PHYSICAL REVIEW VOLUME 143, NUMBER 1 MARCH 1966

Pattern of Current Flow in Nonideal Type-II Superconductors in Longitudinal Magnetic Fields. I*

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(Received 30 August 1965)

The longitudinal magnetization of a wire of a nonideal type-II superconductor in the presence of a stationary longitudinal magnetic field H_a is seen to oscillate between paramagnetic and diamagnetic as a conduction current is impressed and removed. When a conduction current I near the critical value I_c is present, the longitudinal magnetization is paramagnetic and attains a maximum. With $I=0$ *after* passage of a conduction current $I \lesssim I_c$, the longitudinal magnetization is diamagnetic. This diamagnetic magnetization appreciably exceeds the Meissner-effect diamagnetism exhibited by the sample at corresponding fields. This paramagnetism and diamagnetism have been investigated throughout the range $0<H_a<H_{c2}$. The variation of the longitudinal magnetization through paramagnetic and diamagnetic states as the conduction current is isothermally varied through a full cycle starting at $I \lesssim I_c$ has also been measured. All of the observations are satisfactorily described by a simple model which requires that macroscopic conduction and induced currents flow in helical paths which seek to preserve the existing magnetic-flux configuration and maintain a constant inward-directed Lorentz-force density throughout the specimen in the presence of a stationary longitudinal magnetic field.

INTRODUCTION

THE longitudinal magnetization of a wire of a nonideal type-II superconductor in the presence of a stationary longitudinal magnetic field oscillates between paramagnetic and diamagnetic as a conduction current is impressed and removed. The appearance of a paramagnetic magnetization when the conduction current approaches the critical value has been reported recently[1] and indicates that the applied current flows in helical paths of a sense opposite to that expected for free electrons moving in a magnetic field. As the applied current is subsequently varied through zero, the excess flux parallel to the wire axis is completely released and a longitudinal diamagnetic magnetization is generated which exceeds that due to the flux expulsion associated with the partial Meissner effect of the specimen. In this article we describe the variation of the longitudinal magnetization during a cycle of the impressed current and show that both the paramagnetic and diamagnetic behavior can be understood if the macroscopic currents flow in the wire in helical configurations which seek to preserve previous history and maintain a constant Lorentz force throughout the specimen in the presence of a longitudinal magnetic field.

RESULTS

The phenomena we describe have been observed in wires of Nb and NbTa with various histories of anneal. We present data obtained with an as-received 0.125-cm-diam wire of high-purity Nb(50 at.%)Ta. The experimental technique is identical to that described previously.[1] The section of the wire seen by the search coil is maintained at a temperature higher than that of the 4.2°K bath in which the current contacts are immersed to insure that true critical currents are attained. Figure 1 gives the critical current I_c versus the static longitudinal applied field H_a.

In Fig. 2, curves A and K show the maximum diamagnetic and paramagnetic magnetization, with zero conduction current, in increasing and decreasing fields, respectively. Curve D gives the partial Meissner effect exhibited upon cooling through T_c to the chosen temperature. Curve E gives the paramagnetic magnetization observed when the applied current reaches the critical value I_c. This paramagnetism induced by the

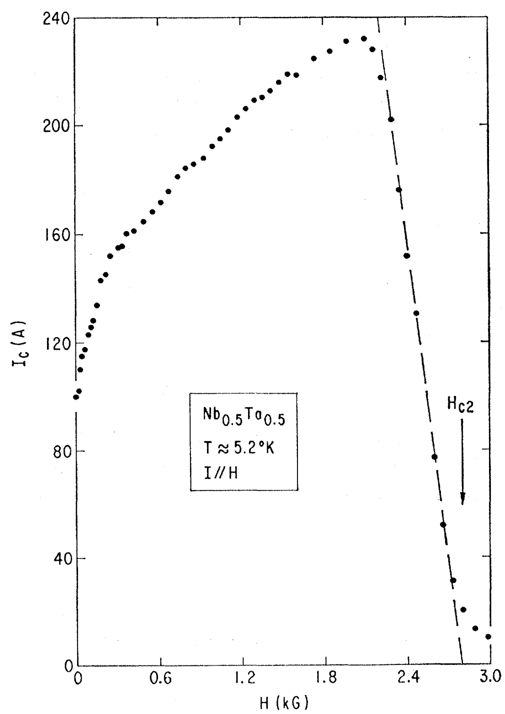

FIG. 1. Critical current versus applied longitudinal magnetic field. $T\approx 5.2$°K.

