [100] and [001] axes of this plane are recorded as solid and broken lines respectively in Fig. 4. Only the prominent and well-resolved peaks are on the graph. We find sets of peaks with the same angular variation as well as range of observation. For example the first, third, and fifth of the peaks observed with H along the [001] axis belong together; likewise the second and fourth peaks observed in this orientation. Each of the peaks of a given series shifts in field position the same fractional amount. Anisotropy curves such as Fig. 4 serve to identify peaks of a given series and confirm the interpretation of the effect as an oscillation.

Curves belonging to different series generally show different angular variation and range of observation. It did not prove possible to fit the observed anisotropies to any simple function of the angle. Khaikin[1] has reported the anisotropy of the low-field signals to be characteristically $H(\theta)=H(0)/\cos\theta$, where θ is the angle measured from the symmetry axis. Even though in some instances this proves to be a reasonable approximation to our data, it is not generally as simple as that. Typical

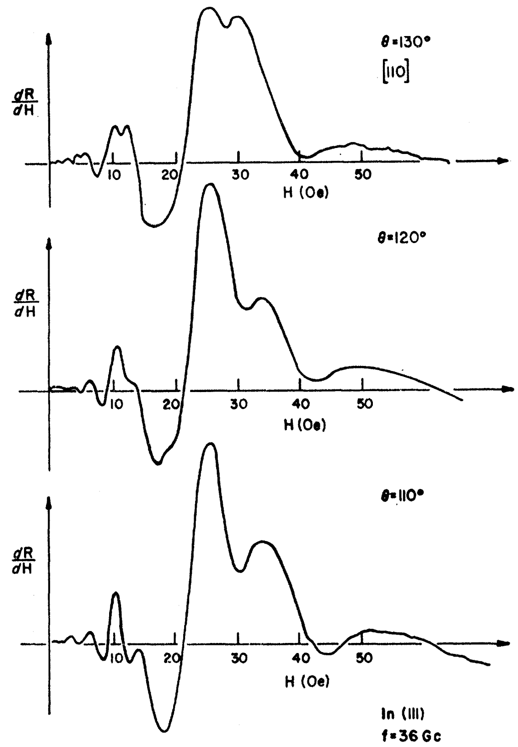

FIG. 3. Sequence of recorder tracings in the In(111) plane illustrating anisotropy of the oscillations with orientation of magnetic field. One series of peaks shifts to increasing magnetic fields, the other is approximately isotropic over this range of angles.

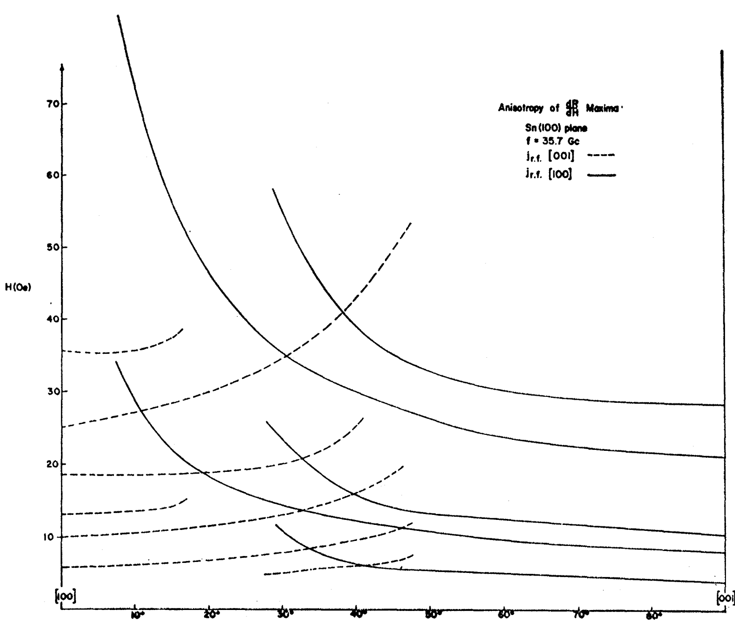

FIG. 4. Plot of the positions in magnetic field of the dR/dH maxima observed in the (100) plane of Sn as a function of orientation of the magnetic field. Only the well-resolved peaks are recorded. The magnetic fields are always increased as we rotate away from a symmetry axis. Peaks originating at the [001] are seen most strongly with the j_{rf}[100] current mode, those originating at the [100] with the j_{rf}[001] current mode.

