Then, we have resonance forms, like $D(1,2,\rho)$. We therefore want to evaluate $(p_1+p_2)^2$. As previously noted, $(p_1+p_2)^2=(P-P_3)^2=-m_3^2-E(E-2E_3)$. Furthermore, there is the energy dependence of the width of the resonance (the width appears through the prescription that we replace m_ρ by $m_\rho-i\Gamma_\rho/2$, where

$$\Gamma_\rho=\frac{2}{3}\frac{g_{\rho\pi\pi}^2}{4\pi}\frac{|\mathbf{p}_\pi|^3}{-p_\rho^2},$$

obtained using the usual $\rho\pi\pi$ coupling and the (off-mass-shell) mass of the ρ). In the case at hand, $p_\rho^2=(p_1+p_2)^2$ which we have just evaluated. $\mathbf{p}_\pi$ is the momentum of the π in the ρ center of mass. Using $(P_1-P_2)^2=2P_1^2+2P_2^2-(P_1+P_2)^2$ and the fact that both π's have the same momentum, one finds

$$4\mathbf{p}_\pi^2=m_3^2+E(E-2E_3)-2(m_1^2+m_2^2)+\frac{(m_1^2-m_2^2)^2}{m_3^2+E(E-2E_3)}.$$

All of the above formulas were constructed with their suitability for computing in mind, and in the above form are immediately programmable.

Numerical integration with a sixth-order polynomial fit was used to obtain results, the program automatically subdividing the integration interval where necessary (e.g., under a resonance peak) until the answer was good to a desired number of significant figures (chosen to be three).

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Coupling Constants in Broken $\tilde{U}(12)$ Symmetry*

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The effects of $SU(3)$ and $\tilde{U}(4)$ breaking on the coupling constants in $\tilde{U}(12)$ symmetry are investigated using the spurion technique. For an $SU(3)$-breaking spurion which is a member of **143**, only two parameters are introduced in addition to the one for the formal symmetry. All 132 baryon-meson coupling constants can be expressed in terms of these three quantities. For vertices involving pions or ρ mesons, only two parameters are relevant. The effects of $\tilde{U}(4)$ breaking as well as simultaneous $\tilde{U}(4)$ and $SU(3)$ breaking are studied with spurions which belong to the representations **143**, **4212**, and **5940** of $\tilde{U}(12)$. The sum rules for the coupling constants which follow from the formalism are in reasonable agreement with experiment.

1. INTRODUCTION

The $\tilde{U}(12)$ scheme[1,2] provides a relativistic framework for the derivation of the $SU(6)$ results.[3] In addition to these results, $\tilde{U}(12)$ also gives an absolute value for the proton magnetic moment which is of the right order of magnitude, and it relates all meson-baryon vertices to a single form factor. Even though the application of formal $\tilde{U}(12)$ symmetry to scattering processes meets with certain difficulties,[4] its success in the case of the vertex function is encouraging. We expect $\tilde{U}(12)$ to be broken in two ways corresponding to its subgroups $SU(3)$ and $\tilde{U}(4)$. The deviations from $SU(3)$ are conventionally described by introducing a spurion which transforms like the eighth component of an $SU(3)$ octet. It is well known that this also gives rise to mass splittings between the members of the $SU(3)$ multiplets. $\tilde{U}(4)$ on the other hand is broken by the equations of motion which give rise to $\tilde{U}(4)$ noncovariant subsidiary conditions for the representations of $\tilde{U}(12)$. In addition, to simulate higher order effects, we shall introduce $\tilde{U}(4)$ breaking spurions,[5] which belong to the representations **143**, **4212**, and **5940** of $\tilde{U}(12)$.

In the second section we give the effective interaction Hamiltonian densities for the meson baryon vertex including spurions. The third section deals with the reduction of the $\tilde{U}(12)$ field operators under $\tilde{U}(4)\otimes SU(3)$, and in the fourth section we study the effects of $SU(3)$ breaking spurions. In the fifth section we investigate $\tilde{U}(4)$ breaking as well as simultaneous $SU(3)$ and $\tilde{U}(4)$ breaking, and in the sixth section we compare the results with the experimental data.

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[3] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964); F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964).

[4] R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters **14**, 518 (1965).

[5] P. G. O. Freund, Phys. Rev. Letters **14**, 803 (1965); R. Oehme, *ibid.* **14**, 866 (1965).