* Work performed under the auspices of the U. S. Atomic Energy Commission; Contract No. (AEC-USC-3).

[1] M. A. R. LeBlanc, B. C. Belanger, and R. M. Fielding, Phys. Rev. Letters 14, 704 (1965); M. W. Williams and C. J. Bergeron, Jr., Bull. Am. Phys. Soc. 10, 60 (1965); M. A. R. LeBlanc, Phys. Letters 16, 30 (1965).

applied current reaches a maximum when I attains I_c and is nearly independent of previous history. Upon withdrawal of an applied current $I \lesssim I_c$ this paramagnetism vanishes and a diamagnetic magnetization is induced in the wire, as shown by curve C. We note that the diamagnetism generated by such a half-cycle in the applied current is appreciably greater than that produced by the Meissner flux expulsion which occurs upon cooling through T_c in the corresponding fields (compare curves C and D). Since curve K is a measure of the flux trapping capacity of the specimen we might expect that a comparable part of the longitudinal flux generated by the applied current would be retained by the wire as the current is reduced to zero.

Figure 3 shows the locus of the magnetization in a stationary field of 0.94 kG during the first half-cycle (solid points) and the second half-cycle (crosses) of the applied current after its initial increase. We note that the frequency of oscillation of the magnetization is twice that of the applied current.[2]

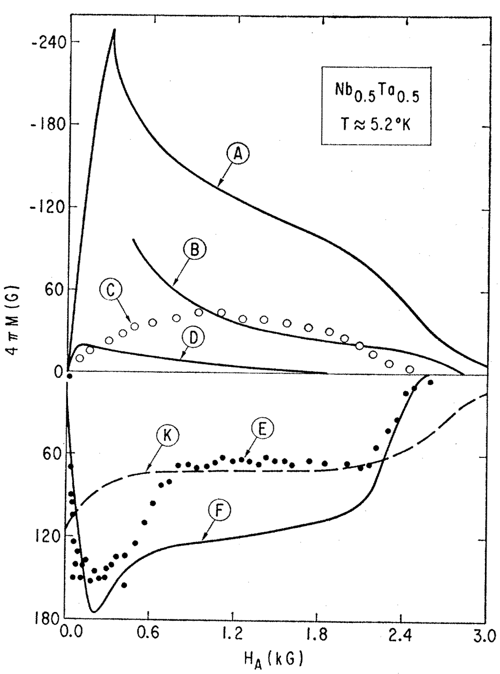

FIG. 2. Curve A: Initial magnetization in increasing magnetic field; curves B and C: Remanent longitudinal diamagnetism of wire after a current just below the critical value is impressed and removed in the presence of a static field, theoretical and experimental, respectively; curve D: Partial Meissner-effect diamagnetism observed upon cooling through T_c in static fields; curves E and F: Longitudinal paramagnetic magnetization as the applied current attains the critical value, experimental and theoretical, respectively; curve K: Maximum paramagnetism (trapped flux) in decreasing fields (no applied currents). Applied magnetic fields are longitudinal and $T \approx 5.2$°K for all data.

[2] Analogous oscillatory behavior of the magnetization of a ferromagnetic wire which carries an alternating current in the presence of a stationary longitudinal magnetic field has been observed [St. Procopiu, J. Phys. Radium **1**, 306 (1930)] and investigated recently [R. Skórski and A. Duracz, J. Appl. Phys. **36**, 511 (1965)].

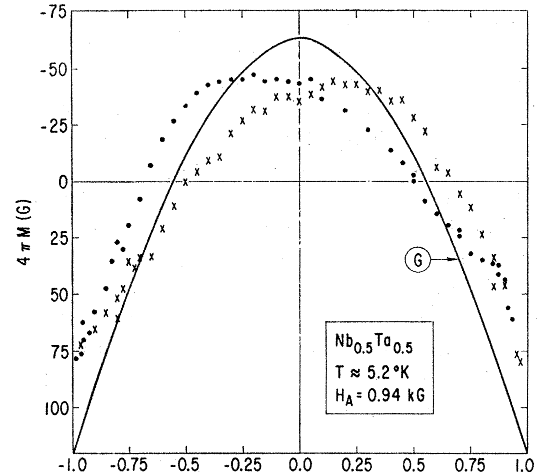

FIG. 3. Behavior of the longitudinal magnetization in a static longitudinal field of 0.94 kG after initial increase of the applied current I to just below the critical value I_c as I is varied from $\approx +I_c$ to $-I_c$ (● data points) and subsequently varied from $\approx -I_c$ to $+I_c$ (X data points). Curve G: behavior calculated from model described in text.

