Neutron Exchange in the Reaction $p+p \rightarrow d+\pi^+$

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Several features of observed $p+p \rightarrow d+\pi^+$ cross sections in the BeV region suggest that a one-particle (in this case, one-neutron) exchange mechanism may play a significant role. We have investigated this possibility. The treatment of the dnp vertex requires some care; the Fourier transform of the deuteron wave function plays a crucial role in the calculations. The resulting predictions for the angular distribution clearly exhibit the forward peaking which is experimentally observed, although the calculated total cross section, as a function of energy, does not agree very well with experiment.

I. INTRODUCTION

THE reaction

$$p+p \rightarrow d+\pi^+ \tag{1}$$

has been of considerable interest, both theoretically[1–3] and experimentally.[4–7] The most conspicuous feature, a prominent peak in the total cross section at a proton laboratory kinetic energy of about 600 MeV, was shown by Mandelstam[1] to follow by a final-state interaction mechanism from the existence of the (3,3) resonance $N^*(1238)$ in the pion-nucleon system. Turkot *et al.*,[4] and more recently Yao,[2] extended this mechanism to higher energies, and showed that the pion-nucleon resonance $N^*(1920)$ should produce a peak in the total cross section for reaction (1) at a laboratory kinetic energy of about 2.8 BeV. If the existence of this latter peak, which the experiment of Turkot *et al.*[4] suggests, is confirmed, it will establish this calculational scheme, involving one-pion exchange plus final-state interaction, as a reasonable approximation scheme.

There is, however, another single-particle exchange which may occur, namely neutron exchange (Fig. 1). This process was briefly discussed by Yao[2]; he remarks that the relevant diagrams are (a) difficult to calculate because the neutron is far from its mass shell, and (b) probably smooth functions of energy and therefore not able to reproduce the observed peaks. We are in general agreement with both of these points, but it is still of interest to attempt the calculation. The differential cross section[5] exhibits forward (and backward) peaking, and the forward peak approaches 0° with increasing energy. This behavior is very suggestive of a one-particle exchange mechanism.

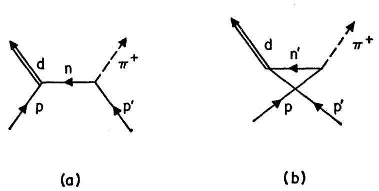

FIG. 1. Feynman diagrams for single neutron exchange in the reaction $p+p \rightarrow d+\pi^+$: (a) "uncrossed" diagram; (b) "crossed" diagram.

In Sec. II we present the general plan of our calculation. In Sec. III we discuss the treatment of the dnp vertex and in Sec. IV we present our numerical results and some remarks. An Appendix contains some calculational details.

II. MATRIX ELEMENTS

The invariant matrix element T for reaction (1) may be written, in our approximation, as

$$T=T_a-T_b, \tag{2}$$

where T_a and T_b are the contributions from the diagrams of Figs. 1(a) and 1(b), respectively.

The dnp vertex has been discussed by Blankenbecler *et al.*[8] We shall rederive the basic result in Sec. III of this paper, because the derivation in Ref. 8 is highly formal and abbreviated, and because of a misprint in the answer [Eq. (33) of Ref. 8]. The result is that the dnp vertex of Fig. 2 is

$$\bar{v}(p)(g_1\gamma_\mu+g_2p_\mu)u(n), \tag{3}$$

where u and v are particle and antiparticle spinors, respectively, and the index μ is to be contracted with the polarization vector of the deuteron, which we treat as an elementary spin-one particle. Explicit expressions for g_1

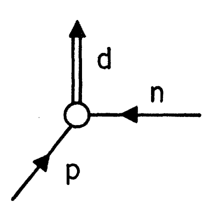

FIG. 2. dnp vertex, with proton and neutron incoming and deuteron outgoing.

