$$B_4=E_3(2Z_3Z_8-Z_9Z_{12}),$$
$$C_1=D_1=E_1(-2Z_4)+E_2(-2Z_5-2Z_6)+E_3(-2Z_2)+E_4(-Z_1Z_3-Z_2Z_4+Z_5Z_6),$$
$$C_2=D_3=E_2[m_p(Z_6-Z_3-Z_4)+m_n(Z_5-Z_1-Z_4)]+E_3[m_p(Z_2-Z_1-Z_6)+m_n(Z_2-Z_3-Z_5)],$$
$$C_3=D_2=E_1[m_p(Z_8-Z_3-Z_{10})+m_n(Z_8-Z_1-Z_9)]+E_2[m_p(Z_{10}-Z_1-Z_7)+m_n(Z_9-Z_3-Z_7)],$$
$$C_4=D_4=E_2(Z_9Z_{10}-Z_1Z_3-Z_7Z_8),$$

where we have made the following definitions:

$$E_1=p_\mu p_\nu D_{\mu\nu}=-m_p^2+(p\cdot d)^2/m_d^2,$$
$$E_2=p_\mu p_\nu' D_{\mu\nu}=-p\cdot p'+(p\cdot d)(p'\cdot d)/m_d^2,$$
$$E_3=p_\mu' p_\nu' D_{\mu\nu}=-m_p^2+(p'\cdot d)^2/m_d^2,$$
$$E_4=\delta_{\mu\nu}D_{\mu\nu}=-3,$$

and

$$Z_1=p'\cdot n-m_pm_n,$$
$$Z_2=p\cdot n+m_pm_n,$$
$$Z_3=p\cdot n'-m_pm_n,$$
$$Z_4=p'\cdot n'+m_pm_n,$$
$$Z_5=p\cdot p'+m_p^2,$$
$$Z_6=n\cdot n'+m_n^2,$$
$$Z_7=p\cdot n-m_pm_n,$$
$$Z_8=p'\cdot n'-m_pm_n,$$
$$Z_9=p\cdot p'-m_p^2,$$
$$Z_{10}=n\cdot n'-m_n^2,$$
$$Z_{11}=n^2-m_n^2,$$
$$Z_{12}=n'^2-m_n^2.$$

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Decays of Baryon Resonances of Any Spin

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General formulas are derived relating coupling constants for the strong decay interactions of a baryon resonance B_s^* of any spin s to the observed decay widths, for the decay processes $B_s^*\to B_{\frac{1}{2}}+P$, $B_s^*\to B_{\frac{3}{2}}^*+P$, and $B_s^*\to B_{\frac{1}{2}}+V$, where P is a pseudoscalar and V a vector meson. The calculations are carried through within the framework of the Rarita-Schwinger formalism, using interaction Lagrangians incorporating derivative couplings of the pseudoscalar or vector-meson field to free spinor fields. The resulting formulas are compared with the well-known potential-theory expressions.

INTRODUCTION

OBSERVATIONS of the strong decays of high-spin baryon resonances afford valuable guidance in assigning these resonances to their correct places in various symmetry schemes. Thus the correct isotopic-spin assignment is likely to be suggested by the experimental branching ratio into the different charge states of particles produced by the decay, while the experimental decay widths provide a means of extracting phenomenological coupling constants whose magnitudes are connected in $SU(3)$ or some higher symmetry scheme.

The purpose of this investigation is to derive, for a baryon resonance B_s^* of arbitrary spin s and mass ω, general formulas relating the coupling constants $g_{s\frac{1}{2}P}$, $g_{s\frac{3}{2}P}$ and $g_{s\frac{1}{2}V}$ to the widths $\Gamma(\omega)$ for their respective decay processes

$$B_s^*\to B_{\frac{1}{2}}+P, \qquad (1)$$
$$B_s^*\to B_{\frac{3}{2}}^*+P, \qquad (2)$$
$$B_s^*\to B_{\frac{1}{2}}+V, \qquad (3)$$

where P is a pseudoscalar and V a vector meson. Examples of each of these processes already exist in nature, reactions of the type (1) being by far the most common mode for a given resonance because of the relatively low masses of the product particles for such reactions. Perturbation-theory relations for $\Gamma(\omega)$ have already been given for the lowest spin values and are well-known[1] and extension to higher spins suggested by analogy with the resonance-theory results of Blatt and Weisskopf.[2]

Lagrangians incorporating derivative couplings of the pseudoscalar or vector-meson field to free spinor fields are employed here, and the calculations carried through in the framework of the Rarita-Schwinger[3] formalism.

[1] J. D. Jackson, Nuovo Cimento **34**, 1644 (1964) [see especially Appendix A].

[2] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, New York, 1952), pp. 332, 361, 406–422. The form of Eq. (49) is retained in the derivation of W. M. Layson, Nuovo Cimento **27**, 724 (1963) by means of the Klein-Gordon equation rather than the Schrödinger equation. A useful summary of results appears in L. D. Roper, University of California Radiation Laboratory Report No. UCRL 14193, 1965 (unpublished).

[3] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

In all cases the decay interactions are assumed to be parity-conserving. The result obtained for reaction (1) is shown to agree in general form, as one would expect, with the resonance theory result in the limit of a point interaction at the decay vertex. The general result for reaction (2) reduces, in the special case $s=\frac{5}{2}$, to the expression obtained in the same formalism by Brudnoy[4] in considering the decay of the $F_{\frac{5}{2}}$ resonances into $P_{\frac{3}{2}}$ resonances, viz.,

$$N_{\frac{5}{2}}^*(1688) \to N_{\frac{3}{2}}^*(1238)+\pi, \qquad (2a)$$

$$Y_{\frac{5}{2}}^*(1815) \to Y_{\frac{3}{2}}^*(1385)+\pi. \qquad (2b)$$

Reactions (2) and (3) possess, respectively, 2 and 3 different couplings, which will lead inevitably to ambiguities in interpreting experimental data. In the absence of an exact theory it will be necessary to create models like the Stodolsky-Sakurai[5] ρ-meson–isovector-photon analogy for future guidance.

II. KINEMATICS AND FREE FIELDS

A. Kinematics

The decay rate for the process $d\to p+k$ is given[6] in the rest frame O_d of d by[7]

$$\Gamma(\omega)=\frac{1}{(2\pi)^2}\int \delta^{(4)}(p+k-d) \times\frac{1}{2\omega}\frac{1}{(2s+1)}\sum|\mathfrak{M}|^2\frac{d^3p}{2p_0}\frac{d^3k}{2k_0}, \qquad (4)$$

where $\mathfrak{M}$ is the Lorentz-invariant amplitude, defined in terms of the S matrix via the general relation

$$S_{fi}=\delta_{fi}-i(2\pi)^4\delta^4(p_f-p_i)(\mathfrak{M}_{fi}/\textstyle\prod_i 2p_{0i})^{1/2}),$$

where a factor $2p_{0i}$ occurs for each initial and final particle. The summation in (4) is over all polarization states of d, p, and k. The integration may be performed, resulting in

$$\Gamma(\omega)=\frac{1}{4\pi}\frac{q}{2\omega^2}\frac{1}{2s+1}\sum|\mathfrak{M}|_0^2, \qquad (5)$$

where the suffix 0 denotes evaluation of the matrix-element modulus squared with $|\mathbf{p}|=|\mathbf{k}|=q$, the common momentum from the decay of d at rest, and with p_0 and k_0 having their mass-shell values. As can be readily shown,

$$p_0=(2\omega)^{-1}[\omega^2+m^2-\mu^2], \qquad (6a)$$

$$k_0=(2\omega)^{-1}[\omega^2+\mu^2-m^2], \qquad (6b)$$

and

$$q=(2\omega)^{-1}[\omega^4-2\omega^2(m^2+\mu^2)+(m^2-\mu^2)^2]^{1/2}. \qquad (7)$$