DISCUSSION

Following concepts due to Gorter, Anderson, and Kim *et al.*[3] we expect that a Lorentz force can be sustained by supercurrent elements in nonideal type-II superconductors due to pinning effects provided by imperfections. Consequently we assume that the macroscopic critical current density j_c is determined by the relation $|\mathbf{F}_L| = |\mathbf{j}_c \times \mathbf{B}| = \beta$ where $\mathbf{B}$ is the average local magnetic field and β is a field-independent, isotropic and homogeneous pinning parameter of the material. Presumably $\beta \rightarrow 0$ in ideal samples and transport currents assume a force-free configuration as suggested by Bergeron.[4] In harmony with the Bean model[5] we also assume that the current density $|j| \equiv (j_z^2 + j_\theta^2)^{1/2} = j_c$ wherever induced and/or conduction currents flow. For a cylinder of infinite length, the Lorentz-force condition becomes

$$|F_L| = |j_\theta B_z - j_z B_\theta| = \beta, \qquad (1)$$

where the net Lorentz-force density can be directed either radially inwards or outwards. We assume that the former situation is always encountered and write

$$j_\theta B_z - j_z B_\theta = -\beta. \qquad (2)$$

With our model this choice always yields results in considerably better agreement with all of our relevant

[3] C. J. Gorter, Nuovo Cimento **6**, 1168 (1957); Physica **23**, 45 (1957); P. W. Anderson, Phys. Rev. Letters **9**, 309 (1962); Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **129**, 528 (1963); **131**, 2486 (1963).

[4] C. J. Bergeron, Jr., Appl. Phys. Letters **3**, 63 (1963).

[5] C. P. Bean, Phys. Rev. Letters **8**, 250 (1962); Rev. Mod. Phys. **36**, 31 (1964).

data than the alternative possibility. The reason for this selection is not clear at present.[6]

The macroscopic critical currents and critical current densities cannot be derived *a priori* at present. We assume for simplicity that the critical longitudinal current density $|j_z|$ is uniform wherever longitudinal currents flow[7] and is determined by the *external* longitudinal field H_a. Hence j_z is obtained from the measured critical currents and the relation $j_z = I_c/\pi R^2$, where R is the wire radius. With an applied current I_c it follows from the assumption of a uniform j_z that the local circular field is given by $B_\theta(r) = I_c r/5R^2$. Introducing Maxwell's equation $\mu j_\theta = -dB_z/dr$ where $\mu = 4\pi/10$ and these expressions for j_z and B_θ into Eq. (2) leads to

$$\frac{d}{dr}(B_z^2) = 2\alpha - \frac{4}{R}\left(\frac{I_c}{5R}\right)^2\left(\frac{r}{R}\right), \tag{3}$$

where $\alpha = \mu\beta$. Since at the surface $B_z = H_a$, integration yields

$$B_z(r) = \left\{ -2\alpha R\left(1-\frac{r}{R}\right) + 2\left(\frac{I_c}{5R}\right)^2\left(1-\left(\frac{r}{R}\right)^2\right) + H_a^2 \right\}^{1/2}. \tag{4}$$

The average longitudinal magnetization can be obtained from the definition

$$4\pi \bar{M}_z = \frac{2}{R^2}\int_0^R (B_z - H_a) r\, dr. \tag{5}$$

The magnetization at I_c versus H_a, calculated using B_z from Eq. (4) and letting $\alpha = 2.5\times 10^6$ A·G/cm² is given by curve F of Fig. 2. Fig. 4(a) shows schematically the pattern of current flow which follows from our model when an applied current $I \approx I_c$ is present.

