On the other hand, from the transformation property of S_i, we can write

$$\langle\pi^+|S_8|\pi^+\rangle = F_{8_1}\begin{pmatrix} 8 & 8 & 8_1 \\ \pi^+ & \eta & \pi^+ \end{pmatrix}. \qquad \text{(A7)}$$

Putting Eqs. (A5) and (A6) into the left-hand side of Eq. (A7), performing tensor calculations, and comparing the left-hand side with the right-hand side, we find $F_{8_1}=\frac{3}{5}\sqrt{3}$.

The other matrix elements can be obtained along the same lines.

PHYSICAL REVIEW VOLUME 143, NUMBER 4 MARCH 1966

Calculation of the Nucleon Magnetic Moments by Dispersion-Theory Methods

HENRY D. I. ABARBANEL,* CURTIS G. CALLAN, JR.,† AND DAVID H. SHARP

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received 25 October, 1965)

The magnetic moments of the neutron and proton are calculated within the framework of the S-matrix perturbation theory recently developed by Dashen and Frautschi. In the present context, this method expresses the magnetic moments in terms of a dispersion integral involving photopion production. Evaluation of this integral in terms of contributions from appropriate low-mass intermediate states yields results for the individual magnetic moments which are larger than the experimental values by about a factor of two. The calculation does, however, give an approximately correct value for the ratio of the isovector moment to the isoscalar moment, and a value for the isoscalar moment that agrees with the experimental value to within about a factor of two.

I. INTRODUCTION

RECENTLY, Dashen and Frautschi[1,2] have suggested a method for finding the changes in the residues and positions of bound-state poles in the S matrix when the strong interactions are perturbed by the addition of another, weaker force. Dashen[3] applied this method to calculate the proton-neutron mass difference and obtained a result in good agreement with experiment. It is our purpose in this paper to apply these methods to calculate the magnetic moments of the nucleons.

To discuss the nucleon magnetic moments from this point of view, we need to study a scattering process in which the magnetic moments appear as a residue of a pole in the scattering amplitude. Photopion production in the $J=\frac{1}{2}^+$, $T=\frac{1}{2}$ channel has a nucleon pole whose residue, apart from kinematic factors, is proportional to the nucleon magnetic moments. Therefore, this is an appropriate process to study. Note, however, that it is the residue of the photoproduction amplitude, not a perturbation on this residue, which contains the quantity we want to calculate.

To understand in what sense this may be regarded as a perturbation calculation, one may consider a two-channel S matrix in which channel 1 is the $J=\frac{1}{2}^+$, $T=\frac{1}{2}\pi N$ state, and channel 2 is the $J=\frac{1}{2}^+$, $T=\frac{1}{2}\gamma N$ state. When electromagnetism is turned off, channel 1 is coupled only to itself, through the strong interactions, and there is no scattering in the 22 amplitude. This defines the unperturbed problem. If the electric forces are turned on, and only terms of *first* order in e are kept, then the S matrix is *changed* only by the appearance of new nonzero matrix elements corresponding to photoproduction, which are of order e. It is in this sense that we speak of carrying out a perturbation calculation here.[4]

It may seem strange to think of the magnetic moments as in any way connected with an electromagnetic perturbation, because it is, of course, true that the magnetic moments are closely related to the nucleon form factors at $q^2=0$, which are determined by the strong interactions alone. It is, therefore, important to note that the basic formula to be used here for the magnetic moments can also be derived in a way which makes it clear that the magnetic moments are not of electromagnetic origin (see Sec. II).

Thus, we consider the $J=\frac{1}{2}^+$, $T=\frac{1}{2}$ partial-wave photoproduction amplitude. One would not normally expect to be able to determine the residue of this amplitude at the nucleon pole by purely S-matrix methods, simply because analyticity alone is compatible with any value whatever for this residue. However, if we *require* a suitably rapid convergence at high energies

* National Science Foundation Graduate Fellow.

† Minnesota Mining and Manufacturing Corporation, Post-Doctoral Fellow.

[1] R. F. Dashen and S. C. Frautschi, Phys. Rev. **135**, B1190 (1964).

[2] R. F. Dashen and S. C. Frautschi, Phys. Rev. **137**, B1318 (1965).

[3] R. F. Dashen, Phys. Rev. **135**, B1196 (1964).

[4] This point is discussed in Ref. 2.

for the scattering amplitude,[5] we will find (in the next section) that the residue itself *is* determined, and therefore, also the magnetic moment. We obtain by these methods values for the proton and neutron magnetic moments that are larger than the experimental values by about a factor of two. The calculation does, however, give an approximately correct value for the ratio of the isovector moment to the isoscalar moment, and a value for the isoscalar moment that agrees with the experimental value to within about a factor of two.

In Sec. II, we shall derive the basic formula for the magnetic moments in terms of a dispersion relation involving the photoproduction amplitude. In Sec. III, we briefly summarize the kinematics of the photoproduction process. Section IV contains a discussion of the analytic structure of the magnetic dipole amplitude $M_{1-}(W)$, in which the nucleon appears as a pole. Section V deals with the contributions to the invariant amplitudes arising from the exchange of the nucleon (N), and the $J=\frac{3}{2}$, $T=\frac{3}{2}\pi N$ resonance (N^*) in the u channel, and of the π, ρ, and ω mesons in the t channel. We assume that it is an adequate approximation to include only the singularities associated with the long-range part of the forces arising from the exchange of these particles. In Sec. VI we take up the important questions of the evaluation of the dispersion integrals, the uncertainties inherent in this phase of this calculation, and the computation of the magnetic moments. Section VII is a summary of the paper, while the Appendix is devoted to a discussion of the determination of the ρ and ω coupling constants which enter into the calculation.