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[1] S. Mandelstam, Proc. Roy. Soc. (London) **A244**, 491 (1958).
[2] T. Yao, Phys. Rev. **134**, B454 (1964).
[3] R. M. Heinz, O. E. Overseth, and M. H. Ross, Bull. Am. Phys. Soc. **10**, 19 (1965).
[4] F. Turkot, G. B. Collins, and T. Fujii, Phys. Rev. Letters **11**, 474 (1963).
[5] O. E. Overseth *et al.*, Phys. Rev. Letters **13**, 59 (1964).
[6] References to earlier experimental work may be found in Refs. 1, 4, and 5.
[7] D. Dekkers, *et al.*, Phys. Letters **11**, 161 (1964).
[8] R. Blankenbecler, M. L. Goldberger, and F. R. Halpern, Nucl. Phys. **12**, 629 (1959).

TABLE I. Deuteron "decay" matrix elements. q denotes the common momentum of the proton and neutron. $F_{p,n}=E_{p,n}+m_{p,n}$ where $E_{p,n}$ is the total energy of the proton or neutron.

Proton spin	Neutron spin	Matrix element
Up	Up	$(2F_pF_n)^{-1/2}[2g_1(F_pF_n+q^2\cos^2\theta)+g_2(F_p+F_n)q^2\sin^2\theta]$
Up / Down	Down / Up	$(2F_pF_n)^{-1/2}[2g_1-g_2(F_p+F_n)]q^2\sin\theta\cos\theta e^{i\phi}$
Down	Down	$(2F_pF_n)^{-1/2}[2g_1-g_2(F_p+F_n)]q^2\sin^2\theta e^{2i\phi}$

and g_2, in terms of the deuteron wave function, will be given in Sec. III.

If we use the dnp vertex (3), the matrix elements T_a and T_b of (2) are

$$\begin{aligned} T_a&=\bar{v}(p)(g_1\gamma_\mu+g_2p_\mu)(n-m_n)^{-1}\sqrt{2}g\gamma_5u(p'),\\ T_b&=\bar{v}(p')(g_1\gamma_\mu+g_2p_\mu')(n'-m_n)^{-1}\sqrt{2}g\gamma_5u(p), \end{aligned}\tag{4}$$

where g is the conventional pion-nucleon coupling constant; $(g^2/4\pi)\approx14.7$. The $p+p\rightarrow d+\pi^+$ differential cross section is then

$$\frac{d\sigma}{d\Omega}=\frac{1}{256\pi^2}\frac{q_2}{q_1}\frac{1}{W^2}\sum|T|^2.\tag{5}$$

q_1, q_2, and W are the initial momentum, final momentum, and total energy in the c.m. system, respectively, and $\sum$ denotes the usual sum over proton and deuteron polarizations. The squaring and summing are performed by standard techniques; details are presented in the Appendix.

III. THE dnp VERTEX

Let us begin by imagining that $m_d>m_p+m_n$ so that the deuteron is unstable and decays into $p+n$. The matrix element describing the decay will be taken to be (3), or more precisely the Hermitian conjugate of (3), since the roles of incoming and outgoing particles have been interchanged.

We first wish to relate the coupling constants g_1 and g_2 to the s- and d-wave decay amplitudes. A straightforward way to do this is to imagine the deuteron to be "spin up"; i.e., the polarization vector is $-(\frac{1}{2}\sqrt{2})(\hat{x}+i\hat{y})$. By inserting explicit expressions for the spinors in (3), we obtain decay matrix elements to the various nucleon spin states. These are given in Table I. Note that we assume the constants g_1 and g_2 to be real throughout.

If a typical matrix element of Table I is denoted by T, the decay rate to that spin state is

$$\Gamma=\frac{q}{32\pi^2m_d^2}\int|T|^2d\Omega.\tag{6}$$

We may alternatively describe the "decay" of the deuteron by giving the amplitudes f_s and f_d for s- and d-wave decay, respectively. Reference to tables of Clebsch-Gordan coefficients and spherical harmonics gives then the decay matrix elements of Table II; again we assume that the initial deuteron is spin up. We fix the normalization of f_s and f_d by defining formula (6) also to hold for the matrix elements of Table II.

TABLE II. Deuteron "decay" matrix elements, in terms of s- and d-wave decay amplitudes.