2. BARYON-MESON VERTICES IN $\tilde{U}(12)$

$\tilde{U}(12)$ has 143 generators. In the defining (twelve-dimensional) representation these generators are simply the direct product of the $\tilde{U}(4)$ generators (Dirac algebra) with the $U(3)$ generators. The representations of $\tilde{U}(12)$ transform like direct products of twelve-dimensional quarks and can be written as tensors in a twelve-dimensional space. The $\tilde{U}(12)$ quark is the direct product of a Dirac spinor and a three-component $U(3)$ quark. The spin-$\frac{1}{2}^+$ baryon octet and the $\frac{3}{2}^+$ decuplet are assigned to the 364-dimensional representation of $\tilde{U}(12)$, which is described by a totally symmetric third-rank tensor ψ_{ABC}. The pseudoscalar meson octet and singlet as well as the vector-meson octet and singlet belong to the 143-dimensional representation of $\tilde{U}(12)$, which is described by a second rank mixed tensor $\phi_B{}^A$. The symmetry-breaking spurions $S_B{}^A$ will transform like a member of the self-adjoint representation **143**, the same one the mesons belong to. We shall also consider spurions belonging to the 4212- and 5940-dimensional representations of $\tilde{U}(12)$. While the representation **4212** is described by an antisymmetric mixed fourth-rank tensor $\phi_{[CD]}{}^{[AB]}$, **5940** is described by a symmetric one $\phi_{\{CD\}}{}^{\{AB\}}$. The effective $\tilde{U}(12)$ symmetric interaction density for the meson baryon vertex is given by

$$\mathcal{H}_0 = g_0\bar{\psi}^{ABC}\psi_{EBC}\phi_A{}^E. \tag{1}$$

Insertion of a spurion $S_B{}^A$ of **143** introduces four additional terms

$$\mathcal{H}_1 = g\bar{\psi}^{ABC}\psi_{ABC}\phi_E{}^D S_D{}^E + \bar{\psi}^{ABC}\psi_{EBC} \times (g_1\phi_A{}^D S_D{}^E + g_1'\phi_D{}^E S_A{}^D) + 2g_2\bar{\psi}^{ABC}\psi_{EFC}\phi_A{}^E S_B{}^F. \tag{2}$$

With a spurion $S_{[CD]}{}^{[AB]}$ of **4212** there is only one coupling

$$\mathcal{H}_2 = h\bar{\psi}^{ABC}\psi_{EBC}\phi_F{}^D S_{[AD]}{}^{[EF]}.$$

The introduction of a spurion $S_{\{CD\}}{}^{\{AB\}}$ of **5940** gives four terms

$$\mathcal{H}_3 = h_1\bar{\psi}^{ABC}\psi_{EBC}\phi_F{}^D S_{\{AD\}}{}^{\{EF\}} + \bar{\psi}^{ABC}\psi_{EFC} \times (h_2\phi_A{}^D S_{\{DB\}}{}^{\{EF\}} + h_2'\phi_D{}^E S_{\{AB\}}{}^{\{DF\}}) + h_3\bar{\psi}^{ABC}\psi_{EFG}\phi_A{}^E S_{\{BC\}}{}^{\{FG\}}. \tag{3}$$

3. REDUCTION OF THE REPRESENTATIONS OF $\tilde{U}(12)$ UNDER $\tilde{U}(4)\otimes S\tilde{U}(3)$

The regular representation of $\tilde{U}(12)$ **143** decomposes under $\tilde{U}(4)\otimes SU(3)$ into

$$\phi_B{}^A = [\phi^i + \gamma_5\phi_5{}^i + \gamma_\mu\gamma_5\phi_{\mu5}{}^i + \gamma_\mu\phi_\mu{}^i + \tfrac{1}{2}\sigma_{\mu\nu}\phi_{\mu\nu}{}^i]_\alpha{}^\beta \times (\lambda_i)_p{}^q \tag{4}$$

where $\delta_\alpha{}^\beta$, γ_5, $\gamma_\mu\gamma_5$, γ_μ, and $\sigma_{\mu\nu}$ represent the sixteen generators of $\tilde{U}(4)$ while λ_i ($i=0,\cdots 8$) are the familiar matrices of $SU(3)$.[6] To each $\tilde{U}(12)$ index on the left there corresponds a pair of indices on the right. Greek letters indicate $\tilde{U}(4)$ indices which run from one to four while Latin indices stand for the $SU(3)$ part and run from one to three. To make (4) represent the physical particles, we have to impose the equations of motion. $\phi_B{}^A$ transforms like the direct product of a quark and an antiquark. The equations of motions amount to applying the Dirac equation to each $\tilde{U}(4)$ index: $\not{p} = p^\mu\gamma_\mu$

$$(\not{p})_\alpha{}^\gamma\phi_{\gamma,p}{}^{\beta,q} = \mu\phi_{\alpha,p}{}^{\beta,q}, \quad (\not{p})_\gamma{}^\beta\phi_{\alpha,p}{}^{\gamma,q} = -\mu\phi_{\alpha,p}{}^{\beta,q}, \tag{5}$$

μ is the meson mass. Clearly Eqs. (5) are not $\tilde{U}(4)$ covariant since the four γ matrices are singled out. A number of $\tilde{U}(12)$ noncovariant subsidiary conditions follow from these equations. The subsidiary conditions eliminate the scalar part of **143** and relate the pseudovector to the pseudoscalar as well as the tensor to the vector part.[1,2] Under these subsidiary conditions expression (4) for $\phi_B{}^A$ reduces to

$$\phi_B{}^A(q) = \left\{\left(1+\frac{q}{\mu}\right)\left[\gamma_5\left(P_q{}^p + \frac{X^0}{\sqrt{3}}\delta_q{}^p\right) + \gamma^\mu\epsilon_\mu{}^{(\lambda)}V^{(\lambda)}{}_q{}^p\right]\right\}_\beta^\alpha \tag{6}$$

which contains only the pseudoscalar-meson octet P and singlet X^0 and the vector-meson nonet V. The matrices P and V are

