Meson Production in Antiproton-Nucleon Annihilation*

MICHAEL PARKINSON†‡§
The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, University of Chicago, Chicago, Illinois

(Received 4 October 1965)

The antiproton-nucleon system at rest and its annihilation are discussed. Final states of two mesons, of many mesons with all but one in a vector resonance, and of two vector resonances are the main concern, although the decay to lepton pairs and other properties of the $\bar{p}$-nucleon system are also considered. The basic assumptions are that annihilations proceed mainly through an intermediate vector meson (${}^3S_1$ decay) or pseudoscalar meson (${}^1S_0$ decay) and also that $SU(3)$ symmetry is valid for obtaining unknown coupling constants. The unknown parameters in the $SU(3)$ couplings are determined by fitting to experiment, with the following interesting results: $g_{\rho NN}\gtrsim g_{\omega NN}$ and $g_{\eta^* NN}\approx -2g_{\pi NN}$, where η^* refers to the 960-MeV $\eta\pi\pi$ resonance, and the above coupling constants for the ρ and ω are the sum of the γ_μ and $\sigma_{\mu\nu}q_\nu$ coefficients. $g_{\rho NN}\gtrsim g_{\omega NN}$ at $q^2=-4m_N^2$ is shown to be consistent with nuclear-force calculations around $q^2=0$, and the above value for $g_{\eta^* NN}$ helps explain the rapid increase with energy of the proton Compton-scattering cross section. The $SU(3)$ $\bar{B}BV$ interaction is determined from the rates for ${}^3S_1$ annihilations of $\bar{p}p$ to $\pi^+\pi^-\pi^0$, $\pi^+\pi^-$, K^+K^-, and K_1K_2, the VVP coupling from $\omega\to 3\pi$ and $\phi\to 3\pi$, and the $\bar{B}BP$ interaction from the ${}^1S_0$ decays of $\bar{p}p$ to $K^*\bar{K}^*$, $\rho^0\rho^0$, and $\rho^0\omega^0$. We discuss briefly the relation of the $SU(3)$ couplings determined here and the results of broken $SU(6)$ and $U(12)$. In particular, we show that $d/f=2$ for $\bar{B}BV$ at $q^2=-4m_N^2$, in contradiction to the value $\frac{3}{2}$ given by $SU(6)$ and $U(12)$; we also conclude that the d/f ratio is not constant for the $\bar{B}BV$ coupling. A table of relative rates and graphs of expected mass spectra are given for final states consisting of a vector resonance plus pseudoscalar meson, and a comparison with the available experimental evidence is made. The approximate agreement expected is observed. The discussion of the vector-vector final states leads to the prediction that $\Gamma(\bar{p}p\to\rho^0\rho^0)\approx\Gamma(\bar{p}p\to\omega^0\omega^0)$, and we also determine that $(\bar{p}p\to e^+e^-)/(\bar{p}p\to\pi^+\pi^-)\approx 2\times 10^{-5}$. Finally we conclude that the assumptions made are thus far consistent with experiment.

1. INTRODUCTION

WE will discuss in this paper both the antiproton-proton system and the related antiproton-neutron system. Experimentally, a true $\bar{p}n$ system is not as yet feasible, but one generally assumes that the deuteron is loosely bound enough so that the neutron is almost free and thus still a neutron.[1]

The antiproton-nucleon system is an interesting one in many ways; its study should shed light on various aspects of particle phsyics. In particular, we will consider the role of resonances in the annihilation process to final states consisting of leptons, and also many mesons. With regard to the meson final states, we will consider those in which there are two pseudoscalar mesons and in which the mesons, except for one, are in a state suitable for the production of a vector-meson resonance; i.e., a vector-pseudoscalar final state. We will also discuss the vector-vector final states.

Our purpose is to give theoretical predictions for the branching ratios and for the mass spectra of the above states.

The reason for considering these meson states can be understood in terms of the old problem of explaining why multimeson final states were so much more frequent than the statistical model predicted.[2] The (predicted) explanation was given in 1960 by Sakurai[3] who suggested that the cause was to be found in the existence and fairly copious production of vector-meson resonances. Such vector mesons obviously make the unadorned statistical model incorrect. A more refined statistical model would tend to treat the vector mesons on an equal footing with the pseudoscalars.[4] This is, of course, in keeping with the "nuclear democracy" of S-matrix theory,[5] although there remains the problem, due to the relatively large widths of some of the resonances, of accounting properly for resonance production in the wings of the peak.

Here, we will not use a statistical approach at all, but will attempt an approach from first principles. We will use dispersion theory to provide the basis for the approximations used and field theory, in the perturbation expansion, for the calculations. Our approximation is perhaps the most obvious one. We will assume that the vector-meson poles in the s channel dominate the amplitude. The residues of the poles (coupling constants) will be determined from other experiments and when necessary from $SU(3)$.

In this way, we can predict branching ratios for all the final states mentioned above (see Table II) and also give their expected mass spectra (see Figs. 3 through 9).

* This work is supported in part by the U. S. Atomic Energy Commission.
† National Science Foundation pre-doctoral fellow.
‡ Thesis submitted to the Department of Physics of the University of Chicago in partial fulfillment of the requirements for the Ph.D. degree.
§ Present address: Department of Physics, University of California, Los Angeles, California.
[1] The proposition that the deuteron ought to be regarded as a bound state of proton and neutron has had public debate. See S. Weinberg, Phys. Rev. **137**, B672 (1965).

[2] E. Segrè, Ann. Rev. Nucl. Sci. **8**, 127 (1958).
[3] J. J. Sakurai, Ann. Phys. **11**, 1 (1960).
[4] See, for example, G. R. Kalbfleisch, Phys. Rev. **127**, 971 (1962) where the results of such a calculation are presented for $\bar{p}p$ in flight.
[5] See G. Chew, Physics **1**, 77 (1965).

The annihilation of antiprotons has been discussed before,[6] but in somewhat different contexts from that here. Lapidus and Shpiz[6] discuss $\bar{p}$–p annihilations in flight with the same assumption of vector-meson dominance, but do not discuss the relation to $SU(3)$. The other papers of Ref. 6 discuss only $SU(3)$ [and in the last paper of Ref. 6 also $SU(4)$] sum rules, but do not consider dynamics.

2. PROPERTIES OF THE $\bar{p}$-NUCLEON SYSTEM

At rest, and in an S state, the $\bar{p}$-nucleon system will behave in its interactions with other particles as though it were either a heavy pseudoscalar meson ($^1S_0$ state) or a heavy vector meson ($^3S_1$). When stopping $\bar{p}$'s in a hydrogen or deuterium bubble chamber, the arguments of Day, Snow, and Sucher[7] apply, indicating that S-state capture, although not necessarily from the lowest energy state, will predominate. One would then expect, on statistical grounds and without taking into account any constraints such as those imposed by C, P, and T invariance, that $^3S_1$:$^1S_0$ annihilation is 3:1. This expectation is not confirmed, since $\rho\pi$ production seems to come primarily, if not only, from the $^3S_1$ state, whereas the nonresonant $\pi^+\pi^-\pi^0$ (no ρ's) final state seems to come primarily from the $^1S_0$ state.[8] The reason for this behavior is not clear.

We will therefore assume that vector-pseudoscalar final states come only from the $^3S_1$ state of the $\bar{p}$-nucleon system.[9] This assumption is not needed for the two pseudoscalar-meson final states, since parity ensures that they cannot come from $^1S_0$.

As far as the vector-vector final states go, they do not seem to come predominantly from the $^3S_1$ state, since $\rho^0\rho^0$ and $\rho^0\omega^0$ are seen and come only from $^1S_0$, not $^3S_1$.[10]

The lepton final states (e^+e^- or $\mu^+\mu^-$) should also come predominantly from $^3S_1$, since then only one photon is needed to produce the lepton pair, whereas $^1S_0 \not\rightarrow \gamma$ by C invariance. One would expect, therefore, that $^3S_1 \rightarrow e^+e^-/^1S_0 \rightarrow e^+e^- \sim 137$.

[6] K. Tanaka, Phys. Rev. **135**, B1186 (1964); I. R. Lapidus and J. M. Shpiz, *ibid.* **138**, B178 (1965); Y. Dothan *et al.*, Phys. Letters **1**, 309 (1962); M. Parkinson, Phys. Rev. Letters **13**, 288 (1964).

[7] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters **3**, 61 (1959) where they considered K^-p. An analysis for $\bar{p}p$ was performed by B. R. Desai, Phys. Rev. **119**, 1385 (1960).

[8] G. B. Chadwick *et al.*, Phys. Rev. Letters **10**, 62 (1963); M. Cresti *et al.*, in *Proceedings of the Sienna International Conference on Elementary Particles, 1963*, edited by G. Bernadini and C. P. Puppi (Società Italina di Fisica, Bologna, 1963), p. 263; N. Gelfand, University of Chicago (private communication) regarding $\rho\pi$ production from $^3S_1$ and $^1S_0$ (to be published). The experimental evidence revolves in part about the simple fact that from $^3S_1$, $\rho^+\pi^-$: $\rho^-\pi^+$: $\rho^0\pi^0=1$:1:1 whereas from $^1S_0$, one would have $\rho^+\pi^-$: $\rho^-\pi^+$: $\rho^0\pi^0=1$:1:0. Equal numbers (within statistics) of ρ^+, ρ^-, and ρ^0 are observed. There are other experimental ways of distinguishing $^1S_0$ final states from $^3S_1$. See Ref. 18 for a discussion.