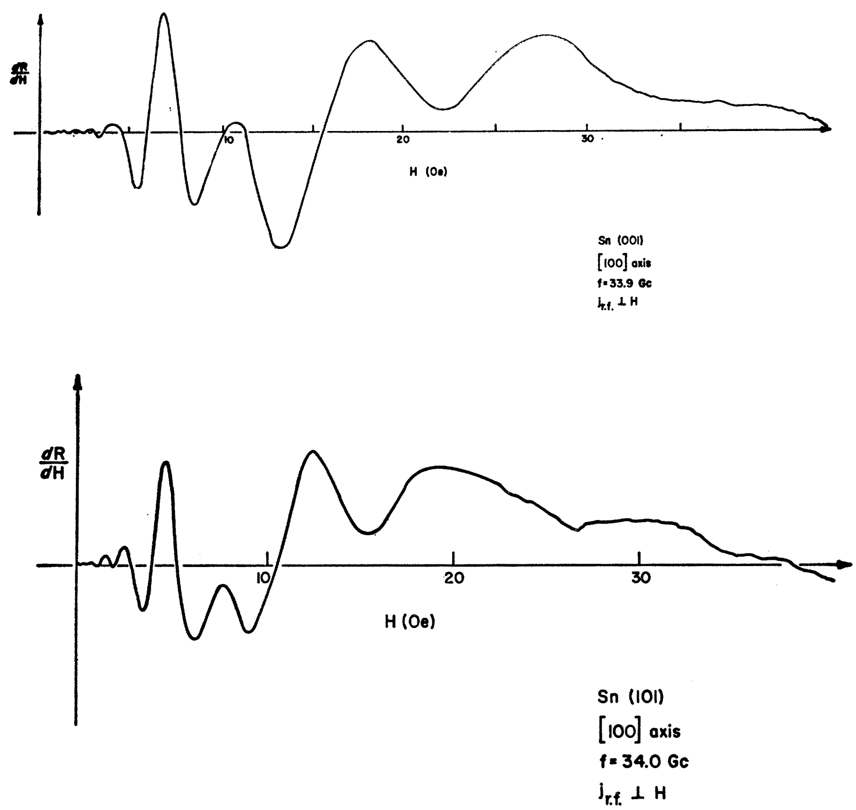

FIG. 5. Sn [100] axis data observed in the (001) and (101) sample planes. Compare also with Fig. 2. The low-field oscillations for a given orientation of magnetic field are found to depend on the sample plane in which this direction is observed.

of the anisotropy curves in Sn, as well as in Al and In, is the shift of the peaks to higher magnetic field with increasing angle away from the symmetry direction. Within the range of observation the field increases by a factor of 2 or 3 over that for the symmetry axis. Especially near the limit of observation, the peaks move up steeply in field and show a marked decrease in amplitude. We have observed signals in some cases that are nearly isotropic, but never any that move to lower fields.

It would be desirable to measure and plot the fields for the absorption maxima instead of the derivative maxima as we have done in the previous discussion. With the complicated spectra of several superposed series, this cannot be done accurately and consistently. Even though it is clear that qualitatively the angular variations of the two maxima would be nearly the same.

3. Anisotropy with Crystal Surface

Characteristic of the weak-field oscillations is the fact that, not only do the observed signals depend on the direction of the magnetic field in the surface, but they also depend on the crystallographic surface in which this direction is contained. As an illustration of this anisotropy, we consider the [100] axis of Sn. This axis lies in the (100), (101), and (001) crystallographic planes. Figure 2 shows data for the [100] axis in the (100) sample. Figure 5 shows data for this same axis but contained in the (001) and (101) symmetry planes.

