Effect of Electron-Damped Dislocations on the Determination of the Superconducting Energy Gaps of Metals

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When account is taken of the damping of dislocations by electrons—i.e., by electron viscosity—it is shown that the energy losses due to this source are appreciable and are different in the normal and in the superconducting states. By subtracting off these two losses for the acoustic attenuation measured for these two states, and using the relation between the residual attenuations for the two states determined from the BCS theory, a much better determination of the energy gaps for superconductors is obtained. For lead the value is $2\epsilon_0=(4.1\pm0.1)kT_c$, in good agreement with other methods for measuring the gap. The electron damping of dislocations accounts also for the amplitude effect in the superconducting range first observed by Love and Shaw. Theoretical calculations by Tsuneto have indicated that the ratio of the attenuation in the superconducting to that in the normal state for longitudinal waves should be the same for the region $ql\ll1$ as that which has been found by BCS in the region $ql\gg1$. When account is taken of the dislocation damping effect, it is shown that existing data confirm this calculation.

I. INTRODUCTION: EXPERIMENTAL DATA INDICATING ELECTRON DAMPING OF DISLOCATIONS

ENERGY-gap determinations for superconducting metals, made by the use of acoustic attenuation measurements, use the equation derived for longitudinal waves by Bardeen, Cooper, and Schrieffer,[1] which has the form

$$\alpha_S/\alpha_N=2/(1+e^{\epsilon/kT}), \tag{1}$$

where α_S and α_N are the attenuations in the superconducting and normal states due to electronic damping, and ϵ is half the energy gap between the two states at the temperature T. In obtaining this ratio it has been usual to subtract off a constant attenuation from both normal and superconducting states equal to the attenuation in the superconducting state measured at the lowest temperature. This procedure neglects the fact that there is another source of attenuation which also changes between the normal and the superconducting state. Hence, to obtain a good measurement, this source—i.e., attenuation caused by dislocations damped by electrons—has to be evaluated.

This effect was probably first indicated by the measurements of Landauer[2] and of Welber and Quimby[3] but was not recognized as a dislocation damping effect. These measurements showed a decrease in Young's modulus of about 30 parts in 10^6 at a measuring frequency of 50 000 cps when a lead sample was taken from the normal state to the superconducting state. A small decrease in the internal-friction parameter Q^{-1} also occurred. This change in modulus is much larger than the expected thermodynamic change (Mason[4]), but is consistent with the immobilization of dislocation loop lengths longer than about 10^{-4} cm by a large electron damping of the dislocations in the normal state. Very pure crystals are expected to have dislocation loops this long as part of the tail of a distribution peaked at around 0.2 to 0.3×10^{-4} cm.

A more definite indication of the presence of electron damping of dislocations was provided by the work of Love and Shaw.[5] As shown by Fig. 1, the attenuation in the superconducting region of lead shows an amplitude-dependent attenuation while that in the normal region—maintained by a magnetic field greater than 800 G—does not show an amplitude effect over a voltage range of 30 to 1. The attenuation loss at the highest stress level is larger than that in the normal state. It was first pointed out by Tittmann and Bömmel[6] that this effect could be explained by a higher electron damping in the normal region than in the superconducting region, but no quantitative results were given. The writer (Mason[7]) first calculated the drag coefficient

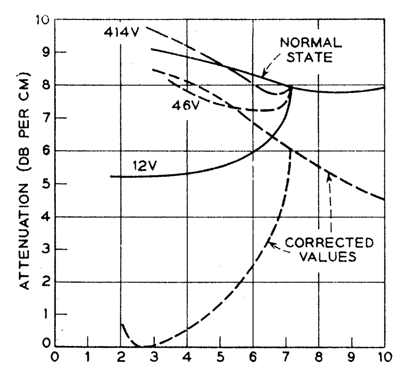

FIG. 1. Attenuation in lead single crystal for longitudinal waves at 50 Mc/sec as a function of the temperature and voltage applied to the transducer (after Love, Shaw, and Fate).

[1] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957); **108**, 1175 (1957).
[2] J. K. Landauer, Phys. Rev. **96**, 296 (1954).
[3] B. Welber and S. Quimby, Acta Met. **6**, 35 (1958).
[4] See W. P. Mason, *Physical Acoustics and the Properties of Solids* (D. Van Nostrand, Inc., Princeton, New Jersey, 1958), Chap. XI.
[5] R. E. Love and R. W. Shaw, Rev. Mod. Phys. **34**, 260 (1964); R. E. Love, R. W. Shaw, and W. A. Fate, Phys. Rev. **138**, A1453 (1965).
[6] B. R. Tittmann and H. E. Bömmel, Phys. Rev. Letters, **14**, 296 (1965).
[7] W. P. Mason, Appl. Phys. Letters **6**, 111 (1965).

from electron-viscosity theory and showed that it accounted quantitatively for the difference in the two regions. The present paper extends these calculations into the low-amplitude region and shows that better energy-gap determinations can be obtained.

