where $L_{\mu\nu}$ is defined by Eq. (34), and $\frac{1}{2}(1+i\gamma_5\gamma_\mu S_\mu')$ is a spin-projection operator. The four-vector S_μ' is obtained from the $\mathbf{S}$ defined above by forming in the electron's rest system the four-vector $(\mathbf{S},0)$. This four-vector is then transformed into the laboratory system by a simple Lorentz transformation without rotation; the result is S_μ'. The tensor $K_{\mu\nu}$ is antisymmetric and

$$K_{12}=\frac{i\mathbf{S}_l\cdot(k-k')}{2\omega\omega' P}(\omega-\omega')m_l-\frac{i\mathbf{S}_l\cdot\mathbf{k}}{2\omega\omega' P}\left[(\omega-m)(\omega'+m)+k'^2-kk'\cos\theta\left(1+\frac{\omega'+m}{\omega+m}\right)\right],$$

$$K_{23}=\frac{i\mathbf{S}_l\cdot\hat{x}}{2\omega\omega'}(\omega-\omega')m_l+\frac{i\mathbf{S}_l\cdot k}{2\omega\omega' P}kk'\sin\theta\left[1-\frac{\omega'+m}{\omega+m}\right],$$

$$K_{31}=\frac{i\mathbf{S}_l\cdot\hat{y}}{2\omega\omega'}(\omega-\omega')m_l.$$

Our theorem can be derived by following exactly the same method as that given in Sec. I.1, but using $(L_{\mu\nu}+K_{\mu\nu})$ in place of $L_{\mu\nu}$.

PHYSICAL REVIEW VOLUME 143, NUMBER 4 MARCH 1966

Boson Masses

R. W. King*
Purdue University, Lafayette, Indiana
AND
D. C. Peaslee
The Australian National University, Canberra, Australia

(Received 8 September 1965; revised manuscript received 19 November 1965)

A gross mass scale is assigned to the baryon-antibaryon model for bosons: for charge octets $m^2(^3L)-m^2(^1L)\approx\frac{1}{2}$ BeV^2. For charge singlets the mass formula is less certain but seems to be quite different from that for octets. The quantum number of A parity is pointed out as an inescapable feature of this model; the empirical values of A then rule out quarks for the simplest realization. Interest is remarked in $K3\pi$ resonances and in possible resolution of the $K\pi$ and $K2\pi$ modes of the $K^*(1430)$.

EMPIRICAL mass relations among elementary particles have been remarked from time to time,[1] with high numerical precision but lacking a relevant physical model. We present here an approach from the other extreme by assigning a preliminary mass scale to the baryon-antibaryon model for bosons. The assignment relies heavily on the set of $J^P=2^+$ mesons that now seems established[2,3]; it is concerned in this instance only with the gross features of J^P ordering. The approach is one of bounded speculation but already yields specific boson interpretations at some variance with those currently popular, and amenable to experimental study, viz., the question of $J^P=1^+$ for the X meson, the survey of $K3\pi$ resonances. Such measurements are feasible but cannot usually be reconstructed from randomly assorted data; in the hope of arousing critical discussion, the elementary considerations below are presented.

The energy scale chosen is so gross that the internal structure of octets and nonets will be of little concern, and details of coupling schemes need scarcely be specified. In order to avoid the opposite extreme of complete formlessness, we impose the following constraints:

(i) The basic Fermion and anti-Fermion are not quarks nor triplets but just the observed baryon charge octet of spin $\frac{1}{2}$ in some "bare" state, which is taken not to differ very substantially from the dressed state. This is a little unfashionable at present but entrains the next restriction, often neglected; confrontation with experiment is discussed below.

(ii) The baryons are taken to obey ordinary Fermi statistics, having never given evidence of parastatistics; that is, the Pauli principle is taken seriously.

(iii) Systematic corrections could be introduced as

---

* Alfred P. Sloan Foundation Fellow.

[1] See, e.g., T. F. Kycia and K. F. Riley, Phys. Rev. Letters **10**, 266 (1963); R. M. Sternheimer, *ibid.* **10**, 309 (1963).