B. Free Fields

In the Rarita-Schwinger formalism a particle of spin $s(=n-\frac{1}{2})$, where n is an integer, is described by a field operator $\Phi_{\alpha_1\cdots\alpha_{n-1}}$ $(\alpha_i=1\cdots 4)$ which is a tensor of rank $n-1$, each component of which is a four-spinor satisfying the Dirac equation

$$(\gamma_\nu\partial_\nu+m)\Phi_{\alpha_1\cdots\alpha_{n-1}}=0 \qquad (8)$$

and the condition

$$\gamma_{\alpha_1}\Phi_{\alpha_1\cdots\alpha_{n-1}}=0. \qquad (9a)$$

Furthermore, Φ is a completely symmetric tensor

$$\Phi_{\cdots\alpha_i\cdots\alpha_j\cdots}=\Phi_{\cdots\alpha_j\cdots\alpha_j\cdots}, \qquad (9b)$$

with the properties

$$\partial_{\alpha_1}\Phi_{\alpha_1\cdots\alpha_{n-1}}=0, \qquad (9c)$$

and

$$\delta_{\alpha_1\alpha_2}\Phi_{\alpha_1\alpha_2\cdots\alpha_{n-1}}=0. \qquad (9d)$$

Equations (8), (9a), and (9b) can be shown to reduce the number of independent components of Φ to $2s+1$, as required for a particle of spin s. Equations (9b), (9c), and (9d) with the Klein-Gordon equation for each tensor component [corresponding to (8)] are the Rarita-Schwinger equations for a particle of integral spin $n-1$.

Following Fronsdal[8] we introduce the orthogonal projection operator Θ with the following properties

$$\Theta_{\cdots\alpha_i\cdots\alpha_j}{}^{\beta_1\cdots}=\Theta_{\cdots\alpha_j\cdots\alpha_i\cdots}{}^{\beta_1\cdots}, \qquad (10a)$$

$$p_{\alpha_1}\Theta_{\alpha_1\cdots}{}^{\beta_1\cdots}=0, \qquad (10b)$$

$$\gamma_{\alpha_1}\Theta_{\alpha_1\cdots}{}^{\beta_1\cdots}=0, \qquad (10c)$$

$$\delta_{\alpha_1\alpha_2}\Theta_{\alpha_1\alpha_2\cdots}{}^{\beta_1\cdots}=0. \qquad (10d)$$

The special value of Θ arises from the property, of great use in trace calculations, that if U is a free-particle wave function satisfying the conditions (9a), (9b) as well as the momentum space analogs of (8) and (9c), i.e., describes a particle of unique spin s, then

$$\sum_{\text{spin}} U_{\alpha_1\cdots}\bar{U}^{\beta_1\cdots}=\Theta_{\alpha_1\cdots}{}^{\beta_1\cdots}\Lambda^+(p)=\Lambda^+(p)\Theta_{\alpha_1\cdots}{}^{\beta_1\cdots},$$

where Λ^+ are the usual positive-energy projection operators of the spin-$\frac{1}{2}$ theory. Explicit expressions for the Θ operators for general spin may be obtained, but for

[4] D. M. Brudnoy, Phys. Rev. Letters 14, 273 (1965).

[5] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters 11, 90 (1963).

[6] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper & Row Publishers, Inc., New York, 1962), p. 486, Eq. (118).

[7] As well as identifying them, d, p, and k are taken as the four-momenta of the corresponding particles, their masses being ω, m, and μ, respectively; thus $d=(\mathbf{d},id_0)$, $d^2=\mathbf{d}^2-d_0^2=-\omega^2$. The γ matrices are Hermitian, with $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$, and satisfy $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu}$. The convention that spinors for particles of mass m and arbitrary spin are normalized to $2m$ has been adopted. The spin-$\frac{1}{2}$ baryon spinors ψ satisfy $(i\gamma\cdot p+m)\psi=0$, and have positive-energy projection operator $\Lambda^+(p)=m-i\gamma\cdot p$. The short-hand notation $\not{p}=\gamma\cdot p$ will be used.

[8] C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958).

$s>\frac{5}{2}$ become vastly complex (see Ref. 4 for the case $s=\frac{5}{2}$) and it is far simpler in trace calculations to exploit certain relations such as[8]

$$\Theta_{\alpha_1\cdots\alpha_{n-1}}{}^{\beta_1\cdots\beta_{n-1}}(s) =\frac{n}{(2n+1)}\gamma_\alpha\gamma_\beta\Theta_{\alpha\alpha_1\cdots\alpha_{n-1}}{}^{\beta\beta_1\cdots\beta_{n-1}}(n), \quad (11)$$

where $\Theta_{\mu_1\cdots\mu_n}{}^{\nu_1\cdots\nu_n}$ is the operator for bosons of spin n. This quantity is useful as it involves no γ matrices and hence may be taken outside a trace expression. A further useful property is

$$\delta_{\alpha\beta}\Theta_{\alpha\alpha_1\cdots\alpha_{n-1}}{}^{\beta\beta_1\cdots\beta_{n-1}}(n) =\frac{2n+1}{2n-1}\Theta_{\alpha_1\cdots\alpha_{n-1}}{}^{\beta_1\cdots\beta_{n-1}}(n-1). \quad (12)$$

A closed expression for Θ in the form of a sum over the $n=1$ operators, which have the form

$$\Theta_\mu{}^\nu=\delta_{\mu\nu}-p_\mu p_\nu/p^2=\delta_{\mu\nu}{}^\perp(p) \quad (13a)$$

has been given,[9] which may be used to obtain certain formulas needed here; these are listed in the Appendix. The symmetry between a top and bottom suffix is peculiar to the $n=1$ case and this object will be called $\delta_{\mu\nu}{}^\perp(p)$ as in any frame it is perpendicular to $p[\delta_{\mu\nu}{}^\perp(p)p_\nu=0]$. It extracts the part of any vector q perpendicular to p and hence we define

$$q_\mu{}^\perp(p)=\delta_{\mu\nu}{}^\perp(p)q_\nu \quad (13b)$$

with

$$q_\mu{}^\perp(p)p_\mu=0.$$

A quantity which will occur frequently is

$$\delta_{\mu\nu}{}^\perp(p)q_\nu{}^\perp(p)q_\mu{}^\perp(p) =q_\mu{}^\perp(p)q_\mu{}^\perp(p)=q^2-(q\cdot p)^2/p^2=q_p{}^2 \quad (13c)$$

which will be written $q_p{}^2$ to indicate the fact that in O_p it has the value $q^2-q_0{}^2=\mathbf{q}^2$, the magnitude of the space momentum squared.

A vector particle is described by a field A_μ satisfying the subsidiary relation $\partial_\mu A_\mu=0$, which, in terms of the polarization vector $\epsilon_\mu{}^{(j)}$ and four-momentum p_μ has the momentum-space analog

$$k_\mu\epsilon_\mu{}^{(j)}=0. \quad (14)$$

A sum over the polarization states j is obtained with the aid of the projection operator (13a); thus

$$\sum_j \epsilon_\mu{}^{(j)}\epsilon_\nu{}^{(j)}=\delta_{\mu\nu}-k_\mu k_\nu/k^2=\delta_{\mu\nu}{}^\perp(k). \quad (15)$$

In the following, the calculational steps crucial to the avoidance of impossibly wieldy expressions have been included for pedagogic reasons. The matrix elements arising in (A) and (C) are closely related to those considered by Fox,[10] whose algebraic methods have been followed where possible.