[9] The papers of Ref. 20 disagree as to whether $^1S_0$ is present in K^*K final states.

[10] C. Baltay *et al.*, Phys. Rev. Letters **15**, 532 (1965); **15**, 597(E) (1965); N. Barash, Columbia University (private communication).

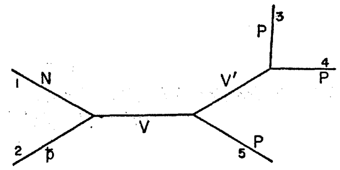

FIG. 1. $\bar{p}$-nucleon annihilation. N is a nucleon, V and V' are vector mesons, and P is a pseudoscalar meson.

The recent observation[11] of K-mesonic x rays indicates the possibility of observing antiprotonic x rays. Such an experiment would give explicit information about the capture process for antiprotons and would also provide, through a measurement of the level shifts due to strong interactions, and evaluation of the various partial-wave scattering lengths.[12]

3. THE ANNIHILATION PROCESS

We have already remarked that the $^3S_1$ $\bar{p}$-nucleon at rest looks very much like a heavy vector meson. Thus, in the decay $\bar{p}p \rightarrow \pi^+\pi^-\pi^0$ one immediately conceives of resonance dominance as in the Gell-Mann–Sharp–Wagner model of the ω^0 decay[13]; i.e., we assume that the $\pi^+\pi^-$ pair are in a ρ^0 resonance and similarly for the other pion pairs. And then we assume that the $\rho\pi$-nucleon amplitude is dominated by the ω^0. The validity of these assumptions is still unknown; the predictions for the annihilation of antiprotons will provide a test.

The Feynman diagram for this model of the annihilation process is given in Fig. 1. We assume the following interaction between the nucleons and vector meson:

$$\bar{\psi}(a\gamma_\mu+(b/2m_N)\sigma_{\mu\nu}P_\nu)\psi V_\mu$$

where ψ is the nucleon field, V is the vector-meson field, and P_μ is the total momentum of the nucleon-antinucleon pair.[14] The vector-vector-pseudoscalar interaction has the form $\epsilon_{\mu\nu\lambda\sigma}\partial_\mu V_\nu \partial_\lambda V_\sigma'\phi$, where V and V' are the vector fields and ϕ is the pseudoscalar field. It is not hard to see that the amplitude of the process diagrammed in Fig. 1 will take the form $\bar{\psi}[a\gamma_\mu+(b/2m_N)\sigma_{\mu\nu}P_\nu]\psi S_\mu$ where S_μ is a vector such that $S\cdot P=0$. Using the identity $ia_\mu\sigma_{\mu\nu}b_\nu=\boldsymbol{ab}-2a\cdot b$, and using the fact that $\boldsymbol{p}u=im_N u$, while $\boldsymbol{p}v=-im_N v$ (u and v being the particle and antiparticle spinors, respectively), one can rewrite the final amplitude to the form: $(a+b)\bar{\psi}\boldsymbol{S}\psi$. This form for the amplitude shows that it is a total magnetic-moment coupling, since if $V_\mu=A_\mu$, the electromagnetic field, $a+b$ gives the total magnetic moment of the nucleon in units of the nuclear magneton; and if vector mesons dominate the electromagnetic form factors, as is

[11] G. R. Burleson *et al.*, Phys. Rev. Letters **15**, 70 (1965).

[12] The expression for the S state level shift is $\Delta E/E=(4/n)a/r_0$, where $a=$ (complex) scattering length, and r_0 is the Bohr radius; that for the P state is more complicated, but still depends essentially only on the P-wave scattering length over the Bohr radius. See T. L. Trueman, Nucl. Phys. **26**, 57 (1961).

[13] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[14] We are using the Pauli-Dirac metric: $p_\mu=(p,iE)$, $\gamma_i=\rho_2\sigma_i$, and $\gamma_4=\rho_3$, where ρ_i and σ_i are the Pauli sigma matrices

$$\sigma_{\mu\nu}=(1/2i)[\gamma_\mu,\gamma_\nu]_-.$$

generally believed to be the case, this description carries right over to them. It is this amplitude which has been used in the calculation, with the appropriate vector S. In Appendix I, we construct the vector S for the various cases of interest.

4. COUPLING CONSTANTS

It now remains to evaluate the coupling constants for each case. Without the aid of some higher principle, this would not be possible unless individual experimental results were available. Fortunately, we now know that higher symmetries such as $SU(3)$ and $SU(6)$ have a certain amount of validity. $SU(3)$ is believed to hold for $\bar{p}$-nucleon annihilation, whereas the applicability of $SU(6)$ has not yet been fully resolved. In fact, since as mass-difference effects become smaller, $SU(3)$ ought to become better, it is possible that $\bar{p}$-nucleon annihilation will conform more closely to the predictions of $SU(3)$ than, say, meson-baryon scattering at moderate energies. This remains to be seen. The relativistic generalization of $SU(6)$, $U(6,6)$, variously called $\tilde{U}(12)$, $SU_{\mathfrak{L}}(12)$, $M(12)$, forbids at-rest decays such as $\bar{p}p\to\pi^+\pi^-$, or $\bar{p}p\to\rho\pi$,[15] which is not borne out by experiment.[8,16] Therefore, we will use the relations generated by $SU(3)$ in order to determine unknown coupling constants. We determine the unknown $SU(3)$ parameters in the following way.

The process $\bar{p}+p\to P+P$ is given, according to the approximations of this paper, by the diagram in Fig. 2. Once again, the amplitude takes the form $(a+b)\bar{\psi}S\psi$, where

$$S_\mu=(P^2+m_V{}^2)^{-1}g_{VVP}Q_\mu \quad \text{and} \quad Q=p_3-p_4\,.$$

Now, the $SU(3)$ VPP coupling has only one unknown parameter, since it has to be f type, and the unitary singlet vector meson, ω_1, cannot couple to two pseudoscalars because of C invariance. This parameter may be determined from any one of three decays: $\rho\to 2\pi$, $K^*\to K\pi$, or $\phi\to K\bar{K}$. These determinations do not quite agree, but this is not disturbing, since we know a symmetry-breaking interaction exists. In particular, $SU(3)$ predicts $g_{\rho\pi\pi}{}^2/g_{K^{*+}K^0\pi^+}{}^2=g_{\rho\pi\pi}{}^2/g_{\phi K^+K^-}{}^2=2$. From experiment, we have $g_{\rho\pi\pi}{}^2/g_{K^{*+}K^0\pi^+}{}^2=1.2\pm0.08$, and $g_{\rho\pi\pi}{}^2/g_{\phi K^+K^-}{}^2=1.7\pm0.4$. (See Appendix II for a derivation of these results.) Thus, we will always use the experimentally determined coupling constant in preference to its $SU(3)$-predicted value. The important point to note here is that the branching ratios $K^+K^-:K_1K_2:\pi^+\pi^-$ from $\bar{p}p$ annihilation will enable us

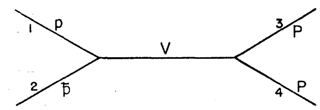

FIG. 2. $\bar{p}$-p annihilation. V is a vector meson, and P is a pseudoscalar meson.

to determine the d/f ratio of the $\bar{B}BV$ coupling, which we may then use to predict the vector-pseudoscalar branching ratios. In fact, from the diagram of Fig. 2, one obtains the following relative rates:

$$\Gamma(\bar{p}p\to K^+K^-)=(\alpha+\beta)^2\times187m_\pi{}^3\,,$$
$$\Gamma(\bar{p}p\to\pi^+\pi^-)=4\alpha^2\times293m_\pi{}^3\,,$$
$$\Gamma(\bar{p}p\to K_1K_2)=(\alpha-\beta)^2\times186m_\pi{}^3\,,$$

where we have used a factor of p^3 to account for phase space and the matrix element. In the above equations, $\alpha=d+f$ and $\beta=-1.35d+3.39f$, where we have taken into consideration the relatively small differences between the ρ^0, ω^0, and ϕ propagators. (See Appendix II for the couplings, ω-ϕ mixing assumed, etc.) The experiment[16] yielded for the above the following relative rates:

$$\Gamma(K^+K^-)=131\pm40\,,$$
$$\Gamma(\pi^+\pi^-)=375\pm30\,,$$
$$\Gamma(K_1K_2)=51\pm8\,.$$

Solving the above equations pairwise, we find three different values for d/f: 1.6, 2.0, 2.4. We will use the mean value of 2.0 in our calculations, which gives a reasonable χ^2 fit to the above data, provided we increase the error bars on the experimental numbers by a factor of two. There is no problem in blaming this increase solely on the imperfection of $SU(3)$.

We would like to emphasize the difference between this d/f ratio and the one at $q^2=0$: $d/f|_{q^2=0}\approx\frac{3}{2}$. The value $\frac{3}{2}$ follows from the vector-meson dominance of electromagnetic form factors, and also is a result of $SU(6)$. If we are so bold as to use $d/f|_{q^2=-4m^2}\approx\frac{3}{2}$, we predict that $\Gamma(K^+K^-)/\Gamma(K_1K_2)\approx12$ rather than its observed value of ≈2. This is precisely the difficulty with the prediction of one kind of broken $U(6,6)$.[17] In other words, the above branching ratios are a sensitive function of the d/f ratio.