II. DESCRIPTION OF THE METHOD

Although the basic formula for the magnetic moments which we shall use can be obtained directly from the Dashen-Frautschi formalism,[2] we shall derive this formula here by a slightly different method.

We start with the pion photoproduction amplitude in the $J=\frac{1}{2}^+$, $T=\frac{1}{2}$ channel, $A(s)$, where s is the square of the total energy in the barycentric system. Assume that $A(s)$ has been defined so that it is free of kinematic singularities. Then $A(s)$ has a pole at $s=M^2$, whose residue R we wish to compute, as well as a right-hand unitarity cut extending from $s=(M+\mu)^2$ to $s=\infty$ (M is the nucleon mass, μ the pion mass). According to the Fermi-Watson theorem,[6] between $s=(M+\mu)^2$ and the inelastic threshold at $s=(M+2\mu)^2$, $A(s)$ has the phase $\delta_{1-}(s)$ of pion-nucleon elastic scattering in the $J=\frac{1}{2}^+$, $T=\frac{1}{2}$ state. We will assume that $\delta_{1-}(s)$ is, in fact, the phase of $A(s)$ all along the right-hand cut. This is the assumption of elastic unitarity which is commonly made in dispersion relation treatments of pion photoproduction. In addition to these singularities $A(s)$ has certain left-hand cuts associated with particle exchanges in the crossed channels.

Now, suppose we can find a function $D(s)$ which has the phase $-\delta_{1-}(s)$ on the right-hand cut, a zero at $s=M^2$, and no other singularities. Then the function $J(s)=D(s)A(s)$ has no right-hand cut, no pole, and the same left-hand cuts as $A(s)$. Furthermore, we see that $J(M^2)=D'(M^2)R$. This is just the approach suggested by Dashen and Frautschi[2] who further inform us that the appropriate $D(s)$ is just the denominator function of the pion-nucleon scattering amplitude $f_{1-}(s)$.

Using the analyticity properties of $J(s)$ we may apply Cauchy's theorem to find at $s=M^2$:

$$J(M^2)=RD'(M^2)=\frac{1}{\pi}\int_L \frac{D(s')\,\mathrm{Im}A(s')ds'}{s'-M^2} +\frac{1}{2\pi i}\int_C \frac{D(s')A(s')ds'}{s'-M^2}, \quad (2.1)$$

where L is a contour enclosing the left-hand cuts, and C is the contour at infinity. When we assume that the integral over C vanishes, we have an equation for R:

$$R=\frac{1}{\pi D'(M^2)}\int_L \frac{D(s')\,\mathrm{Im}A(s')ds'}{s'-M^2}. \quad (2.2)$$

The functions $D(s)$ and $A(s)$ are both assumed known on the left. In practice, of course, approximations must be employed for both of these functions. Once some definite choice is made, Eq. (2.2) provides a method for calculating R, the residue at the nucleon pole in the photoproduction amplitude. This residue will involve the nucleon magnetic moments, so that (2.2) may be looked on as an equation for the magnetic moments.

The derivation of this crucial equation required that the integral on the circular contour at infinity, Eq. (2.1), vanish. This will only be true if $D(s)A(s)\to 0$ sufficiently rapidly as $s\to\infty$.

We also know that the partial-wave dispersion relations for the photoproduction amplitude $A(s)$, can be solved for *any* value of the residue R at the nucleon pole. If we call this solution $A(s;R)$, and remember that all solutions $A(s;R)$ have the same left-hand cuts and discontinuities across these cuts, it may seem paradoxical that we have nevertheless arrived at an equation which purports to calculate R in terms of an integral over specified left-hand cuts. Or could it really be that specifying the left-hand cuts of a scattering amplitude is sufficient to determine the residue at a pole in this amplitude? In general, the answer to this question is no. Let us then see what else is here.

The solution of the paradox, which is the essence of the following work, is that for arbitrary R, $D(s)A(s;R)$ does not go to zero at infinity rapidly enough for the

[5] R. F. Dashen and S. C. Frautschi, Phys. Rev. **137**, B1331 (1965).

[6] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. **4**, 219 (1954).

TABLE I. Values of the isospin functions for physical processes.

	$\gamma+p\rightarrow\pi^0p$	$\gamma+p\rightarrow\pi^++n$	$\gamma+n\rightarrow\pi^0n$	$\gamma+n\rightarrow\pi^-p$
$\mathcal{G}_\beta^{(+)}$	1	0	1	0
$\mathcal{G}_\beta^{(-)}$	0	$\sqrt{2}$	0	$-\sqrt{2}$
$\mathcal{G}_\beta^{(0)}$	1	$\sqrt{2}$	-1	$\sqrt{2}$

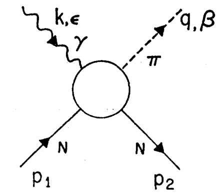

FIG. 1. Photoproduction of pions on nucleons.

integral around the infinite circle in Eq. (2.1) to vanish. In fact, only for the R computed from Eq. (2.2) will this be true.