[9] R. E. Behrends and C. Fronsdal, Phys. Rev. **106**, 345 (1957).
[10] G. C. Fox, University of Cambridge report, 1965 (unpublished).

III. MATRIX ELEMENTS FOR DECAY INTERACTIONS

A. Pseudoscalar-Meson-Baryon Decays

The invariant matrix element for the decay process $B_s^*\to B+P$ will be considered first. If the intrinsic parities of B_s^* and B are P_s and P, respectively, then for $\mathcal{P}=P_sP(-1)^n=+1$ the parity-conserving interaction Lagrangian is (recalling that $n=s+\frac{1}{2}$),

$$\mathcal{L}=\frac{g_{s\frac{1}{2}P}}{m_\pi{}^{(n-1)}}\bar\psi\Phi^{\mu_1\cdots\mu_{n-1}}\partial_{\mu_1}\cdots\partial_{\mu_{n-1}}\phi, \quad (16)$$

but if $\mathcal{P}=-1$ an $i\gamma_5$ must be inserted between the spin-$\frac{1}{2}$ field ψ and the spin $-(n-\frac{1}{2})$ field $\Phi_{\mu_1\cdots\mu_{n-1}}$. The pseudoscalar meson field is ϕ and the factor $1/m_\pi{}^{(n-1)}$ has been introduced to make the coupling constant $g_{s\frac{1}{2}P}$ dimensionless. Going over to momentum space the amplitude corresponding to (16) therefore contains a vertex factor

$$V_{\mu_1\cdots\mu_{n-1}}=\frac{g_{s\frac{1}{2}P}}{m_\pi{}^{n-1}}k_{\mu_1}\cdots k_{\mu_{n-1}}, \quad (17)$$

so that the required amplitude squared and summed over initial and final spins is

$$\sum|\mathfrak{M}|^2=\mathrm{Tr}\{\gamma_4V_{\mu_1'\cdots\mu_{n-1}'}{}^\dagger\gamma_4\Lambda^+(p) \times V_{\mu_1\cdots\mu_{n-1}}\Lambda^+(d)\Theta_{\mu_1'\cdots\mu_{n-1}'}{}^{\mu_1\cdots\mu_{n-1}}(d,s)\}. \quad (18)$$

Now the factor $\gamma_4V^\dagger\gamma_4$ is clearly just V, if defined as in Eq. (17); if $P_sP(-1)^n=-1$, V must be redefined to include the factor $i\gamma_5$, so one has a γ_5 appearing on either side of $\Lambda^+(p)$ $(=m-ip)$ in (18), one of which can be passed through the $\Lambda^+(p)$ to annihilate the other, resulting in a sign change from m to $-m$. Furthermore one can use Eq. (11), giving

$$\sum|\mathfrak{M}|^2=\frac{g_{s\frac{1}{2}P}{}^2}{m_\pi{}^{2(n-1)}}\frac{n}{2n+1}k_{\mu_1}\cdots k_{\mu_{n-1}} \times k_{\mu_1'}\cdots k_{\mu_{n-1}'}\Theta_{\alpha\mu_1'\cdots\mu_{n-1}'}{}^{\beta\mu_1\cdots\mu_{n-1}}(d,n) \times\mathrm{Tr}\{(m'-ip)(\omega-id)\gamma_\alpha\gamma_\beta\},$$

$$m'=\pm m \quad\text{if}\quad P_sP(-1)^n=\pm1. \quad (19)$$

The quantity formed by contracting the k's with Θ, written $\Theta_{\alpha k\cdots k}{}^{\beta k\cdots k}(d,n)$ is listed as (A4) in the Appendix where it is seen to be symmetric under the interchange of suffixes $\alpha\leftrightarrow\beta$. Thus only the part of the trace which is likewise symmetric can make a nonzero contribution, this being $4\delta_{\alpha\beta}(m'\omega-p\cdot d)$. Therefore,

$$\sum|\mathfrak{M}|^2=\frac{g_{s\frac{1}{2}P}{}^2}{m_\pi{}^{2(n-1)}}\frac{n}{2n+1}4(m'\omega-p\cdot d) \times\delta_{\alpha\beta}\Theta_{\alpha k\cdots k}{}^{\beta k\cdots k}(d,n)$$

and from (A1)

$$=(g_{s\frac{1}{2}P}{}^2/m_\pi{}^{2(n-1)})4D_nk_d{}^{2(n-1)}(m_b'm_d-p\cdot d)\,.$$

On calculating this in O_d [in which $k_d=q$, as defined in Eq. (5)] and substituting in Eq. (5) there results the final expression for the decay width, namely,

$$\Gamma_{s\to\frac{1}{2}+P}(\omega)=\frac{g_{s\frac{1}{2}P}{}^2}{4\pi}\frac{D_n}{n}\frac{q}{\omega}\frac{q^{2(n-1)}}{m_\pi{}^{2(n-1)}}(p_0+m')\,. \qquad (20)$$

This result is true for any $n\geqslant 1$, i.e., decays of baryons having spins $\geqslant\frac{1}{2}$. Note that interaction Lagrangians of the type

$$\bar{\psi}\gamma_\lambda\Phi^{\mu_1\cdots\mu_{n-1}}\partial_{\mu_1}\cdots\partial_{\mu_{n-1}}\partial_\lambda\phi \qquad (21)$$

connecting free fields do not have to be considered, for wave functions $\bar{u}(p)$ and $U^{\mu_1\cdots\mu_{n-1}}(d)$ each obey the Dirac equation, and using $d=p+k$ the amplitude may be shown to reduce to that stemming from (16) with suitable choice of coupling constant.

B. Pseudoscalar-Meson-Baryon Resonance Decays

Turning to decays of the type $B_s^*\to B_{\frac{3}{2}}^*+P$ there are two couplings[11] to consider, involving two dimensionless coupling constants:

$$\mathcal{L}=\frac{g_{s\frac{3}{2}P}}{m_\pi{}^{n-2}}\bar{\Phi}_{\mu_1}\Phi^{\mu_1\mu_2\cdots\mu_{n-1}}\partial_{\mu_2}\cdots\partial_{\mu_{n-1}}\phi\,, \qquad (22a)$$

and

$$\mathcal{L}=\frac{g_{s\frac{3}{2}P}{}'}{m_\pi{}^{n}}\bar{\Phi}_{\mu_1}\Phi^{\mu_m\mu_2\cdots\mu_{n-1}}\partial_{\mu_1}\partial_{\mu_m}\partial_{\mu_2}\cdots\partial_{\mu_{n-1}}\phi\,, \qquad (22b)$$

if $\mathcal{P}=P_sP_{\frac{3}{2}}(-1)^{n+1}=+1$. Once again to conserve parity an $i\gamma_5$ is to be inserted between the spinors if $\mathcal{P}=-1$. The vertex factor corresponding to (22a and 22b) is

$$V_{\mu_m\mu_1\cdots\mu_{n-1}}=\frac{g_{s\frac{3}{2}P}}{m_\pi{}^{n-2}}\delta_{\mu_m\mu_1}k_{\mu_2}\cdots k_{\mu_{n-1}}+\frac{g_{s\frac{3}{2}P}{}'}{m_\pi{}^{n}}k_{\mu_m}k_{\mu_1}k_{\mu_2}\cdots k_{\mu_{n-1}} \qquad (23)$$