II. CALCULATION OF ELECTRON DRAG

According to the Granato-Lücke theory of dislocation damping, there is a region—the linear, low-amplitude region— for which the average displacement is proportional to the applied stress. It is generally believed that this motion is the result of the displacement of segments of dislocations, called kinks, which connect straight-line portions of the dislocations which lie in minimum energy positions. These kinks lie across Peierl's energy barriers but they do not require thermal activation to move since the periodic character of the Peierls' energy barrier is smeared out for a dislocation kink of appreciable width. Hence, the total displacement is very similar to that calculated by considering a dislocation as a straight line between pinning points with a line tension which is usually taken to be $\frac{1}{2}\mu b^2$ for isotropic material, where μ is the shearing modulus in the glide plane and b the Burger's vector.

These dislocations have a mass per unit length which is usually taken as ρb^2, where ρ is the density of the material. These dislocations are damped by an effective drag constant B which produces a dragging force proportional to the velocity with which the dislocation is dragged through the material. Since the dislocation is assumed to be traversing perfect material, the interaction has to be with the waves present in the crystal, i.e., the phonons and electrons. Phonon damping has been discussed in several papers and the principal mechanisms suggested for damping are thermoelastic effects (Eshelby[8]), radiation pressure (Leibfried[9]) and phonon viscosity (Mason[10]).

Electrons are scattered by dislocations and the scattering contributes to thermal resistance. However, since electron velocities are about 300 times larger than sound velocities, it does not appear that there should be any appreciable difference in the scattering caused by the motion of the dislocation. If such an effect existed it should be independent of the temperature since the electron numbers and velocities are temperature-independent in a metal. This is not in agreement with the measured drag coefficients of electron damped dislocations. Hence, the only mechanism which appears to account for electron damping of dislocations is one connected with electron viscosity.

As discussed by Pippard[11] the electric and magnetic fields produced by a strain wave in a metal cause the electrons to move in such a way as to establish current neutrality. The result is that the electrons are given a velocity equal to the particle velocity of the acoustic wave. The momentum imparted to the electrons can be exchanged between surfaces moving with slightly different velocities and produces an electron viscosity which can attenuate a longitudinal or shear acoustic wave propagated through the metal. The value of the viscosity has been discussed for a spherical Fermi surface and has been shown by Mason[12] Morse[13] and Pippard[11] to be given by the equation

$$\eta = 9\times 10^{11}\hbar^2(3\pi^2 N)^{2/3}/5e^2\rho, \tag{2}$$

where $\hbar$ is Planck's constant h divided by 2π, N is the number of electrons per cc, e is the electronic charge in cgs units and ρ is the resistivity in ohm cm. As summarized by Mason[4] Morse,[14] and Pippard,[11] this equation is in fair agreement with experiment for a number of metals whose Fermi surfaces can be approximated by the free-electron spherical Fermi surface. These metals include copper, lead, tin, indium, and sodium. On the other hand, it is found that this formula underestimates the attenuation of tungsten and molybdenum for which the Fermi surface departs markedly from the spherical surface (Jones and Rayne[15]).

A dislocation is surrounded by a strain field and as it moves through the crystal the strain at any point changes as a function of the time. By virtue of the viscosity of the electrons this rate of change of strain produces a thermal loss which, when integrated over the whole space surrounding the dislocation, results in a dragging force which is proportional to the velocity of the dislocation. As with the phonon drag coefficient due to phonon viscosity[10] the easiest case to consider is the screw dislocation. This dislocation is surrounded by a shearing strain field of the form

$$S_{\theta z} = b/2\pi r, \tag{3}$$

where b is the Burger's vector and r the distance from the center of the dislocation. If we consider an element of volume at a distance r_1 from the center of the dislocation and at an angle θ with the direction of motion of the dislocation, the rate of change of the shearing strain is given by the equation

$$\frac{S_{\theta z_2} - S_{\theta z_1}}{dt} = \frac{(b/2\pi)(1/r_2 - 1/r_1)}{dt}, \tag{4}$$

where

$$r_2 = r_1[1 - \cos\theta u dt/r_1],$$

u being the velocity of motion of the dislocation and dt is the time between the measurements of r_2 and r_1. Inserting the value of r_2 in (4), the rate of change of the