If one requires $D(s)$ to go to a constant at $s=\infty$, then in order for the integral around the circular contour C at infinity to vanish, one must *assume* that $A(s;R)\rightarrow 1/s^\epsilon$, $\epsilon>0$, as $s\rightarrow\infty$. This assumption must be judged by its consequences, one of which will be a definite value for the magnetic moments, as it cannot be justified on the basis of analyticity in the energy plane alone. The succeeding sections of this paper are devoted to calculating that value.

III. SUMMARY OF THE KINEMATICS OF PHOTOPRODUCTION

In the preceding section, we pointed out that the evaluation of the nucleon magnetic moments will involve a knowledge of the partial-wave photoproduction amplitudes. In this Section, we shall write down these amplitudes, along with the necessary kinematics. The results given here are to be found in the papers of Chew, Goldberger, Low, and Nambu[7] (hereafter called CGLN) and of Ball,[8] whose notation we adopt.

We consider the collision of a photon of momentum k and polarization ϵ with a nucleon of momentum p_1, resulting in a nucleon of momentum p_2 and a pion of momentum q and "type" β. We introduce the Mandelstam variables (see Fig. 1):

$$s=-(p_1+k)^2,\quad t=-(q-k)^2,\quad u=-(p_1-q)^2, \tag{3.1}$$

with $s+t+u=2M^2+\mu^2$. M is the nucleon mass; μ, the pion mass.

In the center-of-mass system the above quantities may be written

$$s=(E_1+k)^2=(\omega+E_2)^2=W^2, \tag{3.2a}$$

$$t=\mu^2-2\omega k+2qkx, \tag{3.2b}$$

$$u=M^2-2E_2k-2qkx, \tag{3.2c}$$

where E and k are the energies of the incoming nucleon and photon, while E_2 and ω are the energies of the outgoing nucleon and pion. Also, q is the magnitude of the meson 3-momentum, and $x=\mathbf{q}\cdot\mathbf{k}/qk$ is the scattering angle.

[7] G. F. Chew, M. L. Goldberger, F. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[8] J. S. Ball, Phys. Rev. **124**, 2014 (1961).

The following relations will also be used:

$$k=(W^2-M^2)/2W,\quad q=\{[W^2-(M+\mu)^2][W^2-(M-\mu)^2]\}^{1/2}/2W, \tag{3.3}$$

$$\omega=(W^2-M^2+\mu^2)/2W,\quad E_1=(W^2+M^2)/2W,\quad E_2=(W^2+M^2-\mu^2)/2W. \tag{3.4}$$

The photopion production T matrix[9] may be written as[7,8]

$$T=T^{(+)}+T^{(-)}+T^{(0)}=\sum_{i=1}^{4}[A_i^{(+)}\mathcal{G}_\beta^{(+)}+A_i^{(-)}\mathcal{G}_\beta^{(-)}+A_i^{(0)}\mathcal{G}_\beta^{(0)}]M_i, \tag{3.5}$$

where $A_i^{(\pm,0)}$ are a set of twelve invariant functions, free of kinematic singularities,[8] which we assume satisfy the Mandelstam representation. The M_i are a set of four gauge-invariant spin matrices[7]:

$$M_1=i\gamma_5\gamma\cdot\mathcal{E}\gamma\cdot k, \tag{3.6a}$$

$$M_2=2i\gamma_5(P\cdot\mathcal{E}q\cdot k-P\cdot kq\cdot\mathcal{E}), \tag{3.6b}$$

$$M_3=\gamma_5(\gamma\cdot\mathcal{E}q\cdot k-\gamma\cdot kq\cdot\mathcal{E}), \tag{3.6c}$$

$$M_4=2\gamma_5(\gamma\cdot\mathcal{E}P\cdot k-\gamma\cdot kP\cdot\mathcal{E}-iM\gamma\cdot\mathcal{E}\gamma\cdot k), \tag{3.6d}$$

where $P=\frac{1}{2}(p_1+p_2)$, and the $\mathcal{G}_\beta^{(\pm,0)}$ are isospin matrices;

$$\mathcal{G}_\beta^{(+)}=\tfrac{1}{2}\{\tau_\beta,\tau_3\}=\delta_{\beta 3}, \tag{3.7a}$$

$$\mathcal{G}_\beta^{(-)}=\tfrac{1}{2}[\tau_\beta,\tau_3], \tag{3.7b}$$

$$\mathcal{G}_\beta^{(0)}=\tau_\beta, \tag{3.7c}$$

and the τ_β act in nucleon charge space. β labels the isotopic spin of the pion.[10] In Table I the matrix elements of the $\mathcal{G}_\beta^{(\pm,0)}$, between states of definite charge, are summarized.

The matrix elements of $T^{(\pm,0)}$ are simply related to those between states of definite total isotopic spin and its z projection[10,11]:

$$\langle\tfrac{3}{2}\ \tfrac{1}{2}|T|\tfrac{1}{2}\ \tfrac{1}{2}\rangle=\langle\tfrac{3}{2}-\tfrac{1}{2}|T|\tfrac{1}{2}-\tfrac{1}{2}\rangle=\sqrt{\tfrac{2}{3}}(T^+-T^-), \tag{3.8a}$$

[9] The photoproduction transition matrix T is related to the S matrix as follows:

$S=-i(2\pi)^4\delta^{(4)}(p_1+k-q-p_2)[M^2/4\omega kE_1E_2]^{1/2}\bar{u}(P_2)Tu(p_1)$.

