Possible Tests of C_{st} and T_{st} Invariances in $l^{\pm}+N \to l^{\pm}+\Gamma$ and $A \to B+e^{+}+e^{-}$†

N. CHRIST* AND T. D. LEE
Department of Physics, Columbia University, New York, New York

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A systematic method to test the C_{st} and T_{st} invariances of the electromagnetic interaction is to use the inelastic scattering $l^{\pm}+N \to l^{\pm}+\Gamma$ where $l^{\pm}$ is the charged lepton, N is the target nucleus (or nucleon), and $\Gamma \neq N$ but, otherwise, can be any system of the strongly interacting particles. General expressions for the various possible C_{st}- and T_{st}-noninvariant effects in such reactions are derived and discussed. Similar considerations are also applied to the decay $A \to B+e^{+}+e^{-}$, where A and B are any complexes of the strongly interacting particles.

I. INTRODUCTION

THE recent discovery[1] of

$$K_2^0 \to \pi^+ + \pi^- \tag{1}$$

has stimulated an extensive re-examination of the experimental foundations of the various discrete space-time symmetries for all the interactions. It was found[2] that there are good evidences that the strong interaction H_{st} is separately invariant under the space inversion P_{st}, the time reversal T_{st} and the particle-antiparticle conjugation C_{st}; there are also strong evidences that the electromagnetic interaction H_γ is invariant under the same space-inversion operation P_{st} and the product $(C_{st}T_{st}P_{st})$. However, at present, there exists no evidence[3] that H_γ is, or is not invariant under C_{st} or T_{st}. Throughout this paper, for clarity, we use the subscript "st" to denote the particular choices of these discrete symmetry operators that are determined by the strong interaction alone. From the observed (π^+,π^-) and (K^+,K^-) symmetries in the $\bar{p}+p$ annihilation experiment,[4] the operator C_{st} must satisfy

$$C_{st}|p\rangle = |\bar{p}\rangle,$$
$$C_{st}|\pi^+\rangle = |\pi^-\rangle,$$

and

$$C_{st}|K^+\rangle = |K^-\rangle, \tag{2}$$

where $|p\rangle$, $|\bar{p}\rangle$, $|\pi^{\pm}\rangle$, and $|K^{\pm}\rangle$ all refer to the various physical single-particle states. From the observed reciprocity relations in the nuclear reactions[5] and the p-p scattering experiments,[6] we conclude that

$$T_{st}|p_{\mathbf{k},\lambda}\rangle = \eta_T|p_{-\mathbf{k},\lambda}\rangle \tag{3}$$

and

$$T_{st}|n_{\mathbf{k},\lambda}\rangle = \eta_T|n_{-\mathbf{k},\lambda}\rangle, \tag{4}$$

where the subscripts $\mathbf{k}$ and λ denote, respectively, the momentum and the helicity of p or n, and η_T is a phase factor.

If H_γ is not invariant under the particle-antiparticle conjugation C_{st}, then we can attribute reaction (1) not to the violation of $C_{st}P_{st}$ by H_{weak}, but to the virtual effect of H_γ. There is, then, a natural explanation of the smallness of the observed amplitude of reaction (1) which is about (α/π) times that of $K_1^0 \to \pi^+ + \pi^-$.

On the other hand, it has been well established, at least for the leptons, that the electromagnetic interaction H_γ is separately invariant under a *charge conjugation* C_γ, a time reversal T_γ, and a space inversion P_γ. Furthermore, we know that the minimal electromagnetic interaction of any system of the spin-0 and spin-$\frac{1}{2}$ particles is always invariant under C_γ, T_γ, and P_γ. It seems, therefore, aesthetically appealing to *assume* that there exists a charge-conjugation operation C_γ under which all electromagnetic currents change sign and that *all electromagnetic interactions*, including that of the nonleptons, are invariant under this *charge-conjugation symmetry* C_γ.

As remarked before, H_γ is already found to be invariant under P_{st} and the product $(C_{st}T_{st}P_{st})$. Thus, if H_γ is invariant under C_γ, it must also be separately invariant under T_γ and P_γ, where T_γ and P_γ for the nonleptons are defined by

$$P_\gamma = P_{st} \tag{5}$$

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964). See also A. Abashian *et al.*, *ibid.* **13**, 243 (1964).

[2] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965).

[3] Recently, there have been several new experiments testing the C_{st} invariance of H_γ: L. R. Price and F. S. Crawford, Phys. Rev. Letters **15**, 123 (1965); J. S. Lindsay and G. A. Smith, *ibid.* **15**, 221 (1965); A. Rittenberg and G. R. Kalbfleisch, *ibid.* **15**, 556 (1965); C. Alff *et al.*, *Proceedings of the 1965 Oxford International Conference on Elementary Particles* (Rutherford High Energy Laboratory, Harwell, England, 1966). The conclusions that can be drawn from these experiments are still, however, extremely limited. In particular, if the $C_{st}=+1$ current K_μ transforms either like a unitary octet or like a unitary singlet, or like an isoscalar; then in the limit of SU_3 symmetry, or isospin rotation symmetry, $\eta^0 \nrightarrow \pi^0+e^++e^-$ in the single-photon-exchange approximation; similarly, there is no time-reversal noninvariant effect in $\Sigma^0 \to \Lambda^0+e^++e^-$, etc. [See the theoretical discussion by J. Bernstein, G. Feinberg, and T. D. Lee, (Ref. 2); N. Cabibbo, Phys. Rev. Letters **14**, 965 (1965); T. D. Lee. Phys. Rev. **140**, B959, B967 (1965); G. Feinberg, *ibid.* **140**, B1402 (1965).

[4] C. Baltay, N. Barash, P. Franzini, N. Gelfand, L. Kirsch, G. Lütjens, J. C. Severiens, J. Steinberger, D. Tycko, and D. Zanello, Phys. Rev. Letters **15**, 591 (1965).

[5] L. Rosen and J. E. Brolley, Jr., Phys. Rev. Letters **2**, 98 (1959); D. Bodansky *et al.*, *ibid.* **2**, 101 (1959).

[6] See e.g., A. Abashian and E. M. Hafner, Phys. Rev. Letters **1**, 255 (1958); C. F. Hwang, T. R. Ophel, E. H. Thorndike, and R. Wilson, Phys. Rev. **119**, 352 (1960).

and

$$C_\gamma T_\gamma P_\gamma = C_{st} T_{st} P_{st}. \tag{6}$$

In this case, the possibility that the electromagnetic interaction may violate the C_{st} symmetry can be simply viewed as a possible mismatch between the two conjugation operators C_{st} and C_γ; i.e.,

$$C_{st} \neq C_\gamma, \tag{7}$$

and therefore

$$T_{st} \neq T_\gamma. \tag{8}$$

We may decompose the electromagnetic current $e\mathcal{J}_\mu$ of all particles, leptons, and nonleptons, into three parts:

$$\mathcal{J}_\mu = j_\mu + J_\mu + K_\mu, \tag{9}$$

where j_μ is the charged leptonic current

$$j_\mu = i \sum_{l=e,\mu} \psi_l^\dagger \gamma_4 \gamma_\mu \psi_l,$$

and J_μ, K_μ are both currents of the nonleptons which satisfy

$$C_{st} J_\mu C_{st}^{-1} = -J_\mu, \tag{10}$$

and

$$C_{st} K_\mu C_{st}^{-1} = +K_\mu. \tag{11}$$

By definition, under the charge conjugation C_γ, all electromagnetic currents must change their signs; i.e.,

$$C_\gamma \mathcal{J}_\mu C_\gamma^{-1} = -\mathcal{J}_\mu. \tag{12}$$

A mismatch between C_{st} and C_γ means that

$$K_\mu \neq 0 \tag{13}$$

and vice versa. The transformation properties of J_μ and K_μ under T_{st} and T_γ are determined by Eqs. (5), (6), and (10)–(12). We find

$$T_\gamma \mathcal{J}_\mu T_\gamma^{-1} = -\mathcal{J}_\mu, \tag{14}$$

$$T_{st} J_\mu T_{st}^{-1} = -J_\mu, \tag{15}$$

but

$$T_{st} K_\mu T_{st}^{-1} = K_\mu. \tag{16}$$

Thus, the existence of K_μ implies also a mismatch between T_{st} and T_γ. Explicit examples of such K_μ have already been given in some previous papers.[7,8]

Independent of these theoretical speculations, the fact that, at present, there does not exist any evidence[2,3] that H_γ is invariant under C_{st} or T_{st} should provide sufficient incentives for further experimental efforts in this direction.

The purpose of this paper is to point out that a systematic study of the question of T_{st} invariance of H_γ, over a wide range of energy momentum transfer, can be made by considering the inelastic scattering

$$l^\pm + N \to l^\pm + \Gamma, \tag{17}$$

where N is any target nucleon (or nucleus), $l=e$ or μ, the final system Γ should be different from N, $\Gamma \neq N$, but otherwise Γ can consist of all possible final states (continuum and resonances) of the same total energy and momentum, or Γ can consist of only certain resonant states. The theoretical aspects of reaction (17) are considered in the next section. A possible test of T_{st} invariance of H_γ can be made by using a *polarized* target nucleus, and by measuring the correlation function

$$\mathbf{S}_{in} \cdot (\mathbf{k} \times \mathbf{k}'),$$

where $\mathbf{S}_{in}$ is the polarization vector of the initial nucleus and $\mathbf{k}$, $\mathbf{k}'$ are, respectively, the initial and the final momenta of $l^\pm$ in the laboratory system.

