Diffusion Constant in the Effective Bloch Equation for Ferromagnetic Resonance in Metals*

JEROME I. KAPLAN
Brown University, Providence, Rhode Island

(Received 14 October 1965)

A method is given for calculating the spin-diffusion constant in a ferromagnetic metal using a spin-transport equation. The result is that the diffusion constant in a ferromagnetic metal is reduced below that for a normal metal by the factor $(1+\omega_{ex}^2T_p^2)^{-1}$, where ω_{ex} is the effective exchange frequency and T_p is the momentum relaxation time. The physical idea for the calculation was first thought of by L. L. Hirst, who has obtained the same results using an alternative method of calculation.

IT has been previously pointed out by the author[1] that the effective spin diffusion constant in a ferromagnetic metal must be down by a factor 10^{-5} from that in a nonferromagnetic metal. The reason is that the inclusion of the diffusion term is found to be unnecessary in order to make a fit to the ferromagnetic line shape, and its effect would be in evidence unless reduced by the vector 10^{-5}. Recently this point was again noticed by Hirst,[2] who developed an explanation of the effect in an unpublished report which he kindly sent to me. The basic motivation of the calculation, i.e., that the effective molecular field reduces the diffusion constant by forcing the magnetic moment of a particular electron to follow the local average magnetization, I believe is correct, but the calculation itself I felt can be put on a more rigorous and less intuitive basis. It turns out that both methods obtain the same results.

It is assumed that the effective magnetic-moment-bearing electrons obey the kinetic equation for the distribution function $\hat{f}$:

$$\frac{\partial\hat{f}}{\partial t}+\mathbf{v}\cdot\nabla_r\hat{f}+\nabla_p\hat{f}\cdot\left\{e\mathbf{E}+\frac{e}{c}(\mathbf{v}\times\mathbf{H})\right\}+\frac{i}{\hbar}[H_s,\hat{f}]+\left(\frac{\partial\hat{f}}{\partial t}\right)_{\text{coll}}=0, \quad (1)$$

which has been used in a number of previous calculations.[3] The Hamiltonian for the spin is given as $H_s=\mu\boldsymbol{\sigma}\cdot\mathbf{H}_{\text{eff}}$ where

$$\mathbf{H}_{\text{eff}}=\mathbf{H}_0+\mathbf{H}_{\text{rf}}+a\mathbf{M}+b\nabla^2\mathbf{M} \quad (2)$$

and

$$\mathbf{H}=\mathbf{H}_0+\mathbf{H}_{\text{rf}}. \quad (3)$$

The a and b in Eq. (2) are constants which arise from expanding the local magnetization about the position of the electron of interest.[4] The collision term has two parts—the momentum relaxation and the spin relaxation. Thus one has

$$\left(\frac{\partial\hat{f}}{\partial t}\right)_{\text{coll}}=\frac{\hat{f}-\langle\hat{f}\rangle}{T_p}+\frac{\hat{f}-\hat{f}^0}{T_s}, \quad (4)$$

where

$$\langle\hat{f}\rangle=\frac{\int v^{-1}\hat{f}ds}{\int v^{-1}ds} \quad (5)$$

and

$$\hat{f}^0=\begin{vmatrix} f_0(\epsilon_0+aM_z) & 0 \\ 0 & f_0(\epsilon_0-aM_z)\end{vmatrix}, \quad (6)$$

where f_0 is the equilibrium Fermi distribution. The form for the spin relaxation is not critical as our results will be essentially independent of T_s.

Equation (1) and the expression for the current and magnetic moment

$$\mathbf{j}=\frac{e}{h^3}\int\mathbf{v}\,\mathrm{Tr}\hat{f}d^3p,$$
$$\mathbf{M}=\frac{\mu}{h^3}\int\mathrm{Tr}\boldsymbol{\sigma}\hat{f}d^3p, \quad (7)$$

are simplified by writing

$$\hat{f}=f_I+\mathbf{f}\cdot\boldsymbol{\sigma}. \quad (8)$$

The spin-dependent part of Eqs. (7) and (1) can then be written as

$$\mathbf{M}=\frac{2\mu}{h^3}\int\mathbf{f}d^3p, \quad (9)$$

where for no saturation

$$M_z=\frac{\mu}{h^3}\int\chi_0d^3p \quad (10)$$

and

$$\frac{\partial\mathbf{f}}{\partial t}+[\mathbf{v}\cdot\nabla_r]\mathbf{f}+\frac{e}{c}[(\mathbf{v}\times\mathbf{H})\cdot\nabla_p]\mathbf{f}+\frac{\mathbf{f}-a\chi_0}{T_s}+\frac{\mathbf{f}-\langle\mathbf{f}\rangle}{T_p}+\mathbf{f}\times\frac{2\mu}{h}\mathbf{H}_{\text{eff}}=0, \quad (11)$$

* This work was supported by Advanced Research Project Agency.

[1] J. I. Kaplan, Phys. Rev. 115, 575 (1959).

[2] L. L. Hirst, Technical Report No. 471, University of Maryland (unpublished).

[3] M. Ya. Azbel, Y. I. Gerasimenko, and I. M. Lifshitz, Zh. Eksperim. i Teor. Fiz. 32, 1212 (1957) [English transl.: Soviet Phys.—JETP 5, 986 (1957)]; M. Ya. Azbel and I. M. Lifshitz, Progr. Low Temp. Phys. 3, 288 (1961).