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta^0}{\sqrt{6}} \end{pmatrix},$$

$$V = \begin{pmatrix} (\rho^0+\omega^0)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (\omega^0-\rho^0)/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \varphi^0 \end{pmatrix},$$

The ω-φ mixing which follows from $\tilde{U}(12)$ and a mass splitting interaction which transforms like our spurion (8) has been incorporated in V. The mesons η^0 and X^0 seem to have only very little mixing.[7] Spurions are not subject to equations of motion. We can choose any desired component or combination of components of a representation to be a spurion depending on what symmetry violation we want to simulate. To preserve Lorentz invariance however a spurion should be a

[6] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[7] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965); **38**, 1945 (1965).

Lorentz scalar and for strong interactions we want parity to be conserved. To form a $\tilde{U}(4)$ breaking spurion,[8] which belongs to the representation **143** of $\tilde{U}(12)$, we contract the permissible Dirac matrices in expression (4) with the two independent four-momenta of the vertex and write $\not{p}=p\cdot\gamma$

$$\Gamma_B{}^A=[a+b\not{p}_1+c\not{p}_2+d\not{p}_1\not{p}_2]_\beta{}^\alpha\cdot\delta_q{}^p. \quad (7)$$

Spurions which belong to higher representations can be formed analogously.

The $SU(3)$-breaking spurion $S_A{}^B$ is a singlet under the transformations of $\tilde{U}(4)$ and transforms like λ_8 under $SU(3)$.

$$S_B{}^A\propto\delta_\alpha{}^\beta\cdot(\lambda_8)_q{}^p. \quad (8)$$

The baryons are described by the fully symmetric tensor ψ_{ABC} of the 364-dimensional representation. It decomposes under $\tilde{U}(4)\otimes SU(3)$ to give

$$\psi_{ABC}=\tfrac{1}{2}(\sqrt{\tfrac{3}{2}})D_{\alpha\beta\gamma,pqr}+\epsilon_{pqr}V_{[\alpha\beta\gamma]}+\frac{1}{2\sqrt{6}}$$
$$\times(\epsilon_{pqs}N_{[\alpha\beta]\gamma,r}{}^s+\epsilon_{qrs}N_{[\beta\gamma]\alpha,p}{}^s+\epsilon_{rps}N_{[\gamma\alpha]\beta,q}{}^s). \quad (9)$$

Use of the Bargmann-Wigner equations shows that the totally antisymmetric singlet $V_{[\alpha\beta\gamma]}$ vanishes. $D_{\alpha\beta\gamma,pqr}$ is fully symmetric in its $\tilde{U}(4)$ as well as in its $SU(3)$ indices and represents the spin-$\frac{3}{2}^+$ baryon decuplet. It can be represented in terms of the Rarita-Schwinger[9] wave function ψ_μ for spin-$\frac{3}{2}$ particles and the decuplet wave function d_{pqr} of $SU(3)$ as

$$D_{\alpha\beta\gamma,pqr}(p)=\frac{1}{m}[(\not{p}+m)\gamma_\mu C]_{\alpha\beta}\psi_\gamma{}^\mu(p)\cdot d_{pqr}, \quad (10)$$

where C is the charge conjugation matrix and insures the right symmetry for the $\tilde{U}(4)$ indices. The decuplet wave function is defined as usual by

$$\begin{aligned}
&d_{111}=N^{*++},\\
&d_{112}=N^{*+}/\sqrt{3}, \quad d_{122}=N^{*0}/\sqrt{3}, \quad d_{222}=N^{*-},\\
&d_{113}=Y^{*+}/\sqrt{3}, \quad d_{123}=Y^{*0}/\sqrt{6}, \quad d_{223}=Y^{*-}/\sqrt{3},\\
&d_{133}=\Xi^{*0}/\sqrt{3}, \quad d_{223}=\Xi^{*-}/\sqrt{3}, \quad d_{333}=\Omega^-.
\end{aligned}$$

$\epsilon_{pqs}N_{[\alpha\beta]\gamma,r}{}^s$ is of mixed symmetry. Dropping the $SU(3)$ indices we have, for instance,

$$N_{[\alpha\beta]\gamma}+N_{[\gamma\alpha]\beta}+N_{[\beta\gamma]\alpha}=0, \quad N_{[\alpha\beta]\gamma}=-N_{[\beta\alpha]\gamma}.$$