Proton spin	Neutron spin	Matrix element
Up	Up	$f_s+(\sqrt{\frac{1}{8}})f_d(3\cos^2\theta-1)$
Up / Down	Down / Up	$(9/8)^{1/2}f_d\sin\theta\cos\theta e^{i\phi}$
Down	Down	$(9/8)^{1/2}f_d\sin^2\theta e^{2i\phi}$

A comparison of Tables I and II now yields the relations

$$\begin{aligned} g_1&=\left(\frac{F_pF_n}{2}\right)^{1/2}\frac{1}{F_pF_n+q^2}\left(f_s+\frac{f_d}{\sqrt{2}}\right),\\ g_2&=(2F_pF_n)^{1/2}\frac{1}{(F_p+F_n)(F_pF_n+q^2)}\\ &\quad\times\left(f_s-\frac{f_d}{\sqrt{8}}\frac{3F_pF_n+q^2}{q^2}\right). \end{aligned}\tag{7}$$

We now consider the effect of the deuteron mass moving to its correct value, below the sum m_p+m_n. The deuteron wave function at large distances changes from the form

$$\psi\sim(e^{iqr}/r)[a_s+a_d\times(d\text{-state angular function})],\tag{8}$$

to

$$\psi\sim(e^{-\alpha r}/r)[a_s+a_d\times(d\text{-state angular function})].\tag{9}$$

The a_s and a_d of (8) are clearly proportional to the f_s and f_d of Table II; the precise relation follows from the observation that the wave function (8) implies a decay rate

$$\Gamma=(4\pi qm_d/E_pE_n)(|a_s|^2+|a_d|^2).\tag{10}$$

A comparison of (6) and (10), using the matrix elements of Table II, shows that

$$f_{s,d}=(32\pi^2m_d^2/E_pE_n)^{1/2}a_{s,d}.\tag{11}$$

We shall *assume* that the deuteron mass may be moved to its correct value without doing violence to the above relations. We thus obtain the coupling constants g_1 and g_2 in terms of the constants a_s and a_d of the asymptotic deuteron wave function (9), by combining relations (7) and (11):

$$\begin{aligned} g_1&=4\pi\left(\frac{F_pF_nm_d^3}{E_pE_n}\right)^{1/2}\frac{1}{F_pF_n+q^2}\left(a_s+\frac{a_d}{\sqrt{2}}\right),\\ g_2&=8\pi\left(\frac{F_pF_nm_d^3}{E_pE_n}\right)^{1/2}\frac{1}{(F_p+F_n)(F_pF_n+q^2)}\\ &\quad\times\left(a_s-\frac{a_d}{\sqrt{8}}\frac{3F_pF_n+q^2}{q^2}\right). \end{aligned}\tag{12}$$

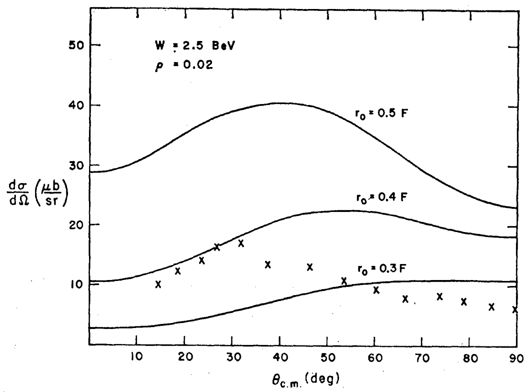

FIG. 3. Calculated c.m. differential cross sections for $r_0=0.3$, 0.4, and 0.5 F. The crosses are experimental points from Ref. 4.

The question arises as to the proper values of $E_{p,n}$, $F_{p,n}$ and q^2 to use in (12). For the values of g_1 and g_2 when all three particles of Fig. 2 are on the mass shell ("at the pole," for the diagrams of Fig. 1), the prescription is clear. We simply use the same relations that hold for $m_d > m_p + m_n$, namely,

$$\begin{aligned}
E_p &= (m_d^2+m_p^2-m_n^2)/2m_d\,,\\
E_n &= (m_d^2+m_n^2-m_p^2)/2m_d\,,\\
F_p &= [(m_d+m_p)^2-m_n^2]/2m_d\,,\\
F_n &= [(m_d+m_n)^2-m_p^2]/2m_d\,,\\
q^2 &= [m_d^4-2m_d^2(m_p^2+m_n^2)+(m_p^2-m_n^2)^2]/4m_d^2\,.
\end{aligned} \qquad (13)$$

However, if we use these values, together with a reasonable asymptotic deuteron wave function, we obtain coupling constants g_1 and g_2 which lead to ridiculously large cross sections for reaction (1). There are at least two reasons for this:

(a) The value of q^2 is anomalously small at the pole, because the deuteron binding energy is so small. This small value of q^2 in (12) causes the d-state contribution (i.e., the contribution from a_d) to be unusually large at the pole. It seems unphysical to expect this effect to persist away from the pole.