[2] S. U. Chung, O. I. Dahl, L. M. Hardy, R. I. Hess, L. D. Jacobs, K. Jirz, and D. H. Miller, Phys. Rev. Letters **15**, 325 (1965).

[3] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

TABLE I. Orbital assignments.

Orbit:		$^1S_0$	$^3S_1$	$^3P_2$	$^3P_1$	$^1P_1$
Octet	$I=1$	$\pi(140)$	$\rho(760)$	$A_2(1320)$	$A_1(1090)$	$B(1215)$
	$I=\frac{1}{2}$	$K(495)$	$K^*(890)$	$K^*(1430)$	$C(1220)[C'?]$	$[K^*(1175)?]$
	$I=0$	$\eta(550)$	$\varphi(1020)$	$f'(1525)$	$D(1280)[E?]$	$[X(960)?]$
A		$-$	$+$	$-$[a]	$-$[b]	$+$[c]
m^2		$0.17\,(\text{BeV})^2$	0.74	1.96	1.39	[1.29]
m'^2		$0.16\,(\text{BeV})^2$	0.80	2.03	1.41	1.20
Singlet		$[X(960)?]$	$\omega(785)$	$f^0(1250)$	$\cdots$	$E(1410)[D?]$
m_0^2		$[0.92\,(\text{BeV})^2]$	0.62	1.56		[2.00]

[a] $A_2\to\rho\pi$; $f'\to\bar{K}K^*$; $K^*(1430)\to K\pi$ is A violating, but there appears (see Ref. 2) to be a substantial fraction of $K^*\to K^*(890)+\pi$, although the available phase space is an order of magnitude less.
[b] $A_1\to\rho\pi$; $C(C')\to K\pi\pi$; $D(E)\to\bar{K}K\pi$.
[c] $B\to\omega\pi$; $X\to\gamma\rho^0$, $X\not\to\gamma\omega$.

(n baryon)-(n antibaryon) terms with $n=2,3,\cdots$. We *assume* that such an expansion is physically meaningful in the sense that the $n=1$ terms are already a fair approximation to much that is observed.

Assumption (iii) implies that higher order combinations like decouplet-antibaryon and decouplet-antidecouplet need not be considered here. In this approach the decouplet is already a (2 baryon)-(1 antibaryon) system, so that decouplet-antibaryon corresponds to $n=2$, and decouplet-antidecouplet to $n=3$.

According to (ii) the baryon-antibaryon pair satisfy the exchange relation

$$P^BP^CP^LP^S=-1\,, \qquad (1)$$

where the spin exchange is $P^S=+1$, -1 for triplet, singlet; space exchange is $P^L=(-1)^L$. For charge exchange we assume that the values $P^C=\pm1$ will be determined by the predominant SU_3 charge symmetry; admixtures of other charge symmetries such as G_2 or R_7 are necessary to lift the mass degeneracy within an octet, but their representations must accord with the dominant P^C. In the decomposition of 8×8, $P^C=+1$ for **1**, **8**, **27** and $P^C=-1$ for **8**$'$, $\mathbf{10}+\overline{\mathbf{10}}$.

The exchange P_B is on the baryon number; because there are just two particles, one baryon and one antibaryon, P^B is identical with the operation of reversing the baryon number for each fermion. That is,

$$P_B\equiv A\,, \qquad (2)$$

where A is just the "antiparticulation" operator[4] or equivalent charge parity.[5] Note that Eq. (1) reduces to an old formula[6] for positronium where the fermions are leptons and have no internal charge coordinates: in that case $P^C\to1$, $A\to C$ (charge conjugation). Equation (2) shows that the charge parity A is an integral part of the model and cannot be overlooked.