[10] A clear derivation of these results is to be found in M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), Sec. 9.2.

[11] The charged pion fields are given by $\pi^\pm=\mp(\pi_1\pm i\pi_2/\sqrt{2})$, and the Condon-Shortley phase conventions are used for the Clebsch-Gordan coefficients.

TABLE II. Allowed multipole transitions in photomeson production.

j	J	l	Parity	Multipole
j	$j-\frac{1}{2}$	$j-1$	$(-1)^j$	$E_{(j-1)+}$
j	$j+\frac{1}{2}$	$j+1$	$(-1)^j$	$E_{(j+1)-}$
j	$j+\frac{1}{2}$	j	$-(-1)^j$	M_{j+}
j	$j-\frac{1}{2}$	j	$-(-1)^j$	M_{j-}

$$\langle \tfrac{1}{2}\ \tfrac{1}{2}|T|\tfrac{1}{2}\ \tfrac{1}{2}\rangle = -\sqrt{\tfrac{1}{3}}(T^+ + 2T^- + 3T^0)\,, \tag{3.8b}$$

$$\langle \tfrac{1}{2} -\tfrac{1}{2}|T|\tfrac{1}{2} -\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}(T^+ + 2T^- - 3T^0)\,. \tag{3.8c}$$

We shall require the partial-wave amplitudes for the photoproduction process. For the purpose of obtaining this partial-wave expansion it is convenient to introduce the amplitude $F(s,t)$ in the Pauli spin space of the nucleons[7,12]:

$$F(s,t) = i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}F_1(s,t) + \frac{(\boldsymbol{\sigma}\cdot\mathbf{q})(\boldsymbol{\sigma}\cdot\mathbf{k}\times\boldsymbol{\varepsilon})}{qk}F_2(s,t) + i\frac{(\boldsymbol{\sigma}\cdot\mathbf{k})(\mathbf{q}\cdot\boldsymbol{\varepsilon})}{qk}F_3(s,t) + i\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})(\mathbf{q}\cdot\boldsymbol{\varepsilon})}{q^2}F_4(s,t)\,. \tag{3.9}$$

(We will suppress the isotopic spin labels in the following considerations.)

The F_i may be expanded in terms of multipole amplitudes and Legendre polynomials.[7,8] These expansions may be inverted to obtain[13]:

$$M_{l+}(s) = \frac{1}{2(l+1)}\int_{-1}^{+1} dx\left\{F_1(s,t)P_l(x) - F_2(s,t)P_{l+1}(x) - F_3(s,t)\frac{[P_{l-1}(x)-P_{l+1}(x)]}{2l+1}\right\}, \quad \text{for } l\geq 1, \tag{3.10a}$$

$$M_{l-}(s) = \frac{1}{2l}\int_{-1}^{+1} dx\left\{-F_1(s,t)P_l(x) + F_2(s,t)P_{l-1}(x) + F_3(s,t)\frac{[P_{l-1}(x)-P_{l+1}(x)]}{2l+1}\right\}, \quad \text{for } l\geq 1, \tag{3.10b}$$

$$E_{l+}(s) = \frac{1}{2(l+1)}\int_{-1}^{+1} dx\left\{F_1(s,t)P_l(x) - F_2(s,t)P_{l+1}(x) + lF_3(s,t)\frac{[P_{l-1}(x)-P_{l+1}(x)]}{2l+1} + (l+1)F_4(s,t)\frac{[P_l(x)-P_{l+2}(x)]}{2l+3}\right\}, \quad \text{for } l\geq 0, \tag{3.10c}$$

and

$$E_{l-}(s) = \frac{1}{2l}\int_{-1}^{+1} dx\left\{F_1(s,t)P_l(x) - F_2(s,t)P_{l-1}(x) - (l+1)F_3(s,t)\frac{[P_{l-1}(x)-P_{l+1}(x)]}{2l+1} - lF_4(s,t)\frac{[P_{l-2}(x)-P_l(x)]}{2l-1}\right\}, \tag{3.10d}$$

for $l\geq 2$. The allowed transitions in photomeson production corresponding to each multipole amplitude[14] are summarized in Table II.

The kinematics of the problem are completed when the F_i are related to our previously introduced A_i. These connections are given by[7,8]:

$$F_1(s,t) = \alpha(s)\left[A_1(s,t) + (W-M)A_4(s,t) - \frac{t-\mu^2}{2(W-M)}(A_3(s,t) - A_4(s,t))\right], \tag{3.11a}$$

[12] The differential cross section is related to F by

$$\frac{d\sigma}{d\Omega}(\gamma+N\rightarrow\pi+N) = \frac{q}{k}\sum_{\text{spin}}|\langle 2|F|1\rangle|^2,$$

where $|1\rangle$ and $|2\rangle$ are Pauli spinors. In writing Eq. (3.9), we have chosen a gauge in which $\mathbf{k}\cdot\boldsymbol{\varepsilon}=0$.

[13] M. L. Goldberger (private communication).