An alternative test can be made by using an *unpolarized* target, provided the final system Γ consists of only a single baryon-meson resonance state N^*. The T_{st} invariance property of H_γ can be tested by measuring the correlation between the spin of N^* and the vector $(\mathbf{k} \times \mathbf{k}')$. The spin of N^* can, in turn, be determined by measuring the final nucleon spin in its decay products. The first method of using a polarized target to test the time-reversal invariance is applicable to any final states of strongly interacting particles; the second method of using an unpolarized target is applicable only if all polarization effects due to final states other than N^* can be properly subtracted out.

T_{st} invariance can also be tested by using an unpolarized target and by analyzing the angular distribution of the decay products of the N^*. Although this method does not require the measurement of any spin direction, the final state must consist of only a single resonant state, and the effects of all other final states must be removed. This particular method of testing T_{st} invariance will be discussed in detail in Appendix A.

With some slight changes, the same formula which is derived for reaction (17) can be directly applied to the decay

$$A \to B + e^+ + e^-. \tag{18}$$

As examples of such decays we may mention either[9]

$$\pi^- + p \to n + e^+ + e^-, \tag{19}$$

[7] As an example of such K_μ, we may write $K_\mu = i\lambda(\partial/\partial x_\nu) \times (\phi_\mu \omega_\nu - \phi_\nu \omega_\mu)$, where ϕ_μ and ω_μ are the field operators of the ϕ^0 and ω^0 particles, and λ is a real constant. The ϕ^0 and ω^0 have the same $C_{st} = -1$ eigenvalues, but the opposite C_γ eigenvalues. Thus, K_μ is even under C_{st} but odd under C_γ. In this case, K_μ transforms like members of the octet representation under the SU_3 transformations. Since $\int K_4 d^3r = 0$, both C_{st} and C_γ anticommute with the charge operator. For more discussions, see T. D. Lee, Phys. Rev. 140, B967 (1965).

[8] Another example of such a current K_μ is the possible existence of a charged, but $C_{st}=1$, particle. See T. D. Lee, Phys. Rev. 140, B959 (1965). In this case, K_μ transforms like a scalar under either the isospin transformations or the SU_3 transformations.

[9] The possible use of reaction (19) to test the T_{st} invariance of H_γ was suggested by L. Lederman and M. Schwartz (private communication). Some theoretical considerations of this reaction have also been made by G. Feinberg (unpublished).

or[10]

$$\Sigma^0 \rightarrow \Lambda^0 + e^+ + e^-. \tag{20}$$

A discussion of these reactions and the related tests of T_{st} invariance is given in Sec. III.

However, if the $C_{st} = +1$ current K_μ transforms like an isoscalar, or if it transforms like a mixture of the singlet and the octet representations under the SU_3 transformations, then reaction (20) is not a suitable candidate for testing the T_{st} invariance. While the possible T_{st}-noninvariant effect in reaction (19) is not reduced by any isospin or SU_3 selection rules, it is greatly restricted by the available limited range of four-momentum transfer. As will be discussed in Sec. III.3, even for a maximal violation of T_{st} invariance, the correlation between the final neutron spin and the decay plane is estimated to remain quite small.

II. INELASTIC SCATTERING $l^\pm + N \rightarrow l^\pm + \Gamma$

Throughout this section, reaction (17) will be considered and the following notations will be used:

$$\hbar = c = 1,$$

$\mathbf{k}, \mathbf{k}'$ = laboratory momenta of the initial and the final $l^\pm$, respectively;

ω, ω' = laboratory energies of the initial and the final $l^\pm$, respectively;

θ = angle between $\mathbf{k}$ and $\mathbf{k}'$;

m_l, m_N = masses of $l^\pm$ and N, respectively;

$\mathbf{P} = \mathbf{k} - \mathbf{k}'$ = total laboratory momentum of Γ;

$E = \omega + m_N - \omega'$ = total laboratory energy of Γ;

$k, k', P = |\mathbf{k}|, |\mathbf{k}'|, |\mathbf{P}|$, respectively;

$m_\Gamma = (E^2 - P^2)^{1/2}$ = "effective mass" of Γ;

q^2 = (four-momentum transfer)$^2 = P^2 - (E - m_N)^2$.

In this problem, there are three independent variables which may be taken as k, q^2, and m_Γ (or k, k', and θ). The P and E are functions of only q^2 and m_Γ;

$$P = (2m_N)^{-1}[(m_\Gamma + m_N)^2 + q^2]^{1/2}[(m_\Gamma - m_N)^2 + q^2]^{1/2} \tag{21}$$

and

$$E = (2m_N)^{-1}[m_\Gamma^2 + m_N^2 + q^2]. \tag{22}$$

1. Polarized Target

We consider first the case that the target nucleus, or nucleon, is polarized and that its spin j_N is

$$j_N = \tfrac{1}{2}. \tag{23}$$

Let $\mathbf{S}_{in}$ be the polarization vector of the target nucleus, where

$$|\mathbf{S}_{in}| = 1 \tag{24}$$

corresponds to a 100% polarization.

The intial lepton is assumed to be unpolarized, and the final polarization of both Γ and $l^\pm$ are not measured. The differential cross section $d\sigma$ of reaction (17), in the single-photon-exchange approximation, is independent of the sign of the charge of $l^\pm$, and it is given by the following theorem:

Theorem:

$$\begin{aligned} d\sigma = 4\pi\alpha^2 k'^2 [(q^2)^2 k\omega' m_\Gamma]^{-1} dk' d(\cos\theta) & \\ \times \{2(\omega\omega' - kk'\cos\theta - 2m_l^2)W_1 & \\ + (\omega\omega' + kk'\cos\theta + m_l^2)W_2 & \\ + [\mathbf{S}_{in}\cdot(\mathbf{k}\times\mathbf{k}')](\omega^2 - \omega'^2) m_N^{-2} W_3\}, & \end{aligned} \tag{25}$$

where α is the fine-structure constant, W_1, W_2, and W_3 are three real and dimensionless functions of q^2 and m_Γ only. These functions satisfy the following inequalities:

$$W_2 \geqq (q^2/P^2) W_1 \geqq 0 \tag{26}$$

and

$$[W_2 - (q^2/P^2)W_1]W_1 \geqq [\tfrac{1}{2} m_N^{-2}(E - m_N) P W_3]^2. \tag{27}$$

Furthermore, if

$$W_3 \neq 0,$$

then H_γ violates the T_{st} invariance,[11] and, therefore, also the C_{st} invariance.

Proof:

If $\mathbf{S}_{in} = 0$, the above expression of $d\sigma$ is well known.[12] The following proof of the theorem, however, follows a somewhat different approach[13] than that used in most of the literature[14] on electron scatterings.

Let us consider the laboratory system and choose the z axis and the y axis to be parallel to $\mathbf{P}$ and $(\mathbf{k}\times\mathbf{k}')$, respectively. It is convenient to first resolve the initial and the final states of N and Γ into the helicity eigenstates[15] $|\lambda_N\rangle$ and $|\lambda_\Gamma\rangle$, where λ_N and λ_Γ denote, respec-

[10] Reaction (20) has been considered in Ref. 2.

[11] If we assume that H_γ is invariant under C_γ and T_γ, then $W_3 \neq 0$ (or, $W_4 \neq 0$) also implies that H_{st} is not invariant under C_γ and T_γ.

[12] J. D. Bjorken (unpublished); R. Von Gehlen, Phys. Rev. **118**, 1455 (1960); M. Gourdin, Nuovo Cimento **21**, 1094 (1961); L. N. Hand, Ph.D. thesis, Stanford University Physics Department, 1961 (unpublished); S. D. Drell and J. D. Walecka, Ann. Phys. (N. Y.) **28**, 18 (1964).

[13] The proof given in this paper follows closely the method used in deriving the general expressions of inelastic neutrino cross sections. See T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962).

[14] See, however, a recent paper by J. D. Bjorken and J. D. Walecka, Ann. Phys. (to be published).