The theoretical branching relations presented above are the same as those derived from a species of broken $SU(6)$.[18] The reason for this is that the assumption that the two $SU(6)$ meson matrices must appear as a commutator automatically ensures octet dominance. Consider the pseudoscalar meson part (P) of the $SU(6)$ meson matrix M. Using $M_1M_2-M_2M_1$ means that we

[15] Y. Hara, Phys. Rev. Letters **14**, 404 (1965); R. Delbourgo *et al.*, *ibid.* **14**, 845 (1965); N. P. Chang and J. M. Sphiz, *ibid.* **14**, 617 (1965); H. Harrari, H. J. Lipkin, and S. Meshkov, *ibid.* **14**, 845 (1965). If one breaks $U(6,6)$, one can get finite results for two-meson final states. The results depend on the particular way it is broken.

[16] R. Armenteros *et al.*, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 351; R. Goldberg, Rutgers University (private communication).

[17] H. Harari and H. J. Lipkin, Phys. Letters **15**, 286 (1965). The same authors (Trieste 1965 unpublished report) using another kind of broken $U(12)$ based on what they call the $U(6)_W$ subgroup, have obtained the prediction $\langle\bar{p}p|K^+K^-\rangle/\langle\bar{p}p|K^0\bar{K}^0\rangle=2$ which is better. However, they also predict $\langle\bar{p}p|K^+K^-\rangle/\langle\bar{p}p|\pi^+\pi^-\rangle=2$, which does not agree with experiment at all (see Ref. 16). I thank H. Harari for a discussion of this.

[18] M. Konuma and E. Remiddi, Phys. Rev. Letters **14**, 1082 (1965).

have $P_1P_2-P_2P_1$ appearing. As is well known, $8\times 8=1+8_a+8_s+10+\overline{10}+27$ for $SU(3)$, where 8_a=antisymmetric octet$=P_{1j}{}^iP_{2k}{}^j-P_{2j}{}^iP_{1k}{}^j$, and 8_s=symmetric octet$=P_{1j}{}^iP_{2k}{}^j+P_{2j}{}^iP_{1k}{}^j$. Now, every term above is symmetric in 1 and 2 with the exception of 8_a. Thus, only the $SU(3)$ octet part of the $\bar{B}B$ product can contribute, leaving us with just two independent amplitudes, from the two 8's in the decomposition of $\bar{B}B$.

The VVP coupling has two unknown parameters: the amount of singlet coupling and the amount of octet coupling (which must be pure d, since f type give a $\rho\rho\pi$ coupling). The rates of $\omega\to\rho\pi$ and $\phi\to\rho\pi$ enable us to determine both.

That leaves only one parameter to be determined: the amount of singlet coupling in $\bar{B}BV$. We may determine it through the use of the branching ratio $(\bar{p}p\to\pi^+\pi^-\pi^0)/(\bar{p}p\to\pi^+\pi^-)\approx 7.5$.[8,16] However, at this point a fork in the road appears. The theoretical result for the above ratio, from the model under consideration, is

$$\frac{\Gamma(\pi^+\pi^-\pi^0)}{\Gamma(\pi^+\pi^-)}\approx\frac{1}{2\pi}\frac{g_{\omega\rho\pi}{}^2}{4\pi}\left(\frac{g_{\omega NN}{}^2}{g_{\rho NN}{}^2}\right)\times I,$$

where I is a certain integral (see Appendix II). By setting $g_{\omega NN}/g_{\rho NN}=0.366$, we obtain the correct answer. This will complete the determination of the $\bar{B}BV$ coupling. However, in deriving the above result, there appeared a factor of the center-of-mass energy squared in the numerator which came from the $\omega\rho\pi$ vertex. This is worrisome, since in the "gedanken" experiment of $\rho\pi$ scattering, it leads to a violation of unitarity. This means that probably $g_{\omega\rho\pi}\neq$constant, but instead decreases with increasing energy. To answer this question accurately would require a complete solution for the $\rho\pi$ scattering amplitude, something not currently available. We will adopt the following ansatz: replace $E_T{}^2$ by $m_V{}^2$, the mass-shell mass of the intermediate vector meson. This, of course, leads to a different $\bar{B}BV$ coupling. Whether or not this is a reasonable step is an open question which we will not try to answer. The results from both modes of calculation will be given. They do not differ greatly, with the exception of the $\bar{K}K\pi$ final states (this has to do with a somewhat unexpected cancellation of the various coupling constants involved), which is not surprising, since $m_\rho{}^2\approx m_\omega{}^2\sim m_\phi{}^2$.

The details of both determinations will be found in Appendix II. For a summary of the experimental information used to determine the coupling constants, see Table I.

TABLE I. Summary of experimental data used to determine unknown parameters.

$SU(3)$ coupling	Unknown parameters	Experimental data used	Ref.
VPP	f	$\rho\to 2\pi$	31
		$K^*\to K\pi$	31
		$\phi\to K\bar{K}$	31
VVP	d,s	$\omega\to 3\pi$	33
		$\phi\to 3\pi$	31,32
$\bar{B}BV$	s	$\bar{p}p\to\pi^+\pi^-\pi^0$	8
	d/f	$\bar{p}p\to K^+K^-$	16
		$\bar{p}p\to K_1K_2$	16
		$\bar{p}p\to\pi^+\pi^-$	16
$\bar{B}BP$	s,f,d	$\bar{p}p\to K^*\bar{K}^*$	10
		$\bar{p}p\to\rho^0\rho^0$	10
		$\bar{p}p\to\rho^0\omega^0$	10

5. EVALUATION OF THE RATES

Having determined the coupling constants, we are in a position to write down the exact form of the amplitude. Take, for example, $\bar{p}p\to\pi^+\pi^-\pi^0$ (see Fig. 1). Our model assumes that $\omega\rho\pi$ will dominate this amplitude, since $g_{\phi\rho\pi}\ll g_{\omega\rho\pi}$. Thus the amplitude becomes $g_{\omega NN}\bar{p}Sp$, where $S_\mu\propto[1/(P^2+m_\omega{}^2)]\epsilon_{\mu\nu\lambda\sigma}P_\nu Q_{\lambda\sigma}$ and

$$Q_{\lambda\sigma}=\left[\frac{(p_3+p_4)_\lambda(p_3-p_4)_\sigma}{(p_3+p_4)^2+m_\rho{}^2}+\frac{(p_4+p_5)_\lambda(p_4-p_5)_\sigma}{(p_4+p_5)^2+m_\rho{}^2}+\frac{(p_5+p_3)_\lambda(p_5-p_3)_\sigma}{(p_5+p_3)^2+m_\rho{}^2}\right].$$

The differential rate, $\partial^2\Gamma/\partial E_1\partial E_2$, comes out proportional to $\mathbf{S}^2$, where $\mathbf{S}=(\mathbf{p}_3\times\mathbf{p}_4)\{[1/(p_3+p_4)^2+m_\rho{}^2]+\text{cyclic permutation}\}$ (that the amplitude must be like $\mathbf{S}\cdot\boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}$ is the polarization vector of the $\bar{p}p$ system, can be seen purely from invariance requirements[19]). We have then integrated $\mathbf{S}^2$ over phase space numerically in order to obtain the total rate. The kinematical relations used and other details are presented in Appendix III.

The results are to be found in Table II. The $\pi^+\pi^-\pi^0\pi^0$ and $\pi^+\pi^-\pi^0\pi^-$ rates were computed as though the $\pi^+\pi^-\pi^0$ were on the ω^0 mass shell always, a good approximation since the ω^0 width is small. For the two-body final states, or in this case, what J. D. Jackson has called

TABLE II. $^3S_1$ $\bar{p}p$ and $\bar{p}n$ annihilation. Only the *resonant* parts of these final states have been considered, or compared against experiment. States obtained by charge conjugation have not been listed explicitly.

Final state	Contributing resonances	Dominant resonances	Relative rates: Normal vertex	Relative rates: "Mass-shell" vertex
$\pi^+\pi^-\pi^0$	ρ	ρ	491	154
$\pi^+\pi^-\eta$	ρ	ρ	25.8	7.71
$K^+\pi^-\bar{K}^0$	K^*,ρ	K^*	22.5	37.9
$K^+K^-\pi^0$	K^*,ρ,ω,ϕ	K^*	27.0	37.5
$K^+K^-\eta$	K^*,ρ,ω,ϕ	ϕ	0.281	4.56
$K^-\pi^0K^0$	K^*,ρ	K^*	36.8	11.0
$K^-\eta K^0$	K^*,ρ	K^*	0.219	0.0656
$K^+K^-\pi^-$	K^*,ϕ	K^*	45.8	13.6
$\pi^+\pi^-\pi^0\pi^0$	ω,ϕ	ω	972	50.8
$\pi^+\pi^-\pi^0\pi^-$	ω,ϕ	ω	1940	102

[19] This feature of these decays was used by C. Bouchiat and G. Flammand, Nuovo Cimento 23, 13 (1962), in order to construct their amplitudes.

a quasi-two-body final state, we have used the standard techniques. A limited comparison with experiment is possible.