Contrasting the spectra in these three planes, we find that although each of them shows the same two prominent series of peaks, the observed periodicities and corresponding positions in magnetic field of the dR/dH maxima are quite different. The (dR/dH) maxima of the fundamentals for the two series are seen to vary by more than a factor of 2 for the various surfaces studied. We note also that the ratio of the periods for the two series varies significantly for the different surfaces, so that we must conclude that each of them shows a different dependence on the crystal plane in which the series is observed. In the (100) plane peaks of the lower field series are nearly twice as strong as those of the high-field series. In the (101) and (001) planes these amplitudes are more nearly equal.

Each data point of this dependence on crystal plane requires preparation of a different sample disk and a more complete anisotropy study would prove exceedingly tedious. The data for the three symmetry planes in Sn does, however, suggest the kind of variation with surface that is to be expected. It also distinguishes the low-field oscillations from some of the other magnetic-field-dependent phenomena, notably cyclotron resonance or the de Haas–van Alphen oscillations in the surface impedance. Observed cyclotron masses and de Haas–van Alphen periods are not dependent on the plane in which the effect is observed.

4. Dependence on the Polarization of rf Currents

For a given direction of the magnetic field in the sample plane, we find that the periodicity of the observed oscillations and corresponding positions in field of the peaks, do not vary with the direction of rf currents. It is only the amplitude of the effect that is dependent on the rf polarization. In particular, for the magnetic field in a symmetry direction, the amplitude is always a maximum with the current perpendicular to this direction, and the signal vanishes entirely in the current parallel mode. For arbitrary directions of current relative to the symmetry axis, the signals can be resolved in both of the current modes. Except for amplitude, they are identical with those observed using the current perpendicular to H.

With the magnetic field in an arbitrary direction in the sample surface, the dependence on rf polarization is no longer so simple. Referring specifically to the Sn (100) plane, for which we have plotted the data in Fig. 4, we have found that the favorable polarization mode for those oscillations that originate at the [001] axis is the j rf [100] mode over the entire range of observation. Even though this is the favorable mode, some weak signals due to these oscillations are also seen in the other mode when we get more than about 40° away from the [001] direction. The data from oscillations that originate at the [100] axis are best observed with the current in the [001] direction. In the plot data taken with the two different modes appear as solid and dashed lines. We conclude that each of the sets of peaks have a characteristic favorable polarization which is a fixed direction in the sample plane. When data are taken in this plane with a cavity where the polarization effectively rotates with the magnetic field, and is either parallel or perpendicular to the field, the two sets of data are scrambled together.

In the case of the tipping experiments that we discuss below the polarization dependence is quite clear cut. As a rule, in both Al and Sn tipping data, optimum signals are seen in the current mode perpendicular to the tipping plane. No signals appear in the current mode parallel to the tipping plane.

5. Magnetic-Field Tipping Measurements

The low field effect can readily be observed with the magnetic field tilted out of the plane on the sample (see Fig. 5). Usually there is no significant decrease in the signal amplitude to about 80° of inclination of the field with respect to the surface. The signals disappear with the field normal to the surface.

We have made detailed tipping runs for the Sn (100) sample as well as the Al (110) crystals. In both of these crystals the symmetry is such that there is a plane normal to the sample surface containing a crystallographically equivalent range of angles. To allow meaningful comparison of data with tipped and parallel magnetic