[15] Throughout the paper, these helicity states are assumed to be normalized; i.e., $\langle\lambda|\lambda\rangle = 1$. The relative phases between the various state vectors $|\lambda\rangle$ with different helicities λ are chosen according to the convention used in A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957). We use also Edmonds' convention for the Clebsch-Gordan coefficients.

tively, the eigenvalues of the spin of N and the total angular momentum of the complex Γ along the direction of $\mathbf{P}$ (i.e., the z axis). Since the components of the electromagnetic current $\mathcal{J}_\mu$ transform like a vector and a scalar under the three-dimensional rotations, the difference $(\lambda_\Gamma-\lambda_N)$ can only be 0 or ±1. We find from the current conservation

$$P\langle\lambda_\Gamma=\lambda_N|\mathcal{J}_z|\lambda_N\rangle=-i(E-m_N)\langle\lambda_\Gamma=\lambda_N|\mathcal{J}_4|\lambda_N\rangle, \quad (28)$$

from the rotation invariance along the z axis

$$\langle\lambda_\Gamma=\lambda_N\pm1|\mathcal{J}_x(0)|\lambda_N\rangle =\pm i\langle\lambda_\Gamma=\lambda_N\pm1|\mathcal{J}_y(0)|\lambda_N\rangle, \quad (29)$$

and from the space-reflection symmetry with respect to the (x,z) plane

$$\langle\lambda_\Gamma|\mathcal{J}_\mu(0)|\lambda_N\rangle=\eta\exp[i\pi(\lambda_N-\lambda_\Gamma)]\langle-\lambda_\Gamma|\mathcal{J}_\mu(0)|-\lambda_N\rangle$$

for $\mu\neq y$, and

$$\langle\lambda_\Gamma|\mathcal{J}_y(0)|\lambda_N\rangle =-\eta\exp[i\pi(\lambda_N-\lambda_\Gamma)]\langle-\lambda_\Gamma|\mathcal{J}_y(0)|-\lambda_N\rangle, \quad (30)$$

where

$$\eta=\mathcal{P}_N\mathcal{P}_\Gamma^{-1}\exp[i\pi(j_N-j_\Gamma)], \quad (31)$$

$\mathcal{P}_N$ and $\mathcal{P}_\Gamma$ are, respectively, the parities of N and Γ, $j_N=\frac{1}{2}$ is the spin of N and j_Γ is the spin, or the total angular momentum, of Γ.

For any given complex Γ, there are, therefore, only three independent nonleptonic matrix elements (called form factors) F_+, F_-, and F_z, where

$$F_\pm=\mp\tfrac{1}{2}\langle\lambda_\Gamma=\tfrac{1}{2}\pm1|\mathcal{J}_x(0)\pm i\mathcal{J}_y(0)|\lambda_N=\tfrac{1}{2}\rangle, \quad (32)$$

and

$$F_z=\langle\lambda_\Gamma=\tfrac{1}{2}|\mathcal{J}_z(0)|\lambda_N=\tfrac{1}{2}\rangle. \quad (33)$$

It is clear that only the nonleptonic current $(J_\mu+K_\mu)$ in $\mathcal{J}_\mu$ contributes to the above matrix elements.

In the expression for $d\sigma$, the leptonic current j_μ appears only through the sum

$$L_{\mu\nu}=\tfrac{1}{2}\sum_{\lambda,\lambda'}\langle\mathbf{k},\lambda|j_\mu(0)|\mathbf{k}',\lambda'\rangle\langle\mathbf{k}',\lambda'|j_\nu(0)|\mathbf{k},\lambda\rangle, \quad (34)$$

where the state $|\mathbf{k},\lambda\rangle$ refers to that of a lepton l^+, or l^-, with a momentum $\mathbf{k}$ and a helicity λ. The sum $L_{\mu\nu}$ is given by

$$L_{\mu\nu}=(2\omega\omega')^{-1}[l_\nu'l_\mu+l_\mu'l_\nu-(m_l^2+l_\alpha l_\alpha')\delta_{\mu\nu}], \quad (35)$$

where $l_\mu=(\mathbf{k},i\omega)$, $l_\mu'=(\mathbf{k}',i\omega')$ and the repeated index is to be summed over. In the particular laboratory system used in this section, we have

$$L_{zz}=-P^{-2}(E-m_N)^2L_{44},$$
$$L_{44}=-(2\omega\omega')^{-1}(\omega\omega'+\mathbf{k}\cdot\mathbf{k}'+m_l^2),$$
$$L_{\mu\mu}=(\omega\omega')^{-1}(\omega\omega'-\mathbf{k}\cdot\mathbf{k}'-2m_l^2),$$
$$L_{zx}=(2\omega\omega'P^2)^{-1}(\omega^2-\omega'^2)(\mathbf{k}\times\mathbf{k}')_y,$$

and

$$L_{yx}=L_{yz}=L_{y4}=0.$$

Equation (25) can now be readily derived by setting the initial proton state to be an appropriate coherent mixture of $|\lambda_N=+\frac{1}{2}\rangle$ and $|\lambda_N=-\frac{1}{2}\rangle$. The three real functions W_1, W_2, and W_3 are related to the three complex form factors $F_\pm$ and F_z by

$$W_1=\sum_\Gamma[|F_+|^2+|F_-|^2]m_\Gamma\delta(\omega+m_N-\omega'-E), \quad (36)$$

$$W_2=\sum_\Gamma[|F_+|^2+|F_-|^2+(E-m_N)^{-2}q^2|F_z|^2]m_\Gamma q^2P^{-2} \times\delta(\omega+m_N-\omega'-E), \quad (37)$$

and

$$W_3=i\eta\sum_\Gamma[F_z{}^*F_--F_-{}^*F_z]m_\Gamma m_N{}^2q^2P^{-2}(E-m_N)^{-2} \times\delta(\omega+m_N-\omega'-E). \quad (38)$$

From these expressions, the inequalities (26) and (27) follow immediately. The functions W_1, W_2, and W_3 clearly depend only on q^2 and m_Γ.

In Eqs. (36)–(38) the sum over Γ can consist of all possible final continuum and resonant states of the same effective mass m_Γ, or it can consist of only a single resonance. In either case, each Γ is an eigenstate of H_{st} and, without any loss of generality, it can also be chosen to be an eigenstate of $[T_{st}\times\exp(-i\pi J_y)]$; i.e.,

$$T_{st}\exp(-i\pi J_y)|\lambda_\Gamma\rangle=\eta_\Gamma|\lambda_\Gamma\rangle,$$

where J_y is the y component of the total angular momentum $\mathbf{J}$ and η_Γ is a phase factor independent of the particular helicity value of the state Γ. Thus, for each final state Γ, the form factors F_+, F_-, and F_z must be relatively real if T_{st} invariance holds for H_γ (i.e., $K_\mu=0$); therefore, $W_3\neq0$ implies that H_γ is not invariant under T_{st}.

It is useful to define an asymmetry parameter a:

$$a=\frac{d\sigma(\uparrow)-d\sigma(\downarrow)}{d\sigma(\uparrow)+d\sigma(\downarrow)}, \quad (39)$$

where $d\sigma(\uparrow)$ and $d\sigma(\downarrow)$ refer to the differential cross sections when the initial polarization $\mathbf{S}_{\text{in}}=\hat{y}$ and $\mathbf{S}_{\text{in}}=-\hat{y}$, respectively, and $\hat{y}$ is a unit vector parallel to $(\mathbf{k}\times\mathbf{k}')$. By using Eq. (25), we find

$$a=\Delta^{-1}[kk'(\omega^2-\omega'^2)m_N{}^{-2}\sin\theta W_3], \quad (40)$$

where

$$\Delta=2(\omega\omega'-kk'\cos\theta-2m_l^2)W_1 +(\omega\omega'+kk'\cos\theta+m_l^2)W_2. \quad (41)$$

The test of T_{st} invariance is to measure the parameter a.

In most cases, it is a good approximation to set $m_l=0$; Eqs. (25) and (40), then, acquire some simpler forms:

$$d\sigma=2\pi\alpha^2\omega'(\omega m_\Gamma q^2)^{-1}d\omega'd\cos\theta\{2W_1+W_2\cot^2(\theta/2) +(\mathbf{S}_{\text{in}}\cdot\hat{y})(\omega^2-\omega'^2)m_N{}^{-2}W_3\cot(\theta/2)\}, \quad (42)$$

and

$$a=[2W_1+W_2\cot^2(\theta/2)]^{-1}\times(\omega^2-\omega'^2)m_N^{-2}W_3\cot(\theta/2). \quad (43)$$

The above considerations can be easily extended to an initial nucleus of arbitrary spin j_N, and the result is given in Appendix B.

It is important to note that Eq. (25) is valid only in the single-photon-exchange approximation. A correlation such as $\mathbf{S}_{\rm in}\cdot(\mathbf{k}\times\mathbf{k}')$ can also be generated by the interference term between the single-photon-exchange process and the two-photon-exchange process, without violating the $T_{\rm st}$ invariance. However, the amount of such $T_{\rm st}$-invariant correlation is necessarily small, since it contains an additional power of the fine structure constant α; furthermore, it is proportional to the sign of the charge of the lepton, while the $T_{\rm st}$-non-invariant correlation, given by Eq. (25), is not.

II. Unpolarized Target and Final-State Polarization

Possible tests of $T_{\rm st}$ invariance can also be given by using an unpolarized target. Let us consider the simple case that the final system consists of only a single resonance state N^*. Reaction (17) becomes simply

$$l^\pm+N\to l^\pm+N^*. \quad (44)$$

By using the same method given in the preceding section, the density matrix D of the resonance N^* can be calculated, where D is a $(2j_{N^*}+1)(2j_{N^*}+1)$ Hermitian matrix and j_{N^*} is the spin of N^*. We discuss the explicit form of D for the case that the spin of the target nucleus (or nucleon) is

$$j_N=\tfrac{1}{2}.$$

After summing over the polarization of N and that of both the initial and final $l^\pm$, the elements of the (unnormalized) matrix D are given by

$$D_{\frac{3}{2},\frac{3}{2}}=D_{-\frac{3}{2},-\frac{3}{2}}=|F_+|^2(L_{xx}+L_{yy}),$$

$$D_{\frac{1}{2},\frac{1}{2}}=D_{-\frac{1}{2},-\frac{1}{2}}=|F_-|^2(L_{xx}+L_{yy})+|F_z|^2[q^2(E-m_N)^{-2}]^2L_{zz},$$

$$D_{\frac{1}{2},\frac{3}{2}}=-D_{-\frac{3}{2},-\frac{1}{2}}{}^*=q^2(E-m_N)^{-2}F_+{}^*F_zL_{zx},$$

$$D_{-\frac{1}{2},\frac{3}{2}}=D_{-\frac{3}{2},\frac{1}{2}}{}^*=-F_+{}^*F_-(L_{xx}-L_{yy}),$$

and

$$D_{-\frac{1}{2},\frac{1}{2}}=[q^2(E-m_N)^{-2}][F_-{}^*F_z-F_-F_z{}^*]L_{zx}, \quad (45)$$

where $L_{\mu\nu}$ is given by Eq. (35) and the subscripts λ, λ' in $D_{\lambda,\lambda'}$ denote the helicities of N^*. All other matrix elements of D, except those which are related to the above one by the Hermitian conjugation, are zero.