The same is true for the corresponding $SU(3)$ indices. $N_{[\alpha\beta]\gamma,r}{}^s$ represents the spin-$\frac{1}{2}^+$ baryon octet, and can be expressed in terms of ordinary Dirac spinors and the usual $SU(3)$ baryon octet wave function $b_r{}^s$ as

$$N_{[\alpha\beta]\gamma,r}{}^s(p)=\frac{1}{m}[(\not{p}+m)\gamma_5 C]_{\alpha\beta}\psi_\gamma(p)\cdot b_r{}^s \quad (11)$$

$$b_r{}^s=\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}.$$

4. THE EFFECT OF $SU(3)$ BREAKING

$\tilde{U}(12)$ drastically reduces the number of coupling parameters for the meson baryon vertex. With Lorentz invariance and charge independence 132 coupling constants are needed to describe all the vertices between the eight-spin-$\frac{1}{2}^+$ baryons N, the ten-spin-$\frac{3}{2}^+$ baryons D, the nine pseudoscalar mesons P, and the nine vector mesons V. Formal $\tilde{U}(12)$ relates all these 132 coupling constants to a single coupling parameter. These interactions are contained in the effective interaction density (1). We expect some deviations from the symmetry scheme. In particular, we know already that there are deviations from $SU(3)$ symmetry, which is a subgroup of $\tilde{U}(12)$. We want to study how this reflects itself in the $\tilde{U}(12)$ scheme. Usually the $SU(3)$ breaking term is assumed to transform like the $Y=0$, $I=0$ component of an octet. Correspondingly we break $\tilde{U}(12)$ by the component of its self-adjoint representation **143** which is a singlet in $\tilde{U}(4)$ space and the $Y=0$, $I=0$ component of an octet in $SU(3)$ space. Thus we insert the spurion (8) into the interaction density (2). Due to charge conjugation invariance, g_1 equals g_1'. Further we find that the first term in (2) vanishes for the spurion (8). It follows that in broken $\tilde{U}(12)$ we can express the 132 coupling constants of the meson baryon vertices in terms of only three parameters; g_0, g_1, and g_2. This goes far beyond broken $SU(3)$. There one already needs seven parameters just to interrelate the twelve NNP coupling constants,[10] five more parameters for the twelve DNP coupling constants,[11] and so on.

Explicit calculation gives for the NNP interaction[12]

$$M_{NNP}=\frac{1}{12}\bar{u}(p_2)\gamma_5 u(p_1)\frac{(m_1+m_2)^2-q^2}{m_1m_2}\times\left(1+\frac{m_1+m_2}{\mu}\right)G_{NNP}, \quad (12)$$

[8] The $\tilde{U}(4)\otimes SU(3)$ content of higher representations of $\tilde{U}(12)$ is given by A. Salam, R. Delbourgo, M. A. Rashid, and J. Strathdee, Proc. Roy. Soc. (London) **A285**, 312 (1965).

[9] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

[10] M. Muraskin and S. L. Glashow, Phys. Rev. **132**, 482 (1963).

[11] V. Gupta and V. Singh, Phys. Rev. **135**, B1442 (1964); C. Becchi, E. Eberle, and G. Morpurgo, *ibid.* **136**, B808 (1964).

[12] The labeling of the coupling constants and matrix elements stands for baryon 1 $\to$ baryon 2+ meson. m_1, m_2 are the masses of baryons 1 and 2, respectively, and μ is the meson mass.

where $q=p_1-p_2$ is the meson momentum. In formula (12) the matrix element is multiplied by an expression depending on the masses. G_{NNP} contains only Clebsch-Gordan coefficients and the three coupling parameters. The conventional coupling constants g_{NNP} are related to G_{NNP} by

$$g_{NNP}=\frac{1}{12}\frac{(m_1+m_2)^2-\mu^2}{m_1m_2}\left(1+\frac{m_1+m_2}{\mu}\right)G_{NNP}\,, \quad (13)$$

By evaluation of a number of $SU(3)$ traces we find the Eqs. (14).[13] All other NNP coupling constants can be obtained from the 16 following ones by charge independence[12]:

$$\begin{aligned}
G_{np\pi^-}&=5(g_0+2g_1+2g_2)\,,\\
G_{\Xi^-\Xi^0\pi^-}&=g_0+2g_1-4g_2\,,\\
(\surd 6)G_{\Sigma^-\Lambda\pi^-}&=6(g_0+2g_1-g_2)\,,\\
\sqrt{2}G_{\Sigma^-\Sigma^0\pi^-}&=4(g_0+2g_1-g_2)\,,\\
\sqrt{2}G_{p\Sigma^0K^+}&=g_0-g_1+2g_2\,,\\
(\surd 6)G_{p\Lambda K^+}&=-9(g_0-g_1+2g_2)\,,\\
\sqrt{2}G_{\Sigma^0\Xi^-K^+}&=5(g_0-g_1-g_2)\,,\\
(\surd 6)G_{\Lambda\Xi^-K^+}&=3(g_0-g_1-g_2)\,,\\
(\surd 6)G_{pp\eta}&=3(g_0+6g_1+2g_2)\,,\\
(\surd 6)G_{\Xi^0\Xi^0\eta}&=-9(g_0-(14/3)g_1-\tfrac{4}{3}g_2)\,,\\
(\surd 6)G_{\Sigma^0\Sigma^0\eta}&=6(g_0+2g_1)\,,\\
(\surd 6)G_{\Lambda\Lambda\eta}&=-6(g_0-6g_1+2g_2)\,,\\
G_{ppX^0}&=\sqrt{3}(g_0+2g_1+2g_2)\,,\\
G_{\Xi\Xi X^0}&=\sqrt{3}(g_0-6g_1)\,,\\
G_{\Sigma\Sigma X^0}&=\sqrt{3}(g_0+4g_1-2g_2)\,,\\
G_{\Lambda\Lambda X^0}&=\sqrt{3}(g_0-4g_1+2g_2)\,.
\end{aligned} \quad (14)$$