[8] J. D. Eshelby, Proc. Roy. Soc. (London) **A197**, 396 (1949).
[9] G. Leibfried, Z. Physik **127**, 344 (1950).
[10] W. P. Mason, J. Appl. Phys. **35**, 2779 (1964).
[11] A. B. Pippard, Phil. Mag. **46**, 1104 (1955); Advan. Phys. **9**, 176 (1960).
[12] W. P. Mason, Phys. Rev. **97**, 557 (1955).
[13] R. W. Morse, Phys. Rev. **97**, 1716 (1955).
[14] R. W. Morse, *Progress in Cryogenics* (Academic Press Inc., New York, 1959) pp. 221–259.
[15] C. K. Jones and J. Rayne, Phys. Letters **14**, 282 (1964).

shearing strain is

$$\dot{S}_{\theta z}=bu\cos\theta/2\pi r^2. \quad (5)$$

The energy dissipated is the stress $T_{\theta z}$ times the strain rate or

$$T_{\theta z}\dot{S}_{\theta z}=\eta(\dot{S}_{\theta z})^2. \quad (6)$$

Performing the integration over a cylinder of unit length surrounding the dislocation we find

$$\dot{W}=\frac{b^2u^2\eta}{4\pi^2}\int_{a_0}^{\infty}rdr\int_0^{2\pi}\frac{\cos^2\theta}{r^4}d\theta=\frac{b^2u^2\eta}{8\pi a_0^2}. \quad (7)$$

To determine the drag coefficient, denoted by the letter B, we note that the velocity attained by the dislocation is determined by the equation

$$F=T_{13}b=uB, \quad (8)$$

where F is the force per unit length on the dislocation determined by the product of the shearing stress T_{13} times the Burger's vector b. The energy dissipated is equal to the force times the velocity u and hence we have

$$\dot{W}=u^2B=b^2u^2\eta/8\pi a_0^2; \quad (9a)$$

hence

$$B=b^2\eta/8\pi a_0^2. \quad (9b)$$

A very similar expression is obtained for edge dislocations, namely,

$$B=\tfrac{3}{4}(b^2\eta/8\pi(1-\sigma)^2a_0^2), \quad (10)$$

where σ is Poisson's ratio.

The value of B depends on the cutoff radius a_0. Since the strain near the dislocation becomes very large, we can expect nonlinearities in the deformation potential which cause a saturation of the drag effect (Blount[16]). To calculate the radius exactly requires a knowledge of the Fermi surface and the changes in the surface with respect to the Brillouin zone due to the applied strain. These quantities are not known for lead. In the case of copper, for which the Fermi surface is known,[11] it is evident that strains as high as 5×10^{-2}—corresponding to radii $r_0=10^{-7}$ cm—and deformation potentials in the order of 4.5 eV (those usually associated with a spherical surface) will cause a considerable part of the Fermi surface to be pushed through the Brillouin zone. Since the effect of this is equivalent to bringing the displaced part inside the first zone, it can be seen that the effect saturates. The same conclusions are obtained from the calculations of Blount,[16] which indicates that saturation occurs for the condition $ql_eSE/mv^2\approx1$. For the conditions holding for lead, this results in a strain S of about 5×10^{-2} equivalent to $r_0=10^{-7}$ cm.

Hence it appears that 10^{-7} cm is a reasonable cutoff radius. With this value the drag coefficient becomes

$$B=4.9\times10^{-3}\eta=4.2\times10^{-11}/\rho \text{ for lead}, \quad (11)$$

[16] E. I. Blount, Phys. Rev. **114**, 418 (1959).

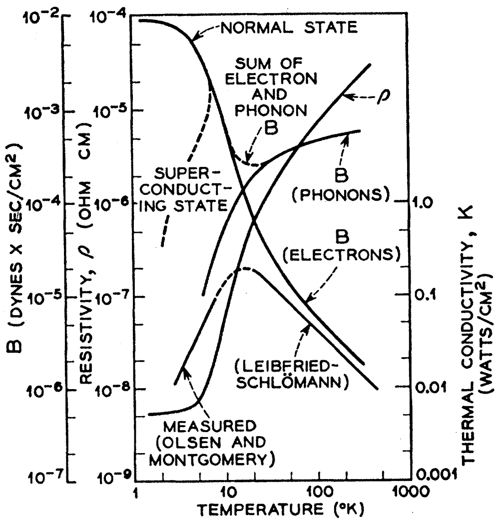

FIG. 2. Drag coefficients for electron damping in the normal and superconducting ranges. Other curves show resistivity of lead and phonon damping of dislocations calculated from the thermal conductivity curve shown.

assuming that the number of electrons equals the number of atoms, i.e., 3.3×10^{22} per cc. Figure 2 shows a plot of the electron and also the phonon drag coefficients for lead as a function of the temperature. The sum of the electron and phonon damping produces a plateau between 15 and 40°K which is in agreement with recent measurements.[17]

In the superconducting range, the number of electrons which can transfer momentum decreases rapidly as the temperature becomes lower than the superconducting temperature. Since the attenuation in the superconducting range determines the amount of momentum transfer, the rate for which the drag coefficient should decrease is the same as the attenuation decrease. Using the ratios shown by Fig. 3, the drag coefficient in the superconducting range is shown by the dashed line of Fig. 2.