From the density matrix, it is straightforward to calculate the average of the spin vector $\mathbf{j}_{N^*}$ of the resonance. We find

$$\langle\mathbf{j}_{N^*}\rangle=\Delta^{-1}(\mathbf{k}\times\mathbf{k}')(\omega^2-\omega'^2)m_N^{-2}\eta\times[(j_{N^*}+\tfrac{1}{2})W_3+(j_{N^*}-\tfrac{1}{2})^{1/2}(j_{N^*}+\tfrac{3}{2})^{1/2}W_4], \quad (46)$$

where η and Δ are given by Eqs. (31) and (41), respectively,

$$W_4=i\eta(F_z{}^*F_+-F_zF_+{}^*)m_\Gamma m_N{}^2q^2P^{-2}(E-m_N)^{-2}\times\delta(\omega+m_N-\omega'-E) \quad (47)$$

and W_1, W_2, W_3 are given by Eqs. (36)–(38) in which the sum over Γ consists now only of one term $\Gamma=N^*$. The $\delta(\omega+m_N-\omega'-E)$ function in the various W_i $(i=1,\cdots,4)$, of course, cancels each other in the final expression for $\langle\mathbf{j}_{N^*}\rangle$. All the above momenta and energies are measured in the laboratory system, but the spin $\langle\mathbf{j}_{N^*}\rangle$ is measured in a particular rest system of N^* which can be reached from the laboratory system by making a Lorentz transformation in the (z,t) subspace where the z axis is parallel to $\mathbf{P}$. From Eq. (46), it follows that if $\langle\mathbf{j}_{N^*}\rangle\neq0$ then either W_3, or W_4, or both, must be different from zero; this means that H_γ is not invariant[11] under $T_{\rm st}$ and, therefore, also $C_{\rm st}$.

The spin state of N^* can be analyzed by measuring the spin of the final nucleon N in its subsequent decay

$$N^*\to N+\pi. \quad (48)$$

The distribution of the polarization vector $\mathbf{S}_f$ of the final nucleon can be calculated by using the density matrix D of N^*. For example, the value of $\mathbf{S}_f$ averaged over the different directions of the momentum of the final nucleon is given by

$$\tfrac{1}{2}\langle\mathbf{S}_f\rangle=\pm(2l+1)^{-1}\langle\mathbf{j}_{N^*}\rangle, \quad (49)$$

where l is the orbital angular momentum of the decay product $(N+\pi)$, and the $+$ and $-$ signs are for $l=j_{N^*}-\frac{1}{2}$ and $j_{N^*}+\frac{1}{2}$, respectively. [$|\mathbf{S}_f|=1$ corresponds to a 100% polarization for the final nucleon along the direction $\mathbf{S}_f$.]

A detection of the correlation

$$\mathbf{S}_f\cdot(\mathbf{k}\times\mathbf{k}'), \quad (50)$$

therefore, can also be used as a test of $T_{\rm st}$ invariance, provided the background events which are due to states other than the resonance N^* can be neglected.

It should be emphasized that there always exist, in addition to N^*, other continuum final states. The interference terms between these continuum (i.e., nonresonant) states and N^*, as well as the interference terms between the different continuum states can produce a background correlation between $\mathbf{S}_f$ and $(\mathbf{k}\times\mathbf{k}')$ without any violation of $T_{\rm st}$ invariance. The angular average over the final nucleon momentum directions can eliminate those interference terms that are between $(N+\pi)$ states of different l values, such as $p_{3/2}$ and $d_{3/2}$, etc. The remaining background correlation must be subtracted out in order to use the final-state polarization as a test of $T_{\rm st}$ invariance.

Remarks

(i). In the above derivations for Eqs. (25) and (46) the initial $l^\pm$ is assumed to be unpolarized and the two spin states of the final $l^{\pm}$ are summed over. For $l^{\pm}=\mu^{\pm}$, the initial muon, in most cases, is longitudinally polarized; its spin lies in the (x,z) plane. The correlation between the y axis [i.e., the direction $(\mathbf{k}\times\mathbf{k}')$] and $\mathbf{S}_{\text{in}}$ remains given by Eq. (25), and the correlation between the y axis and $\langle \mathbf{j}_{N^*}\rangle$ remains given by Eq. (46). Both correlations can still be served as tests of T_{st} invariance. A detailed discussion of the consequences of the polarized muon is given in Appendix C.

(ii). So far, we have not discussed the isotopic spin selection rules of the $C_{st}=+1$ current K_μ. If K_μ exists and if it satisfies the $|\Delta\mathbf{I}|=0$ rule,[8] then, for an initial proton target, it is best to detect the asymmetry parameter a, or the average $\langle \mathbf{j}_{N^*}\rangle$, for the final complex Γ which contains one of the $I=\frac{1}{2}$ resonances.

(iii). If the final state $\Gamma=N^*$ is of spin $\frac{1}{2}$, then

$$F_+=0. \tag{51}$$

In this case, the y component of $\langle \mathbf{j}_{N^*}\rangle$ is related to the asymmetry parameter a, defined by Eq. (39). We find

$$\langle \mathbf{j}_{N^*}\rangle_y=\eta a. \tag{52}$$

(iv). If the final state Γ is of spin $\frac{1}{2}$ and parity $\mathcal{P}_\Gamma=\mathcal{P}_N$, the matrix element of $\mathcal{J}_\mu(0)$ can also be written as

$$\langle\Gamma|\mathcal{J}_\mu(0)|N\rangle=iU_\Gamma^\dagger\gamma_4[\gamma_\mu f+i(\Gamma_\mu+N_\mu)f' +i(\Gamma_\mu-N_\mu)f'']U_N, \tag{53}$$

where N_μ, Γ_μ are, respectively, the four-momenta of states $|N\rangle$ and $|\Gamma\rangle$, U_N and U_Γ are spinor solutions of the free-particle Dirac equations with the same four-momenta as the physical N and Γ, the form factors f, f', and f'' are functions of q^2, which are relatively real if T_{st} invariance holds.

From current conservation, these functions satisfy the relation

$$f=(m_N+m_\Gamma)f'-(m_\Gamma-m_N)^{-1}q^2f''. \tag{54}$$

By using Eqs. (32) and (33), the form factors $F_\pm$ and F_z are related to f, f', and f'' by

$$F_+=0,$$
$$F_-=-[2E(m_\Gamma+E)]^{-1/2}Pf,$$

and

$$F_z=[2E(m_\Gamma+E)]^{-1/2}P[f-(m_\Gamma+E)(f'+f'')]. \tag{55}$$

(v). If the final state Γ is of spin $\frac{1}{2}$ but parity $\mathcal{P}_\Gamma=-\mathcal{P}_N$, then Eq. (53) is replaced by

$$\langle\Gamma|\mathcal{J}_\mu(0)|N\rangle=iU_\Gamma^\dagger\gamma_4[\gamma_\mu f+i(\Gamma_\mu+N_\mu)f' +i(\Gamma_\mu-N_\mu)f'']\gamma_5U_N. \tag{56}$$

The current conservation gives, instead of Eq. (54),

$$f=(m_\Gamma-m_N)f'-(m_\Gamma+m_N)^{-1}q^2f''; \tag{57}$$

the form factors $F_\pm$, F_z are given by

$$F_+=0,$$
$$F_-=-[(m_\Gamma+E)/(2E)]^{1/2}f,$$

and

$$F_z=-[(m_\Gamma+E)/(2E)]^{1/2}[f+(E-m_\Gamma)(f'+f'')], \tag{58}$$

instead of Eq. (55).

III. $A\rightarrow B+e^++e^-$

In this section we apply the method developed in the last section to the reaction

$$A\rightarrow B+e^++e^-, \tag{59}$$

where A and B are two arbitrary states of the strongly interacting particles. For simplicity, only the formula for the special case that A is unpolarized and

$$\text{spin of } B=\tfrac{1}{2} \tag{60}$$

will be given explicitly. The spin of A is, however, arbitrary. Special examples of such reactions are given by (18)–(20).