The $K^{\pm}\pi^{\mp}K^0$ rate has been determined as 1.42×10^{-3} and 0.97×10^{-3}.[20] The $\pi^+\pi^-\pi^0$ rate is 2.7×10^{-2}.[8] Thus we predict that $K^{\pm}\pi^{\mp}K^0/\pi^+\pi^-\pi^0=1/11$ whereas something like 1/20 is observed. The "mass shell" vertex yields a prediction of $\frac{1}{5}$ which is not in as good agreement with experiment. Experimentally, $(\bar{p}n\to\omega\pi^-)/(\bar{p}n\to\phi\pi^-)=10\pm5$,[21] which is not in agreement with our prediction, which is (except for phase space) $(\bar{p}n\to\omega\pi^-)/(\bar{p}n\to\phi\pi^-)=g^2{}_{\omega\rho\pi}/g^2{}_{\phi\rho\pi}\approx700$. This means that, owing to the smallness of $g^2{}_{\phi\rho\pi}$, we are not justified in assuming that the ρ^- pole dominates the $\phi\pi^-$ production. For example, baryon exchange in the cross channel is probably significant.

The theoretical mass distributions are given in Figs. 3 through 9. The experimental mass distributions for $K^{\pm}\pi^{\mp}$ and $K^0\pi^{\pm}$ in the $K^{\pm}\pi^{\mp}K^0$ final state are shown in Figs. 10 and 11.[22] They do not indicate clearly which vertex is to be preferred.

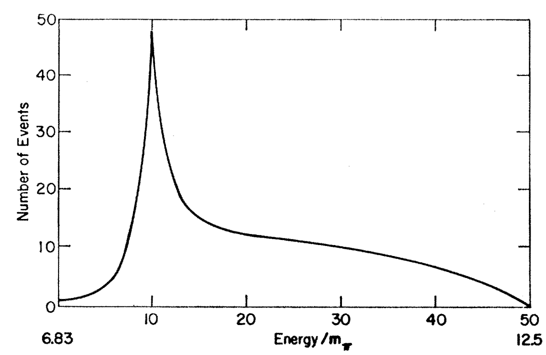

FIG. 3. $\pi^+\pi^-\pi^0$ final state: $\pi^+\pi^-$ total energy spectrum, for both kinds of vertex.

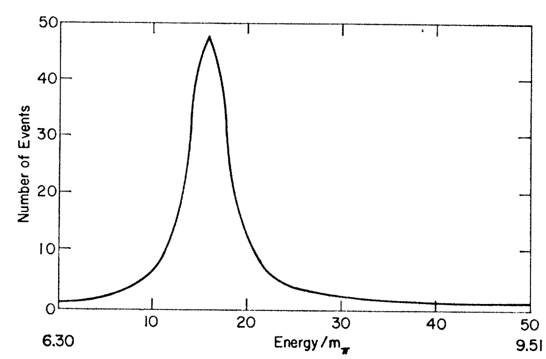

FIG. 4. $\pi^+\pi^-\eta$ final state: $\pi^+\pi^-$ total energy spectrum, for both kinds of vertex.

For the decays $\bar{p}p\to e^+e^-$ and $\mu^+\mu^-$,[23] we expect that the diagram of Fig. 12 dominates. This may be contrasted with the decay $\bar{p}p\to\pi^+\pi^-$. One obtains the result (neglecting the lepton mass)

$$\frac{\bar{p}p\to e^+e^-}{\bar{p}p\to\pi^+\pi^-}=\frac{\bar{p}p\to\mu^+\mu^-}{\bar{p}p\to\pi^+\pi^-}=R,$$

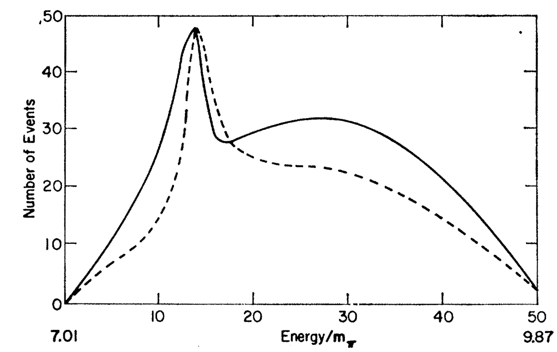

FIG. 5. $K^+\pi^-\bar{K}^0$ final state: $K^+\pi^-$ or $K^0\pi^-$ total energy spectrum. —— normal vertex; - - - "mass shell" vertex. Experimental distribution is seen in Figs. 10 and 11.

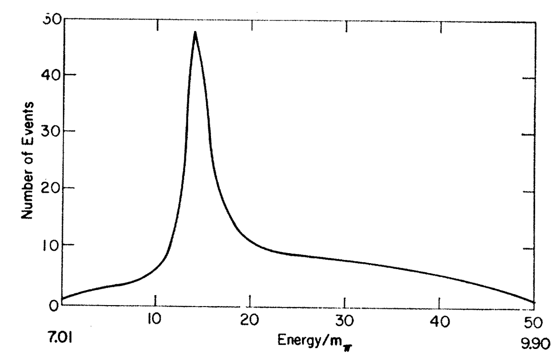

FIG. 6. $K^+K^-\pi^0$ final state: $K^+\pi^0$ total energy spectrum for both kinds of vertex. No ϕ peak appears in the K^+K^- spectrum.

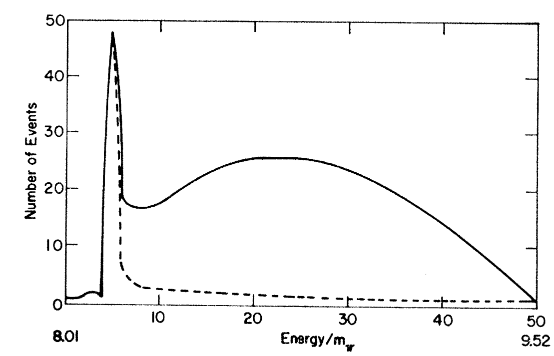

FIG. 7. $K^+K^-\eta$ final state: K^+K^- total energy spectrum. —— normal vertex; - - - "mass shell" vertex.

---

[20] N. Barash *et al.*, Nevis Cyclotron report No. 133 (unpublished); R. Armenteros *et al.*, Phys. Letters **17**, 170 (1965); P. Franzini, Columbia University (private communication). The number given is that for those final states in which a K^* is present.

[21] T. Kalogeropoulos, University of Rochester (private communication).

[22] These figures are reproduced from the first paper of Ref. 20.

[23] These annihilations have also been discussed by A. Zichichi *et al.*, Nuovo Cimento **24**, 170 (1962).

where

$$R=\frac{e^4}{\gamma^2/4\pi}\,\frac{1}{g_{\rho\pi\pi}{}^2/4\pi}\,\frac{[m_\rho{}^2+\frac{1}{3}(D_\rho/D_\omega)(g_{\omega NN}/g_{\rho NN})m_\omega{}^2+\frac{1}{3}\sqrt{2}(D_\rho/D_\phi)(g_{\phi NN}/g_{\rho NN})m_\phi{}^2]^2}{16m_N(m_N{}^2-m_\pi{}^2)^{3/2}}.$$

(γ is the f-type $\bar{B}BV$ coupling at $q^2=0$. $\gamma^2/4\pi\sim1$. D_ρ, D_ω, and D_ϕ are the denominators of the ρ, ω, and ϕ propagators.)

Inserting the proper values (see Appendix II), one finds:

$$R\approx9.6\times10^{-6}\quad\text{(Normal vertex)},$$
$$R\approx1.9\times10^{-5}\quad\text{("Mass shell" vertex)}.$$

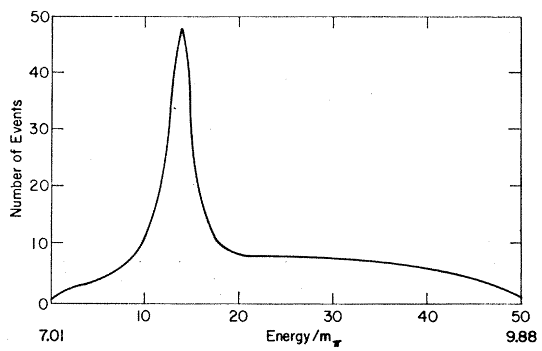

FIG. 8. $K^-\pi^0K^0$ final state: $K^-\pi^0$ or $K^0\pi^0$ total energy spectrum for both kinds of vertex.

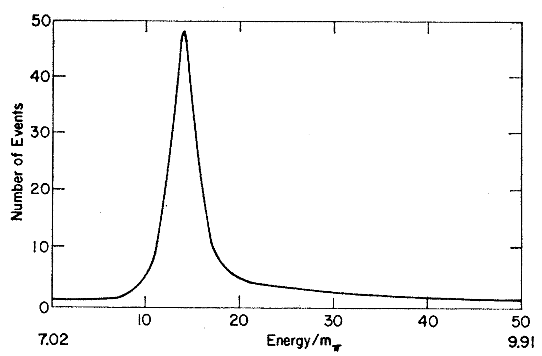

FIG. 9. $K^+K^-\pi^-$ final state: $K^+\pi^-$ total energy spectrum for both kinds of vertex. No ϕ peak appears in the K^+K^- spectrum.