The chief empirical difficulties with A center around octet-singlet mixing, which is supposed to be large in order to account for the failure of the SU_3 mass formula for all but the pseudoscalar octet. Since it is generally true that the octet and singlet have opposite A values, such mixing represents a serious violation of A. Our qualitative escape from this difficulty has been indicated above: Ascribe the octet mass failures to admixture of G_2 and R_7 symmetries[7]; then octet-singlet mixing is slight and A remains a fairly good quantum number. There is no difficulty about reproducing either octet P^C in this way; for $(\mathbf{7}+\mathbf{1})\times(\mathbf{7}+\mathbf{1})=\mathbf{1}+\mathbf{1}'+\mathbf{7}+\mathbf{7}'+\cdots$. With this assumption of minimal octet-singlet mixing, satisfactory assignments of A can be given to all but one of the 16 or more bosons listed in Table I.

To illustrate the importance of A as a physical parameter, note that it distinguishes clearly in favor of baryons over quarks on the present model. For quarks the charge symmetries are $P^C=+1$ for **8**, $P^C=-1$ for **1**; then by Eq. (1) the $^1S_0$ pseudoscalar octet would have $A=+1$, the $^3S_1$ vector octet would have $A=-1$, so that $V\to(P_V{}^S)^2$ would be A-forbidden. The empirical assignment[5] is just the opposite, indicating that the pseudoscalar and vector octets are both **8**$'$ with $P^C=-1$. The necessity of ignoring Fermi statistics is a frequent feature of simple quark models.

Since we assume singlet-octet mixing to be small, average (mass)^2 values are taken direct from the octets:

$$m^2\equiv\tfrac{1}{8}[m_1^2+4m_2^2+3m_3^2]\,,$$
$$m'^2\equiv\tfrac{1}{2}[m_1^2+m_3^2]\,, \qquad (3)$$

where the subscript is $(2I+1)$. The first Eq. (3) is just a simple average over the octet; the second is derived from the first by use of the SU_3 mass formula, $3m_1^2+m_3^2=4m_2^2$. Since we ignore G_2 and R_7 mixing, $m'^2\neq m^2$ in general. These (mass)^2 values for known and suggested octets are shown in Table I; values m_0^2 for some possible corresponding singlets are included.

The assignments in Table I run in decreasing order of reliability; the last column is admittedly suspect but correspondingly provides the greatest experimental interest.

From the m^2 and m'^2 entrices in Table I we find $m^2(^3S)-m^2(^1S)\approx0.6$ BeV^2. In the absence of a well-

[4] D. C. Peaslee, Phys. Rev. **117**, 873 (1960).
[5] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).
[6] L. Michel, Nuovo Cimento **10**, 319 (1953).
[7] D. C. Peaslee, J. Math. Phys. **4**, 910 (1963).

established $^3P_0$ octet, we cannot find $m^2(^3P)$ precisely; but the missing octet has a relative weight of only $\frac{1}{9}$, so we take $m^2(^3P)\approx\frac{3}{8}m^2(^3P_1)+\frac{5}{8}m^2(^3P_2)$. Then $m^2(^3P)-m^2(^1P)\approx 0.5$ to 0.6 BeV^2. This consistency suggests that the ^1P states in Table I are in about the right position, and that the singlet-triplet mass splitting is roughly independent of L

$$m^2(^3L)-m^2(^1L)\approx 0.5 \quad \text{to} \quad 0.6\ \text{BeV}^2. \tag{4}$$

The corresponding relation for singlet masses is much more doubtful; but the suggestions from Table I are

$$m_0{}^2(^3L)-m_0{}^2(^1L)\approx -0.3 \quad \text{to} \quad -0.4\ \text{BeV}^2, \tag{4'}$$

emphasizing the great difference in character between octet and singlet mesons. It turns out that all the octets in Table I are **8'**, so that each singlet has the opposite value of A from its corresponding octet. Absence of octets **8** suggests the dominance of F-type SU_3 coupling.