which gives for the squared-and-summed amplitude

$$\sum|\mathfrak{M}|^2=\mathrm{Tr}\{\gamma_4V_{\mu_m\mu_1\cdots\mu_{n-1}}{}^\dagger\gamma_4\Lambda^\dagger(p)\Theta_{\mu_1}{}^{\nu_1}(p,\tfrac{3}{2})V_{\nu_m\nu_1\cdots\nu_{n-1}}\Lambda^\dagger(d)\Theta_{\nu_m\nu_2\cdots\nu_{n-1}}{}^{\mu_m\mu_2\cdots\mu_{n-1}}(d,s)\}$$
$$=\frac{2}{5}\frac{n}{2n+1}\Theta_{\alpha\mu_1}{}^{\beta\nu_1}(p,2)\Theta_{\kappa\nu_m\nu_2\cdots\nu_{n-1}}{}^{\tau\mu_m\mu_2\cdots\mu_{n-1}}(d,n)\frac{1}{m_\pi{}^{2(n-2)}}\mathrm{Tr}\Big\{\Big(g\delta_{\mu_m\mu_1}k_{\mu_2}\cdots k_{\mu_{n-1}}+\frac{g'}{m_\pi{}^2}k_{\mu_m}k_{\mu_1}\cdots k_{\mu_{n-1}}\Big)$$
$$\times(m'-ip)\gamma_\alpha\gamma_\beta\Big(g\delta_{\nu_m\nu_1}k_{\nu_2}\cdots k_{\nu_{n-1}}+\frac{g'}{m_\pi{}^2}k_{\nu_m}k_{\nu_1}\cdots k_{\nu_{n-1}}\Big)(\omega-id)\gamma_\kappa\gamma_\tau\Big\}\,, \qquad (24)$$

where $m'=\pm m$ if $\mathcal{P}=\pm 1$, by an argument similar to that preceding Eq. (19).

This expression will now be evaluated taking g^2, g'^2, and gg' terms separately.

(i) $g_{s\frac{3}{2}P}{}^2$ term: This is

$$\frac{1}{m_\pi{}^{2(n-2)}}\frac{2}{5}\frac{n}{2n+1}\Theta_{\alpha\mu}{}^{\beta\nu}(p,2)\Theta_{\kappa\nu k\cdots k}{}^{\tau\mu k\cdots k}(d,n)\,\mathrm{Tr}\{(m'-ip)\gamma_\alpha\gamma_\beta(\omega-id)\gamma_\kappa\gamma_\tau\}\,. \qquad (25)$$

In calculating the trace, terms in p_α, p_β, d_τ, and d_κ can be neglected since they will vanish by (10b); it then resolves itself into two parts, the first being symmetric under either of the interchanges $\alpha\leftrightarrow\beta$ and $\kappa\leftrightarrow\tau$, and the second antisymmetric. The first part has magnitude $4\delta_{\alpha\beta}\delta_{\kappa\tau}(m'\omega-p\cdot d)$, so its contribution to (25) is

$$\frac{2}{5}\frac{n}{2n+1}\Theta_{\alpha\mu}{}^{\alpha\nu}(p,2)\Theta_{\kappa\nu k\cdots k}{}^{\kappa\mu k\cdots k}(d,n)\times 4(m'\omega-p\cdot d)$$
$$=\frac{4}{3}\frac{D_n}{(n-1)}\frac{k_d{}^{2(n-2)}}{m_\pi{}^{2(n-2)}}(m'\omega-p\cdot d)\delta_{\mu\nu}{}^{\perp}(p)\Big[n\delta_{\mu\nu}{}^{\perp}+\frac{(n-2)}{k_d{}^2}k_\mu{}^{\perp}k_\nu{}^{\perp}\Big]\quad\text{by (A2)},$$
$$=\frac{4}{3}\frac{D_n}{(n-1)}\frac{k_d{}^{2(n-2)}}{m_\pi{}^{2(n-2)}}(m'\omega-p\cdot d)\Big[n\Big(2+\frac{(p\cdot d)^2}{p^2d^2}\Big)+(n-2)\Big(1-\frac{(p\cdot k^{\perp})^2}{k_d{}^2p^2}\Big)\Big]\,. \qquad (26)$$

[11] As the coupling of free fields only is considered here, the number of independent couplings is obtained by asking how many irreducible components of the form C_s exist in the direct product $C_{s_1}\times C_{s_2}$ of irreducible, unitary representations of the proper orthochronous inhomogeneous Lorentz group. J. S. Lomont, J. Math. Phys. 1, 237 (1960) states that the multiplicity of C_s for $s\geqslant s_1+s_2$ is $(2s_1+1)(2s_2+1)$; the requirement of parity conservation will halve this number. Thus for reactions (1), (2), and (3) there will be 1, 2, and 3 independent couplings, respectively. Following J. D. Jackson, Nuovo Cimento 33, 906 (1964) and G. C. Fox (Ref. 10), for reaction (3) the highest order coupling, $\bar{\psi}\partial_\lambda\Phi^{\mu_1\cdots\mu_{n-1}}\partial_{\mu_1}\cdots\partial_{\mu_{n-1}}A_\lambda$, will here be neglected for simplicity.

The antisymmetric part of the trace can be shown to be

$$4(m'\omega-p\cdot d)(\delta_{\alpha\tau}\delta_{\beta\kappa}-\delta_{\alpha\kappa}\delta_{\beta\tau})+4p_\kappa(\delta_{\alpha\tau}d_\beta-\delta_{\beta\tau}d_\alpha)-4p_\tau(\delta_{\alpha\kappa}d_\beta-\delta_{\beta\kappa}d_\alpha)\,, \tag{27}$$

which is to be contracted with

$$\mathcal{A}\Theta_{\alpha\mu}{}^{\beta\nu}(p,2)\,\mathcal{A}\Theta_{\nu\kappa k\cdots k}{}^{\mu\tau k\cdots k}(d,n)=\frac{5(2n+1)}{24n(n-1)}D_n k_d{}^{2(n-2)} \times\left(\left[\delta_{\alpha\tau}{}^{\perp\perp}(p)\delta_{\beta\kappa}{}^{\perp\perp}(p)+\frac{(n-2)}{k_d{}^2}\{\delta_{\alpha\tau}{}^{\perp\perp}(p)k_\beta{}^{\perp\perp}(p)k_\kappa{}^{\perp}+\delta_{\beta\kappa}{}^{\perp\perp}(p)k_\alpha{}^{\perp\perp}(p)k_\tau{}^{\perp}\}\right]-[\alpha\leftrightarrow\beta]\right), \tag{28}$$

the latter equation following from (A3) and (A7a) and (A7b). The resulting contribution to (25) may be shown by direct calculation to be

$$\frac{4}{3}\frac{D_n}{(n-1)}\frac{k_d{}^{2(n-2)}}{m_\pi{}^{2(n-2)}}\left[(m'\omega-p\cdot d)\left\{\left(1+2\frac{(p\cdot d)^2}{p^2d^2}\right)+(n-2)\left(1+\frac{(p\cdot d)^2}{p^2d^2}-\frac{(p\cdot k^\perp)^2}{k_d{}^2p^2}\right)\right\} -\frac{p\cdot d}{p^2}k_d{}^2\left\{(2n-2)+(n-2)\frac{k_d{}^2}{p^2}\left(1-\frac{(p\cdot k^\perp)^2}{k_d{}^4}\right)\right\}\right]. \tag{29}$$

On evaluating (26) and (29) in O_d one obtains the partial decay width due to the $(n-2)$-fold derivative coupling:

$$\Gamma_{s\to\frac{3}{2}+P}(\omega)=\frac{g_{s\frac{3}{2}P}{}^2}{4\pi}\frac{D_n}{n}\frac{q^{2(n-2)}}{m_\pi{}^{2(n-2)}}\frac{q}{\omega}\left[\frac{(2n-1)}{(n-1)}(p_0+m')+\frac{2q^2}{3m^2}(p_0+2m')\right]. \tag{30}$$