1. Density Matrix of B

Let the energy and the momentum of $e^\pm$, measured in the laboratory system (i.e., the rest system of A), be $\omega_\pm$ and $\mathbf{k}_\pm$, respectively. The z axis is, again, chosen to be parallel to the laboratory momentum $\mathbf{P}$ of the particle B where

$$\mathbf{P}=-(\mathbf{k}_++\mathbf{k}_-), \tag{61}$$

and the y axis is parallel to $(\mathbf{k}_+\times\mathbf{k}_-)$. Just as in Eqs. (32) and (33), there are only three form factors which are defined by

$$G_\pm=\mp\tfrac{1}{2}\langle\lambda_B=\tfrac{1}{2}|\mathcal{J}_x(0)\pm i\mathcal{J}_y(0)|\lambda_A=\tfrac{1}{2}\mp1\rangle \tag{62}$$

and

$$G_z=\langle\lambda_B=\tfrac{1}{2}|\mathcal{J}_z(0)|\lambda_A=\tfrac{1}{2}\rangle, \tag{63}$$

where λ_A and λ_B denote, respectively, the helicities of A and B (i.e., the eigenvalues of the spin operators of A and B along the z axis). Clearly, $G_\pm$ and G_z are functions of q^2 only, where q^2 is the (four-momentum)^2, and is given by

$$q^2=P^2-(E-m_A)^2,$$
$$E=(P^2+m_B{}^2)^{1/2},$$

and

$$P=|\mathbf{P}|.$$

The density matrix D of the particle B can be written as

$$D=T+\boldsymbol{\sigma}\cdot\mathbf{S}, \tag{64}$$

where $\boldsymbol{\sigma}$ are the usual (2×2) Pauli spin matrices. By following exactly the same methods used in the previous section, we find, in the single-photon-exchange approximation,

$$T=(\omega_+\omega_-)^{-1}[2(\omega_+\omega_- - k_+k_-\cos\theta+2m_e^2)W_1 + (\omega_+\omega_- + k_+k_-\cos\theta - m_e^2)W_2] \quad (65)$$

and

$$\mathbf{S}=(m_A^2\omega_+\omega_-)^{-1}(\mathbf{k}_+\times\mathbf{k}_-)(\omega_+^2-\omega_-^2)W_3, \quad (66)$$

where m_A and m_e are, respectively, the masses of A and e, θ is the angle between $\mathbf{k}_+$ and $\mathbf{k}_-$, and $k_\pm$ denotes the magnitude $|\mathbf{k}_\pm|$. The functions W_1, W_2, and W_3 are related to the form factors $G_\pm$, G_z by

$$W_1=|G_+|^2+|G_-|^2, \quad (67)$$

$$W_2=(q^2/P^2)[|G_+|^2+|G_-|^2+(E-m_A)^{-2}q^2|G_z|^2], \quad (68)$$

and

$$W_3=i\eta q^2 m_A^2 P^{-2}(E-m_A)^{-2}(G_+G_z^*-G_zG_+^*), \quad (69)$$

where

$$\eta=\mathcal{P}_A\mathcal{P}_B^{-1}\exp[i\pi(j_A-j_B)], \quad (70)$$

$\mathcal{P}_A$, $\mathcal{P}_B$ are, respectively, the parities of A and B, $j_B=\frac{1}{2}$ is the spin of B, and j_A is the spin of A. The W_1, W_2, W_3 satisfy the inequalities (26) and (27).

In the case that the spin of B is summed over, the trace of the density matrix, $2T$, is related to the reaction rate by

$$\text{Rate}(A\to B+e^++e^-)=\alpha^2\pi^{-3}(2j_A+1)^{-1} \times\int d^3k_+d^3k_-\delta(\omega_++\omega_-+E-m_A)(q^2)^{-2}T. \quad (71)$$

The vector $T^{-1}\mathbf{S}$ gives the polarization of B, measured in a particular rest system of B which can be reached from the rest system of A by making a Lorentz transformation in the (z,t) subspace.

If the electromagnetic interaction satisfies the T_{st} invariance, then $G_\pm$ and G_z are all relatively real and, consequently, W_3 should be zero. Thus, a possible test of the T_{st} invariance is to measure whether the polarization vector $T^{-1}\mathbf{S}$ is zero or not.

We note that the density matrix is symmetric with respect to the interchange between $\mathbf{k}_+$ and $\mathbf{k}_-$. It is convenient to define the normal vector $\hat{n}$ of the decay plane to be the unit vector parallel to

$$\hat{P}\times(\hat{k}_++\hat{k}_-), \quad (72)$$

where $\hat{P}$, $\hat{k}_\pm$ are the unit vectors along $\mathbf{P}$ and $\mathbf{k}_\pm$, respectively. The normal vector $\hat{n}$ is, then, symmetric with respect to the interchange between $\mathbf{k}_+$ and $\mathbf{k}_-$. For a given orientation of the decay plane, the density matrix depends on two independent variables which may be chosen to be q^2 and ϵ, where

$$\epsilon=(\omega_+-\omega_-) \quad (73)$$

and q^2 is related to the sum $(\omega_++\omega_-)$ by

$$q^2=m_A^2-m_B^2-2m_A(\omega_++\omega_-). \quad (74)$$

Let $d^2N_\uparrow$ and $d^2N_\downarrow$ be the partial-decay rates of reaction (59) in which the spin of B is, respectively, parallel and antiparallel to the normal vector $\hat{n}$. By using Eqs. (65)–(71), we find[16]

$$\begin{aligned}d^2N_\uparrow+d^2N_\downarrow &=[-q^2\pi m_A(2j_A+1)]^{-1}\alpha^2E[d\epsilon d(-q^2)]\\ &\times\{[1+(\epsilon/p)^2-(4m_e^2/q^2)][|G_+|^2+|G_-|^2]\\ &+[1-(\epsilon/p)^2][-q^2/(m_A-E)^2]|G_z|^2\}\end{aligned} \quad (75)$$

and

$$\begin{aligned}d^2N_\uparrow-d^2N_\downarrow &=[-q^2\pi m_A(2j_A+1)P^2(m_A-E)]^{-1}\\ &\times 2i\eta\alpha^2Ek_+k_-\epsilon\sin\theta[d\epsilon d(-q^2)]\\ &\times[G_zG_+^*-G_+G_z^*],\end{aligned} \quad (76)$$

where ϵ varies from

$$-[1+(4m_e^2/q^2)]^{1/2}P \quad\text{to}\quad +[1+(4m_e^2/q^2)]^{1/2}P$$

and $(-q^2)$ varies from $4m_e^2$ to $(m_A-m_B)^2$. The P and E are functions of q^2, and are given by

$$P=\frac{1}{2m_A}[(m_A+m_B)^2+q^2]^{1/2}[(m_A-m_B)^2+q^2]^{1/2}, \quad (77)$$

$$E=\frac{1}{2m_A}[m_A^2+m_B^2+q^2]. \quad (78)$$

In Eq. (76), $\sin\theta>0$ if $\epsilon>0$ and $\sin\theta<0$ if $\epsilon<0$.

In order to detect the T_{st}-noninvariant term $(d^2N_\uparrow-d^2N_\downarrow)$, one should choose events for which $-q^2\gg 4m_e^2$. Let $dN_\uparrow$ and $dN_\downarrow$ be, respectively, the integrals $\int_\epsilon d^2N_\uparrow$ and $\int_\epsilon d^2N_\downarrow$ integrated over the entire ϵ range for a given q^2. The ϵ integration can be easily carried out, and we obtain, after neglecting $-(4m_e^2/q^2)$ as compared to unity,

$$\begin{aligned}dN_\uparrow+dN_\downarrow &=[3(-q^2)m_A\pi(2j_A+1)]^{-1}4\alpha^2EPd(-q^2)\\ &\times\{2[|G_+|^2+|G_-|^2]+[-q^2/(m_A-E)^2]|G_z|^2\}\end{aligned} \quad (79)$$

and

$$\begin{aligned}dN_\uparrow-dN_\downarrow &=[3m_A\pi(2j_A+1)(m_A-E)]^{-1}\\ &\times(-q^2)^{-1/2}d(-q^2)[2i\alpha^2EP\eta][G_zG_+^*-G_+G_z^*].\end{aligned} \quad (80)$$

It is useful to define the asymmetry function $A(q^2)$ by

$$\begin{aligned}A(q^2) &\equiv(dN_\uparrow+dN_\downarrow)^{-1}(dN_\uparrow-dN_\downarrow)\\ &=\{|G_+|^2+|G_-|^2+\tfrac{1}{2}[-q^2/(m_A-E)^2]|G_z|^2\}^{-1}\\ &\times\tfrac{1}{4}i(m_A-E)^{-1}(-q^2)^{1/2}\eta(G_zG_+^*-G_+G_z^*).\end{aligned} \quad (81)$$

[16] Expressions similar to Eq. (75) have been discussed in the literature for reactions in which the spin of B is not measured. See N. Kroll and W. Wada, Phys. Rev. **98**, 1359 (1955) for a discussion on $\pi^-+p\to n+e^++e^-$. The reaction $\Sigma\to\Lambda^0+e^++e^-$ has been analyzed by G. Feinberg, Phys. Rev. **109**, 1019 (1958); G. Feldman and T. Fulton, Nucl. Phys. **8**, 106 (1958).

Thus, independent of the forms of $G_\pm$ and G_z, we have

$$|A(q^2)| \leqq (2\sqrt{2})^{-1}. \tag{82}$$

If $A(q^2) \neq 0$ in the single-photon-exchange approximation, then T_{st} invariance is violated.