III. ATTENUATION IN THE HIGH-AMPLITUDE RANGE DUE TO ELECTRON DAMPING

The attenuation introduced by dislocations has several amplitude ranges, as shown by Fig. 4, due to Granato and Lücke. In the low amplitude region, dislocations are pinned by both impurity atoms and network joins. The effect of a stress is to bow out the dislocations as shown by Figs. 4(B) and 4(C). In this region attenuation is caused by the conversion of dislocation motion to heat by interaction with phonons or electrons. This interaction determines the drag coefficient B calculated for electrons in the last section. As the applied stress becomes larger, the dislocation is bent at such an angle with respect to the impurity that enough force is exerted to pull the dislocation away

[17] K. Lücke (private communication).

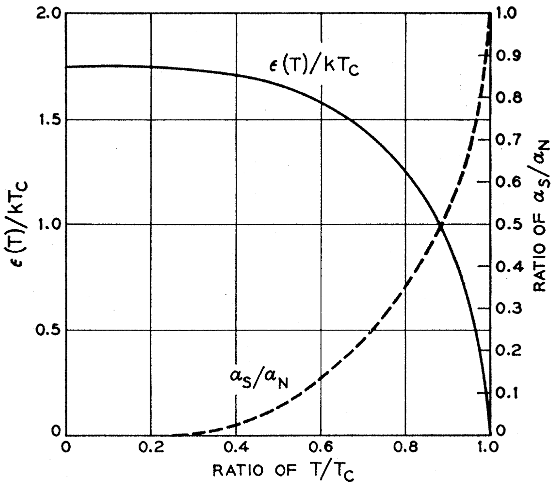

FIG. 3. Energy-gap curve and corresponding ratio of α_S/α_N from BCS theory plotted as a ratio of T/T_c.

from the pinning atom. When this occurs for one loop, the free loop becomes larger and the dislocation will then be torn away up to the network pinning points. It was shown by Granato and Lücke[18] that this breakaway was the source of an acoustic loss which varies with amplitude and it is this loss which causes the nonlinear attenuation occurring in Fig. 1. The force required to produce this breakaway is higher in the normal region since on account of the higher drag coefficient B it takes a larger force to bow the dislocation out to the critical angle than it does in the relatively undamped superconducting state.

This effect can be shown from the well known equations for the dislocation-string model[4,18]

$$M\frac{\partial^2 x}{\partial t^2}+B\frac{\partial x}{\partial t}-T\frac{\partial^2 x}{\partial y^2}=\text{Force}=T_{13}b, \tag{12}$$

where M is the mass per unit length, usually taken as ρb^2, where ρ is the density of the material and b the Burger's vector, B is the drag coefficient and T is the tension usually taken as $\frac{1}{2}\mu b^2$ for an isotropic material, where μ is the shearing modulus. For all the frequencies considered here the mass M can be neglected. For a static or slowly varying shear stress T_{13} in the glide plane, the displacement x at any distance y from a pinning point, separated from the next by the distance l is

$$x=A\,(ly-y^2). \tag{13}$$

Substituting this expression in (12) and integrating with respect to y from 0 to l, the constant A takes the form for a sinusoidally applied stress

$$\mu b^2Al+\tfrac{1}{6}j\omega BAl^3=T_{13}bl. \tag{14}$$

[18] A. Granato and K. Lücke, J. Appl. Phys. **27**, 583 (1956); **27**, 789 (1956).

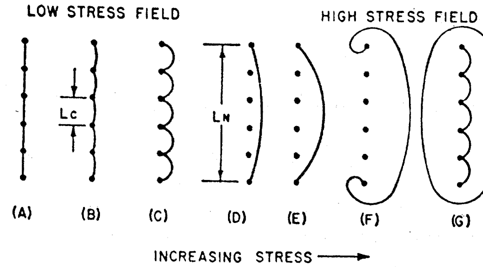

FIG. 4. Dislocation processes occurring at various stress amplitudes (after Granato and Lücke).