(ii) g'^2 term: From (24) this is

$$\frac{1}{m_\pi{}^{2n}}\frac{2}{5}\frac{n}{(2n+1)}\Theta_{\alpha k}{}^{\beta k}(p,2)\,\Theta_{\kappa k\cdots k}{}^{\tau k\cdots k}(d,n)\ \mathrm{Tr}\{(m'-ip)\gamma_\alpha\gamma_\beta(\omega-id)\gamma_\kappa\gamma_\tau\}\,. \tag{31}$$

One sees from (A4) that the Θ's are symmetric in their suffixes so that only the symmetric part of the trace contributes, giving

$$\frac{1}{m_\pi{}^{2n}}\times\tfrac{2}{5}\delta_{\alpha\beta}\Theta_{\alpha k}{}^{\beta k}(p,2)\delta_{\kappa\tau}\Theta_{\kappa k\cdots k}{}^{\tau k\cdots k}(d,n)4(m'\omega-p\cdot d)=(8/3)D_n\frac{k_d{}^{2(n-1)}}{m_\pi{}^{2(n-1)}}\frac{k_p{}^2}{m_\pi{}^2}(m'\omega-p\cdot d)\,,$$

where $k_p{}^2=\delta_{\alpha\beta}{}^{\perp}(p)k_\alpha{}^{\perp}(p)k_\beta{}^{\perp}(p)$, consistent with the the definition (13c); note that this quantity has magnitude $(\omega^2/m^2)q^2$ in O_d. The partial decay width for n-fold derivative coupling is therefore

$$\Gamma_{s\to\frac{3}{2}+P}{}'(\omega)=\frac{g_{s\frac{3}{2}P}{}'^2}{4\pi}\frac{2D_n}{3n}\frac{q^{2n}}{m_\pi{}^{2n}}\frac{q}{m}\frac{\omega}{m}(p_0+m')\,. \tag{32}$$

(iii) gg' term: From (24) one has

$$\frac{1}{m_\pi{}^{2(n-1)}}\frac{2}{5}\frac{n}{2n+1}\Theta_{\alpha\mu_1}{}^{\beta\nu_1}(p,2)\,\Theta_{\kappa\nu_m\nu_2\cdots\nu_{n-1}}{}^{\tau\mu_m\mu_2\cdots\mu_{n-1}}(d,n)\ \mathrm{Tr}\{\delta_{\mu_m\mu_1}k_{\mu_2}\cdots k_{\mu_{n-1}}k_{\nu_m}k_{\nu_1}\cdots k_{\nu_{n-1}} \times(m'-ip)\gamma_\alpha\gamma_\beta(\omega-id)\gamma_\kappa\gamma_\tau+\delta_{\nu_m\nu_1}k_{\nu_2}\cdots k_{\nu_{n-1}}k_{\mu_m}k_{\mu_1}\cdots k_{\mu_{n-1}}(m'-ip)\gamma_\alpha\gamma_\beta(\omega-id)\gamma_\kappa\gamma_\tau\}\,. \tag{33}$$

By swapping upper and lower indices of the Θ's and interchanging dummy labels for the first term in the trace, this becomes

$$\frac{1}{m_\pi{}^{2(n-1)}}\times\frac{2}{5}\frac{n}{2n+1}\Theta_{\alpha k}{}^{\beta\nu_1}(p,2)\,\Theta_{\kappa\nu_1 k\cdots k}{}^{\tau k\cdots k}(d,n)\ \mathrm{Tr}\{m'-ip)\gamma_\beta\gamma_\alpha(\omega-id)\gamma_\tau\gamma_\kappa+(m'-ip)\gamma_\alpha\gamma_\beta(\omega-id)\gamma_\kappa\gamma_\tau\}\,. \tag{34}$$

The two halves of the trace are equal since they are individually symmetric under the simultaneous interchange,

$\alpha \leftrightarrow \beta$ and $\tau \leftrightarrow \kappa$, as may be seen from (27). Taking the part of the trace in $\delta_{\alpha\beta}\delta_{\tau\kappa}$ first, and using (A5) one has

$$\frac{2}{m_\pi^{2(n-1)}}\times(8/3)D_n k_d^{2(n-2)} k_b^2 (m'\omega - p\cdot d)(1-d\cdot k/d^2) \tag{35}$$

while the antisymmetric part gives, with the aid of (A6),

$$\frac{2}{m_\pi^{2(n-1)}}\frac{D_n}{24}k_d^{2(n-2)}\{[\delta_{\alpha\tau}^{\perp\perp}(p)k_{\beta}^{\perp}(p)k_\kappa^{\perp}+\delta_{\beta\kappa}^{\perp\perp}(p)k_\alpha^{\perp}(p)k_\tau]-[\alpha\leftrightarrow\beta]\}\,\mathfrak{A}\,\mathrm{Tr}$$

$$=\frac{2}{m_\pi^{2(n-1)}}\frac{2}{3}D_n k_d^{2(n-2)}k_b^2\left\{(m'\omega-p\cdot d)\left(1-\frac{d\cdot k}{d^2}+\frac{p\cdot d}{d^2}\right)+p\cdot k^{\perp}-k_d^2\right\}. \tag{36}$$

Equations (35) and (36) when evaluated in O_d together give the decay-width contribution due to interference between the n-fold and $(n-2)$-fold derivative couplings as

$$\Gamma_{s\to\frac{3}{2}+P}{}^{\mathrm{interf.}}(\omega)=\frac{g_{s\frac{3}{2}P}g_{s\frac{3}{2}P}'}{4\pi}\frac{2D_n}{3n}\frac{q^{2(n-1)}}{m_\pi^{2(n-1)}}\frac{q}{m^2}[3p_0(p_0+m')-q^2]. \tag{37}$$

The expressions (30), (32), and (37) hold for $s\geqslant\frac{3}{2}$ provided the Lagrangian (22a) is interpreted to have reduced to the simple nonderivative form $\bar{\Phi}\cdot\Phi\phi$ in the special case $s=\frac{3}{2}$. The case $\frac{1}{2}\to\frac{3}{2}+P$ is readily shown to have the same matrix element as the reaction $\frac{3}{2}\to\frac{1}{2}+P$ so the width is given by an expression of the same form as Eq. (20) for $n=2$ with the choice $m'=\pm m$ according as $P_{\frac{1}{2}}P_{\frac{3}{2}}=\pm1$.

Finally one may observe that Lagrangians differing from (22) by the insertion of a γ_μ between the spinor fields and contracted either with a spinor index or a derivative index must vanish, either because of (9a), or by an argument similar to that following Eq. (21) if one considers free fields.