An observational difficulty associated with the $^3P_0$ states is that they should all be relatively narrow and decay primarily by modes that are in some way inhibited. The simplest fully allowed decay for $J^P=0^+\rightarrow(0^-)^n$, $A=-1\rightarrow(-1)^n$, is with $n=5$. The phase-space restrictions at moderate initial masses are so severe that $0^+\rightarrow(0^-)^2$ is much more likely with violation of charge symmetry rules. The members of this octet could then be the $(K\bar{K})$ at $\sim$1040 [or possibly the ϵ at 720], the κ at $\sim$725 and the $\zeta(575)$ with a width $<$0.1 MeV because of a further α^2 factor. Obviously any such assignment is very tentative, but illustrates the great uncertainties to be expected in establishing the $^3P_0$ octet. Strong forward peaking in η and X^0 production on protons by π^- and K^- suggests exchange of one boson, for which the $J^P=0^+$ octet is an obvious candidate.

Extrapolation of $m^2(S)$, $m^2(P)\cdots$ by an $L(L+1)$ law and combining with Eq. (4) suggests the incidence of ^1D ($J^P=2^-$) states in the mass region around 1.7 BeV; the corresponding ^3D states center around 1.9 BeV. The partial indications from ^3P are for a strong L-S coupling and a weaker tensor term; accordingly, the $^3D_1$ states may appear below the ^1D states. They will be 1^- mesons indistinguishable in external quantum numbers, including A, from the $(\rho K^*\zeta,\omega)$; because of the weakness of the tensor force, not much mixing is expected between the $^3S_1$ and $^3D_1$ vector mesons. A number of these states should cause large D-wave phase shifts in N-$\bar{N}$ scattering at low energies; the large absorption cross section is mostly S wave, since it follows a $1/v$ law.

Assignment of the $X(960)$ in Table I is ambiguous. There is yet no incontrovertible evidence distinguishing 1^+ from 0^- for the X; this is clearly a question of prime importance for the present model. In either case, however, the model uniquely predicts $A=+1$. The corresponding K^* with $J^P=1^+$ and $A=+$ should have $K3\pi$ as its minimal allowed decay mode, but in this mass range it would probably have a prominent $K2\pi$ mode by A violation. A direct search for $K3\pi$ resonances would be of great interest in this connection.

The only serious difficulty with A parity is raised by the $K^*(1430)$, which is reported to have a width of order 100 MeV for $K^*\rightarrow K+\pi$, although with perhaps[8] a substantial minor decay mode $K2\pi$. One possibility is that the $K^*(1430)$ should ultimately be resolved into two resonances, a broad one with the $K\pi$ mode (and $J^P=1^+?$), a narrower one with $K2\pi$ mode and $J^P=2^+$.

The f^0 appears as the 2^+ singlet; absence of mixing with the octet and invocation of A forbiddenness would alter Table I of Ref. 3 in a way not violating observation: Terms like $(F\cos\theta-G\cos\theta)^2$ are replaced by G^2, those like $(2F\cos\theta-G\sin\theta)^2$ by "A forbidden," and the predictions would be essentially unchanged. The validity of our A assignment could in principle be checked by comparing the degree of one-pion peripheral photoproduction: ideally $\gamma+p\rightarrow p+f'$ by this process, but $\gamma+p\rightarrow p+f^0$ is A forbidden for one-pion exchange.

The $\epsilon^0(720)$ has a substantial width for 2π decay but cannot have $A=+1$ and $J^P=0^+$ as a baryon-antibaryon combination. It is accordingly a candidate as the first member of a (2 baryon)-(2 antibaryon) series. Such states must exist somewhere, and the simplest configuration under P^L will certainly be the $I=Y=0$ singlet, $A=G=+1$. The ϵ^0 on this picture will have strong $\pi\pi$ components but is not directly coupled to bare N-N vertices. Higher $n=2$ states like $J^P=2^+$ should mix strongly with the $n=1$ series, viz., the f^0.

[8] *Note added in proof.* Recent measurements by J. Badier *et al.*, Phys. Letters **19**, 612 (1965), indicate that $K\pi\pi$ is the major decay mode. Comparison with the $K\pi$ mode is uncertain because of the difference in final spin states, but it is not hard to infer from their data an inhibition factor of order 5 to 10 for $K\pi$ decay.