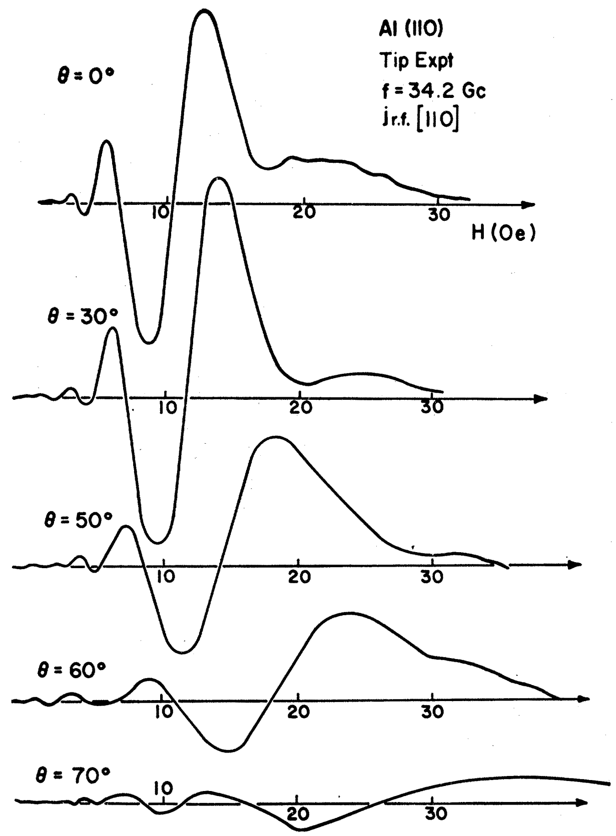

FIG. 6. Sequence of recorder tracings of tipping data in the (110) plane of Al. The angles marked represent the inclination of the magnetic field with respect to the sample surface in the (110) tipping plane. Whereas the in-surface data shows that the series of peaks is nearly isotropic, the tip data shows that the series of peaks shifts to increasing fields as $1/\cos\theta$.

fields, we take tipped field data only in this equivalent plane. For the tipping experiment the two rf current modes are fixed in the surface and are either perpendicular or parallel to the tipping plane.

In the case of the Sn (100) sample we have studied the low-field oscillations in the range of angles starting at the [001] axis in the surface to the [100] direction perpendicular to the surface. In this tipping plane we observe only the oscillations that appear as solid lines in Fig. 4, i.e., the oscillations that originate at the [001] axis. They can be observed with the rf current along the [100] in the surface. Peak positions and amplitudes are identical with the surface signals. The peaks indicated by the broken line are entirely absent in the tipping plane in both modes of polarization.

For the Al (110) crystal we have searched for signals in the range between the [100] axis in the surface and the [110] perpendicular to this surface. In this tipping plane we observe data as in Fig. 6. The series of peaks is found to shift to increasing field very nearly as $1/\cos\theta$, with angle of tipping. The in-surface data in this sample for the equivalent range of angles apparently shows several superimposed signals. The series of peaks seen along the [100] axis, as far as it can be identified in the surface data, seems to be approximately isotropic. At any rate, it does not show the $1/\cos\theta$ variation that is observed for the tipping geometry. In contrast with the results in Sn, the data on this series of peaks in Al appears tip-dependent. The polarization dependence however is exactly as for the Sn (100) sample. The optimum polarization is the mode where the rf current is perpendicular to the tipping plane.

This disagreement between results in the Sn and Al tip experiment still presents a puzzle at the moment. We have taken some preliminary data in a (110) crystal of Sn, where as in the case of the (100) plane those signals that can be seen in the tipping plane are identical with corresponding surface signals. The Al data seems to be the exception, but more extensive measurements in Al will be necessary to make a good comparison between data taken in the surface and in the tipping geometry.

6. Frequency Dependence

The period and corresponding position in magnetic field of the peaks depend on the microwave frequency. We have studied this dependence in the range of frequencies between 28 and 70 Gc/sec.

With increased frequency we find that the series of peaks shift to higher values of magnetic field. We have studied these shifts both for the (dR/dH) maxima and R maxima with identical results. A log-log plot of peak position versus frequency gives a straight line with slope 1.52±0.05, or equivalently $H\propto\omega_{\rm rf}^{1.52\pm0.05}$. Within limits of experimental accuracy, we conclude that the frequency dependence of the $R(H)$ and dR/dH maxima is $H\propto\omega_{\rm rf}^{3/2}$.

It was important to determine the frequency scaling accurately enough to distinguish between a possible $\omega_{\rm rf}^{5/3}$ scaling. To avoid errors due to possible misorientation of the sample for runs at different frequencies, we used cavities with several modes resonating at frequencies in a desired range. For frequencies between 28 and 40 Gc/sec, the sample was mounted in the spectrometer and not moved between successive frequency runs. For comparison with frequencies in the 70 Gc/sec range, it was remounted to another cavity and microwave spectrometer. By using the data from an axis of symmetry, the possible misorientation errors were minimized. With these precautions we could readily distinguish between the two frequency-scaling laws in question.