[14] The multipole amplitudes introduced here are related to the partial-wave T-matrix elements for photomeson production by a photon of angular momentum j, multipole type ϕ [ϕ=electric E, or magnetic M] into a pion-nucleon state of angular momentum l, total angular momentum J, $T_{l,i,\phi}(s)$, by

$$M_{l+}(s) = -4\pi T_{l,l,M}{}^{l+\frac{1}{2}}(s)/[l(l+1)]^{1/2},$$
$$M_{l-}(s) = -4\pi T_{l,l,M}{}^{l-\frac{1}{2}}(s)/[l(l+1)]^{1/2},$$
$$E_{l+}(s) = +4\pi T_{l,l+1,E}{}^{l+\frac{1}{2}}(s)/[(l+1)(l+2)]^{1/2},$$
$$E_{l-}(s) = -4\pi T_{l,l-1,E}{}^{l-\frac{1}{2}}(s)/[l(l-1)]^{1/2}.$$

Isospin indices are suppressed as usual.

$$F_2(s,t)=\beta(s)\left[-A_1(s,t)+(W+M)A_4(s,t)-\frac{t-\mu^2}{2(W+M)}(A_3(s,t)-A_4(s,t))\right], \tag{3.11b}$$

$$F_3(s,t)=q\alpha(s)[(W-M)A_2(s,t)+(A_3(s,t)-A_4(s,t))], \tag{3.11c}$$

$$F_4(s,t)=q\beta(s)[-(W+M)A_2(s,t)+(A_3(s,t)-A_4(s,t))], \tag{3.11d}$$

where

$$\alpha(s)=(W-M)[(E_2+M)(E_1+M)]^{1/2}/8\pi W \quad \beta(s)=q\alpha(s)/(E_2+M).$$

The explicit appearance of $W=\sqrt{s}$ in these formulas will require us to work in the W plane in order to avoid the square-root branch point at $s=0$. In the W plane some interesting and very useful reflection properties hold among the partial-wave amplitudes,[8,13] namely;

$$M_{l+}(-W)=\frac{1}{l+1}[(l+2)M_{(l+1)-}(W)+E_{(l+1)-}(W)], \tag{3.12a}$$

$$M_{l-}(-W)=\frac{1}{l}[(l-1)M_{(l-1)+}(W)+E_{(l-1)+}(W)], \tag{3.12b}$$

$$E_{(l-1)+}(-W)=\frac{1}{l}[M_{l-}(W)-(l-1)E_{l-}(W)], \tag{3.12c}$$

and

$$E_{(l+1)-}(W)=\frac{1}{l+1}[M_{l+}(W)-(l+2)E_{l+}(W)]. \tag{3.12d}$$

We see from Table II, and as was pointed out in Sec. II, that the magnetic dipole amplitude for photomeson production in the $J=\frac{1}{2}^+$ channel will contain the nucleon pole and be of central importance in the remainder of the calculation. For this reason we will close this section by writing down its expression in terms of the $A_i(s,t)$:

$$\begin{aligned}2M_{1-}(W)=&-\alpha(W)\Big\{A_1^1(W)+(W-M)A_4^1(W)\\&+\frac{\omega k}{W-M}(A_3^1(W)-A_4^1(W))-\frac{qk}{(W+M)}\frac{[2(A_3^2(W)-A_4^2(W))+A_3^0(W)-A_4^0(W)]}{3}\Big\}\\&+\beta(W)\Big\{-A_1^0(W)+(W+M)A_4^0(W)+\frac{\omega k}{W+M}[A_3^0(W)-A_4^0(W)]-\frac{qk}{W+M}[A_3^1(W)-A_4^1(W)]\Big\}\\&+\frac{q\alpha(W)}{3}\Big\{(W-M)[A_2^0(W)-A_2^2(W)]+A_3^0(W)-A_4^0(W)-A_3^2(W)+A_4^2(W)\Big\},\end{aligned} \tag{3.13}$$

where we have introduced the partial-wave amplitudes

$$A_i^l(W)=\int_{-1}^{+1}A_i(s,t)P_l(x)dx. \tag{3.14}$$

Recall that isospin indices $(\pm,0)$ should be added to all these amplitudes.

In the following section, we will discuss the analytic structure of the M_{1-} amplitude.

IV. ANALYTIC STRUCTURE OF THE M_{1-} AMPLITUDE

One may see from an inspection of Eq. (3.13) that $M_{1-}(W)$ has a kinematic pole at $W=0$ and kinematic branch cuts associated with $q(W)$. An amplitude $M_{1-}(W)$ which is free of these kinematic singularities may be introduced as follows:

$$\tilde{M}_{1-}^{(\pm,0)}(W)=\frac{4\pi(W+M)[(W+M)^2-\mu^2]^{1/2}}{k}M_{1-}^{(\pm,0)}(W). \tag{4.1}$$

That this amplitude is free of kinematic singularities may be checked by explicit calculation.[15]

[15] The verification that there are no branch cuts associated with factors of $q(W)$ is aided by the observation that an odd power of q always multiplies a factor $Q_0(-(E_2/q))$ or $Q_2(-(E_2/q))$, for example, while an even power of q turns out to multiply a $Q_1(-(E_2/q))$. Consequently, we will find near $q=0$ only *even* powers of q arising in the products of q and Q_l, and no branch cuts coming from factors of q.