C. Vector-Meson–Baryon Decays

For the decay $B_s^*\to B_{\frac{1}{2}}+V$ the two interaction Lagrangians considered[11] are

$$\mathcal{L}=\frac{g_{s\frac{1}{2}V}}{m_\pi^{(n-2)}}\bar{\psi}\Phi^{\mu_1\cdots\mu_{n-1}}\partial_{\mu_2}\cdots\partial_{\mu_{n-1}}A_{\mu_1}, \tag{38a}$$

and

$$\mathcal{L}=\frac{g_{s\frac{1}{2}V}'}{m_\pi^{(n-1)}}\bar{\psi}\gamma_\lambda\Phi^{\mu_1\cdots\mu_{n-1}}\partial_{\mu_1}\cdots\partial_{\mu_{n-1}}A_\lambda \tag{38b}$$

for $\mathcal{P}=P_sP_{\frac{1}{2}}(-1)^{n+1}=+1$, with the usual $i\gamma_5$ insertion for the opposite parity combination. The resulting amplitude, squared, and summed over both baryon spins and vector-meson polarization is

$$\sum|\mathfrak{M}|^2=\mathrm{Tr}\{\gamma_4V_{\mu_1\cdots\mu_{n-1}\lambda}{}^{\dagger}\gamma_4\Lambda^+(p)V_{\nu_1\cdots\nu_{n-1}\rho}\times\Lambda^+(d)\Theta_{\nu_1\cdots\nu_{n-1}}{}^{\mu_1\cdots\mu_{n-1}}(d,s)\}\delta_{\lambda\rho}{}^{\perp}(k), \tag{39}$$

where

$$V_{\mu_1\cdots\mu_{n-1}\lambda}=\frac{g_{s\frac{1}{2}V}}{m_\pi^{(n-2)}}\delta_{\mu_1\lambda}k_{\mu_2}\cdots k_{\mu_{n-1}}+\frac{ig_{s\frac{1}{2}V}'}{m_\pi^{(n-1)}}\gamma_\lambda k_{\mu_1}\cdots k_{\mu_{n-1}}. \tag{40}$$

Proceeding in the usual way by applying Eq. (11) and taking g^2, g'^2, and gg' terms separately, one has the following, in which $m'=\pm m$ according as

$$P_sP_{\frac{1}{2}}(-1)^{n+1}=\pm1:$$

(i) g^2 term: This is

$$\frac{1}{m_\pi^{2(n-2)}}\frac{n}{2n+1}\Theta_{\kappa\rho k\cdots k}{}^{\tau\lambda k\cdots k}(d,n)\delta_{\lambda\rho}{}^{\perp}(k)\times\mathrm{Tr}\{(m'-ip)(\omega-id)\gamma_\kappa\gamma_\tau\}.$$

Because of the presence of the vector-meson operator which is symmetric in $\lambda\leftrightarrow\rho$, only symmetric parts of $\Theta(d)$ and the trace contribute, so on using (A2) this gives

$$\frac{2D_n}{(n-1)}\frac{k_d^{2(n-2)}}{m_\pi^{2(n-2)}}(m'\omega-p\cdot d)\left[n\left(2+\frac{(d\cdot k)^2}{d^2k^2}\right)+(n-2)\frac{(d\cdot k)^2}{k_d^2d^2}\left(1-\frac{(d\cdot k)^2}{d^2k^2}\right)\right], \tag{41}$$

which on evaluating in O_d gives the partial decay width due to $(n-2)$-fold derivative coupling:

$$\Gamma_{s\to\frac{1}{2}+V}(\omega)=\frac{g_{s\frac{1}{2}V}{}^2}{4\pi}\frac{D_n}{n}\left(\frac{q}{m_\pi}\right)^{2(n-2)}\times\frac{q}{\omega}(p_0+m')\left(\frac{2n-1}{n-1}+\frac{q^2}{\mu^2}\right). \tag{42}$$

(ii) g'^2 term: Equation (39) leads to

$$-\frac{1}{m_\pi^{2(n-1)}}\frac{n}{2n+1}\Theta_{\kappa k\cdots k}{}^{\tau k\cdots k}(d,n)\delta_{\lambda\rho}{}^{\perp}(k)\times\mathrm{Tr}\{\gamma_\lambda(m'-ip)\gamma_\rho(\omega-id)\gamma_\kappa\gamma_\tau\}.$$

TABLE I. Decay widths for baryon resonances of spin $s(=n-\frac{1}{2})$ in terms of dimensionless coupling constants g, g' defined by the Lagrangians (16), (22), and (38). The masses of decaying baryon, product baryon, and meson are ω, m, and μ, respectively; q is the decay momentum common to product baryon and meson in the center-of-mass system, p_0 and k_0 are their total energies, and $m'=(-1)^{\mathcal{P}}m$ with $\mathcal{P}$ as given in the first column. $[D_n=(n!)^2 2^n/(2n)!]$.

Decay mode \ $\Gamma(\omega)$	$\frac{g^2}{4\pi}\frac{D_n}{n}\times$	$\frac{g'^2}{4\pi}\frac{D_n}{n}\times$	$\frac{gg'}{4\pi}\frac{D_n}{n}\times$
1. $B_s^*\to B_{1/2}+P$ $\mathcal{P}=P_sP_{1/2}(-1)^n$	$\left(\frac{q}{m_\pi}\right)^{2(n-1)}\frac{q}{\omega}(p_0+m')$	$\cdots$	$\cdots$
2. $B_s^*\to B_{3/2}^*+P$ $\mathcal{P}=P_sP_{3/2}(-1)^{n+1}$	$\left(\frac{q}{m_\pi}\right)^{2(n-2)}\frac{q}{\omega}\left[\left(\frac{2n-1}{n-1}\right)(p_0+m')+\frac{2}{3}\frac{q^2}{m^2}(p_0+2m')\right]$	$\left(\frac{q}{m_\pi}\right)^{2n}\frac{q}{\omega}\frac{\omega^2}{m^2}\frac{2}{3}(p_0+m')$	$\left(\frac{q}{m_\pi}\right)^{2(n-1)}\frac{q}{m^2}[2p_0(p_0+m')-\frac{2}{3}q^2]$
3. $B_s^*\to B_{1/2}+V$ $\mathcal{P}=P_sP_{1/2}(-1)^{n+1}$	$\left(\frac{q}{m_\pi}\right)^{2(n-2)}\frac{q}{\omega}(p_0+m')\left[\left(\frac{2n-1}{n-1}\right)+\frac{q^2}{\mu^2}\right]$	$\left(\frac{q}{m_\pi}\right)^{2(n-1)}\frac{q}{\omega}\left[3(p_0-m')+2\frac{q^2}{\mu^2}\omega\right]$	$-\left(\frac{q}{m_\pi}\right)^{2n-3}\frac{2q}{\omega}\left[q+\frac{k_0q}{\mu^2}(\omega+m')\right]$

Because of the symmetry of Θ and $\delta_{\lambda\rho}{}^1$ only the part of the trace symmetric in $\tau\leftrightarrow\kappa$ and $\lambda\leftrightarrow\rho$ contributes, giving

$$4D_n\frac{k_d{}^{2(n-1)}}{m_\pi{}^{2(n-1)}}\left[2\left(p\cdot d-\frac{p\cdot kd\cdot k}{k^2}\right)-3(m\omega+p\cdot d)\right], \quad (43)$$

which leads to the $(n-1)$-fold coupling width

$$\Gamma_{s\to\frac{1}{2}+V}'(\omega)=\frac{g_{s\frac{1}{2}V}'^2}{4\pi}\frac{D_n}{n}\left(\frac{q}{m_\pi}\right)^{2(n-1)}\times\frac{q}{\omega}\left[3(p_0-m')+\frac{2q^2}{\mu^2}\omega\right]. \quad (44)$$

(iii) gg' term: Adopting the technique used to simplify Eq. (33), this part of Eq. (39) becomes

$$\frac{n}{2n+1}\Theta_{\kappa\nu k\cdots k}{}^{\tau k\cdots k}(d,n)\times\mathrm{Tr}\{\delta_{\nu\rho}(\gamma_\lambda d\gamma_\kappa\gamma_\tau m'+m_d\gamma_\lambda p\gamma_\kappa\gamma_\tau)+(\rho\leftrightarrow\lambda,\,\kappa\leftrightarrow\tau)\}\delta_{\lambda\rho}{}^1(k). \quad (45)$$