(b) It is well known[9] that the Fourier transform of a wave function is closely related to the corresponding three-particle Green's function. In other words, the Fourier transform of the deuteron wave function provides a rapidly decreasing form factor which should be used to reduce our mass shell estimates.

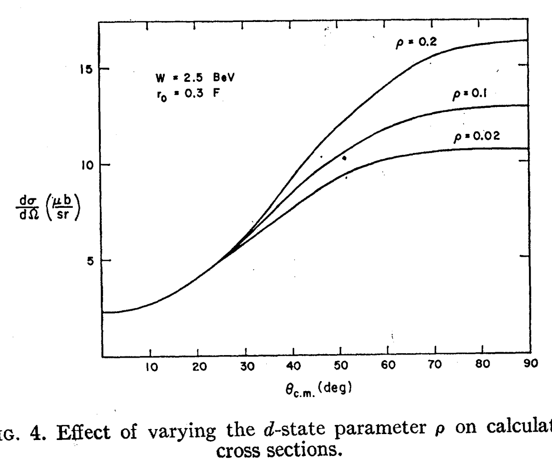

FIG. 4. Effect of varying the d-state parameter ρ on calculated cross sections.

Specifically, we have modified Eqs. (12) as follows:

(a) q^2 is computed from (13), but with m_n^2 replaced by n^2. Thus, on the mass shell ($n^2=m_n^2$), $-q^2$ has its very small value α^2, but it increases rapidly as n^2 departs from m_n^2. The remaining kinematic variables in (12), namely $E_{p,n}$ and $F_{p,n}$, we have given their mass-shell values. We are not certain what the correct treatment of these variables is, but in any case they are relatively slowly varying.

(b) From (9) it follows that

$$a_{s,d} = \lim_{q^2\to-\alpha^2} [(q^2+\alpha^2)/4\pi]\phi_{s,d}(q^2)\,, \qquad (14)$$

where $\phi_{s,d}$ is the Fourier transform of the s-wave (d-wave) radial function of the deuteron. We have replaced relation (14) by

$$a_{s,d}(q^2) = [(q^2+\alpha^2)/4\pi]\phi_{s,d}(q^2)\,. \qquad (15)$$

That is, for a given n^2, q^2 is computed from (13), with m_n^2 replaced by n^2. Then $a_{s,d}$ are computed from (15) and inserted into (12) to obtain the appropriate values of g_1 and g_2.

Finally, we must choose some reasonable deuteron

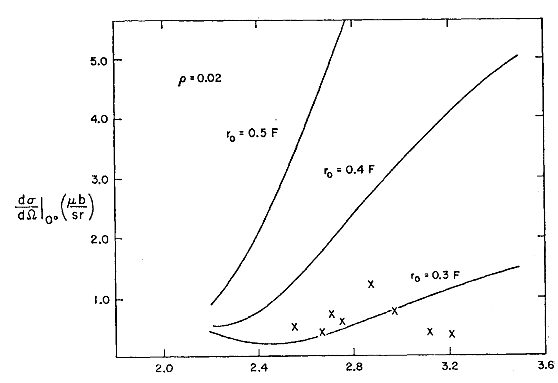

FIG. 5. Zero-degree differential cross sections, as functions of total c.m. energy W. The crosses are experimental points taken from Ref. 2.