Hence

$$A=\frac{T_{13}}{\mu b}\left[\frac{1}{1+j\omega d}\right], \tag{15a}$$

where

$$d=\frac{Bl^2}{6\mu b^2}. \tag{15b}$$

The slope of the dislocation at the pinning points is

$$dx/dy)_{y=0\text{ or }l}=\pm Al. \tag{16}$$

Introducing the value of A from (15), the critical stress to produce breakaway is proportional to

$$T_{13}\alpha(\mu b/l)[1+j\omega Bl^2/6\mu b^2]. \tag{17}$$

For an exponential loop-length distribution, significant differences in the shearing stress occur when the constant in the denominator is 0.5 rather than 6.0. For a damping constant of $B=9\times10^{-3}$, $\omega=2\pi f=3.14\times10^8$; $l\approx10^{-4}$ cm; $\mu=7.3\times10^{10}$ dynes/cm^2, and $b=3.5\times10^{-8}$ cm, the last term of Eq. (17) is over 50 times larger than unity. In the superconducting range the damping constant B is much less as can be seen from Fig. 2 and hence it requires a smaller stress and a smaller voltage applied to the transducer to produce breakaway effects. The ratio of about 30 to 1, shown by Fig. 1, is in the order of that to be expected. Tittmann and Bömmel[6] have made a Granato-Lücke plot of strain-amplitude times the decrement plotted against the inverse strain

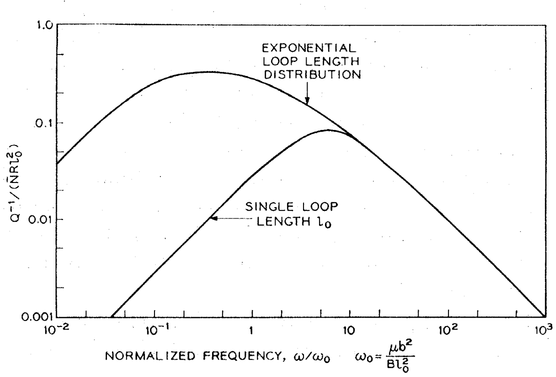

FIG. 5. Ratio of Q^{-1} to $\bar{N}Rl_0^2$ plotted as a function of ω/ω_0. Single-loop-length model and exponential-loop model are shown (after Oen, Holmes, and Robinson).

and find nearly a straight line. Hence, there appears little doubt that the amplitude effect observed is due to dislocation breakaway and that the difference is due to the different damping constants found for the normal and superconducting states.

Equation (17) indicates that at lower frequencies the difference between breakaway loss in the normal and the superconducting states should be much smaller. This is confirmed by the measurements of Welber and Quimby[3] made at 50 kc/sec. The difference between the onset of breakaway loss in the normal and superconducting ranges is only a few percent and is probably accounted for by the tail of the dislocation distribution with loop lengths longer than 10^{-4} cm.

IV. EFFECT OF LOW-AMPLITUDE DISLOCATION DAMPING ON SUPERCONDUCTING ENERGY-GAP DETERMINATIONS

By adding the plastic strain due to the dislocation motion to the elastic strain and calculating the resulting internal friction factor it is readily shown[18] that the internal friction for one slip system energized by the shearing strain in this system is given by

$$Q^{-1}=\frac{\bar{N}l}{6}\left[\frac{(\omega Bl^2/6\mu b^2)}{1+(\omega Bl^2/6\mu b^2)^2}\right]. \qquad (18)$$

To take account of other types of stresses and the distribution of Burger's vectors it is usual to introduce an orientation factor R which depends on the type and direction of propagation. The factor R usually varies from about 0.08 to 0.2. Inserting this factor and defining a relaxation angular frequency ω_0 equal to

$$\omega_0=\mu b^2/Bl^2 \qquad (19)$$

this expression becomes

$$Q^{-1}=\frac{\bar{N}Rl^2(\omega/6\omega_0)}{6[1+(\omega/6\omega_0)^2]}. \qquad (20)$$

A plot of this equation is shown by Fig. 5.

Actually, the dislocation loop distribution is not in the form of a single loop since the pinning positions are random and in general it is assumed (Koehler[19]) that the loop-length distribution takes the form

$$N(l)dl=(\bar{N}/l_A{}^2)e^{-l/l_A}dl, \qquad (21)$$

where l_A is now the average pinned-loop length. The integral of this equation shows that $Nl_A=\bar{N}$, where N is the total number of loops of all lengths and $\bar{N}$ the total length of dislocations per cubic cm.

The effect of an exponential distribution of loop lengths on the internal friction has been investigated by Oen, Holmes, and Robinson[20] and they were able to obtain a solution in closed form, the results of which are shown plotted on Fig. 5, where l_0 is now equal to l_A. The result of this distribution is to spread the internal-fraction peak over a broader frequency range and to increase the value at low frequencies by a factor of 100.