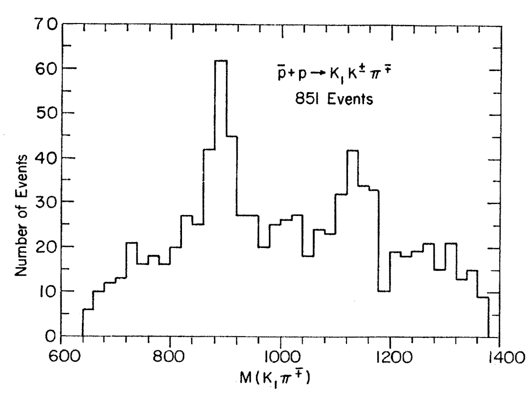

FIG. 10. $(K_1\pi)^+$ effective mass distribution from the reaction $\bar{p}+p\to K_1+K^\pm+\pi^\mp$. Theoretical distribution is seen in Fig. 5.

This branching ratio is perhaps the cleanest test of the validity of the vector-meson-dominance model. For if the $\pi^+\pi^-$ final state does not come predominantly from the ρ^0, the above ratio could take on almost any value.

As for the vector-vector final states, the results[10] leave some doubt as to whether they all proceed only from $^1S_0$. The $\rho^0\rho^0$ and $\rho^0\omega^0$ observed certainly do, because of C invariance, and thus we expect the $K^*\bar{K}^*$ states to be predominantly $^1S_0$ production also. This point has not yet been checked experimentally. The quoted results[10] for the various final states are:

$$\rho^0\rho^0:\quad(3.8\pm3.0)\times10^{-3},$$
$$\rho^0\omega^0:\quad(7.0\pm3.0)\times10^{-3},$$
$$K^{*+}K^{*-}:\quad(1.3\pm0.5)\times10^{-3},$$
$$K^{*0}\bar{K}^{*0}:\quad(2.9\pm0.5)\times10^{-3},$$

where the numbers given are branching ratios with respect to all annihilations.

The experimental results quoted do indicate that at least some of the $K^*\bar{K}^*$ produced come from the $^1S_0$ state. Assume for the moment that they all came from $^3S_1$. Then since both the VPP and VVV couplings have to be f-type and thus are alike, the dominance of the intermediate vector meson s-channel pole implies that

$$\frac{\Gamma(K^+K^-)}{\Gamma(K^0\bar{K}^0)}=\frac{\Gamma(K^{*+}K^{*-})}{\Gamma(K^{*0}\bar{K}^{*0})}.$$

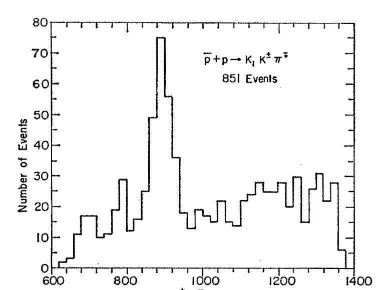

FIG. 11. $(K^\pm\pi^\pm)$ effective mass distribution from the reaction $\bar{p}+p\to K_1+K^\mp+\pi^\pm$. Theoretical distribution is seen in Fig. 5.

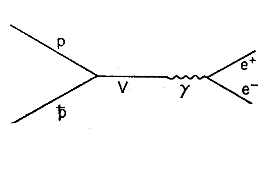

FIG. 12. $\bar{p}p$ annihilation into a lepton pair. l stands for either e or μ.

However, experimentally the right-hand side is just inverted, the left-hand side being approximately 2.[16]

It is possible to distinguish ${}^1S_0$ from ${}^3S_1$ K^*K^* final states in the same way one uses Dalitz pairs to determine if the pion is 0^-. For the ${}^1S_0$ state of $\bar{p}p$ is 0^- and the K^*'s are just heavy photons. Therefore, the K^* polarizations tend to be perpendicular. Furthermore the difference of the three momenta of the resonant pair of particles "remembers" the polarization; and thus from the ${}^1S_0$ state, the angle between $\mathbf{p}_1-\mathbf{p}_2$ and $\mathbf{p}_3-\mathbf{p}_4$ follows a $\sin^2\theta$ distribution, where we have assumed particles 1 and 2 and particles 3 and 4 are the resonant ones.

From the ${}^1S_0$ state, there is a pseudoscalar meson pole in the s channel; its dominance, in keeping with the spirit of this paper, produces the following expressions for the relative rates:

$$\Gamma(\rho^0\rho^0)=\tfrac{1}{2}[1.11(f-\tfrac{1}{3}d)+1.35rs]^2\times 7.79m_\pi{}^5,$$

$$\Gamma(\rho^0\omega^0)=4(d+f)^2\times 7.66m_\pi{}^5,$$

$$\Gamma(K^{*+}K^{*-})=\tfrac{1}{4}[0.0875f-1.36d+1.35rs]^2\times 2.24m_\pi{}^5,$$

$$\Gamma(K^{*0}\bar{K}^{*0})=\tfrac{1}{4}[-2.09f-0.637d+1.35rs]^2\times 2.24m_\pi{}^5,$$

where we have used the traditional $SU(3)$ f and d couplings for $\bar{B}BP$,[24] have taken the η^*(960-MeV $\eta\pi\pi$ resonance) couplings as $S\eta^*\,\mathrm{Tr}(\bar{B}B)$ and $S'\eta^*\,\mathrm{Tr}(VV)$, have set $r=2g_{\eta^*\rho^0\rho^0}/g_{\omega\rho\pi}$ ($g_{\eta^*\rho^0\rho^0}=S'$), have taken into account the π^0, η, and η^* propagator differences, and have used a weighting factor of

$$[(4m_N{}^2-m_1{}^2-m_2{}^2)^2-4m_1{}^2m_2{}^2]\times\left[4m_N{}^2-2(m_1{}^2+m_2{}^2)+\frac{(m_1{}^2-m_2{}^2)^2}{4m_N{}^2}\right]^{1/2}$$

account for the matrix element and phase space.

The quantity r above can be determined from the $\eta^*\to\pi^+\pi^-\gamma$ decay. Theoretically, one would expect this to be dominated by the ρ^0 as indicated in Fig. 13(a); experimentally, this is verified.[25] One obtains the following expression for the rate:

$$\Gamma(\eta^*\to\pi^+\pi^-\gamma)=\frac{e^2}{16m_{\eta^*}{}^2}\frac{g_{\eta^*\rho^0\rho^0}{}^2}{4\pi\zeta^2}(m_{\eta^*}{}^2-m_\rho{}^2)^2\left[m_{\eta^*}{}^2-2m_\rho{}^2+\frac{m_\rho{}^4}{m_{\eta^*}{}^2}\right]^{1/2},$$

where ζ is the rationalized f-type $\bar{B}BV$ coupling ($\zeta\sim1$). Using $\Gamma_{\eta^*\to\pi^+\pi^-\gamma}/\Gamma_{\eta^*}\approx\frac{1}{4}$ and $\Gamma_{\eta^*}\sim1$ MeV,[25] one finds $g_{\eta^*\rho^0\rho^0}{}^2/4\pi\sim0.3m_\pi{}^{-2}$. This implies $r\sim1$. As a

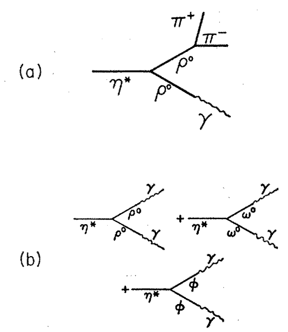

FIG. 13. (a) η^*(960-MeV $\eta\pi\pi$ resonance) $\to\pi^+\pi^-\gamma$; (b) $\eta^*\to\gamma+\gamma$.

byproduct of this calculation, with the aid of Fig. 13(b), one determines that

$$\frac{\Gamma_{\eta^*\to\gamma+\gamma}}{\Gamma_{\eta^*\to\pi^+\pi^-\gamma}}=\frac{4e^2}{9}\frac{m_{\eta^*}{}^5}{(m_{\eta^*}{}^2-m_\rho{}^2)^2}\left[m_{\eta^*}{}^2-2m_\rho{}^2+\frac{m_\rho{}^4}{m_{\eta^*}{}^2}\right]^{-1/2},$$

Inserting $m_{\eta^*}{}^2=47.2m_\pi{}^2$ and $m_\rho{}^2=29.9m_\pi{}^2$, we find

$$\frac{\eta^*\to\gamma+\gamma}{\eta^*\to\pi^+\pi^-\gamma}\approx0.07.$$ [26]

When we least-squares-fit the theoretical expressions for the $\bar{p}+p\to V+V$ rates to the data given above, we find two acceptable solutions:

(A) $\chi^2=0.00971$ ($\approx90\%$ probability),
$rs=-1.32\pm0.17$,
$f=0.666\pm0.155$,
$d=-0.182\pm0.176$.

(B) $\chi^2=0.00966$ ($\approx90\%$ probability),
$rs=-0.587\pm0.167$,
$f=1.35\pm0.16$,
$d=-0.862\pm0.176$.

Solution A implies $d/f=-0.27\pm0.27$, whereas solution B gives $d/f=-0.64\pm0.13$, both of which are quite different from the usually assumed $d/f\approx3$.[27]

The values obtained above for s, d, and f imply (using $r\sim1$) for both solutions that $g_{\eta^*NN}\sim-2g_{\pi^0NN}$; this magnitude and sign for g_{η^*NN} may help to explain the rapid increase with energy of the proton Compton scattering cross section.[28]

---

[24] M. Gell-Mann, California Institute of Technology Report No. CTSL-20 (unpublished).