A large number of sum rules follow from Eqs. (14). In particular, we find that the following relations from formal $\tilde{U}(12)$ symmetry still hold in the broken symmetry:

$$\begin{aligned}
G_{\Sigma^-\Sigma^0\pi^-}&=(2/\sqrt{3})G_{\Sigma^-\Lambda\pi^-}\,,\\
G_{p\Lambda K^+}&=-3\sqrt{3}G_{p\Sigma^0K^+}\,,\\
G_{\Sigma^0\Xi^-K^+}&=(5/\sqrt{3})G_{\Lambda\Xi^-K^+}.
\end{aligned} \quad (15)$$

Relations (14) also obey the sum rules of broken $SU(3)$.[10] One can evaluate g_1 from $G_{np\pi^-}$ and $G_{p\Lambda K^+}$, which are known from experiment:

$$g_1=(1/15)G_{np\pi^-}+\tfrac{1}{9}(\surd\tfrac{2}{3})G_{p\Lambda K^+} \quad (16)$$

The other two parameters shall be expressed in terms of the spin-$\frac{3}{2}^+$ resonance widths. To this end we consider the DNP coupling constants. From (2) the corresponding matrix element is derived to be[12]

$$M_{DNP}=(q^\mu/m_2)\bar{u}(p_2)u_\mu(p_1)g_{DNP}\,, \quad (17)$$

$$g_{DNP}=\left(1+\frac{m_1+m_2}{\mu}\right)G_{DNP}\,. \quad (18)$$

Evaluating the $SU(3)$ algebra for G_{DNP}, we find the following relations[12]:

$$\begin{aligned}
(\surd 6)G_{\Xi^{*-}\Xi^-\pi^0}&=-g_0-2g_1+4g_2\,,\\
(\surd 6)G_{Y_1^{*-}\Sigma^-\pi^0}&=-g_0-2g_1+g_2\,,\\
\sqrt{2}G_{Y_1^{*-}\Lambda\pi^-}&=-g_0-2g_1+g_2\,,\\
G_{N^{*-}n\pi^-}&=g_0+2g_1+2g_2\,,\\
G_{N^{*-}\Sigma^-K^0}&=-g_0+g_1-2g_2\,,\\
\sqrt{3}G_{Y_1^{*-}nK^-}&=g_0-g_1+2g_2\,,\\
\sqrt{2}G_{Y_1^{*-}\Sigma^-\eta}&=g_0-2g_1+g_2\,,\\
\sqrt{3}G_{Y_1^{*-}\Xi^-K^0}&=-g_0+g_1+g_2\,,\\
\sqrt{2}G_{\Xi^{*-}\Lambda K^-}&=-g_0+g_1+g_2\,,\\
(\surd 6)G_{\Xi^{*-}\Sigma^0K^-}&=g_0-g_1-g_2\,,\\
\sqrt{2}G_{\Xi^{*-}\Xi^-\eta}&=g_0-2g_1-2g_2\,,\\
G_{\Omega^-\Xi^-K^0}&=g_0-g_1-4g_2\,,\\
G_{\Xi^{*-}\Xi^-X^0}&=G_{Y_1^{*-}\Sigma^-X^0}=\sqrt{3}(2g_1-g_2)\,.
\end{aligned} \quad (19)$$

All other DNP coupling constants are obtained from the fourteen above through charge independence. Relations (19) satisfy the sum rules derived in broken $SU(3)$ symmetry.[11] However, broken nonrelativistic $SU(6)$ leads to different results.[14] The first four coupling constants in the relations (19) are related to the observed resonance widths. Now we can express the remaining parameters

$$g_2=\tfrac{1}{3}[G_{N^{*-}n\pi^-}+\sqrt{2}G_{Y_1^{*-}\Lambda\pi^-}]\,, \quad (20)$$

$$g_0+2g_1=\tfrac{1}{3}[G_{N^{*-}n\pi^-}-2\sqrt{2}G_{Y_1^{*-}\Lambda\pi}]\,. \quad (21)$$