In evaluating the effect of dislocation damping, it is assumed that most of the residual loss in the superconducting range is due to this effect. This seems to be reasonable since measurements on silicon, germanium and quartz in the low-temperature range give nearly a zero loss. Hence, seal losses and sound-diffraction losses must be very small. It is also seen that below about 3°K the attentuation in the superconducting range becomes very flat. Since we do not know the distribution of loop lengths or the number of dislocations, about the only process available is to fit the curve in the flat region by the dislocation loss exhibited in Fig. 5 and see whether the average loop length and number of dislocations is similar to that obtained by other processes.

The data of Love and Shaw, given by Fig. 1—low-amplitude region—can be fitted reasonably well by assuming $\omega/\omega_0=0.4$ at 3°K where the drag coefficient B is 1.5×10^{-4} dynes$\times$sec/cm^2 from Fig. 2. Since $\omega=3.14\times10^8$, $\omega_0=7.85\times10^8$ and

$$l_A{}^2=\frac{\mu b^2}{\omega_0 B}=\frac{7.2\times10^{10}\times12.25\times10^{-16}}{7.85\times10^8\times1.5\times10^{-4}}=7.5\times10^{-10}\text{ cm}^2$$

or

$$l_A=2.74\times10^{-5}\text{ cm}, \qquad (22)$$

which appears to be a reasonable value for a strained crystal. The attenuation at 3°K, i.e., 5.25 dB, determines the dislocation density according to the equation

$$\alpha=\frac{8.68\times Q^{-1}\omega}{2V}\text{ dB/cm}$$
$$=\frac{8.68\times\bar{N}Rl_A{}^2\times0.33\times3.14\times10^8}{2\times2.35\times10^5}\text{ dB/cm}$$
$$=5.25\text{ dB/cm}, \qquad (23)$$

where the velocity along the $\langle111\rangle$ direction of measurement is 2.35×10^5 cm/sec. Solving this equation for $\bar{N}R$, using $l_A{}^2=7.5\times10^{-10}$, we find $\bar{N}R=3.8\times10^6$. With a reasonable value of $R=0.1$ this corresponds to a dislocation density of 3.8×10^7 per cc which is typical of a strained sample. Table I shows the values of the correction terms for both the normal and the superconducting states using the drag coefficients shown by Fig. 2.

The effect of these correction terms is shown by the dashed lines of Fig. 1. Using the dashed lines as the corrected attenuation due to the energy-gap effect, the energy gap as a function of temperature is shown by

[19] J. S. Koehler, in *Imperfections in Nearly Perfect Crystals*, edited by W. Shockley, J. H. Hollomon, R. Mauer, and F. Seitz (John Wiley & Sons, Inc., New York, 1952), Chap. 7.

[20] O. S. Oen, D. K. Holmes, and M. T. Robinson, U. S. Atomic Energy Commission Report ORNL-3017, 1960, p. 3 (unpublished).

TABLE I. Correction terms for ultrasonic attenuation in lead.

	Superconducting					Normal				
T(°K)	B	ω_0	ω/ω_0	F	α (dB/cm)	B	ω_0	ω/ω_0	F	α (dB/cm)
2.5	9.1×10^{-5}	1.29×10^{9}	0.242	0.325	5.06	8.5×10^{-3}	1.4×10^{7}	22.6	0.038	0.592
3.0	1.5×10^{-4}	7.85×10^{8}	0.4	0.33	5.14	7.5	1.57×10^{7}	20.0	0.042	0.655
4.0	2.94×10^{-4}	4.0×10^{8}	0.785	0.305	4.75	6.15	1.91	16.4	0.05	0.78
5.0	4.4	2.67×10^{8}	1.18	0.27	4.2	4.55	2.58	12.1	0.064	1.0
5.5	5.6	2.1×10^{8}	1.5	0.24	3.74	3.74	3.14	10.0	0.076	1.18
6.0	7.25	1.62×10^{8}	1.94	0.215	3.35	3.0	3.92	8.0	0.091	1.42
6.5	1×10^{-3}	1.175×10^{8}	2.68	0.18	2.8	2.38	4.94	6.36	0.106	1.66
7.0	1.43×10^{-3}	8.2×10^{7}	3.83	0.15	2.33	2.13	5.51	5.7	0.115	1.79
7.2	2×10^{-3}	5.86×10^{7}	5.35	0.12	1.87	2.0	5.86	5.36	0.12	1.87

Fig. 6. All the points lie very close to the theoretical curve with $\epsilon_0/kT_c=2.0$ at $T=0$°K. If we had subtracted off a constant attenuation, the indicated value would be about $\epsilon_0/kTc=2.84$ with the points not being in very good agreement with the theoretical curve.