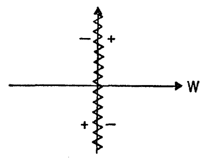

FIG. 2. Cuts due to $Q_l(-E_2/q)$. Signs are those of $\mathrm{Im}Q_l$ as one approaches the cut.

Now let us examine the singularities which remain in $\tilde{M}_{1-}(W)$. There are, first of all, poles at $W=\pm M$. These arise from nucleon exchange in the direct photoproduction channel (s channel), as well as from one-pion exchange in the t channel. The latter is generally regarded as a "kinematic" pole in the sense that it is introduced as a result of imposing gauge invariance. One can not get rid of this singularity with the same device that we used to get rid of the other kinematic singularities, but we will see how to treat this point in Sec. VI.

By writing out the Born amplitudes for nucleon and pion exchange, we can read off the residues of the pole at $W=+M$ in $\tilde{M}_{1-}^{(\pm,0)}(W)$.

These turn out to be

$$R^{(0)}=-\tfrac{4}{3}Mg_r(\mu_p'+\mu_n)\,, \qquad (4.2a)$$

$$R^{(+)}=-\tfrac{4}{3}Mg_r(\mu_p'-\mu_n)\,, \qquad (4.2b)$$

$$R^{(-)}=-\tfrac{2}{3}Mg_r(\mu_p'-\mu_n)-e_rg_r+e_rg_r\left(\frac{\pi}{2}\frac{M}{\mu}-1\right). \qquad (4.2c)$$

In the above expressions, μ_p and μ_n are the total magnetic moments of the proton and neutron, while μ_p' is the anomalous magnetic moment of the proton.[16] Also e_r and g_r are the rationalized charge and pion-nucleon coupling constants; $e_r^2/4\pi=1/137$, $g_r^2/4\pi\approx 14$. The term $e_rg_r[\frac{1}{2}\pi M/\mu-1]$ is the one-pion-exchange contribution.

The residues at the pole at $W=-M$ are of order $(\mu/M)^2$ times the above, and the pole at $W=-M$ can, therefore, be safely ignored.

Now let us investigate the cuts arising from single-particle exchange.

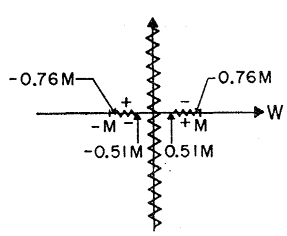

FIG. 3. N^* exchange cuts with appropriate signs for $\mathrm{Im}Q_l$ as one approaches the cuts.

[16] The appearance of the anomalous moment here is a result of our particular choice of the form factors F_1 and F_2 in terms of which to write the γNN vertex. Had we instead used the form factors $G_E=F_1+(t/4M^2)F_2$ and $G_M=F_1+F_2$, then only the total proton and neutron moments would have appeared. It is clear that the entire calculation can be regarded equally well as a computation of the total proton and neutron magnetic moments, or of the anomalous moments.

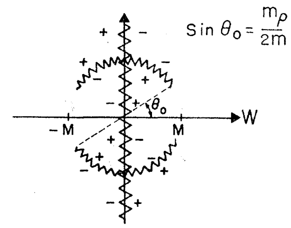

FIG. 4. ρ-exchange cuts with signs of $\mathrm{Im}Q_l$ as one approaches the cuts.

(i) Nucleon exchange in the crossed-photoproduction channel (u channel): The cuts arise from terms of the form:

$$\int_{-1}^{+1} dz\frac{P_l(z)}{u-M^2}=-\frac{1}{qk}Q_l[-(E_2/q)]. \qquad (4.3)$$

The dynamical cuts come from $Q_l[-(E_2/q)]$, which has a branch cut when $-1\leq-(E_2/q)\leq 1$. This branch cut runs along the imaginary axis from $W=-i\infty$ to $W=+i\infty$. Approaching the cut from one side or the other, one finds that $\mathrm{Im}Q_l\rightarrow\pm\frac{1}{2}\pi P_l[-(E_2/q)]$. The correct signs are shown in Fig. 2.

(ii). N^* exchange, u channel: The cuts generated by N^* exchange are those of $Q_l((-2kE_2+M^2-M^{*2})/2qk)$. Here $M^*=$ mass of the N^* resonance. There are two short cuts in the W plane extending from $W=M[2-(M^*/M)^2]^{1/2}\approx 0.51M$ to $W=M^2/M^*\approx 0.76M$ and from $W\approx-0.51M$ to $W\approx-0.76M$. There is also a cut all along the imaginary axis. The signs of the imaginary part of Q_l are shown in Fig. 3.

(iii) ρ and ω exchange, t channel[17] The cuts generated by ρ exchange are those of $Q_l((m_\rho^2-\mu^2+2\omega k)/2qk)$. They consist of the entire imaginary axis plus two arcs of a circle of radius M. These cuts are shown in Fig. 4. ω exchange produces identical cuts, except that $\sin\theta_{0\omega}=m_\omega/2M$ (see Fig. 4).

(iv). π exchange, t channel. The cuts generated by π exchange come from $Q_l(\omega/q)$. This gives a cut along the imaginary axis plus a complete circle of radius $(M^2-\mu^2)^{1/2}$. These are shown in Fig. 5.