Only the combination g_0+2g_1 can be determined from the resonance widths, since g_0 and g_1 appear in this same combination in all processes involving pions or ρ mesons. Combining (16) and (21) we can obtain g_0 alone. We also note the relations

$$\begin{aligned}
G_{Y_1^*\Lambda\pi^-}&=\sqrt{3}G_{Y_1^*\Sigma^-\pi^0}\,,\\
G_{N^{*-}\Sigma^-K^0}&=\sqrt{3}G_{Y_1^{*-}nK^-}\,,\\
G_{\Xi^{*-}\Lambda K^-}&=-\sqrt{3}G_{\Xi^{*-}\Sigma^0K^-}=(\surd\tfrac{3}{2})G_{Y_1^{*-}\Xi^-K^0}\,,
\end{aligned} \quad (22)$$

and the sum rule

$$G_{N^{*-}n\pi^-}+2(\surd 6)G_{Y_1^{*-}\Sigma^-\pi^0}=(\surd 6)G_{\Xi^{*-}\Xi^-\pi^0}. \quad (23)$$

For formal $\tilde{U}(12)$ symmetry there is no DNX^0 coupling. The next-order spurions belongs to the representations

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[13] Performing the $SU(3)$ calculation the relation

$$\epsilon^{pqr}\epsilon_{lmn}A_p{}^lB_q{}^mC_r{}^n=Sp(ABC)+Sp(ACB)-Sp(A)\cdot Sp(BC)$$
$$-Sp(B)\cdot Sp(AC)-Sp(C)\cdot Sp(AB)+Sp(A)\cdot Sp(B)\cdot Sp(C)$$

is useful.

[14] C. Chan and A. Sarker, Phys. Rev. **139**, B626 (1965) and R. Ferrari and M. Konuma, Phys. Rev. Letters **14**, 378 (1965). We do not agree with M. Konuma and E. Remiddi, Nuovo Cimento **38**, 662 (1965).

4212 and **5940** of $\tilde{U}(12)$. Choosing these spurions to be singlets in $\tilde{U}(4)$ space and the $Y=0$, $I=0$ components of an octet in $SU(3)$ we get nothing new except a contribution to the singlet and eighth component of the octet-meson coupling.

Next, we turn to the NNV interaction. There we have two coupling constants[12]

$$M_{NNV}=f_{NNV}\frac{P^\mu}{2(m_1m_2)^{1/2}}\bar{u}(p_2)u(p_1)\epsilon_\mu(q)+g_{NNV}\bar{u}(p_2)\frac{r^\mu}{4m_1m_2}u(p_1)\epsilon_\mu(q)\,, \quad (24)$$

where $P=p_1+p_2$, $q=p_1-p_2$, and $r^\mu=\epsilon^{\mu\nu k\lambda}P_\nu q_k\gamma_\lambda\gamma_5$. The connection between the invariants in (24) and the ones usually used is

$$\bar{u}(p_2)r^\mu u(p_1)=[P^2-(m_1-m_2)^2]\bar{u}\gamma_\mu u-(m_1+m_2)\times P^\mu\bar{u}u-(m_1-m_2)q^\mu\bar{u}u\,,$$

$$\bar{n}(p_2)\sigma^{\mu\nu}q_\nu u(p_1)=\bar{u}(p_2)[(m_1+m_2)\gamma_\mu-P_\mu]u(p_1)\,.$$

The invariants in (24) are convenient for electromagnetic interactions. Their coefficients f_{NNV} and g_{NNV} are directly related to the total charge and magnetic moment.

The pure $SU(3)$ parts F_{NNV} and G_{NNV} defined by

$$f_{NNV}=\frac{m_1+m_2+q^2/\mu}{2(m_1m_2)^{1/2}}F_{NNV}\,, \quad (25)$$

$$g_{NNV}=\frac{1}{3}\left(1+\frac{m_1+m_2}{\mu}\right)G_{NNV}\,, \quad (26)$$

are worked out to be[12]

$$\begin{aligned}
F_{np\rho^-}&=g_0+2g_1-2g_2\,,\\
F_{\Xi^-\Xi^0\rho^-}&=-g_0-2g_1-4g_2\,,\\
F_{\Sigma^-\Lambda\rho^-}&=0\,,\\
\sqrt{2}F_{\Sigma^-\Sigma^0\rho^-}&=2(g_0+2g_1+g_2)\,,\\
\sqrt{2}F_{p\Sigma^0K^{*+}}&=-g_0+g_1+2g_2\,,\\
(\sqrt{6})F_{p\Lambda K^{*+}}&=-3(g_0-g_1-2g_2)\,,\\
\sqrt{2}F_{\Sigma^0\Xi^-K^{*+}}&=g_0-g_1+g_2\,,\\
(\sqrt{6})F_{\Lambda\Xi^-K^{*+}}&=3(g_0-g_1+g_2)\,,\\
F_{pp\varphi}&=4(g_1+g_2)\,,\\
F_{\Xi^0\Xi^0\varphi}&=2(g_0-2g_1-g_2)\,,\\
F_{\Sigma^-\Sigma^-\varphi}&=g_0-2g_2\,,\\
F_{\Lambda\Lambda\varphi}&=g_0-\tfrac{2}{3}g_2\,,\\
\sqrt{2}F_{pp\omega}&=3g_0+2g_1+2g_2\,,\\
\sqrt{2}F_{\Xi^-\Xi^-\omega}&=g_0-2g_1-4g_2\,,\\
\sqrt{2}F_{\Sigma^-\Sigma^-\omega}&=2(g_0+g_2)\,,\\
\sqrt{2}F_{\Lambda\Lambda\omega}&=2(g_0+\tfrac{1}{3}g_2)\,,
\end{aligned} \quad (27)$$