[9] R. Blankenbecler and L. F. Cook, Phys. Rev. **119**, 1745 (1960); R. Blankenbecler, Nucl. Phys. **14**, 97 (1960); M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961); R. D. Amado, Phys. Rev. **127**, 261 (1962).

wave function. We have taken for the s-wave function

$$\psi_s(r)=(N/r)[e^{-\alpha(r-r_0)}-e^{-\beta(r-r_0)}], \quad (r>r_0)$$
$$=0, \quad (r<r_0). \quad (16)$$

This function has the Fourier transform

$$\phi_s(q)=\int d^3x\ \psi_s(r)e^{-i\mathbf{q}\cdot\mathbf{x}}$$
$$=\frac{4\pi N(\beta^2-\alpha^2)}{(q^2+\alpha^2)(q^2+\beta^2)}\left[\cos qr_0+\frac{\alpha\beta-q^2}{q(\alpha+\beta)}\sin qr_0\right]. \quad (17)$$

The constants N and β were evaluated from the normalization condition and the effective range r_0. Neglecting the small ($\sim 5\%$) d-state probability, we obtain

$$N^2=\frac{\alpha}{2\pi}\frac{\beta(\beta+\alpha)}{(\beta-\alpha)^2}=\frac{\alpha}{2\pi}\frac{e^{-2\alpha r_0}}{1-\alpha r_e}.$$

For the effective range, we have taken[10]

$$r_e=1.82\ \mathrm{F}=9.22\ \mathrm{BeV}^{-1}.$$

After making modification (a) above, the d-state part of the deuteron wave function turns out to have very little influence on the final results. We have therefore taken for the d-state function simply a constant ρ times the s-state function. In order to fit the observed quadrupole moment of the deuteron, ρ must be chosen near 0.02[10]; on the other hand, the d-state probability requires a much larger value of ρ. The discrepancy, of course, simply reflects the inaccuracy of our d-state wave function.

IV. NUMERICAL RESULTS AND DISCUSSION

We have calculated numerical values for the differential cross section of reaction (1), using the formulas of Sec. II and the Appendix, and the modifications discussed in Sec. III. The numerical calculations were performed on the IBM 1620 computer of the Indian Institute of Technology, Kanpur.

In Fig. 3 we show the c.m. differential cross section at a total c.m. energy $W=2.5$ BeV (proton lab kinetic energy $=1.46$ BeV). The three curves show the effect of varying the hard-core radius r_0 from 0.3 F to 0.5 F, while holding the d-state parameter ρ fixed at 0.02. The crosses are experimental points, taken from Ref. 5. It is apparent that we cannot fit simultaneously the observed magnitude of the differential cross section and the observed amount of its forward peaking.

In Fig. 4 we show the effect of varying ρ. The cutoff radius is held fixed at $r_0=0.3$ F, and $W=2.5$ BeV.

In Figs. 5 and 6 we show the zero-degree differential cross section and the total cross section, respectively, as functions of W, for $r_0=0.3$, 0.4, and 0.5 F. In no case is there good agreement with the experimental data which we have taken from Refs. 2 and 5.

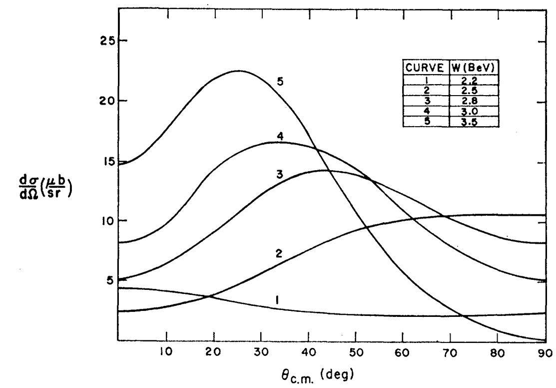

FIG. 7. Calculated differential cross sections for various energies, showing increasing forward peaking with increasing energy. $r_0=0.3$ F; $\rho=0.02$.

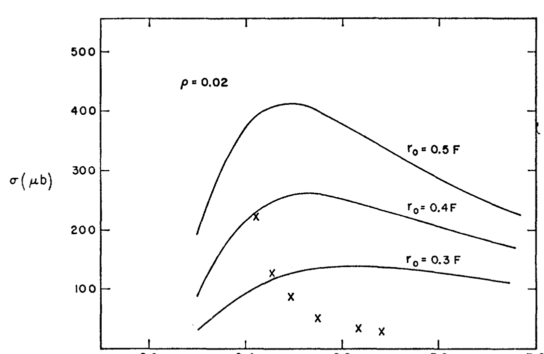

FIG. 6. Total cross sections for various values of r_0. The crosses are experimental points from Ref. 4.