The dependence of the effect on frequency also provides a ready explanation for the much lower magnetic fields (0–4 Oe) at which Khaikin[1] sees the weak field effect in his work at 9.4 Gc/sec. The frequency scaling as above requires that the field be increased by a factor of 7.5 between 9.4 and 36 Gc/sec. Allowing for this increase, we find that we get exactly the range of fields in which we see the effect.

7. Temperature Dependence

The low field effect signals in Al are insensitive to temperature in the range 4.2 to 1.5°K. Signals in Sn and In seem to increase somewhat in amplitude as the temperature is decreased from 4.2°K to just above the superconducting transition temperatures. There is no change in the period or peak position of the oscillations. As the sample warms up from 4.2°K, the effect gradually disappears.

We have examined Sn and In samples below the superconducting transition temperature. In the superconducting state, the oscillations disappear. There are observed however some broad peaks in the surface resistance derivative that are anisotropic and frequency-dependent. At present it is not certain whether or not these are related to the normal-state signals.

IV. DISCUSSION OF THE MODEL AND COMPARISON WITH EXPERIMENTAL RESULTS

We consider a plane metal surface in the presence of a microwave-frequency electromagnetic field. A static magnetic field is applied parallel to the surface. As characteristic of our experiments we take the frequency $\omega_{\rm rf}\approx 10^{11}$ sec^{-1}. The range of penetration of the high-frequency field into the metal is $\delta\approx 10^{-5}$ cm. At low temperature and for the high-purity samples the electron mean free path λ is on the order of 10^{-1} cm. Since $\lambda\gg\delta$ we expect that the surface impedance is well in the anomalous skin effect regime. The Fermi velocity is $v_F\approx 10^8$ cm/sec. The cyclotron radius in a field of H Oe is $R_c=(hc/eH)\rho$, where ρ is the radius of curvature of the Fermi surface and is typically 10^8 cm^{-1}.

When the magnetic field H is equal to zero, electrons move in straight lines. Those responsible for the surface currents are moving nearly parallel to the surface. On the Fermi surface these are electron states contained in a narrow belt on the surface, i.e., the effective zone of the anomalous skin effect theory. In the presence of a magnetic field the electron trajectories are curved. If the field is weak electrons can complete only a small portion of a cyclotron orbit between successive scatterings. For magnetic fields considerably less than 1 Oe, the cyclotron radius is so large that the electron will not leave the skin layer in one mean free path. In this regime of very weak magnetic fields, Azbel and Kaner,[9] with the assumption of diffuse reflection of electrons from the surface as well as a single ellipsoidal energy surface, predict an initial quadratic variation of the impedance with magnetic field. The regime 1–100 Oe represents that region of field, where λ is considerably larger than the section of orbit that lies in the skin layer. In this intermediate or weak-field limit the electrons make a single traversal of the skin region without scattering, but still do not complete orbits. At fields

[9] M. Ya. Azbel' and E. A. Kaner, J. Phys. Chem. Solids 6, 113 (1958).

greater than about 100 Oe, electrons complete cyclotron orbits and make repeated traversals of the skin depth. In this high magnetic-field limit $\omega_c\tau > 1$, and cyclotron resonance can be observed. We note also that, whereas in the cyclotron resonance region the electron moves through the skin in a time $\Delta t < 1/\omega_{\text{rf}}$, in the weak field the orbit radius is so large that $\Delta t \geqslant 1/\omega_{\text{rf}}$ and the electron experiences an electric field varying in time.