for the term in (25). Its g_0 part is of F type. G_{NNV} is obtained from Eqs. (14) by the substitutions:

$$\pi\to\rho\,,\quad K\to K^*\,,\quad \eta\to(1/\sqrt{3})\omega-(\sqrt{\tfrac{2}{3}})\phi\,,\quad X^0\to(\sqrt{\tfrac{2}{3}})\omega+(1/\sqrt{3})\phi \quad (28)$$

Its g_0 part is of $3D+2F$ type, as was the NNP interaction. Substitution (28) reflects the ω-ϕ mixing.

The DDP, DDV, and DNV vertices contain more than one resonance and shall not be considered here.

5. $\tilde{U}(4)$ BREAKING

To make the representations of $\tilde{U}(12)$ correspond to representations of the inhomogeneous Lorentz group which can be identified with physical particles, we imposed on them the equations of motion. But since the equations of motion are not covariant under $\tilde{U}(4)$, this introduced a number of noncovariant subsidiary conditions. These subsidiary conditions eliminated some components of the respective representation of $\tilde{U}(12)$ and interrelated others. We then used the truncated expressions (5), (10), and (11) to evaluate the interaction Hamiltonian (1). This procedure we shall call formal $\tilde{U}(12)$ invariance. Diagrams involving dynamical quantities like propagators will, however, give rise to further symmetry violation. To simulate such higher order effects we shall introduce $\tilde{U}(4)$-breaking spurions. Experimental indication for $\tilde{U}(4)$ breaking comes from the polarization of the outgoing particles in certain scattering processes[4] which should not occur according to the formal symmetry. In the presence of $\tilde{U}(4)$-breaking spurions such polarization effects are no longer forbidden.[5,15] The same is true for meson-pair production from baryon-antibaryon annihilation at rest. In the latter case one needs higher order spurions.

Considering spurions which belong to the self-adjoint representation **143** of $\tilde{U}(12)$, we arrive at expression (7) using the arguments in Sec. 3. Inserting (7) in (2), we see that except for the first term in (2), spurion and meson are sandwiched between baryon expressions demanding $\mathbf{p}_1=m_1$ and $\mathbf{p}_2=m_2$, respectively. The last term in (2) is most easily handled. It only contributes to the over-all form factor. The g_1 term contributes differently to the two form factors of the vector-meson vertex which could be used to adjust the absolute value of the proton magnetic moment precisely to its experimental value.[16] The magnetic-moment ratio however is not affected since the D/F ratio remains unchanged.[15] The first term only contributes for the vector-meson singlet.[5] It gives

$$M_{NNV}=F(q^2,m_1,m_2,\mu)\bar{u}(p_2)u(p_1)(p_1+p_2)^\mu\epsilon_\mu Sp(\bar{b}b)v_0\,.$$

For the vertices involving pseudoscalar mesons, spurion (6) gives a change in the over-all form factor. A variety of $\tilde{U}(4)$-breaking spurions can be constructed

[15] R. Oehme, Phys. Rev. Letters **14**, 664 (1965); Enrico Fermi Institute of Nuclear Studies report EFINS-63-37 (unpublished).

[16] Another way to reduce the expression for the proton magnetic moment is by specific use of the pole model. See P. G. O. Freund and R. Oehme, Phys. Rev. Letters **14**, 1085 (1965); D. Flamm, Nuovo Cimento **38**, 291 (1965).

TABLE I. Resonance width Γ and effective coupling constants.[a]

Decay	m_1 (MeV)	m_2 (MeV)	$\lvert p\rvert$ (MeV)	Γ (MeV)	$g_{DNP}/\sqrt{(4\pi)}$
$N^{*-}\to n+\pi^-$	1236.0	939.6	229.6	120	4.1
$Y_1^{*-}\to\Lambda+\pi^-$	1382.1	1115.4	205.0	48 ±2	−3.6
$Y_1^{*-}\to\Sigma^-+\pi^0$	1382.1	1197.1	117.8	1.5±0.7	−1.5
$\Xi^{*-}\to\Xi^-+\pi^0$	1529.7	1321.0	147.4	2.5±0.6	−1.5

[a] See Ref. 22.

which belong to the representations **4212** and **5940** of $\tilde{U}(12)$. In the frame of these representations we can form also momentum-independent spurions by contracting the Lorentz indices of two Dirac matrices. However, since there is only one matrix element in Lorentz space and one in $SU(3)$ space for the DNP-vertex, even an arbitrary $\tilde{U}(4)$-breaking spurion merely changes the over-all form factor. Nevertheless it is possible to split the NNP from the DNP-coupling constants. With spurion (29) of **5940**, for instance, the last term of Hamiltonian (3) only contributes to the NNP and not to the DNP vertex.

$$S_{\{BC\}}{}^{\{DE\}}=\Sigma(\gamma_5)_\beta{}^\delta(\gamma_5)_\gamma{}^\epsilon\cdot\delta_p{}^r\delta_q{}^s, \qquad (29)$$

where Σ stands for symmetrization of all index pairs. The N^* width calculated from $\tilde{U}(12)$ symmetry using the pion nucleon coupling constant as input turns out to be about 20% smaller than its experimental value. Spurion (29) may account for this discrepancy.