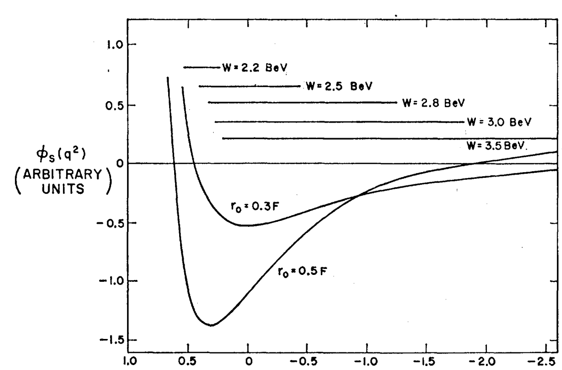

FIG. 8. Fourier transform (17) of our assumed deuteron wave function (16) as a function of n^2, the squared four-momentum of the virtual neutron. The horizontal bars denote the range of n^2 in the uncrossed diagram, corresponding to angles $\theta_{c.m.}$ from 0° to 90°; 0° is at the left.

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[10] R. Wilson, *The Nucleon-Nucleon Interaction* (Interscience Publishers, Inc., New York, 1963).

On the other hand, one characteristic feature of the experimental results,[5] namely the increasing sharpness of forward peaking with increasing energy, appears quite clearly in our model. In Fig. 7 we show differential cross sections calculated for various values of W, with $r_0=0.3$ F, $\rho=0.02$. For $r_0=0.5$ F, the effect is even more pronounced; at $W=3.5$ BeV, the maximum in the angular distribution is at 0°.

The reason for this forward peaking is of some interest. Although such angular distributions are typical in one-*pion*-exchange calculations, we are here dealing with *nucleon* exchange, and a simple calculation of the momentum transfer involved will show one that such sharp peaking is not expected.

In fact, the reason for these angular distributions is our Eq. (15). The $a_{s,d}$ and therefore the effective coupling constants $g_{1,2}$ are proportional to the Fourier transforms of the deuteron wave function. As we vary the angle θ, and therefore the momentum transfer n^2, the variations in $d\sigma/d\Omega$ to a considerable degree simply reflect the variations in the Fourier transform $\phi_s(q^2)$.

To illustrate this point, we have drawn Fig. 8. This shows the Fourier transform $\phi_s(q^2)$ for $r_0=0.3$ F. Above the Fourier transform we have drawn horizontal bars indicating the range of values assumed by n^2 for the uncrossed diagram, as θ goes from 0° to 90°. It is clear from this figure that the presence of the maxima in Fig. 7, moving to smaller and smaller angles as W increases, simply follows from the presence of the maximum in $|\phi_s(q^2)|^2$ near $n^2=0$. The behavior of curve 1 in Fig. 7, showing a *minimum* in $d\sigma/d\Omega$ for $W=2.2$ BeV, can also be understood from Fig. 8. For $W=2.2$ BeV, the zero of $\phi_s(q^2)$ near $n^2=0.45$ BeV^2 occurs for a real scattering angle between 0° and 90°. The fact that angular distributions calculated for $r=0.5$ F exhibit sharper forward peaking is also easily read off from Fig. 8.

We conclude that our one-neutron exchange mechanism is at least partially successful in reproducing the observed character of $p+p\rightarrow d+\pi^+$ cross sections, particularly the forward peaking which increases with energy. The agreement with experiment might very well be improved by the inclusion of absorptive effects[11] in the present calculation. We also suggest that the intimate relation between Fourier transforms of bound-state wave functions and angular distributions, familiar to nuclear physicists from stripping theory,[12] may occur in the present case, as well as others, in high-energy particle physics.

Incidentally, a calculation[13] of the cross section for $p+p\rightarrow d+$ vector boson by Nearing, assuming one-neutron exchange, was made "at the pole," and we would therefore expect it to overestimate the actual cross section enormously because of his omission of the Fourier transform of the deuteron wave function.[14]

ACKNOWLEDGMENTS

One of the authors (JM) would like to acknowledge the hospitality of the Indian Institute of Technology, and also a valuable conversation with Professor M. Banerjee on the subject of Fourier transforms and form factors.