Another set of data was obtained by Tittmann and Bömmel[6] which requires less correction. The measured and corrected values are shown by Fig. 7. The best fit is obtained by letting

$$l_A=3.56\times10^{-5}\text{ cm};\quad \bar{N}R=5.85\times10^{5};$$
$$\bar{N}\doteq5.85\times10^{6}. \qquad (24)$$

This increase of the value of l_A and the decrease in the number of dislocations are both consistent with less deformation of the crystal by cold work. Figure 6 shows the corrected value of the energy gap which is best matched by a value $\epsilon_0=2.1\ kT_c$. Hence to the order of accuracy obtained here, the energy gap is

$$\epsilon_0=(2.05\pm0.05)kT_c. \qquad (25)$$

This is larger than the BCS theoretical energy gap but is consistent with other determinations such as infrared (Leslie and Ginsberg[21]), tunneling (Giaever[22] *et al.* and thermal conductivity (Morris[23] *et al.*).

If one plots the correction curves to lower temperatures it appears that the correction losses should go down again which is somewhat contrary to measurements carried out at the lowest temperatures. These indicate that the added loss remains constant for as low temperatures as have been measured. It is believed that the dislocation is damped by irregularities and

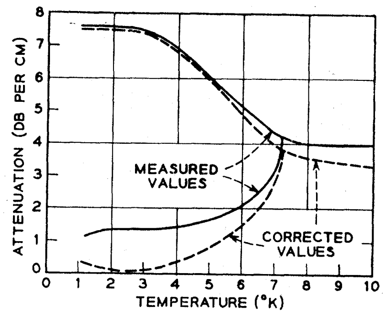

FIG. 7. Measurements of Tittmann and Bömmel and corrections for dislocation damping.

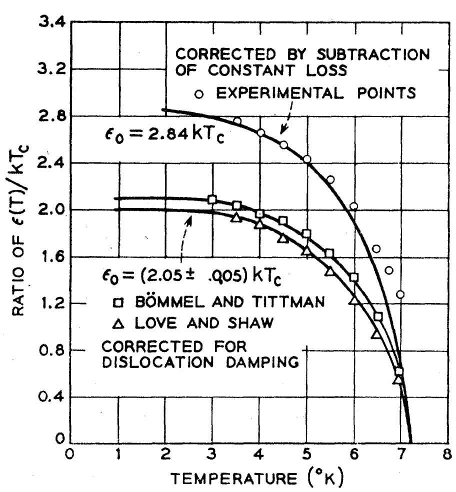

FIG. 6. Evaluation of energy gap taking account of dislocation damping corrections. Top curve shows effect of subtracting a constant loss from both normal and superconducting states.

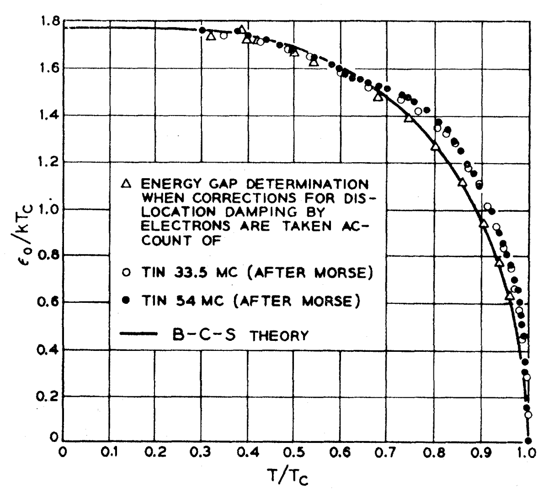

FIG. 8. Comparison of drop in attenuation in the superconducting range with that predicted from BCS theory. Triangles show effect of taking account of dislocation damping.

[21] J. D. Leslie and D. M. Ginsberg, Phys. Rev. **133**, A362 (1964).
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[23] D. E. Morris and M. Tinkham, Phys. Rev. **134**, A1154 (1964).