Because the electron makes only a single transit of the skin layer between successive scatterings, we expect that the energy absorbed from the microwave field is that acquired by the interaction with the field during this transit. If we crudely approximate the electric field $E(z)$ in the metal as a step function, i.e., $E(z)=E(0)$ for $z<\delta$ and $E(z)=0$ for $z>\delta$, we can readily express the energy acquired in terms of the time spent in traveling over that portion of orbit that lies in the skin layer. A maximum of energy absorbed results when the time of exposure Δt is one-half of an rf period T_{rf}. With decreasing magnetic field the cyclotron radius increases and consequently the electron spends a longer time in the skin layer. A second maximum in the microwave absorption arises, when $\Delta t = \frac{3}{2}T_{\text{rf}}$, and successive resistance maxima are expected when $\Delta t = \frac{5}{2}, \frac{7}{2}$, etc., T_{rf}. For the orbit that just touches the surface at its highest point,

$$\Delta t = 2(2R\delta)^{1/2}/v. \tag{1}$$

The values of magnetic field where we expect the resistance maxima to occur are

$$H_n = \frac{1}{(2n+1)^2}\left(\frac{8hc}{\pi^2 e}\right)\omega^2\delta\left(\frac{\rho}{v^2}\right). \tag{2}$$

Both ρ and v are to be taken in the plane perpendicular to the magnetic field and at the point on the Fermi surface where the electrons traverse the skin layer. For values of ω, δ, ρ, and v applicable in these experiments we have for the field of the first of the series of peaks, $H_0 \approx 50$ Oe. Successive peaks would be at 1/9, 1/25, etc., of this value. We seen that even though we can well account for the magnitude of field where we observe the oscillation, we do not get the odd submultiple periodicity found in the experiments. Experimentally observed peaks occur at $\frac{1}{3}$, $\frac{1}{5}$, etc., of the field of the fundamental.

This rather violent disagreement is not too surprising in view of the oversimplification that we introduce in the derivation. In particular, it is assumed above that the electric field is a step function of distance into the metal and the only electron orbits used are those that come in tangent to the surface. More realistically, the interaction of the electron with the microwave fields should be evaluated as an integral of the electric field experienced by the electron over the entire path using an expression for electric field $E(z,t)$ appropriate for the A.S.E. region. It is also necessary to sum over contributions from orbits with different orbit centers, as well as to average over the phases of the electric field encountered by the electron in the skin region.

We are tempted to suggest a somewhat different way to look at this problem. The theory of the anomalous skin effect gives the result that electrons responsible for the surface resistance are those that travel into the metal with the phase velocity of the electric field, or equivalently those making an angle $\theta \approx \omega\delta/V_F$ with respect to the surface. This defines the center of the effective zone. The width of the zone is also on the order of θ. In the presence of the weak magnetic field the electron near the top of its orbit is continually changing its direction with respect to the surface. It does this in a time comparable to or greater than T_{rf}. If we take Δt as the time to traverse the range of angle θ we have

$$\Delta t = R\omega\delta/v^2. \tag{3}$$

Values of the field at which resistance maxima are to occur

$$H_n = \frac{1}{(2n+1)}\left(\frac{hc}{\pi e}\right)\omega^2\delta\left(\frac{\rho}{v^2}\right). \tag{4}$$

This expression gives precisely the same dependence on the frequency and skin depth, as well as on the Fermi surface parameters ρ and v. As in the case of Eq. (2) the predicted field for the fundamental maximum is on the order of 50 Oe. Successive maxima would occur at $\frac{1}{3}$, $\frac{1}{5}$, etc., of this value, in agreement with the experimentally observed periodicity. Even though the results appear attractive, in this way of looking at the problem the relationship between Δt and the resistance maxima is not at all obvious. We feel that a detailed calculation of surface impedance is needed to resolve the problem and that the proposed model can at best only make the appearance of oscillations in the weak field regime be plausible.

We can however explore some of the consequences of the model and compare with the experimental results. The magnetic field values at which a resistance maximum occurs depends on the Fermi surface parameters ρ and v. Both ρ and v are values for the point on the Fermi surface where the electron travels through the skin region. For an arbitrary Fermi surface, we expect to see an oscillation whenever ρ/v^2 has a stationary value. Only then will there be a significant number of electrons with the same value of ρ/v^2. An exceptionally strong signal would be expected for a cylindrical surface with constant Fermi velocity. Of course, since we are looking at only that small portion of orbit lying in the skin region we need not necessarily have a cylinder, nor even closed orbits to observe the low-field oscillations. The cylinder with an elliptical cross section is however a reasonably general model to consider and serves well to illustrate some of the expected features of the effect.