[25] P. M. Dauber *et al.*, Phys. Rev. Letters **13**, 449 (1964), give $\Gamma_{\eta^*}<4$ MeV and $\Gamma_{\eta^*\to\pi^+\pi^-\gamma}/\Gamma_{\eta^*}=0.25\pm0.14$.

[26] These decays have also been discussed by L. M. Brown and H. Faier, Phys. Rev. Letters **13**, 73 (1964); S. K. Kundo and D. C. Peaslee, Nuovo Cimento **36**, 277 (1965).

[27] R. Cutkosky, Ann. Phys. **23**, 415 (1963); A. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

[28] S. K. Kundu and M. Yonezawa, Nucl. Phys. **44**, 499 (1963); A. C. Hearn and E. Leader, *Nucleon Structure, Proceedings of the International Conference at Stanford University, 1963* (Stanford University Press, Stanford, California, 1964), p. 314. I thank R. Köberle for a discussion of this point.

Perhaps the most distinctive prediction of this model of $\bar{p}+p\rightarrow V+V$ is that $\Gamma(\rho^0\rho^0)\approx\Gamma(\omega^0\omega^0)$ independent of s, d, and f, the sole difference being phase space. The production of two ϕ mesons, which does depend on s, d, and f, has the misfortune of violating energy conservation for $\bar{p}p$ at rest. And when the $\bar{p}$ processes sufficient energy, higher partial waves will contribute, so that S-state $\phi\phi$ production is not experimentally clean.

6. SUMMARY AND CONCLUSIONS

We have used a resonance-dominance model of $\bar{p}N$ annihilation, with the aid of $SU(3)$ to fix the undetermined coupling constants, and obtain results consistent with the known experimental results as far as can be expected. We would like to call attention to the fact that the d/f ratio for the $\bar{B}BV$ coupling used in this calculation was not $\frac{3}{2}$ (which is its approximate value at $q^2=0$ if we make the usual assumption that the vector mesons dominate the electromagnetic form factors), but 2.0. This difference is not insignificant, since the results of $\bar{p}p$ annihilation into two pseudoscalar mesons is very sensitive to the d/f ratio. We may conclude that the d/f ratio is not a constant.

If the remaining predictions of this paper turn out to be (approximately) true, one will have strong supporting evidence for two extremely useful concepts: $SU(3)$ symmetry and resonance dominance. We do not expect, of course, much better than approximate agreement in view of the treatment used here; the burning question of how good these predictions should be is one that is not easily answered and is at present unanswerable except by experiment.

ACKNOWLEDGMENTS

I would like to acknowledge frequent and helpful discussions with Norman Gelfand, Joel Yellin, R. T. Torgerson, and particularly J. J. Sakurai, whose patient help was invaluable in the completion of this work.

APPENDIX I

Here we tabulate in Table III the vector S for the different final states of interest. The following abbreviations will be used:

$$D_\omega=P^2+m_\omega^2,$$
$$D_\phi=P^2+m_\phi^2,$$
$$D_\rho=P^2+m_\rho^2,$$

where P is the total four-momentum of the nucleon-antinucleon pair. Also we will use

$$D(1,2,\alpha)=1/[(p_1+p_2)^2+m_\alpha^2].$$

In each case below, $\mathbf{S}=(\mathbf{p}_1\times\mathbf{p}_2)F(p_1,p_2,p_3)$; therefore only F will be given. The particles in the final states will be listed in order; i.e., $\pi^+\pi^-\pi^0$ means that $\pi^+=$particle 1, $\pi^-=$particle 2, and $\pi^0=$particle 3.

TABLE III. The factor F in the vector s for various final states.

Final state	F
$\pi^+\pi^-\pi^0$	$\dfrac{g_{\omega NN}g_{\omega\rho\pi}g_{\rho\pi\pi}}{D_\omega}[D(1,2,\rho)+D(2,3,\rho)+D(3,1,\rho)]$
$K^+K^-\pi^0$	$\left[\dfrac{g_{\rho NN}g_{\rho K^{*+}K^-}}{D_\rho}+\dfrac{g_{\omega NN}g_{\omega K^{*+}K^-}}{D_\omega}+\dfrac{g_{\phi NN}g_{\phi K^{*+}K^-}}{D_\phi}\right]$
	$\times g_{K^{*+}K^+\pi^0}[D(1,3,K^*)+D(2,3,K^*)]$
	$+\left[\dfrac{g_{\rho NN}g_{\rho\omega\pi}g_{\omega K^+K^-}}{D_\rho}\right]D(1,2,\omega)$
	$+\left[\dfrac{g_{\rho NN}g_{\phi\rho\pi}g_{\phi K^+K^-}}{D_\rho}\right]D(1,2,\phi)$
	$+\left[\dfrac{g_{\omega NN}g_{\omega\rho\pi}}{D_\omega}+\dfrac{g_{\phi NN}g_{\phi\rho\pi}}{D_\phi}\right]g_{\rho K^+K^-}D(1,2,\rho)$
$\pi^+\pi^-\eta$	$\dfrac{2g_{\rho NN}g_{\rho\rho\eta}g_{\rho\pi\pi}}{D_\rho}D(1,2,\rho)$
$K^+\pi^-\bar{K}^0$	$\left[\dfrac{g_{\rho NN}g_{\rho K^{*-}K^+}}{D_\rho}+\dfrac{g_{\omega NN}g_{\omega K^{*-}K^+}}{D_\omega}+\dfrac{g_{\phi NN}g_{\phi K^{*-}K^+}}{D_\phi}\right]$
	$\times g_{K^{*-}\bar{K}^0\pi^-}D(2,3,K^*)+\left[\dfrac{g_{\rho NN}g_{\rho K^{*0}\bar{K}^0}}{D_\rho}+\dfrac{g_{\omega NN}g_{\omega K^{*0}\bar{K}^0}}{D_\omega}\right.$
	$\left.+\dfrac{g_{\phi NN}g_{\phi K^{*0}\bar{K}^0}}{D_\phi}\right]g_{K^{*0}K^+\pi^-}D(1,2,K^*)$
	$+\left[\dfrac{g_{\omega NN}g_{\omega\rho\pi}}{D_\omega}+\dfrac{g_{\phi NN}g_{\phi\rho\pi}}{D_\phi}\right]g_{\rho^+K^+\bar{K}^0}D(1,3,\rho)$
$K^+K^-\eta$	$\dfrac{2g_{\rho NN}g_{\rho\rho\eta}g_{\rho K^+K^-}}{D_\rho}D(1,2,\rho)+\dfrac{2g_{\omega NN}g_{\omega\omega\eta}g_{\omega K^+K^-}}{D_\omega}D(1,2,\omega)$
	$+\dfrac{2g_{\phi NN}g_{\phi\phi\eta}g_{\phi K^+K^-}}{D_\phi}D(1,2,\phi)+\left[\dfrac{g_{\rho NN}g_{\rho K^{*+}K^-}}{D_\rho}\right.$
	$\left.+\dfrac{g_{\omega NN}g_{\omega K^{*+}K^-}}{D_\omega}+\dfrac{g_{\phi NN}g_{\phi K^{*+}K^-}}{D_\phi}\right]g_{K^{*+}K^+\eta}D(1,3,K^{*+})$
	$+$last term with $+,-$ interchange.
$K^-\pi^0K^0$	$\dfrac{g_{\bar{p}n\rho^-}g_{\rho^-K^-K^{*0}}}{D_\rho}g_{K^{*0}K^0\pi^0}D(2,3,K^*)$
	$+\dfrac{g_{\bar{p}n\rho^-}g_{\rho^-K^0K^{*-}}}{D_\rho}g_{K^{*-}K^-\pi^0}D(1,2,K^*)$
$K^-\eta K^0$	Replace the π^0 by an η directly above
$K^+K^-\pi^-$	$\dfrac{g_{\bar{p}n\rho^-}g_{\omega\rho\pi}}{D_\rho}g_{\omega K^+K^-}D(1,2,\omega)$
	$+\dfrac{g_{\bar{p}n\rho^-}g_{\phi\rho\pi}g_{\phi K^+K^-}}{D_\rho}D(1,2,\phi)$
	$+\dfrac{g_{\bar{p}n\rho^-}g_{\rho^-K^{*0}K^-}g_{K^{*0}K^+\pi^-}}{D_\rho}D(1,3,K^*)$

APPENDIX II

Here we discuss the determination of the necessary coupling constants.