2. Special Case: $j_A=\frac{1}{2}$ and $\mathcal{P}_A=\mathcal{P}_B$

If $j_A=\frac{1}{2}$ and the states A and B are of the same parity, then, by using Eq. (53), the matrix element of $\mathcal{J}_\mu(0)$ is given by

$$\langle B|\mathcal{J}_\mu(0)|A\rangle = iU_B{}^\dagger\gamma_4[\gamma_\mu f + i(B_\mu+A_\mu)f' + i(B_\mu-A_\mu)f'']U_A. \tag{83}$$

The current conservation, Eq. (54), gives

$$f=(m_A+m_B)f'+(m_A-m_B)^{-1}q^2f''. \tag{84}$$

The form factors $G_\pm$ and G_z are given by

$$G_-=0, \tag{85}$$

$$G_+=-[2E(E+m_B)]^{-1/2}Pf, \tag{86}$$

and

$$G_z=[2E(E+m_B)]^{-1/2}[P(E-m_A)/q^2]g, \tag{87}$$

where

$$g=[(m_A+m_B)f-2m_A(m_B+E)f']. \tag{88}$$

By using Eq. (84), we find

$$g=0 \quad \text{at} \quad q^2=0. \tag{89}$$

In terms of f and g, the asymmetry function becomes $(m_e=0)$

$$A(q^2)=(2|g|^2-4q^2|f|^2)^{-1}i(-q^2)^{1/2}(gf^*-fg^*). \tag{90}$$

For small q^2, f and g are given by

$$f=f_0+O(q^2),$$

and

$$g=(dg/dq^2)_0q^2+O[(q^2)^2].$$

Thus,

$$A \to \tfrac{1}{4}i(-q^2)^{1/2}|f_0|^{-2}[f_0(dg/dq^2)_0{}^*-\text{c.c.}]$$

as $q^2 \to 0$.

Some discussions of the density matrix have been given[17] in Ref. 2 for the decay $\Sigma^0 \to \Lambda^0+e^++e^-$.

[17] In the approximation $m_e=0$, Eqs. (65) and (66) become

$$T=2[|G_+|^2+|G_-|^2][1-(\hat{P}\cdot\hat{k}_+)(\hat{P}\cdot\hat{k}_-)] + P^{-2}(E-m_A)^{-2}(q^2)^2|G_z|^2[1+(\hat{k}_+\cdot\hat{k}_-)],$$

$$S=i\eta q^2P^{-1}(E-m_A)^{-1}[\hat{P}\times(\hat{k}_++\hat{k}_-)](G_+G_z{}^*-G_zG_+{}^*).$$

For the special case $\mathcal{P}_A=\mathcal{P}_B$ and $j_A=\frac{1}{2}$, these equations reduce to

$$T=[2E(E+m_B)]^{-1}\{2|f|^2P^2[1-(\hat{P}\cdot\hat{k}_+)(\hat{P}\cdot\hat{k}_-)] + |g|^2[1+(\hat{k}_+\cdot\hat{k}_-)]\},$$

and

$$\mathbf{S}=i[2E(E+m_B)]^{-1}[\mathbf{P}\times(\hat{k}_++\hat{k}_-)][gf^*-fg^*],$$

where f and g are given by Eqs. (84) and (88). The functions F and G in Ref. 2 are identical with the above f and g; the T and $\mathbf{S}_A$ in Ref. 2 differ from the above T and S by a factor $[2E(E+m_B)]^{-1}$.

3. Special Case: $j_A=\frac{1}{2}$ and $\mathcal{P}_A=-\mathcal{P}_B$

Next, we consider the special case $j_A=\frac{1}{2}$ and the states A and B are of the opposite parity. As an example of such reactions we may mention the s-state capture of $\pi^-+p \to n+e^++e^-$. In this case, the matrix element of $\mathcal{J}_\mu$ can be written as [cf. Eqs. (56) and (57)]

$$\langle B|\mathcal{J}_\mu(0)|A\rangle = iU_B{}^\dagger\gamma_4[\gamma_\mu f+i(B_\mu+A_\mu)f' + i(B_\mu-A_\mu)f'']\gamma_5U_A. \tag{91}$$

The current conservation gives

$$f=-(m_A-m_B)f'-(m_A+m_B)^{-1}q^2f''. \tag{92}$$

The form factors $G_\pm$ and G_z are related to the f, f', and f'' by

$$G_-=0, \tag{93}$$

$$G_+=[(E+m_B)/(2E)]^{1/2}f, \tag{94}$$

and

$$G_z=-[(E+m_B)/(2E)]^{1/2}[f+(E-m_B)(f'+f'')]. \tag{95}$$

By using Eqs. (65), (66), (75), (76) and the above formulas we can easily express the functions T, $\mathbf{S}$, $d^2N_\uparrow$, and $d^2N_\downarrow$ in terms of f, f', and f''.

Sometimes, it may be useful to eliminate f and express G_+ and G_z in terms of f' and f'' only. Equations (94) and (95) can be written in the alternative form

$$G_+=-[(E+m_B)/(2E)]^{1/2}(m_A-m_B) \times[f'+(m_A{}^2-m_B{}^2)^{-1}q^2f''], \tag{96}$$

and

$$G_z=[(E+m_B)/(2E)]^{1/2}(m_A-E) \times[f'-(m_A+m_B)^{-1}(m_A-m_B)f'']. \tag{97}$$

Thus,

$$(G_+G_z{}^*-G_+{}^*G_z)=[E(m_A+m_B)]^{-1} \times(m_A-E)P^2m_A[f'f''^*-f''f'^*]. \tag{98}$$

In the approximation of setting $m_e=0$, the asymmetry function $A(q^2)$ is given by Eq. (81). It follows from Eqs. (94) and (95) that $G_+=-G_z$ as $-q^2$ reaches its maximal value $(m_A-m_B)^2$. Thus,

$$A(q^2)=0$$

at both the minimal and the maximal values of $(-q^2)$.

For the s-state capture of

$$\pi^-+p \to n+e^++e^- \tag{99}$$

the possible magnitude of the asymmetry function $A(q^2)$ may be roughly estimated by noting that the single-pion-pole contribution to the form factors has a quite different q^2 dependence from the rest. Let f_π' and f_π'' be, respectively, the single-pion-pole contributions to the form factors f' and f''. (See Fig. 1.) It can be readily shown that

$$f_\pi'=[q^2+2m_\pi{}^2+(m_\pi{}^3/m_N)]^{-1} \times K(2m_N+m_\pi)(m_\pi/m_N) \tag{100}$$

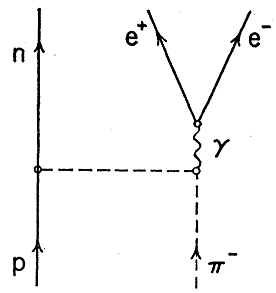

FIG. 1. Diagram for the single-pion-pole contribution to $\pi^- + p \to n + e^+ + e^-$.

and

$$f_\pi'' = [q^2 + 2m_\pi^2 + (m_\pi^3/m_N)]^{-1} K (2m_N + m_\pi), \quad (101)$$

where m_π and m_N are, respectively, the pion and the nucleon masses. The dimensionless constant K is $\sim 3.4 \times 10^{-4}$, and is given explicitly by

$$K = (g^2/\pi)^{1/2} (2m_N + m_\pi)^{-1} m_\pi \alpha^{3/2}, \quad (102)$$

where $(g^2/4\pi) \cong 15.7$ is the square of the pion-nucleon coupling constant. In obtaining the above expressions we have used the fact that the initial pion is captured in the $1s$ orbit. From current conservation, Eq. (92), these single-pion-pole terms necessitate a corresponding term f_π in the form factor f where

$$f_\pi = -K. \quad (103)$$

The form factors f, f', and f'' can now be written as

$$f = f_\pi + K\delta,$$
$$f' = f_\pi' + m_N^{-1} K \delta',$$

and

$$f'' = f_\pi'' + m_N^{-1} K \delta''. \quad (104)$$

The single-pion-pole contributions f_π, f_π', and f_π'' necessarily satisfy the requirement of T_{st} invariance; therefore, they are all real functions. The functions $\delta(q^2)$, $\delta'(q^2)$, and $\delta''(q^2)$ can be complex, if the T_{st} invariance is violated, but they are expected to be slowly varying functions of q^2. In a perturbation calculation, $\delta' = +1$, $\delta'' = -1$, and $\delta = -(m_\pi/m_N)$ at $q^2 = 0$. For the present case, we assume that the phase angles of δ' and δ'' can be arbitrary, but their magnitudes remain of the order of unity; i.e.,

$$|\delta'(q^2)| \sim |\delta''(q^2)| \sim O(1), \quad (105)$$

Consequently, from Eq. (92), $|\delta| \sim O(m_\pi/m_N)$.