Taking parts symmetric and antisymmetric in $\kappa\leftrightarrow\tau$ separately and using (A5), (A6) this leads to

$$8D_n\frac{k_d{}^{2(n-1)}}{m_\pi{}^{2n-3}}\left[\omega\left(1+\frac{2p\cdot k^1}{k_d{}^2}\right)-\frac{d\cdot k}{k^2}(m'+\omega)\right], \quad (46)$$

so the interference-term contribution to the decay width is

$$\Gamma_{s\to\frac{1}{2}+V}{}^{\mathrm{interf.}}(\omega)=\frac{-g_{s\frac{1}{2}V}g_{s\frac{1}{2}V}'}{4\pi}\frac{2D_n}{n}\left(\frac{q}{m_\pi}\right)^{2n-3}\times\frac{q}{\omega}\left\{q+\frac{k_0q}{\mu^2}(m'+\omega)\right\}. \quad (47)$$

The expressions (42), (44), and (47) hold for $s\geq\frac{3}{2}$ provided the Lagrangian (38a) is interpreted as $\bar\psi\Phi\cdot A$ for $s=\frac{3}{2}$. For the case $\frac{1}{2}\to\frac{1}{2}+V$ only one Lagrangian (38b) is relevant, for which Eq. (44) with $n=1$ gives the appropriate width.

As with case B, Lagrangians differing from (38) by the insertion of a γ_μ between the spinor fields may be neglected, as may those incorporating $\cdots\partial_{\mu_i}\cdots A_{\mu_i}$ by the subsidiary condition on the vector field A_μ. The calculated decay widths are summarized in Table I.

IV. DISCUSSION

The decay width for $B_s^*\to B_{\frac{1}{2}}+P$, Eq. (20), can be written in the form $[\mathcal{P}=P_sP_{\frac{1}{2}}(-1)^n]$:

$\mathcal{P}=+1(l=s-\frac{1}{2})$,

$$\Gamma_{(\omega)}=\frac{g^2}{4\pi}\frac{D_n}{n}\frac{1}{2m_\pi{}^{2l}}q^{2l+1}\frac{[(\omega+m)^2-\mu^2]}{\omega^2}, \quad (20a)$$

$\mathcal{P}=-1(l=s+\frac{1}{2})$,

$$\Gamma_{(\omega)}=\frac{g^2}{4\pi}\frac{D_n}{n}\frac{1}{m_\pi{}^{2l-2}}q^{2l+1}\frac{1}{[(\omega+m)^2-\mu^2]}, \quad (20b)$$

which generalizes the result quoted by Jackson[1] for $s=\frac{3}{2}\pm$ resonances. The term in square brackets is just $2\omega(p_0+m)$ which is $\approx 4\omega m$ for $q^2\ll m^2$ giving

$$\Gamma\propto g^2(q^{2l+1}/\omega)m^{\pm1},\quad (l=s\mp\tfrac{1}{2}),\quad q^2\ll m^2. \quad (48)$$

Here l is the orbital angular momentum for the decay process inferred from angular momentum and parity conservation, and the q^{2l+1} may be interpreted as the familiar angular-momentum barrier factor (q^{2l}) times a phase-space factor (q) by analogy with the nonrelativistic potential-theory result.[2] As Dürr and Pilkuhn[12] stress, there are ambiguities in apportioning Feynman amplitudes into orbital angular momentum parts and spin or purely kinematic parts in a relativistic treatment. However, to pursue the analogy, potential theory states that

$$\Gamma \propto \gamma q r_0 V_l(q r_0)\,, \qquad (49)$$

where γ is the "reduced width," r_0 the range of interaction, and $V_l(kr_0)$ a barrier-penetration factor. Some values of $V_l(qr_0)$ are[2]

$$V_0(qr_0)=1\,,\quad V_1(qr_0)=q^2r_0^2/(1+q^2r_0^2)\,,$$
$$V_2(qr_0)=q^4r_0^4/(9+3q^2r_0^2+q^4r_0^4)\,,$$

and in the limit $qr_0 \to 0$, $V_l(qr_0)\sim(qr_0)^{2l}D_n/n(n!)$; thus there is the expected correspondence to the point-interaction field-theoretical result of (20a), (20b). The "reduced width" γ encompasses effects due to the internal properties of the resonance, and would correspond to higher-order terms in a perturbation-theory treatment, as would the denominator of $V_l(qr_0)$ which expresses directly the degree of departure from a point-interaction picture. In choosing an expression $\Gamma\propto(m/\omega)\times q|q^2/(q^2+X^2)|^l$ for the SU_3 analysis of resonance decays, Glashow and Rosenfeld[13] allowed for such a departure empirically by the introduction of a form factor $(q^2+X^2)^{-l}$.

Recent theories of SU_3-breaking suggest[14,15] linear relations among the coupling constants for the decays of members of the same multiplet into a baryon and a pion, such as the following for the $\frac{3}{2}^+$ decuplet

$$2g(N^*\to N\pi)+3\sqrt{2}g(Y^*\to\Lambda\pi)$$
$$-\sqrt{3}g(Y^*\to\Sigma\pi)+2\sqrt{2}g(\Xi^*\to\Xi\pi)=0\,,$$

and

$$g(N^*\to N\pi)=-\sqrt{2}g(Y^*\to\Lambda\pi)$$

which are satisfied by recent data to within experimental error, when the relation (20b) for $l=1$, $\Gamma\propto g^2q^3m/\omega$, is employed. Becchi, Eberle, and Morpurgo[15] determined the effect on the derived coupling constant ratios of introducing the $(q^2+X^2)^{-1}$ factor of Glashow and Rosenfeld,[13] and found it was only 2% for the choice $X=350$ MeV.

Notwithstanding the difficulty of isolating symmetry-breaking effects from form factor effects, it would appear that the unmodified relations (20a), (20b) or (48) are adequate for the moment. Whether the coupling constants for decays of the proposed $\frac{3}{2}^-$ octet containing the $N^*(1520)$, $Y^*(1660)$, and $\Xi^*(1820)$ are satisfactory in this respect is a question awaiting experimental clarification.[16,17] Brudnoy[4] has shown that the decays of the $\frac{5}{2}^+$ resonances $N^*(1688)$ and $Y^*(1820)$ into various $B_{\frac{1}{2}}P$ states are consistent with their being members of an SU_3 octet, using the relation (20) with $n=3$, $\mathcal{P}=-1$ [which is the same as (20b) with $l=3$] without a form factor. Experimental checks on the allocation to multiplets of the higher nucleon resonances such as $N^*(1920)$, $N^*(2190)$, and $N^*(2360)$ must await the identification of co-members.

Turning to $B_s^*\to B_{\frac{3}{2}}^*+P$ decays, the experimental widths for the reactions (2a), (2b) have been shown to be consistent[4] with the identification $N_\alpha^{\mathrm{II}}(1688)\to\Delta_\delta(1238)+\pi$, $\Lambda_\alpha^{\mathrm{II}}(1815)\to\Sigma_\delta(1385)+\pi$ provided the lowest order single-derivative coupling (30) is employed in preference to the threefold derivative coupling (32). Decays of the $\frac{3}{2}^-$ octet members to the corresponding δ-decuplet members have been considered[13,17] but only upper limits to the widths are available, derived from the inelastic (2π) decay modes.

One example of the decay $B_s^*\to B_{\frac{1}{2}}+V$ is known to the author, namely, that of the $Y^*(2300)$ $I=1$ resonance recently identified[18] in 3.5-GeV/c K^-p interactions:

$$Y^*(2300)\to N+K^*(890)\,.$$

Both the spin-parity and multiplet assignment of this resonance await identification.