First, we give the $SU(3)$ couplings we need. We will use the following abbreviations: V, vector octet; P, pseudoscalar octet; B, baryon octet; $\bar{B}$, antibaryon octet, all in matrix form; ω_1, unmixed unitary singlet vector meson; ω_8, unmixed isosinglet member of the vector octet; H.c., Hermitian conjugate.

$$
\begin{aligned}
VPP:\quad &\mathrm{Tr}(V[P,P']_-)=\rho^0(K^+K^- - K^0\bar{K}^0+2\pi^+\pi^-)\\
&+(\omega-\sqrt{2}\phi)(K^+K^-+K^0\bar{K}^0)+\{K^{*+}[K^-\pi^0+\sqrt{2}\bar{K}^0\pi^-+\tfrac{1}{2}\sqrt{3}K^-\eta]+\text{H.c.}\}\\
&+\{K^{*0}[\sqrt{2}K^-\pi^+-\bar{K}^0\pi^0+\tfrac{1}{2}\sqrt{3}\bar{K}^0\eta]+\text{H.c.}\}+\{\rho^+[2\pi^-\pi^0+\sqrt{2}K^0K^-]+\text{H.c.}\}\\
VVP:\quad &\mathrm{Tr}[P\{V,V'\}_+]=\{\pi^+[\sqrt{2}\rho^-\omega+K^{*0}K^{*-}]+\text{H.c.}\}+[K^+(K^{*-}\{\phi+[(\omega^0+\rho^0)/\sqrt{2}]\}+K^{*0}\rho^-)+\text{H.c.}]\\
&+[K^0(\bar{K}^{*0}\{\phi+[(\omega^0-\rho^0)/\sqrt{2}]\}+K^{*-}\rho^+)+\text{H.c.}]\\
&+(\pi^0/\sqrt{2})[2\omega^0\rho^0+K^{*+}K^{*-}-K^{*0}\bar{K}^{*0}]\\
&+(\eta/\sqrt{6})[\rho^0\rho^0+\omega^0\omega^0-2\phi\phi+2\rho^+\rho^- - K^{*+}K^{*-}-K^{*0}\bar{K}^{*0}],
\end{aligned}
$$

where we have taken

$$
V=\begin{pmatrix}(\omega^0+\rho^0)/\sqrt{2} & \rho^+ & K^{*+}\\ \rho^- & (\omega^0-\rho^0)/\sqrt{2} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \phi\end{pmatrix}
$$

which is the unmixed vector octet with $\omega_1/\sqrt{3}$ added to it.[29] We have set $\phi=-(\sqrt{\tfrac{2}{3}})\omega_8+(\sqrt{\tfrac{1}{3}})\omega_1$ and $\omega=(\sqrt{\tfrac{1}{3}})\omega_8+(\sqrt{\tfrac{2}{3}})\omega_1$. Adding this much singlet reduces $g_{\phi\rho\pi}$ to zero, which is approximately true ($g_{\phi\rho\pi}\ll g_{\omega\rho\pi}$). If one considers electromagnetic breaking of $SU(3)$, one can then roughly account for $g_{\phi\rho\pi}$ when its $SU(3)$-symmetric value is zero.

The amount of singlet in the vector octet is immaterial in the VPP coupling, since $\omega_1 \nrightarrow P+P$. But it does make a difference in the $\bar{B}BV$ coupling. We will only concern ourselves with the $\bar{p}p$ coupling, since $\bar{p}n\to\rho^-$ only and $g_{\bar{p}n\rho^-}=\sqrt{2}g_{\bar{p}p\rho^0}$ by isospin alone.

For the moment, let us use V for the unmixed vector octet which has $(\sqrt{\tfrac{1}{6}})\omega_8$, $(\sqrt{\tfrac{1}{6}})\omega_8$, $-(\sqrt{\tfrac{2}{3}})\omega_8$, down the diagonal instead of $\omega/\sqrt{2}$, $\omega/\sqrt{2}$, ϕ.

Then we have for the independent $\bar{B}BV$ couplings:

$$
\begin{aligned}
d\,\mathrm{Tr}(V\{\bar{B},B\}_+)&=d\bar{p}p(\rho^0-\omega_8/3)+\cdots,\\
f\,\mathrm{Tr}(V[\bar{B},B]_-)&=f\bar{p}p(\rho^0+\sqrt{3}\omega_8)+\cdots,\\
s\,\mathrm{Tr}(\omega_1\bar{B}B)&=s\bar{p}p\omega_1+\cdots.
\end{aligned}
$$

(The d, f, and s above stand for the $SU(3)$-symmetric parts of the total magnetic moment coupling given in the main text.) At this point, we can choose $s=2d/\sqrt{3}$ which is equivalent to using the vector nonet written above, and has the virtue of a certain beauty. In that case, we have

$$
d\bar{p}p(\rho^0+\omega^0+\sqrt{2}\phi)+f\bar{p}p(\rho^0+\omega^0-\sqrt{2}\phi).
$$

We then need only to determine d/f. For this, we use the experimental results for $\bar{p}p\to\pi^+\pi^-$, K^+K^-, K_1K_2. Since ω_1 does not contribute to this process, it gives us an unambiguous answer for the d/f ratio. We have discussed in the main text the details of this determination. The result obtained is $d/f=2.0\pm0.3$.

Now, let us examine $(\bar{p}p\to\pi^+\pi^-\pi^0)/(\bar{p}p\to\pi^+\pi^-)$. This branching ratio is known to be approximately 7.5.[8,16] Theoretically, using the model described in the body of this paper, we get

$$
\frac{\Gamma(\pi^+\pi^-\pi^0)}{\Gamma(\pi^+\pi^-)}\approx\frac{1}{2\pi}\frac{g_{\omega\rho\pi}}{4\pi}\left(\frac{g_{\omega NN}}{g_{\rho NN}}\right)^2\times I,
$$

where

$$
I=\int dE_1dE_2(\mathbf{p}_1\times\mathbf{p}_2)^2|D(1,2,\rho)+D(2,3,\rho)+D(3,1,\rho)|^2.
$$

[See Appendix I for the definition of $D(1,2,\rho)$.]

Evaluating I numerically (see Appendix III), we find $8I=525m_\pi^2$. Using $g_{\omega\rho\pi}^2/4\pi=0.672$ (see below for this determination), we obtain the result that $g_{\omega NN}/g_{\rho NN}=0.366$. This means that $s\neq2d/\sqrt{3}=1.16d$ and in fact it becomes $0.330d$. This finally gives us a coupling of $\bar{p}p(\rho^0+0.366\omega+0.0226\phi)$.

However, as pointed out in the main text, there may be good reason to evaluate the $\omega\rho\pi$ vertex on the ω mass shell. If we do this, we find that, with $g_{\omega NN}=g_{\rho NN}$, $\Gamma(\pi^+\pi^-\pi^0)/\Gamma(\pi^+\pi^-)\approx10$ which is close enough for our purposes. So, with a "mass shell" vertex our coupling will be $\bar{p}p(\rho^0+\omega^0+\sqrt{2}\phi/3)$.

To summarize:

$\bar{p}p(\rho^0+0.366\omega+0.0226\phi)$	normal vertex
$\bar{p}p(\rho^0+\omega^0+\sqrt{2}\phi/3)$	"mass-shell" vertex.

It is of interest to compare these couplings with those at $q^2=0$. There, it is well known that $g_{\omega NN}^2\sim10g_{\rho NN}^2$ as determined from nuclear force calculations.[30] However, this result refers only to the γ_μ coupling and not to the $\sigma_{\mu\nu}q_\nu$ coupling which is unimportant for small q^2. But in the cross channel, the anomalous coupling is not

[29] S. Okubo, Phys. Letters 5, 165 (1963). This result is also obtained in $SU(6)$.

[30] A. Wong and D. Y. Scotti, Phys. Rev. Letters 10, 142 (1963); R. A. Bryan and B. L. Scott, Phys. Rev. 135, B434 (1964). See also the table and references in K. Kawarabayashi, *ibid.* 134, B877 (1964).

negligible. In fact, as we saw in the main text, if the coupling is $\bar{\psi}(a\gamma_\mu+(b/2m_N)\sigma_{\mu\nu}q_\nu)\psi V_\mu$, the effective coupling is $(a+b)\bar{\psi}\gamma_\mu\psi V_\mu$, what we have called "total magnetic moment" coupling, which means the anomalous coupling is equally important. The nuclear force calculations, as we would expect, do not determine $b_{\rho NN}$ and $b_{\omega NN}$ very well, except to tell us that $b_{\omega NN}\approx 0$, and $b_{\rho NN}\sim a_{\omega NN}$.[30] Thus, it is consistent with the data at $q^2=0$ that, at $q^2=-4m^2$, we may have $g_{\rho NN}\geq g_{\omega NN}$. Moreover, if one is willing to assume that the a's and b's above are true constants and do not change with q^2, then $\bar{p}p$ annihilation data may well provide a better determination of $b_{\rho NN}$ and $b_{\omega NN}$ than has been possible previously.

To conclude this discussion of coupling constants, let us consider the determination from experiment of $g_{\omega\rho\pi}$, $g_{\phi\rho\pi}$, $g_{\rho\pi\pi}$, $g_{K^*K\pi}$, and $g_{\phi KK}$. These decays can be divided into two classes: $V\to P+P$ and $V\to V+P$. We will take them in order.