By using the above expressions and Eq. (81), the asymmetry function $A(q^2)$ can be expressed in terms of δ' and δ''. After neglecting (m_π/m_N) as compared to unity, we find

$$A(q^2) = -\tfrac{1}{2} |\delta'| (\sin\phi)(m_\pi/m_N)[4 - \tfrac{7}{2}x^2 + x^4]^{-1}$$
$$\times x(1-x^2)(2-x^2) + O(m_\pi^2/m_N^2), \quad (106)$$

where

$$x^2 = -q^2/m_\pi^2 \quad (107)$$

and

$$\delta' = |\delta'| \exp(i\phi). \quad (108)$$

Thus,

$$\left| \frac{A(q^2)}{\delta'(q^2)} \right| \leqq 0.11 (m_\pi/m_N), \quad (109)$$

and the maximum of $|A/\delta'|$ occurs at $(-q^2/m_\pi^2) = 0.39$ and $\sin\phi = \pm 1$. Since $|\delta'|$ is of the order of unity, this means that even for the maximal T_{st} noninvariance (i.e., $\phi = \pm\pi/2$) the magnitude of the asymmetry function $A(q^2)$ is only $\lesssim 1.6\%$.

In the above estimation, the form factors are dominated by the T_{st}-invariant single-pion-pole contributions. Thus, we have $|f''/f'| \sim (m_N/m_\pi) \gg 1$, and the realtive phase angle between f' and f'' is only of the order of (m_π/m_N).

It seems worthwhile also to examine the asymmetry function $A(q^2)$ for the alternative, but less probable, possibility that the pion-pole contribution may turn out to be unimportant. In such a case, the relative phase between f' and f'' may be quite large. We note that if the pion-pole contribution is not important, then there is no reason to expect that $|f''|$ should be much greater than $|f'|$. Therefore, we may neglect $|m_\pi f'|$ as compared to $|m_N f''|$. The asymmetry function is, then, given approximately by

$$A(q^2) \cong \tfrac{1}{2} [m_N (2m_\pi^2 - q^2)]^{-1} (-q^2)^{1/2} (m_\pi^2 + q^2) \xi, \quad (110)$$

where

$$\xi(q^2) = \tfrac{1}{2} i |f'|^{-2} (f' f''^* - f'' f'^*). \quad (111)$$

Equation (110) is valid if $|f''|$ is of the same order of magnitude as, or at least not much greater than, $|f'|$. It is easy to see that Eq. (110) implies that $|A/\xi| \leqq 0.084 \times (m_\pi/m_N)$. In this case, a maximal violation of T_{st} invariance means that $|\xi| \sim 1$; the upper limit of $A(q^2)$ is $\lesssim 1.2\%$ which is about the same order of magnitude as that given by (109).

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APPENDIX A

In this Appendix, we discuss a possible test of T_{st} invariance by using reaction (44) with an unpolarized target. If the final state is a single meson-baryon resonance N^* and if no spin polarizations are determined, an analysis of the angular distribution shown by the decay $N^* \to N + \pi$ can still serve as a test for T_{st} invariance. Define Θ and Φ as the polar and azimuthal angles, respectively, of the $N - \pi$ relative momentum in the N^* rest frame. The coordinate axes in this reference frame are obtained from those previously chosen in the laboratory system by a simple Lorentz transformation without rotation, in the z direction. If the spin of the target nucleus is $j_N=\frac{1}{2}$, we have the following theorem:

Theorem:

$$d\sigma=\frac{4\pi\alpha^2k'^2dk'd(\cos\theta)}{\omega'k|q^2|^2}\delta(E+\omega'-m_N-\omega)\Bigg\{|F_+|^2\bigg[2(\omega\omega'-\mathbf{k}\cdot\mathbf{k}'-2m_l^2)+\frac{q^2}{P^2}(\omega\omega'+\mathbf{k}\cdot\mathbf{k}'+m_l^2)\bigg]$$
$$\times[(P_{j'-1/2}^2)^2+(j'+\tfrac{3}{2})^2(P_{j'-1/2}^1)^2]\frac{(j'-\frac{3}{2})!}{(j'+\frac{3}{2})!}+\bigg[2|F_-|^2(\omega\omega'-\mathbf{k}\cdot\mathbf{k}'-2m_l^2)+\bigg(|F_-|^2+\frac{q^2}{(E-m_N)^2}|F_z|^2\bigg)$$
$$\times\frac{q^2}{P^2}(\omega\omega'+\mathbf{k}\cdot\mathbf{k}'+m^2)\bigg]\bigg[(P_{j'-1/2}^0)^2(j'+\tfrac{1}{2})+\frac{(P_{j'-1/2}^1)^2}{j'+\frac{1}{2}}\bigg]+\cos\Phi(F_+F_z^*+F_+^*F_z)\frac{q^2(\omega^2-\omega'^2)kk'\sin\theta}{P^2(E-m_N)^2}$$
$$\times[P_{j'-1/2}^0P_{j'-1/2}^1(j'+\tfrac{3}{2})(j'+\tfrac{1}{2})+P_{j'-1/2}^2P_{j'-1/2}^1]\bigg(\frac{(j'-\frac{3}{2})!}{(j'+\frac{3}{2})!(j'+\frac{1}{2})}\bigg)^{1/2}+\cos2\Phi(F_+F_-^*+F_+^*F_-)\frac{2k'^2k^2\sin^2\theta}{P^2}$$
$$\times\bigg[(P_{j'-1/2}^1)^2\frac{j'+\frac{3}{2}}{j'+\frac{1}{2}}-P_{j'-1/2}^2P_{j'-1/2}^0\bigg](j'+\tfrac{3}{2})^{-1/2}(j'-\tfrac{1}{2})^{-1/2}\Bigg\}\frac{d\Phi d(\cos\Theta)}{4\pi},$$

where

$$P_l^m=P_l^m(\cos\Theta),\quad P_l^m(x)=\frac{(-1)^m}{2^l l!}(1-x^2)^{m/2}\frac{d^{l+m}}{dx^{l+m}}(x^2-1)^l,$$

and j' is the spin of the N^* resonance.

Proof:

Measuring $d\sigma(\Theta,\Phi)$ corresponds to determining the expectation value of a certain operator $O(\Theta,\Phi)$. If we choose a representation where the final state is labeled by the z component of its angular momentum m, then

$$O_{m'm}(\Theta,\Phi)=\sum_{\lambda=-1/2}^{1/2}(j'-\tfrac{1}{2},\tfrac{1}{2},j',m'|j'-\tfrac{1}{2},m'+\lambda,\tfrac{1}{2},-\lambda)Y_{j'-1/2,m'+\lambda}(\Theta,\Phi)$$
$$\times(j'-\tfrac{1}{2},\tfrac{1}{2},j',m|j'-\tfrac{1}{2},m+\lambda,\tfrac{1}{2},-\lambda)Y_{j'-1/2,m+\lambda}^*(\Theta,\Phi),$$

where $(j_1,j_2,j,m|j_1,m_1,j_2,m_2)$ are the usual Clebsch-Gordan coefficients and the $Y_{lm}(\Theta,\Phi)$ are spherical harmonics.[15] It is interesting to note that $O(\Theta,\Phi)$ does not depend on the orbital angular momentum, $l=j'\pm\frac{1}{2}$, of the resonance state. This is an example of the Minami ambiguity.[18] Using the density matrix for the N^* resonance $D_{m',m}$ given in Sec. II2, we obtain

$$d\sigma=\frac{8\pi\alpha^2k'^2dk'd(\cos\theta)}{k\omega'[q^2]^2}\sum_{mm'}O(\Theta,\Phi)_{m'm}D_{m,m'}d\Phi d(\cos\Theta)$$

which reduces to the result stated in the theorem.

An examination of $d\sigma$ reveals that the coefficients of $|F_+|^2$, $|F_-|^2$, $|F_z|^2$, $F_+F_z^*+F_+^*F_z$, and $F_+F_-^*+F_+^*F_-$ are independent functions of k, Θ, and Φ. Therefore, if $d\sigma(\Theta,\Phi)$ is measured for events with a constant q^2 but for two or more values of k the above five quantities can be determined. T_{st} invariance implies two relations among these quantities

$$|F_+F_z^*+F_+^*F_z|=2|F_+||F_z|,$$
$$|F_+F_-^*+F_+^*F_-|=2|F_+||F_-|.$$

These two equations are tests of T_{st} invariance, provided, of course, that only events due to the N^* resonance are analyzed. Because of the Minami ambiguity, the form of $d\sigma(\Theta,\Phi)$ is independent of the N^*'s parity. Thus, if any measurement of $d\sigma(\Theta,\Phi)$ is to serve as a test of T_{st} invariance, additional conditions must be imposed to ensure that the final states measured are only those with the isotopic spin and parity of the N^*.