APPENDIX

We begin with the invariant matrix element for reaction (1) in the form (2)

$$T=T_a-T_b,$$

with T_a and T_b given by (4). The squared and summed matrix element is

$$\sum|T|^2=\sum|T_a|^2+\sum|T_b|^2-\sum T_a^*T_b-\sum T_b^*T_a.$$

We write

$$\begin{aligned}\sum|T_a|^2&=[2g^2/(n^2-m_n^2)^2]4A,\\ \sum|T_b|^2&=[2g^2/(n'^2-m_n^2)^2]4B,\\ \sum T_a^*T_b&=[2g^2/(n^2-m_n^2)(n'^2-m_n^2)]4C,\\ \sum T_b^*T_a&=[2g^2/(n^2-m_n^2)(n'^2-m_n^2)]4D,\end{aligned}$$

and further decompose A, B, C, D into four parts each:

$$\begin{aligned}A&=|g_1|^2A_1+g_1^*g_2A_2+g_1g_2^*A_3+|g_2|^2A_4,\\ B&=|g_1'|^2B_1+g_1'^*g_2'B_2+g_1'g_2'^*B_3+|g_2'|^2B_4,\\ C&=g_1^*g_1'C_1+g_1^*g_2'C_2+g_1'g_2^*C_3+g_2^*g_2'C_4,\\ D&=g_1'^*g_1D_1+g_1'^*g_2D_2+g_1g_2'^*D_3+g_2'^*g_2D_4.\end{aligned}\tag{1A}$$

We must remember that, following Sec. III, we are modifying the coupling constants at the dnp vertex, so that g_1 and g_2 take on different values accordingly as the four-momentum of the exchanged neutron is n or n'. We have therefore distinguished these alternatives in Eqs. (A1) by using a prime (no prime) to denote a coupling constant for which the virtual neutron has momentum $n'(n)$.

The quantities A_i, $\cdots$, D_i are evaluated by the usual trace and projection-operator techniques. For example,

$$A_1=\tfrac{1}{4}\,\mathrm{Tr}\gamma_\mu(n+m_n)\gamma_5(p'+m_p)\times\gamma_5(n+m_n)\gamma_\nu(p-m_p)D_{\mu\nu},$$

where $D_{\mu\nu}$ is the spin-one projection operator

$$D_{\mu\nu}=-\delta_{\mu\nu}+d_\mu d_\nu/m_d^2.$$

The results of the traces are

$$\begin{aligned}A_1&=E_1(-4Z_1)+E_2(-2Z_{11})+E_4(Z_5Z_{11}-2Z_1Z_2),\\ A_2&=A_3=E_1[m_p(2Z_1-Z_{11})+m_n(2Z_1)]+E_2(m_pZ_{11}),\\ A_4&=E_1(2Z_1Z_7-Z_9Z_{11}),\\ B_1&=E_2(-2Z_{12})+E_3(-4Z_3)+E_4(Z_5Z_{12}-2Z_3Z_4),\\ B_2&=B_3=E_2(m_pZ_{12})+E_3[m_p(2Z_3-Z_{12})+m_n(2Z_3)],\end{aligned}$$

[11] M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 627 (1964).
[12] S. T. Butler, *Nuclear Stripping Reactions* (John Wiley & Sons, Inc., New York, 1957).
[13] J. Nearing, Phys. Rev. **132**, 2323 (1963).
[14] J. Nearing, Phys. Rev. **135**, AB2 (1964).

$$B_4=E_3(2Z_3Z_8-Z_9Z_{12}),$$
$$C_1=D_1=E_1(-2Z_4)+E_2(-2Z_5-2Z_6)+E_3(-2Z_2)+E_4(-Z_1Z_3-Z_2Z_4+Z_5Z_6),$$
$$C_2=D_3=E_2[m_p(Z_6-Z_3-Z_4)+m_n(Z_5-Z_1-Z_4)]+E_3[m_p(Z_2-Z_1-Z_6)+m_n(Z_2-Z_3-Z_5)],$$
$$C_3=D_2=E_1[m_p(Z_8-Z_3-Z_{10})+m_n(Z_8-Z_1-Z_9)]+E_2[m_p(Z_{10}-Z_1-Z_7)+m_n(Z_9-Z_3-Z_7)],$$
$$C_4=D_4=E_2(Z_9Z_{10}-Z_1Z_3-Z_7Z_8),$$