$V\to P_1+P_2$

The coupling, which applies except for isotopic spin factors, to $\rho\pi\pi$, $K^*K\pi$, and ϕKK, is $gV_\mu[P_1^\dagger(\partial_\mu P_2)-(\partial_\mu P_1^\dagger)P_2]$. The decay rate calculated from this is

$$\Gamma=\frac{g^2}{4\pi}\frac{1}{12m_V^2}\left\{m_V^2+\frac{(m_1^2-m_2^2)^2}{m_V^2}-2(m_1^2+m_2^2)\right\}^{3/2}.$$

Using $\Gamma_\rho=106$ MeV, $\Gamma_{K^*}=50$ MeV, and $\Gamma_\phi=3.1$ MeV,[31] we obtain:

$$g_{\rho\pi\pi}^2/4\pi=2.07\pm0.10$$

$$g_{K^{*+}K^0\pi^+}{}^2/4\pi=1.72\pm0.07$$

(we have used the theoretical result that

$$\Gamma(K^{*+}\to K^+\pi^0)/\Gamma(K^{*+}\to K^0\pi^+)=\tfrac{1}{2})$$

and

$$g_{\phi K^+K^-}{}^2/4\pi=1.24\pm0.3$$

(we have used the experimental result that

$$\Gamma(\phi\to K^+K^-)/\Gamma(\phi\to \text{all})=0.46).^{32}$$

$V_1\to V_2+P$

Here, the coupling which applies to both $\omega\rho\pi$ and $\phi\rho\pi$ is $g_{V_1V_2P}\epsilon_{\mu\nu\lambda\sigma}\partial_\mu V_{1\nu}\partial_\lambda V_{2\sigma}P$. Note that $[g]=m^{-1}$. We assume the decays $\omega\to 3\pi$ and $\phi\to 3\pi$ are dominated by the ρ. For the ρ decay, we use the coupling above, thus obtaining

$$\Gamma(V_1\to 3P)=\frac{m_{V_1}}{3\pi}\left(\frac{g^2{}_{V_1V_2P}}{4\pi}\right)\left(\frac{g^2{}_{V_2PP}}{4\pi}\right)\times I,$$

where I is just as before for $\bar{p}p\to\pi^+\pi^-\pi^0$ (see above). Evaluating I numerically again, one finds $8I=0.835m_\pi^2$ for the ω decay and $8I=21.5m_\pi^2$ for the ϕ decay. Using the widths $\Gamma_\omega=13.1$ MeV,[33] $\Gamma_\phi=3.1$ MeV,[31] and taking $\Gamma(\phi\to 3\pi)/\Gamma(\phi\to\text{all})$ as 18%,[32] and $\Gamma(\omega\to 3\pi)/\Gamma(\omega\to\text{all})=90\%$,[33] we find:

$$g_{\omega\rho\pi}^2/4\pi=0.672m_\pi^{-1}$$

and

$$g_{\phi\rho\pi}^2/4\pi=9.59\times10^{-4}m_\pi^{-1}.$$

[31] W. H. Barkas *et al.*, University of California Radiation Laboratory Report UCRL 8030, 1964 (unpublished).

[32] G. Smith *et al.*, Bull. Am. Phys. Soc. **10**, 502 (1965).

APPENDIX III

Here we discuss kinematics and other details in the actual evaluation of the rates.

Kinematics for Three-Body Final States

From $(p_1+p_2)^2=(P-p_3)^2$, where $P=(0,iE)$, one obtains

$$\cos\theta_{12}=\frac{m_1^2+m_2^2-m_3^2+E^2-2E(E_1+E_2)+2E_1E_2}{2[(E_1^2-m_1^2)(E_2^2-m_2^2)]^{1/2}}.$$

We will express everything in terms of $E_s=E_1+E_2$ and $E_D=E_1-E_2$. The boundary of phase space, as is well known, is given by $\theta=0$ or π. Thus, for a given E_s, we can find the boundary points E_{D+} and E_{D-} by solving the following quadratic: $AE_D^2+BE_D+C=0$, where

$$A=E(E-2E_s)-m_3^2,$$

$$B=2E_s(m_1^2-m_2^2),$$

and

$$C=4m_1^2m_2^2-(m_1^2+m_2^2)E_s^2-(A+m_1^2+m_2^2)(A+m_1^2+m_2^2+E_s^2).$$

(The equation was obtained from that for $\cos\theta_{12}$.) In this way, we can iterate our integral over phase space. doing the E_D integration first for a given value of E_s. In order to do this, we need to determine E_s(min) and E_s(max), our integral then taking the form

$$\int_{E_s(\min)}^{E_s(\max)}dE_s\int_{E_{D-}}^{E_{D+}}dE_D|M|^2,$$

where M is the appropriate matrix element. E_s(max) clearly occurs when particle 3 is at rest: $E_s(\max)=E-m_3$. E_s(min) occurs when particles 1 and 2 come off together, for then E_3 is at a maximum. Expressing this requirement as $[p^2+(m_1+m_2)^2]^{1/2}+(p^2+m_3^2)^{1/2}=E$, one finds

$$E_s(\min)=(1/2E)[E^2+(m_1+m_2)^2-m_3^2].$$

Kinematics for the Matrix Element

First, we have a $(\mathbf{p}_1\times\mathbf{p}_2)^2$ term. On the boundary, this is zero. Thus, we are led to suspect that which we may derive without any prior suspicions: $(\mathbf{p}_1\times\mathbf{p}_2)^2\propto AE_D^2+BE_D+C$. The constant of proportionality turns out to be $\frac{1}{4}$.

[33] D. Miller, Columbia University, Ph.D. thesis, Nevis Cyclotron Report (unpublished).

Then, we have resonance forms, like $D(1,2,\rho)$. We therefore want to evaluate $(p_1+p_2)^2$. As previously noted, $(p_1+p_2)^2=(P-P_3)^2=-m_3^2-E(E-2E_3)$. Furthermore, there is the energy dependence of the width of the resonance (the width appears through the prescription that we replace m_ρ by $m_\rho-i\Gamma_\rho/2$, where

$$\Gamma_\rho=\frac{2}{3}\frac{g_{\rho\pi\pi}^2}{4\pi}\frac{|\mathbf{p}_\pi|^3}{-p_\rho^2},$$

obtained using the usual $\rho\pi\pi$ coupling and the (off-mass-shell) mass of the ρ). In the case at hand, $p_\rho^2=(p_1+p_2)^2$ which we have just evaluated. $\mathbf{p}_\pi$ is the momentum of the π in the ρ center of mass. Using $(P_1-P_2)^2=2P_1^2+2P_2^2-(P_1+P_2)^2$ and the fact that both π's have the same momentum, one finds

$$4\mathbf{p}_\pi^2=m_3^2+E(E-2E_3)-2(m_1^2+m_2^2)+\frac{(m_1^2-m_2^2)^2}{m_3^2+E(E-2E_3)}.$$

All of the above formulas were constructed with their suitability for computing in mind, and in the above form are immediately programmable.

Numerical integration with a sixth-order polynomial fit was used to obtain results, the program automatically subdividing the integration interval where necessary (e.g., under a resonance peak) until the answer was good to a desired number of significant figures (chosen to be three).

PHYSICAL REVIEW VOLUME 143, NUMBER 4 MARCH 1966

Coupling Constants in Broken $\tilde{U}(12)$ Symmetry*

D. Flamm

Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 22 October 1965)

The effects of $SU(3)$ and $\tilde{U}(4)$ breaking on the coupling constants in $\tilde{U}(12)$ symmetry are investigated using the spurion technique. For an $SU(3)$-breaking spurion which is a member of **143**, only two parameters are introduced in addition to the one for the formal symmetry. All 132 baryon-meson coupling constants can be expressed in terms of these three quantities. For vertices involving pions or ρ mesons, only two parameters are relevant. The effects of $\tilde{U}(4)$ breaking as well as simultaneous $\tilde{U}(4)$ and $SU(3)$ breaking are studied with spurions which belong to the representations **143**, **4212**, and **5940** of $\tilde{U}(12)$. The sum rules for the coupling constants which follow from the formalism are in reasonable agreement with experiment.

1. INTRODUCTION

THE $\tilde{U}(12)$ scheme[1,2] provides a relativistic framework for the derivation of the $SU(6)$ results.[3] In addition to these results, $\tilde{U}(12)$ also gives an absolute value for the proton magnetic moment which is of the right order of magnitude, and it relates all meson-baryon vertices to a single form factor. Even though the application of formal $\tilde{U}(12)$ symmetry to scattering processes meets with certain difficulties,[4] its success in the case of the vertex function is encouraging. We expect $\tilde{U}(12)$ to be broken in two ways corresponding to its subgroups $SU(3)$ and $\tilde{U}(4)$. The deviations from $SU(3)$ are conventionally described by introducing a spurion which transforms like the eighth component of an $SU(3)$ octet. It is well known that this also gives rise to mass splittings between the members of the $SU(3)$ multiplets. $\tilde{U}(4)$ on the other hand is broken by the equations of motion which give rise to $\tilde{U}(4)$ noncovariant subsidiary conditions for the representations of $\tilde{U}(12)$. In addition, to simulate higher order effects, we shall introduce $\tilde{U}(4)$ breaking spurions,[5] which belong to the representations **143**, **4212**, and **5940** of $\tilde{U}(12)$.

In the second section we give the effective interaction Hamiltonian densities for the meson baryon vertex including spurions. The third section deals with the reduction of the $\tilde{U}(12)$ field operators under $\tilde{U}(4)\otimes SU(3)$, and in the fourth section we study the effects of $SU(3)$ breaking spurions. In the fifth section we investigate $\tilde{U}(4)$ breaking as well as simultaneous $SU(3)$ and $\tilde{U}(4)$ breaking, and in the sixth section we compare the results with the experimental data.

* Researches herein reported have been supported by the Atomic Energy Commission through AEC Contract AT(30-1)-2098.

[1] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, *ibid.* **14**, 48 (1965); M. A. B. Bég and A. Pais, *ibid.* **14**, 267 (1965).

[2] B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965).

[3] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964); F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964).

[4] R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters **14**, 518 (1965).

[5] P. G. O. Freund, Phys. Rev. Letters **14**, 803 (1965); R. Oehme, *ibid.* **14**, 866 (1965).