This completes our survey of nearby cuts. We shall ignore in our calculation all cuts which lie outside a circle of radius M around the pole at $W=+M$. The justification for this assumption will be discussed in Sec. VI.

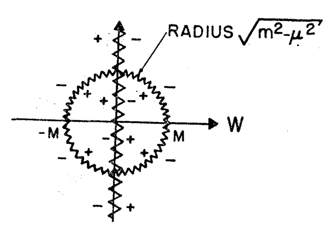

FIG. 5. π-exchange cuts with signs of $\mathrm{Im}Q_l$ as one approaches the cuts.

[17] In these calculations, we ignore the effects of the instability of the ρ and ω resonant states. This is no doubt quite a reasonable approximation for the ω and perhaps a tolerable one for the ρ.

TABLE III. Coupling constants of ρ and ω which enter the calculation of μ_p' and μ_n.

$f_{\rho\pi\gamma}\approx 0.16(e_r/\mu)$	$f_{\rho\pi\gamma}(F_{1\rho NN}/(4\pi)^{1/2})\approx 3(e_r/2M)$
$F_{1\rho NN}/(4\pi)^{1/2}\approx\sqrt{2}$	$f_{\rho\pi\gamma}(F_{2\rho NN}/(4\pi)^{1/2})\approx 5.6(e_r/2M^2)$
$F_{2\rho NN}/(4\pi)^{1/2}\approx\sqrt{2}(\mu_p'-\mu_n)/e_r$	
$f_{\omega\pi\gamma}\approx -0.45(e_r/\mu)$	$f_{\omega\pi\gamma}(F_{1\omega NN}/(4\pi)^{1/2})\approx 1.34(e_r/\mu)$
$F_{1\omega NN}/(4\pi)^{1/2}\approx -3.0$	$f_{\omega\pi\gamma}(F_{2\omega NN}/(4\pi)^{1/2})\approx -0.16(e_r/2M\mu)$
$F_{2\omega NN}/(4\pi)^{1/2}\approx -3.0(\mu_p'+\mu_n)/e_r$	

V. CONSTRUCTION OF THE INVARIANT AMPLITUDES

To perform the integrals entering into the evaluation of the magnetic moments, we require some approximation to the discontinuity of the partial-wave amplitude $\tilde{M}_{1-}^{(\pm,0)}(W)$ on the left-hand or unphysical cuts. We shall replace the true amplitudes there by their Born approximations, including only π, N, N^*, ρ, and ω as one-particle intermediate states. Then we take the discontinuity of these amplitudes as our value for $\text{disc}(\tilde{M}_{1-}^{(\pm,0)}(W))$ on the left-hand cuts. We begin by constructing the invariant amplitudes $A_i(s,t)$ in the Born approximation, then form $A_i^l(W)$, Eq. (3.14), and finally combine the appropriate $A_i^l(W)$'s to build $\tilde{M}_{1-}(W)_{\text{Born}}$.

First we consider the $A_i^{(0)}(s,t)$. This amplitude describes transitions induced by an isoscalar photon. The isotopic spins of the particles we allow are such that only N exchange in the s and u channels and ρ exchange in the t channel may contribute. It is a simple matter to compute $A_i^{(0)}(s,t)$ for these graphs:

$$A_1^{(0)}(s,t)=\tfrac{1}{2}e_rg_r\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)+\frac{tf_{\rho\pi\gamma}F_{2\rho NN}}{t-m_\rho^2},\quad(5.1\text{a})$$

$$A_2^{(0)}(s,t)=\frac{e_rg_r}{(s-M^2)(u-M^2)}-\frac{f_{\rho\pi\gamma}F_{2\rho NN}}{t-m_\rho^2},\quad(5.1\text{b})$$

$$A_3^{(0)}(s,t)=-\tfrac{1}{2}g_r(\mu_p'+\mu_n)\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right),\quad(5.1\text{c})$$

$$A_4^{(0)}(s,t)=-\tfrac{1}{2}g_r(\mu_p'+\mu_n)\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)\times\frac{-f_{\rho\pi\gamma}F_{1\rho NN}}{t-m_\rho^2}.\quad(5.1\text{d})$$

The couplings e_rg_r, μ_p', and μ_n have all been defined in Sec. IV. $f_{\rho\pi\gamma}$, $F_{1\rho NN}$, and $F_{2\rho NN}$ are defined by the following expression for the ρ-exchange contribution:

$$(\rho\text{ exchange})=f_{\rho\pi\gamma}\epsilon_{\mu\nu\sigma\tau}\epsilon_\mu k_\nu q_\sigma\left(\frac{-1}{t-m_\rho^2}\right)\bar{u}(p_2)\times\{\gamma_\tau F_{1\rho NN}+\tfrac{1}{2}iF_{2\rho NN}[p_2-p_1,\gamma_\tau]\}\tau_\beta u(p_1)\phi_\beta^*.\quad(5.2)$$

The kinematics are defined in Sec. III, while ϕ_β^* is the isospin part of the pion-wave function. We see that $F_{1\rho NN}$ and $F_{2\rho NN}$ may be looked upon as the "charge" and "magnetic" couplings of the ρ to the nucleon. As the ρ couplings are very important to the calculation, we have devoted Appendix A to a discussion of them. The values we find are listed in Table III.