where we have made the following definitions:

$$E_1=p_\mu p_\nu D_{\mu\nu}=-m_p^2+(p\cdot d)^2/m_d^2,$$
$$E_2=p_\mu p_\nu' D_{\mu\nu}=-p\cdot p'+(p\cdot d)(p'\cdot d)/m_d^2,$$
$$E_3=p_\mu' p_\nu' D_{\mu\nu}=-m_p^2+(p'\cdot d)^2/m_d^2,$$
$$E_4=\delta_{\mu\nu}D_{\mu\nu}=-3,$$

and

$$Z_1=p'\cdot n-m_pm_n,$$
$$Z_2=p\cdot n+m_pm_n,$$
$$Z_3=p\cdot n'-m_pm_n,$$
$$Z_4=p'\cdot n'+m_pm_n,$$
$$Z_5=p\cdot p'+m_p^2,$$
$$Z_6=n\cdot n'+m_n^2,$$
$$Z_7=p\cdot n-m_pm_n,$$
$$Z_8=p'\cdot n'-m_pm_n,$$
$$Z_9=p\cdot p'-m_p^2,$$
$$Z_{10}=n\cdot n'-m_n^2,$$
$$Z_{11}=n^2-m_n^2,$$
$$Z_{12}=n'^2-m_n^2.$$

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Decays of Baryon Resonances of Any Spin

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General formulas are derived relating coupling constants for the strong decay interactions of a baryon resonance B_s^* of any spin s to the observed decay widths, for the decay processes $B_s^*\to B_{\frac{1}{2}}+P$, $B_s^*\to B_{\frac{3}{2}}^*+P$, and $B_s^*\to B_{\frac{1}{2}}+V$, where P is a pseudoscalar and V a vector meson. The calculations are carried through within the framework of the Rarita-Schwinger formalism, using interaction Lagrangians incorporating derivative couplings of the pseudoscalar or vector-meson field to free spinor fields. The resulting formulas are compared with the well-known potential-theory expressions.

INTRODUCTION

OBSERVATIONS of the strong decays of high-spin baryon resonances afford valuable guidance in assigning these resonances to their correct places in various symmetry schemes. Thus the correct isotopic-spin assignment is likely to be suggested by the experimental branching ratio into the different charge states of particles produced by the decay, while the experimental decay widths provide a means of extracting phenomenological coupling constants whose magnitudes are connected in $SU(3)$ or some higher symmetry scheme.

The purpose of this investigation is to derive, for a baryon resonance B_s^* of arbitrary spin s and mass ω, general formulas relating the coupling constants $g_{s\frac{1}{2}P}$, $g_{s\frac{3}{2}P}$ and $g_{s\frac{1}{2}V}$ to the widths $\Gamma(\omega)$ for their respective decay processes

$$B_s^*\to B_{\frac{1}{2}}+P, \quad (1)$$
$$B_s^*\to B_{\frac{3}{2}}^*+P, \quad (2)$$
$$B_s^*\to B_{\frac{1}{2}}+V, \quad (3)$$

where P is a pseudoscalar and V a vector meson. Examples of each of these processes already exist in nature, reactions of the type (1) being by far the most common mode for a given resonance because of the relatively low masses of the product particles for such reactions. Perturbation-theory relations for $\Gamma(\omega)$ have already been given for the lowest spin values and are well-known[1] and extension to higher spins suggested by analogy with the resonance-theory results of Blatt and Weisskopf.[2]

Lagrangians incorporating derivative couplings of the pseudoscalar or vector-meson field to free spinor fields are employed here, and the calculations carried through in the framework of the Rarita-Schwinger[3] formalism.

[1] J. D. Jackson, Nuovo Cimento **34**, 1644 (1964) [see especially Appendix A].

[2] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, New York, 1952), pp. 332, 361, 406–422. The form of Eq. (49) is retained in the derivation of W. M. Layson, Nuovo Cimento **27**, 724 (1963) by means of the Klein-Gordon equation rather than the Schrödinger equation. A useful summary of results appears in L. D. Roper, University of California Radiation Laboratory Report No. UCRL 14193, 1965 (unpublished).

[3] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).