We shall now concentrate on the baryon resonance width and investigate simultaneous $\tilde{U}(4)$ and $SU(3)$ breaking. A spurion which belongs to the representation **143** of $\tilde{U}(12)$ and whose $SU(3)$ part transforms like λ_8 does not give anything new. With the variety of spurions contained in **4212** and **5940** however we can essentially reduce the symmetry to the direct product of the Lorentz group and broken $SU(3)$. Let us, for example, consider spurion (30) of **5940**:

$$S_{\{BC\}}{}^{\{DE\}}=\Sigma(\gamma^\mu)_\beta{}^\delta(\gamma_\mu)_\gamma{}^\epsilon(\lambda_8)_p{}^r\delta_q{}^s, \qquad (30)$$

where Σ again stands for symmetrization of the index pairs. Spurion (30) can also be used for the construction of the baryon and meson mass operator in $\tilde{U}(12)$.[2,15] If inserted in the second term of Hamiltonian (3) it gives a new combination of the two $SU(3)$ matrix elements. We still get a sum rule for the four resonance width and it is equal to the sum rule which follows from broken $SU(3)$ symmetry.[11]

$$G_{N^{*-}n\pi^-}+(3/\sqrt{2})G_{Y_1^{*-}\Lambda\pi^-}+(\sqrt{\tfrac{3}{2}})G_{Y_1^{*-}\Sigma^-\pi^0} = (\sqrt{6})G_{\Xi^{*-}\Xi^-\pi^0}. \qquad (31)$$

6. COMPARISON WITH EXPERIMENTAL DATA

Besides decay processes, several models and extrapolation procedures are used to extract coupling constants from experimental data. The renormalized pion-nucleon coupling constant, for instance, can be found from scattering data using the static model.[17] Another way to determine it is from photopion production.[18] Similarly the $p\Lambda K$-coupling constant can be obtained from photoproduction data.[19] If a scattering reaction in a certain region is dominated by one-particle exchange the coupling constants involved can be found by extrapolating the scattering amplitude to the pole.[20] From decay processes the coupling constant is most easily found. The baryon resonances provide a good example for this case. For the spin-$\frac{3}{2}$ resonance width the matrix element (18) leads to the formula[21,12]

$$\Gamma=\frac{1}{\tau}=\frac{g_{DNP}{}^2}{4\pi}\frac{2}{3}\frac{p^3}{m_2{}^2}\frac{m_2}{m_1}\tfrac{1}{2}[1+(1+p^2/m_2{}^2)^{1/2}]. \qquad (32)$$

Equation (32) depends very strongly on the center of mass momentum p of the pion

$$p^2=(1/4m_1{}^2)(m_1{}^2-m_2{}^2+\mu^2)^2-\mu^2.$$

The effective coupling constants obtained from Eq. (32) and the experimental width[22] are given in Table I. The coupling constants satisfy the sum rule (23).[23] Comparing with Eq. (22) we find that $\sqrt{3}g_{Y_1^{*-}\Sigma^-\pi^0}$ is about 27% smaller than $g_{Y_1^*\Lambda\pi^-}$, which is quite reasonable, since the measurement of the $Y_1^{*-}\to\Sigma^-+\pi^0$ width is within errors of 50%.

In conclusion the sum rules for the resonance width are reasonably well satisfied. These sum rules persist to hold for $\tilde{U}(4)$-breaking spurions which belong to the representation **143.** Besides the resonance width only the pion-nucleon constant is well known. As discussed in Sec. 5 there may be indications for higher order spurions.

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[19] M. Moravcsik, Phys. Rev. Letters **2**, 352 (1959); G. Morpurgo, Ann. Rev. Nucl. Sci. **11**, 41 (1961).

[20] G. F. Chew, Phys. Rev. **112**, 1380 (1958) and G. F. Chew and F. E. Low, *ibid.* **113**, 1640 (1959).

[21] For projection operators of higher spin fields see, for instance, D. M. Brudnoy, Phys. Rev. Letters **14**, 273 (1965).

[22] The experimental data are taken from A. H. Rosenfeld, A. Barbo-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **36**, 977 (1964); and University of California Radiation Laboratory Report UCRL-8030, 1965 (unpublished).

[23] The sum rules for G_{DNP} also hold for g_{DNP} if we use degenerate masses. Using physical masses in the matrix element and at the same time spurion (8) would correspond to a higher order in the $SU(3)$-breaking interaction. Moreover, the mass dependence in Eq. (18) would be altered by $\tilde{U}(4)$ breaking.