In considering the decay of one resonance into another, the effect of the finite width of the daughter resonance on the decay width of the parent resonance should be considered. The expression for $\Gamma(\omega,m,\mu)$ derived above should more correctly be replaced by a suitable average over the mass of the daughter resonance such as

$$\bar\Gamma(\omega,m_0,\mu)=\frac{m_0}{\pi}\int_0^\infty\Gamma(\omega,m,\mu)\frac{\Gamma'(m_0)}{(m^2-m_0^2)^2+m_0^2\Gamma'^2(m_0)}dm^2$$

if the daughter resonance has mass m_0 and width $\Gamma'(m_0)$. Or, as a rough guess, one could try the usual trick of substituting a complex mass $m^2\to m_0^2+im_0\Gamma'$, and taking

$$\bar\Gamma(\omega,m_0^2,\mu)\approx\mathrm{Re}\,\Gamma(\omega,\,m_0^2-im_0\Gamma',\,\mu)\,,$$

which amounts to an effect $\sim(\Gamma'/m_0)^2$, negligible for existing resonances.

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APPENDIX

The general form of $\Theta_{\mu_1\cdots\mu_n}{}^{\nu_1\cdots\nu_n}$, the spin-$n$ boson positive-energy projection operator, has been given by Behrends and Fronsdal[9] as follows:

$$\Theta_{\mu_1\cdots\mu_n}{}^{\nu_1\cdots\nu_n}=\frac{1}{(n!)^2}\sum_{p(\mu),p(\nu)}\Bigg[\prod_{i=1}^{n}\delta_{\mu_i\nu_i}{}^{\perp}+a_1\delta_{\mu_1\mu_2}{}^{\perp}\delta_{\nu_1\nu_2}{}^{\perp}\prod_{i=3}^{n}\delta_{\mu_i\nu_i}{}^{\perp}+\cdots+\begin{cases}a_{n/2}\delta_{\mu_1\mu_2}{}^{\perp}\delta_{\nu_1\nu_2}{}^{\perp}\cdots\delta_{\mu_{n-1}\mu_n}{}^{\perp}\delta_{\nu_{n-1}\nu_n} & \text{for even } n\\ a_{(n-1)/2}\delta_{\mu_1\mu_2}{}^{\perp}\delta_{\nu_1\nu_2}{}^{\perp}\cdots\delta_{\mu_{n-2}\mu_{n-1}}{}^{\perp}\delta_{\nu_{n-2}\nu_{n-1}}{}^{\perp}\delta_{\mu_n\nu_n}{}^{\perp} & \text{for odd } n\end{cases}\Bigg],$$

where the sum runs over all permutations of the n suffixes $\mu_1\cdots\mu_n$, and separately over all permutations of the n suffixes $\nu_1\cdots\nu_n$. The significance of the $n=1$ operator (Eq. 13a) has been referred to in the text. The explicit form for $n=2$ is

$$\Theta_{\mu_1\mu_2}{}^{\nu_1\nu_2}(2)=\tfrac{1}{2}\delta_{\mu_1\nu_1}{}^{\perp}\delta_{\mu_2\nu_2}{}^{\perp}+\tfrac{1}{2}\delta_{\mu_1\nu_2}{}^{\perp}\delta_{\mu_2\nu_1}{}^{\perp}-\tfrac{1}{3}\delta_{\mu_1\mu_2}{}^{\perp}\delta_{\nu_1\nu_2}{}^{\perp}.$$

The following useful properties of the Θ's when some of their suffixes are contracted with a four-vector k_μ (then written $\Theta_{\cdots\mu\cdots}{}^{\cdots}k_\mu=\Theta_{\cdots k\cdots}{}^{\cdots}$) can be derived from the above definition [some of these have already been given in Fox[10]];

$$\Theta_{\alpha k\cdots k}{}^{\alpha k\cdots k}(d,n)=\frac{2n+1}{n}D_n k_d{}^{2(n-1)},\qquad\text{(A1)}$$

$$\Theta_{\alpha\mu_1k\cdots k}{}^{\alpha\nu_1k\cdots k}(d,n)=\frac{(2n+1)}{2n}\frac{D_n}{(n-1)}\times\left[n\delta_{\mu_1\nu_1}{}^{\perp}+(n-2)\frac{k_{\mu_1}{}^{\perp}k_{\nu_1}{}^{\perp}}{k_d{}^2}\right]k_d{}^{2(n-2)},\qquad\text{(A2)}$$

$$\mathcal{A}_{\mu_1\leftrightarrow\nu_1,\mu_2\leftrightarrow\nu_2}\Theta_{\mu_1\mu_2k\cdots k}{}^{\nu_1\nu_2k\cdots k}(d,n)=\frac{(2n+1)}{2n}\frac{D_n}{2(n-1)}[\delta_{\mu_1\nu_2}{}^{\perp}\delta_{\mu_2\nu_1}{}^{\perp}+((n-2)/k_d{}^2)\times(\delta_{\mu_1\nu_2}{}^{\perp}k_{\nu_1}{}^{\perp}k_{\mu_2}{}^{\perp}+\delta_{\mu_2\nu_1}{}^{\perp}k_{\mu_1}{}^{\perp}k_{\nu_2}{}^{\perp})-(\mu_1\leftrightarrow\nu_1)],\qquad\text{(A3)}$$

$$\Theta_{\mu_1k\cdots k}{}^{\nu_1k\cdots k}(d,n)=\frac{D_n}{2n}\left[(n+1)\delta_{\mu_1\nu_1}{}^{\perp}+(n-1)\frac{k_{\mu_1}{}^{\perp}k_{\nu_1}{}^{\perp}}{k_d{}^2}\right]k_d{}^{2(n-1)},\qquad\text{(A4)}$$

$$\Theta_{\alpha\nu k\cdots k}{}^{\alpha k\cdots k}(d,n)=((2n+1)/n)D_nk_\nu{}^{\perp}k_d{}^{2(n-2)},\qquad\text{(A5)}$$

$$\mathcal{A}_{\mu_1\leftrightarrow\nu_1}\Theta_{\mu_1\mu_2k\cdots k}{}^{\nu_1k\cdots k}(d,n)=\frac{2n+1}{4n}D_n[k_{\mu_1}{}^{\perp}\delta_{\nu_1\mu_2}{}^{\perp}-k_{\nu_1}{}^{\perp}\delta_{\mu_1\mu_2}{}^{\perp}],\qquad\text{(A6)}$$

where

$$D_n=(n!)^2 2^n/(2n)!.$$

These expressions refer to particle d, so that $k_d{}^2$ is the space-momentum-squared of k_μ in O_d, according to the definition (13c). Strictly speaking $\delta_{\mu\nu}{}^{\perp}(d)$ and $k_\mu{}^{\perp}(d)$, rather than $\delta_{\mu\nu}{}^{\perp}$ and $k_\mu{}^{\perp}$ should appear, but for conciseness this label for particle d has been dropped in the text; it is retained for particle p however [see, for example, Eq. (28)].

Certain quantities appear in the text which are contractions of $n=1$ projection operators referring to different particles, defined as follows:

$$\delta_{\mu\nu}{}^{\perp}(p)\delta_{\nu\lambda}{}^{\perp}=\delta_{\mu\lambda}{}^{\perp\perp}(p),\qquad\text{(A7a)}$$

$$\delta_{\mu\nu}{}^{\perp}(p)k_\nu{}^{\perp}=k_\mu{}^{\perp\perp}(p).\qquad\text{(A7b)}$$