[4] C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, Inc., New York, 1957), 2nd ed., Appendix O.

where **a** is a unit vector in the H_0 direction,

$$\chi_0=\tfrac{1}{2}[f_0(\epsilon_0+aM_z)-f_0(\epsilon_0-aM_z)].$$

To simplify the problem, we will take $H_0=H_z$ to be perpendicular to the surface of the ferromagnet. Then assuming that the rf radiation is circularly polarized, of a single frequency ω, and that one can ignore saturation effects Eq. (11) can be written as

$$\left(1/t-i\frac{2\mu}{h}aM_z+v_z\frac{\partial}{\partial z}\right)\mathfrak{f}-\frac{\langle\mathfrak{f}\rangle}{T_p}+i\frac{2\mu}{h}\mathfrak{IC}_{\text{eff}}=0, \quad (12)$$

where

$$\begin{aligned}1/t&=1/T_p+1/T_s+i(\omega-\omega_c),\\ \mathfrak{f}&=(f_x+if_y)/\chi_0,\\ \mathfrak{IC}_{\text{eff}}&=H_{x\text{ eff}}+iH_{y\text{ eff}},\\ &=\mathfrak{IC}_{\text{rf}}+a\mathfrak{M}+b\frac{\partial^2}{\partial z^2}\mathfrak{M}. \end{aligned} \quad (13)$$

Considering only the Fourier component of $\mathfrak{M}$ and $\mathfrak{f}$ going as e^{iqz} one has from Eqs. (12) and (13)

$$\mathfrak{f}=\frac{\langle\mathfrak{f}\rangle/T_p-i(2\mu/h)(\mathfrak{IC}_{\text{rf}}+a\mathfrak{M}-bq^2\mathfrak{M})}{1/t+iqv\cos\theta-i(2\mu/h)aM_z}. \quad (14)$$

Integrating over energy one has

$$\mathfrak{M}(\theta)=\frac{\mathfrak{M}/T_p-i(2\mu/h)(\mathfrak{IC}_{\text{rf}}+a\mathfrak{M}-bq^2\mathfrak{M})}{1/t+iqv_0\cos\theta-i(2\mu/h)aM_z}, \quad (15)$$

where $\mathfrak{M}$ is the angular average of $\mathfrak{M}(\theta)$ and v_0 is an effective average velocity in the region about the Fermi surface. After averaging Eq. (15) over θ one finds that

$$\mathfrak{M}=\frac{\mathfrak{M}/T_p-i(2\mu/h)M_z(\mathfrak{IC}_{\text{rf}}+a\mathfrak{M}-bq^2\mathfrak{M})}{2iv_0q}\times\ln\left[\frac{1/t-iM_z(a2\mu/h)+iqv_0}{1/t-i(M_za2\mu/h)-iqv_0}\right], \quad (16)$$

or solving for $\mathfrak{M}$ one has

$$\mathfrak{M}=\frac{[-i(2\mu/h)M_z\mathfrak{IC}_{\text{rf}}/2iqv_0]\ln\{[1/t-iM_za(2\mu/h)+iqv_0]/[1/t-iM_za(2\mu/h)-iqv_0]\}}{1-[1/T_p-i(2\mu/h)M_z(a-bq^2)]/(2iqv_0)\ln\{[1/t-i(M_za2\mu/h)+iqv_0]/[1/t-i(M_za2\mu/h)-iqv_0]\}}. \quad (17)$$

To find the resonance character of $\mathfrak{M}$ one expands the denominator in powers of q up to q^2 and obtains

$$\left\{1/t-1/T_p-i\frac{2\mu}{h}M_zbq^2-(iqv_0)^2\times\left[1/T_p-i\frac{2\mu}{h}M_za\right]\Big/3\left(1/t-iM_za\frac{2\mu}{h}\right)^2\right\}, \quad (18)$$

which near resonance becomes approximately

$$1/T_s+i(\omega-\omega_0)-i\frac{2\mu M_zbq^2}{h}+\frac{q^2v_0^2T_p}{3}\frac{1}{(1-i\omega_{ex}T_p)}, \quad (19)$$

where $\omega_{ex}=M_za(2\mu/h)$ is the effective molecular field. If one takes the effective Bloch equation[5] to be

$$dM/dt=\gamma(\mathbf{M}\times\mathbf{H}_{\text{eff}})+D\nabla^2\mathbf{M}-\mathbf{M}/T_s, \quad (20)$$

where

$$\mathbf{H}_{\text{eff}}=\mathbf{H}_{\text{rf}}+\mathbf{H}_0+B\nabla^2\mathbf{M}, \quad (21)$$

then corresponding to Eq. (19) one would obtain

$$1/T_s+i(\omega-\omega_0)-\frac{i2\mu M_zBq^2}{h}-q^2D. \quad (22)$$

Thus by comparing Eqs. (19) and (22) one sees that

$$D=\frac{D_0}{1+\omega_{ex}^2T_p^2},$$

where $D_0=v_0^2T_p/3$ is the spin diffusion constant in a nonferromagnetic metal. This is the same result as obtained by Hirst.[2] For a Curie point of the order of 500°, one has $\omega_{ex}\simeq5\times10^{13}$ and thus with $T_p\sim10^{-11}$

$$D=D_0\times4\times10^{-6}.$$

The addition of the diffusion term as given in Eq. (20) maybe needed to make a theoretical resonance fit for ferromagnetic metals with a very low Curie point.

[5] C. Herring and C. Kittel, Phys. Rev. **81**, 869 (1951).