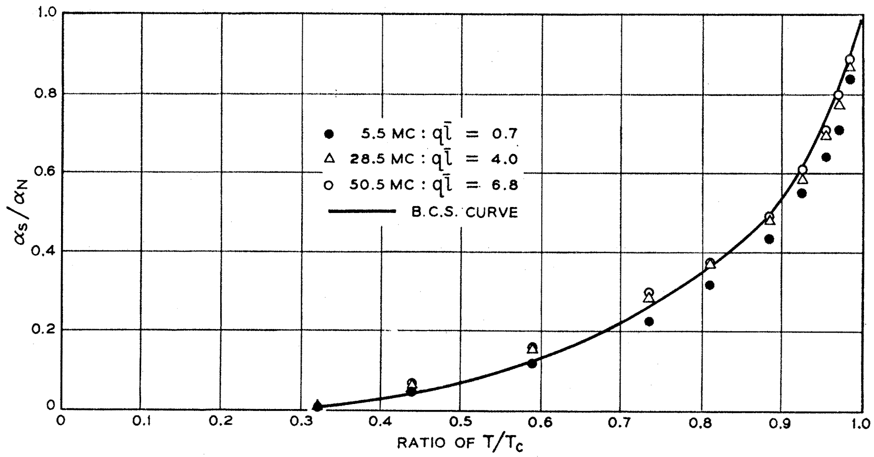

FIG. 9. Calculated ratio of α_S/α_N for three values of the quantity ql_e.

imperfections in the solid. This is consistent with the fact that the attenuation at 1000 Mc/sec is in the order of 0.05 dB per cm for such materials as crystal quartz in the temperature range from 2.0 to 20°K. An effective scattering viscosity of 10^{-4} poises is indicated with a scattering drag coefficient of $B\approx 10^{-5}$ (dyne sec/cm^2). In order to obtain a constant loss down to low temperatures, the order of magnitude of the constant drag would be $B=5\times10^{-5}$ for the deformed sample of Love and Shaw and about 10^{-5} for the pure sample of Tittmann and Bömmel.

The electron drag correction for the attenuation can also account for the difference found between the acoustically determined energy gap and the theoretical BCS gap, which is evident from the curves of Fig. 8.[24] At low ratios of T/T_c, where the subtracted loss and the dislocation damping loss are both independent of the temperature, the calculated curve and the measured points agree. However, for values above $T/T_c=0.6$, the measured points appear to be above the theoretical values. If, however, the corrections due to dislocation damping of the form shown by Figs. 1 and 7 are taken account of, the measured points agree much better with the theoretical curve. The squares of Fig. 8 were obtained from the data of Morse by fitting a dislocation damping correction to the residual loss assuming values of the damping constant for tin consistent with $ql_e>10$ and $l_A\approx2\times10^{-5}$ cm. The loss at the highest point determines the product $\bar{N}R=3\times10^6$. For $R=0.1$, this gives a dislocation density of $\bar{N}=3\times10^7$. Hence it is evident that the difference between the dislocation damping by electrons in the normal state and in the superconducting state has to be taken account of in determining the energy gap in superconducting metals.

Theoretically[25] the ratio of the attenuation of longitudinal waves in the superconducting state to that in the normal state should be independent of the product of ql_e, where l_e is the mean free path for electrons, and should be given by Eq. (1). The best experimental evidence[26] indicates that the drop off from the BCS theory is larger when ql_e is small than when it is large. These data were obtained for polycrystalline indium for the cases that $ql_e=0.7$ and $ql_e=6.8$. To see if the damping by dislocations can account for this effect, the following constants were assumed. From the conductivity measurements, B can be calculated to be about 2×10^{-2} dynes sec/cm^2 in the normal state. Taking $l_A=3.5\times10^{-5}$ cm, $\bar{N}R=6.2\times10^5$ and using the measured data from Morse,[26] Fig. 9 shows the calculated ratio of α_S/α_N. It is evident that the three values calculated do not differ much and are probably within the experimental error with respect to the BCS theoretical curve.

Since it is difficult to evaluate the total dislocation damping without knowing the loop-length distribution and the number of dislocations, it appears that the best way to minimize this effect is by going to higher frequencies. As seen from Eq. (20), and from Fig. 5, the internal friction decreases as $1/\omega$ when $\omega\gg\omega_0$. Hence, the attenuation approaches a constant equal to

$$\alpha=(\bar{N}R\mu b^2/2BV)\ \text{Np/cm}. \qquad (26)$$

Hence, for any of the drag coefficient values given by Fig. 2, the attenuation is in the order of 5 dB or less. On the other hand, the acoustic loss due to electron damping of acoustic waves may be as high[27] as 400 bB/cm for frequencies in the 1000-Mc/sec range. Hence, the correction due to the dislocation attenuation becomes quite small and energy gaps approach their theoretical values, with some deviations for anisotropy.[27]

[24] See Ref. 4, Fig. 11.11.

[25] T. Tsuneto, Phys. Rev. **121**, 402 (1960); see also, J. R. Schrieffer, *Theory of "Superconductivity"* (W. A. Benjamin, Inc., New York, 1964), pp. 62–69.

[26] See Ref. 14, Figs. 10, 12, and 16.

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