APPENDIX B

T_{st} invariance can be tested in the general reaction $l^\pm+N\rightarrow l^\pm+\Gamma$, N and Γ having total angular momentum j_N and j_Γ, respectively. Let the density matrix for the spin distribution of the target nucleus be given by

$$D_N=(1+\mathbf{S}_N\cdot\mathbf{J})/(2j_N+1),$$

[18] S. Minami, Progr. Theoret. Phys. (Kyoto) **11**, 213 (1954).

where $\mathbf{J}$ is the angular momentum operator. In this general case, Eq. (25) is still valid; however, W_1, W_2, and W_3 must be redefined. Let us define

$$F_-(m)=\tfrac{1}{2}\langle\lambda_\Gamma=m-1|J_x(0)-iJ_y(0)|\lambda_N=m\rangle,\quad \text{if } |m-1|\leqslant j_\Gamma$$
$$=0,\quad \text{if } |m-1|>j_\Gamma$$
$$F_z(m)=\langle\lambda_\Gamma=m|J_z(0)|\lambda_N=m\rangle,\quad \text{if } |m|\leqslant j_\Gamma$$
$$=0,\quad \text{if } |m|>j_\Gamma$$

where $|\lambda_N\rangle$ and $|\lambda_\Gamma\rangle$ are helicity eigenstates; λ_N and λ_Γ denote, respectively, the eigenvalue of the spin of N and the total angular momentum of Γ along the direction of $\mathbf{P}$.

$$W_1=\sum_{m=-j_N}^{j_N}\sum_\Gamma |F_-(m)|^2 m_\Gamma\delta(\omega+m_N-\omega'-E)\frac{2}{2j_N+1},$$

$$W_2=\sum_{m=-j_N}^{j_N}\sum_\Gamma \left\{2|F_-(m)|^2+|F_z(m)|^2\frac{q^2}{(E-m_N)^2}\right\}\times\frac{m_\Gamma q^2}{P^2}\delta(\omega+m_N-\omega'-E)\frac{1}{2j_N+1},$$

$$W_3=i\sum_{m=-j_N}^{j_N-1}\sum_\Gamma [(j_N-m)(j_N+m+1)]^{1/2}\times\{F_-(m+1)^*F_z(m)-F_-(m+1)F_z(m)^*\}\times\frac{q^2m_\Gamma m_N^2}{P^2(E-m_N)^2}\delta(\omega+m_N-\omega'-E)\frac{2}{2j_N+1}.$$

Using this definition for W_i, Eq. (25) describes the scattering of leptons from objects of total angular momentum j_N with polarization $\mathbf{S}_N$. As before, the observation of the term $\mathbf{S}_N\cdot(\mathbf{k}\times\mathbf{k}')$ and the consequent nonvanishing of W_3 is an indication of T_{st} noninvariance.

APPENDIX C

Under certain experimental conditions, the initial lepton in the reaction $l^\pm+N\to l^\pm+\Gamma$ may be polarized. Muons produced by the decay $\pi\to\mu+\nu$ have this property. The theorem in Sec. I.1 can be generalized to include this possibility.

Theorem:

$$d\sigma=\frac{4\pi\alpha^2k'dk'd(\cos\theta)}{k\omega'[q^2]^2m_\Gamma}\Bigg\{2(\omega\omega'-\mathbf{k}\cdot\mathbf{k}'-2m_l^2)W_1+(\omega\omega'+\mathbf{k}\cdot\mathbf{k}'+m_l^2)W_2+\mathbf{S}_N\cdot(\mathbf{k}\times\mathbf{k}')\frac{\omega^2-\omega'^2}{m_N^2}W_3$$
$$+\mathbf{S}_N\cdot\mathbf{S}_l m_l(\omega-\omega')W_5+\mathbf{S}_N\cdot\left[\frac{(\mathbf{k}\times\mathbf{k}')\times(\mathbf{k}-\mathbf{k}')}{P^2}\right]\mathbf{S}_l\cdot\mathbf{k}\left(1-\frac{\omega'+m_l}{\omega+m_l}\right)W_5-\mathbf{S}_N\cdot\frac{(\mathbf{k}-\mathbf{k}')}{P}$$
$$\times\left[\mathbf{S}_l\cdot\frac{(\mathbf{k}-\mathbf{k}')}{P}m_l(\omega-\omega')(W_5+W_6)-\frac{\mathbf{S}_l\cdot\mathbf{k}}{P}\left((\omega-m)(\omega'+m)+k'^2-kk'\cos\theta\left[1+\frac{\omega'+m_l}{\omega+m_l}\right]\right)W_6\right]\Bigg\},$$

where

$$W_5=\sum_\Gamma\frac{q^2m_\Gamma\eta}{(\omega-\omega')^2}(F_zF_-^*+F_z^*F_-)\delta(\omega+m_N-E-\omega'),$$
$$W_6=\sum_\Gamma 2m_\Gamma(|F_+|^2-|F_-|^2)\delta(\omega+m_N-E-\omega'),$$

and $\mathbf{S}_l$ is the polarization of the initial lepton as measured in its rest frame. The particular coordinate system in which the components of $\mathbf{S}_l$ are to be determined is obtained from that chosen in the laboratory by a simple Lorentz transformation without rotation, in the $\mathbf{k}$ direction. If the initial lepton is completely polarized, $|\mathbf{S}_l|=1$. The W_i satisfy the following relations:

$$2W_1\geqslant|W_6|;\ (2W_1-W_6)\left(\frac{P^2}{q^2}W_2-W_1\right)\geqslant[P^2W_3(E-m_N)/m_N^2[q^2]^{1/2}]^2,$$

$$W_2\geqslant\frac{q^2}{P^2}W_1;\ (2W_1-W_6)\left(\frac{P^2}{q^2}W_2-W_1\right)\geqslant[W_5(E-m_N)/[q^2]^{1/2}]^2.$$

Proof:

In the present case, the initial muon is polarized, but the two final muon spin states are summed over. The lepton current j_μ enters the cross section only in the sum

$$\tfrac{1}{2}\sum_{\lambda\lambda'}\langle k\lambda|j_\mu(0)|k'\lambda'\rangle\langle k'\lambda'|j_\nu(0)(1+i\gamma_5\gamma_\mu S_\mu')|k\lambda\rangle=L_{\mu\nu}+K_{\mu\nu},$$

where $L_{\mu\nu}$ is defined by Eq. (34), and $\frac{1}{2}(1+i\gamma_5\gamma_\mu S_\mu')$ is a spin-projection operator. The four-vector S_μ' is obtained from the $\mathbf{S}$ defined above by forming in the electron's rest system the four-vector $(\mathbf{S},0)$. This four-vector is then transformed into the laboratory system by a simple Lorentz transformation without rotation; the result is S_μ'. The tensor $K_{\mu\nu}$ is antisymmetric and

$$K_{12}=\frac{i\mathbf{S}_l\cdot(k-k')}{2\omega\omega' P}(\omega-\omega')m_l-\frac{i\mathbf{S}_l\cdot\mathbf{k}}{2\omega\omega' P}\left[(\omega-m)(\omega'+m)+k'^2-kk'\cos\theta\left(1+\frac{\omega'+m}{\omega+m}\right)\right],$$

$$K_{23}=\frac{i\mathbf{S}_l\cdot\hat{x}}{2\omega\omega'}(\omega-\omega')m_l+\frac{i\mathbf{S}_l\cdot k}{2\omega\omega' P}kk'\sin\theta\left[1-\frac{\omega'+m}{\omega+m}\right],$$

$$K_{31}=\frac{i\mathbf{S}_l\cdot\hat{y}}{2\omega\omega'}(\omega-\omega')m_l.$$

Our theorem can be derived by following exactly the same method as that given in Sec. I.1, but using $(L_{\mu\nu}+K_{\mu\nu})$ in place of $L_{\mu\nu}$.

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Boson Masses

R. W. King*
Purdue University, Lafayette, Indiana
AND
D. C. Peaslee
The Australian National University, Canberra, Australia

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A gross mass scale is assigned to the baryon-antibaryon model for bosons: for charge octets $m^2(^3L)-m^2(^1L)\approx\frac{1}{2}$ BeV^2. For charge singlets the mass formula is less certain but seems to be quite different from that for octets. The quantum number of A parity is pointed out as an inescapable feature of this model; the empirical values of A then rule out quarks for the simplest realization. Interest is remarked in $K3\pi$ resonances and in possible resolution of the $K\pi$ and $K2\pi$ modes of the $K^*(1430)$.

EMPIRICAL mass relations among elementary particles have been remarked from time to time,[1] with high numerical precision but lacking a relevant physical model. We present here an approach from the other extreme by assigning a preliminary mass scale to the baryon-antibaryon model for bosons. The assignment relies heavily on the set of $J^P=2^+$ mesons that now seems established[2,3]; it is concerned in this instance only with the gross features of J^P ordering. The approach is one of bounded speculation but already yields specific boson interpretations at some variance with those currently popular, and amenable to experimental study, viz., the question of $J^P=1^+$ for the X meson, the survey of $K3\pi$ resonances. Such measurements are feasible but cannot usually be reconstructed from randomly assorted data; in the hope of arousing critical discussion, the elementary considerations below are presented.

The energy scale chosen is so gross that the internal structure of octets and nonets will be of little concern, and details of coupling schemes need scarcely be specified. In order to avoid the opposite extreme of complete formlessness, we impose the following constraints:

(i) The basic Fermion and anti-Fermion are not quarks nor triplets but just the observed baryon charge octet of spin $\frac{1}{2}$ in some "bare" state, which is taken not to differ very substantially from the dressed state. This is a little unfashionable at present but entrains the next restriction, often neglected; confrontation with experiment is discussed below.

(ii) The baryons are taken to obey ordinary Fermi statistics, having never given evidence of parastatistics; that is, the Pauli principle is taken seriously.

(iii) Systematic corrections could be introduced as

* Alfred P. Sloan Foundation Fellow.

[1] See, e.g., T. F. Kycia and K. F. Riley, Phys. Rev. Letters **10**, 266 (1963); R. M. Sternheimer, *ibid.* **10**, 309 (1963).

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[3] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).