The variation of the fields of the resistance maxima, with the direction of the magnetic field in a given plane is expected to be as $1/\cos\theta$, where θ is the angle measured from the axis of the cylinder. As in Fig. 4, this is not a bad approximation to oscillations observed in the (100) plane of Sn. The fact that each of these has its own characteristic dependence on angle, and none exactly as $1/\cos\theta$, probably reflects the fact that we are not dealing with perfect cylinders or that the Fermi velocity is not strictly constant.

If we change the surface in which a given axis is contained, we effectively change the region of the cylinder that is exposed to the rf fields. For a cylinder with noncircular cross section, we then have a different value of the radius of curvature ρ. The dependence of the signals on the sample plane as observed in the experiments is wholly expected.

The maximum amplitude is to be expected when the direction of the surface current j_{rf} is the same as the velocity of the electrons at the region on the Fermi surface that gives rise to the oscillation. If, in fact, we are dealing with a cylindrical surface the expected optimum polarization will be at right angles to the axis of the cylinder. In Fig. 4 we show the (100) plane data in Sn taken with current j_{rf} along either [100] or [001] axes. We note that the peaks originating at the [001] axis in this plane are seen most strongly with the j_{rf} [100] current mode over the entire angular range where they can be observed. Since the minimum value of H for these peaks does occur at the [001] axis, we can be sure that this is also the axis of the cylinder. The polarization dependence in this instance confirms the assumption of cylindrical Fermi surface regions in the (100) plane of Sn.

We can also understand the magnetic-field tipping results in Sn in terms of the cylindrical Fermi surface section. We found that tip data in the (100) plane perpendicular to the sample surface is identical with the surface data, except that oscillations centered at the [100] in the surface were completely absent from the tip data. For a cylinder parallel to the [001] axis, the orbit in real space will always be in the plane perpendicular to the [001], regardless of the orientation of magnetic field relative to the [001] axis. With this in mind, it is seen that H tipped at angle θ in the (100) plane normal to the sample surface, is entirely equivalent to an orientation of θ degrees away from the [001] in the surface. The real space orbit through the skin layer is exactly the same in the two cases and the resulting oscillations are expected to be identical for in and out of surface data. On the other hand, we do not expect to see data from cylinders arranged along the [100] axis in the tip geometry. This is because in the tipping plane H is always perpendicular to the [100] axis in the surface and consequently there are no electron orbits on a cylinder parallel to this direction. Orbits due to the [100] axis that is normal to the surface likewise should give no signals, because electrons then are moving parallel to the surface never leaving the surface region.

As we have seen, the anisotropy at the weak-field oscillation, as well as the polarization and tip dependence are a function of the Fermi surface geometry. The observations on the (100) plane of Sn very strongly suggest that we are dealing with cylindrical sections. It is interesting to compare these results with what one would expect for the case of a spherical Fermi surface section with constant velocity. The expected signals would be due to the central cross section of the sphere, for which ρ/v^2 is a minimum. The resulting oscillations would be completely isotropic with rotation of the magnetic field in the sample surface. The optimum polarization would not be along some fixed direction as is the case for the cylinder, but instead would be always perpendicular to the applied magnetic field. As the field is tipped out of the sample plane, the orbit is now likewise tipped. The effective skin depth becomes $\delta/\cos\theta$, where θ is the angle of H with respect to the surface, and there should result a shift of the peak position to increased field as $1/\cos\theta$. It is quite likely that this is the explanation for the Al (110) tipping results, suggesting that the oscillation observed in Al is essentially due to a spherical section.