Next we consider the behavior as the applied current is cycled after an excursion to near the critical value. In keeping with the laws of magnetic induction we require that the specimen seek to preserve the existing configuration of flux consistent with the change in I. This requirement is approximately satisfied if the existing configuration of the *circular* flux is maintained consistent with the change in I when, as in the present sample at intermediate fields, the volume average of the circular field $\langle |B_\theta| \rangle_{av} = \int |B_\theta| dv/V \gg \langle |B_z - H_a| \rangle_{av} \equiv \int |B_z - H_a| dv/V$. The circular current j_θ is thus considered free to adjust to meet the Lorentz force condition given by Eq. (2). To attempt to preserve the circular flux configuration, the applied current I must now be the resultant of two concentric and oppositely flowing longitudinal currents denoted by I_i and I_0. Thus we may write $I = |I_i| - |I_0|$, where I_i and I_0 fill the regions $0 < r < R_1$ and $R_1 < r < R$, respectively. The distribution of the longitudinal currents as I is cycled is seen to be similar to that encountered in a wire of a normal metal carrying a high frequency alternating current.[8] From the assumption that $|j_z|$ is uniform it follows that $B_\theta = I_c r/5R^2$ and $B_\theta = I_c\{(1+I/I_c) - (r/R)^2\}/5r$, for the regions occupied by I_i and I_0, respectively. Introducing these expressions into Eq. (2) and integrating yields

$$B_z = \left\{ -2\alpha R\left(\frac{R_1}{R} - \frac{r}{R}\right) + 2\left(\frac{I_c}{5R}\right)^2\left(\left(\frac{R_1}{R}\right)^2 - \left(\frac{r}{R}\right)^2\right) + B_z^2(R_1) \right\}^{1/2}, \tag{6}$$

$$B_z = \{-2\alpha R(1-r/R) + 2(I_c/5R)^2(1-(r/R)^2) + 4(I_c/5R)^2(1+I/I_c)\ln(r/R) + H_a^2\}^{1/2}, \tag{7}$$

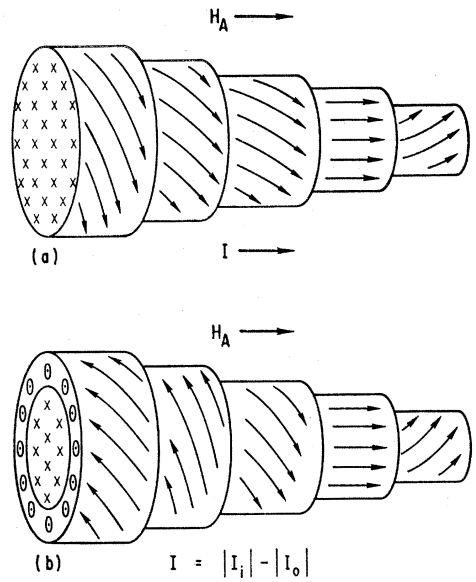

FIG. 4. Pattern of macroscopic current flow at various radii when (a) an applied current just below the critical value is present in the wire, and (b) an applied current near the critical value has been impressed and is being withdrawn.

[6] Equation (1) should include a radially outward directed inertial (centrifugal) force term $F_c = nmv_\theta^2/r = \lambda_B^2 j_\theta^2/r$, where $\lambda_B = (m/ne^2)^{1/2}$ is the Bean penetration depth and n is the density of macroscopic current carriers. In our specimen, this term is negligible except for an infinitesimal core region of radius $r \lesssim 10^{-5}$ cm. This force, however may lead to the instability and elimination of current elements experiencing an outward Lorentz force.

[7] This assumption is probably a fair approximation at intermediate fields. Solutions of the form $j_z \alpha J_0(r/\lambda_B)$ and $I_0(r/\lambda_B)$, where J_0 and I_0 are Bessel functions and λ_B is the Bean penetration depth, can be obtained from a force-free configuration (Refs. 1 and 4) and from minimizing the energy $\int (B^2/8\pi + \lambda_B^2 j^2/2) dv$, respectively. When $\lambda_B > R$ these solutions do not deviate appreciably from our assumption.

[8] N. W. McLachlan, *Bessel Functions for Engineers* (Oxford University Press, New York, 1955), 2nd ed., pp. 153–162.

for these two regions, respectively, where $R_1/R = ((I_c+I)/2I_c)^{1/2}$. The pattern of current flow when the applied current is reduced after an excursion to $I \approx I_c$ is shown schematically in Fig. 4(b).