We can readily predict the dependence of the magnetic field for a given resistance maximum on the experimental frequency ω_{rf}. From Eq. (2) as well as Eq. (4) we have $H_n \propto \omega^2\delta$. The theory of A.S.E. in the absence of magnetic fields has $\delta \propto \omega_{rf}^{-1/3}$. The expected scaling becomes $H_n \propto \omega_{rf}^{5/3}$. The experiments on frequency scaling definitely showed $H_n \propto \omega_{rf}^{3/2}$, in disagreement with this prediction. It is tempting, although probably incorrect, to take $\delta \propto \omega_{rf}^{-1/2}$ as is the case for the normal skin effect. This would then lead to agreement with the experimentally determined frequency scaling. The frequency dependence of the weak-field oscillation is very different from that of many other phenomena associated with electron dynamics in a magnetic field. The cyclotron resonance effect for example has $H \propto \omega_{rf}$, whereas quantum oscillations in the surface impedance are expected to be independent of frequency.

V. CONCLUSIONS

We expect that the study of oscillations in a weak magnetic field can in principle yield some very detailed information on the Fermi surface and related parameters. In contrast to many other Fermi surface experiments, where the measured quantity is an orbit or possibly a cross-sectional area, the low-field effect is essentially a point property of the Fermi surface.

At the same time, we feel that an adequate understanding of the effect will require a detailed calculation of surface impedance for this regime of magnetic fields. We hope that the experiments serve to outline the important features of the low-field oscillations as well as to stimulate some interest in developing a theory to explain the effect. We want to emphasize again that the outstanding difficulties in terms of our explanation for the oscillations, are the questions of periodicity and frequency scaling. On the other hand we are quite successful in accounting for observations related to Fermi-surface geometry, such as the anisotropy of the effect with rotation of H as well as its dependence on the sample plane, and the polarization and tipping observations. We are not exactly intoxicated with the success of our explanations, but feel that they do define the problem and serve as a reasonable working model.

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Combined Effects of Sound and Electric Fields upon the Conduction Electrons in Solids

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The Boltzmann equation for the electronic distribution function in the presence of ultrasonic sound and a dc electric field is solved to produce a unified picture of the different effects resulting from the combined action of these two external disturbances. The response of the electronic system is described by the sum of various dc and ac current densities. Specifically it is shown which parts of the distribution function give rise to ultrasonic attenuation and to the acoustoelectric effect for both zero and nonzero external dc field. The Weinreich relation is found to be valid except for small correction terms due to collision drag. The orders of magnitude of the acoustoelectric field and the acoustoresistive effect are discussed for metals and semiconductors.

I. INTRODUCTION

FROM measurements of the ultrasonic attenuation due to conduction electrons much valuable information concerning the electron-phonon interaction can be obtained. In semiconductors, however, this effect is usually only a small contribution to the total attenuation,[1] the main part of which arises from the interaction of sound with thermal phonons and defects. The electronic part can only be determined separately when the amount of doping is changed. Information can also be obtained from the evaluation of experimental results of the mobility of electrons scattered by phonons of the acoustic branches. This, however, can give only average values over the whole phonon spectrum and over-all directions of propagation and polarization.[2]

To obtain a more detailed picture of the electron-phonon interaction, one should therefore look for cross effects which are caused by the combined action of sound and a dc electric field upon the system of conduction electrons in metals or semiconductors. Such effects are:

(1) the effect of sound upon an electric current under the action of an applied dc electric field,

(2) the effect of an applied dc electric field upon the propagation of sound,

and as a special case of (1):

(3) the generation of a dc electric current or field merely by the action of sound.

Case (1) is an acoustoresistive effect; practically nothing is known about it. Case (2) has been studied extensively in piezoelectric semiconductors in connection with the amplification of sound by a strong electric field,[3] and with the saturation of current.[4] Less work has been done in connection with sound amplification in nonpiezoelectric semiconductors, such as germanium and silicon.[5,6] Case (3) is known as the acoustoelectric effect and has been treated by many authors.[7–13]

The acoustoelectric effect and the electronic part of the ultrasonic attenuation are connected by the so-called Weinreich relation which was primarily derived from basic considerations concerning the transfer of momentum.[14] There have been many discussions both of the exactness and of the general validity of this relation.[11,13,15,16]

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