Introducing Eqs. (6) and (7) in Eq. (5) we have calculated $4\pi\bar{M}_z$ by numerical integration as a function of I/I_c with α as given above for various H_a. The remanent longitudinal magnetization calculated for $I=0$ after an excursion to I_c is shown versus H_a by curve B of Fig. 2. At low fields the calculated values deviate considerably from the data since the condition $\langle|B_\theta|\rangle_{\text{av}} \gg \langle|B_z-H_a|\rangle_{\text{av}}$ is not satisfied. Curve G of Fig. 3 shows the behavior of $4\pi\bar{M}_z$ calculated as I varies from $\pm I_c$ to $\mp I_c$ in a static field. We note that the calculated curve is symmetric whereas the experimental data yield asymmetric curves. In practice, the applied current can only be raised close to I_c when cycled. When this fact is taken into account in our model and appropriate expressions are derived, curves of $4\pi\bar{M}_z$ versus I/I_c exhibiting an asymmetry in qualitative agreement with the data are obtained. A discussion of this feature is beyond the scope of this preliminary report.

CONCLUSION

The longitudinal magnetization induced in cylindrical nonideal type-II superconductors by applied currents in the presence of static longitudinal fields is satisfactorily accounted for by a model which requires that (i) the net critical Lorentz force density is constant throughout the specimen and is directed radially inwards and (ii) the existing circular flux configuration is preserved consistent with the change in the applied current. From this model, paramagnetic and diamagnetic helical configurations of conduction and induced currents are thought to arise and coexist in a cylindrical wire when an applied current is cycled in the presence of a static longitudinal field. The helical circulating currents which remain after passage of the applied current and fill the bulk of the specimen can give rise to an appreciable longitudinal diamagnetic moment.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the assistance of J. A. Szabo and Dr. D. J. Griffiths in the measurements and the collaboration of C. T. M. Chang in the calculations.

PHYSICAL REVIEW VOLUME 143, NUMBER 1 MARCH 1966

Selective Absorption of Circularly Polarized Light in Broad Bands by the Zeeman Components of Divalent Thulium in Calcium Fluoride*

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(Received 22 September 1965)

The paramagnetic circular dichroism of the $4f$-$5d$ absorption bands of CaF_2:Tm^{2+} was investigated at 2.0°K and 9.0 kG. Using group theory it is shown that the bands can be separated into $E_{5/2}$ and $G_{3/2}$ character. Experimentally it was found that in this case the bands are primarily $G_{3/2}$ in character. Optical pumping experiments were performed where changes as large as $\pm 20\%$ were easily observed in the electron-paramagnetic-resonance absorption signal at X-band frequencies and 1.5°K by changing the polarization of the pump light from right to left circular. A method for optically monitoring the difference in the population of the two Zeeman components of the ground state is also reported on.

I. INTRODUCTION

At the first Quantum Electronics Conference, Brossel made the observation that if the magnetic sublevels of the ground state of a paramagnetic ion in a solid absorbed polarized light in broad absorption bands by different amounts, one had a system very suitable for achieving large population changes in these levels by optical pumping.[1] Margerie[2] demonstrated that the four sublevels in the ground state of Cr^{3+} in ruby did have slightly different absorption coefficients for circularly polarized light in the $^4A_2$-$^4F_2$ green band. Karlov[3] with Margerie then demonstrated that the induced circular dichroism in the F center in KBr was as large as 9% in some regions of the bands and demonstrated that the spins could be inverted by optically pumping with circularly polarized light. Since then, Margerie[4] and others have made measurements on a number of other alkali halides with F centers.

There has also been a renewed interest in Faraday

* The research reported in this paper was sponsored by the Department of Defense and RCA Laboratories, Princeton, New Jersey.

[1] J. Brossel, *Quantum Electronics* (Columbia University Press, New York, 1960), p. 81.

[2] J. Margerie, Comp. Rend. **257**, 2634 (1963).

[3] N. V. Karlov, J. Margerie, and V. Merle D'Aubigne, J. Phys. **24**, 717 (1963).

[4] J. Margerie and R. Romestain, Comp. Rend. **258**, 4490 (1964). R. Romestain and J. Margerie, *ibid.* **258**, 2525 (1964). J. Gareyte and V. Merle D'Aubigne, *ibid.* **258**, 6393 (1964).