The $A_i^{(\pm)}(s,t)$ are rather more complicated. They describe all the transitions induced by an isovector photon. This means that N, N^*, ω, and π exchange all contribute. G-parity conservation forbids the ρ to enter here. The contributions of π, ω, and N are not difficult to construct. The N^* contribution is more involved. We have chosen to approach the problem through the Mandelstam representation for the $A_i^{(\pm)}(s,t)$ as given in Eqs. (8.29) to (8.32) of Ball's paper.[8] These dispersion relations are dominated by $M_{1+}^{3/2}(W)$ multipole amplitude which contains contributions from N^* in the direct channel. The contribution of the N^* to $M_{1+}^{3/2}(W)$ in the Born approximation has been given by Gourdin and Salin.[18] They find

$$M_{1+}^{3/2}(W)/qk=R/(W^2-M^{*2}+iM^*\Gamma),\quad(5.3)$$

where R is a constant evaluated by comparison with experiment. We then make the approximation that the N^* is a sharp resonance (Γ/M^* small) and write for the imaginary part of $M_{1+}^{3/2}(W)$

$$\text{Im}M_{1+}^{3/2}(W)\approx-\pi R\delta(W^2-M^{*2}).\quad(5.4)$$

When this expression is substituted into the dispersion integrals given by Ball[8] and the N, π, and ω contributions are added, we find for the Born approximation to $A_i^{(+)}(s,t)$:

$$A_1^{(\pm)}(s,t)=\tfrac{1}{2}e_rg_r\left(\frac{1}{s-M^2}\pm\frac{1}{u-M^2}\right)\pm\binom{2}{1}h\left(\frac{M^{*2}-M^2-\mu^2}{2M^*}+t+\mu^2\right)\left(\frac{1}{s-M^{*2}}\pm\frac{1}{u-M^{*2}}\right)+\binom{1}{0}\frac{tf_{\omega\pi\gamma}F_{2\omega NN}}{t-m_\omega^2},\quad(5.5\text{a})$$

$$A_2^{(\pm)}(s,t)=\frac{-e_rg_r}{t-\mu^2}\left(\frac{1}{s-M^2}\pm\frac{1}{u-M^2}\right)\mp\binom{2}{1}3h\left(\frac{1}{s-M^{*2}}\pm\frac{1}{u-M^{*2}}\right)-\binom{1}{0}\frac{f_{\omega\pi\gamma}F_{2\omega NN}}{t-m_\omega^2},\quad(5.5\text{b})$$

[18] M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 (1963); 27, 309 (1963).

$$A_3^{(\pm)}(s,t)=-\tfrac{1}{2}g_r(\mu_p'-\mu_n)\left(\frac{1}{s-M^2}\mp\frac{1}{u-M^2}\right)\pm\binom{2}{1}h$$
$$\times\left(\frac{3}{2}\frac{t-\mu^2}{M^*+M}+\frac{M^{*2}-M^2+\mu^2}{2M^*}-(M^*+M)\right)\left(\frac{1}{s-M^{*2}}\mp\frac{1}{u-M^{*2}}\right),\quad(5.5c)$$

$$A_4^{(\pm)}(s,t)=-\tfrac{1}{2}g_r(\mu_p'-\mu_n)\left(\frac{1}{s-M^2}\pm\frac{1}{u-M^2}\right)\pm\binom{2}{1}h$$
$$\times\left(\frac{3}{2}\frac{t-\mu^2}{M^*+M}+\frac{M^{*2}-M^2+\mu^2}{2M^*}+2(M^*+M)\right)\left(\frac{1}{s-M^{*2}}\pm\frac{1}{u-M^{*2}}\right)-\binom{1}{0}\frac{f_{\omega\pi\gamma}F_{1\omega NN}}{t-m_\omega^2},\quad(5.5d)$$

where

$$h=\frac{e_r}{18}[(M^*+M)^2-\mu^2]^{-1/2}\left\{\frac{3M^{*2}+4MM^*+M^2}{2M^*\mu}c_3+\frac{M^{*2}-M^2}{\mu^2}c_4\right\}\frac{\lambda_1}{\mu}$$

and c_3, c_4, and λ_1 are defined and evaluated in the papers of Gourdin and Salin.[18] The particular quantity

$$c=+3\pi h\approx+11.8\pi(e_r/M^2)\qquad(5.6)$$

will appear often in the next section. $f_{\omega\pi\gamma}$, $F_{1\omega NN}$, and $F_{2\omega NN}$ are defined for the ω as in Eq. (5.2) for the ρ.

We may now compute $\tilde{M}_{1-}(W)$, using Eqs. (3.13), (3.14), and (4.1), and likewise its discontinuity across the unphysical cuts using the results of Sec. IV. This brings us to the task of evaluating the dispersion integral of Eq. (2.2), to which we turn in the next section.

VI. EVALUATION OF THE DISPERSION INTEGRALS AND CALCULATION OF THE NUCLEON MAGNETIC MOMENTS

This section will describe the evaluation of the dispersion integrals and the calculation of the magnetic moments. The dispersion integrals may be evaluated in two ways; first using the formula (2.2) as is, and then in a form in which a subtraction is made at $W=-(M+\mu)$. This choice of the subtraction point enables us to evaluate the subtraction constant analytically. Comparison of the results obtained with the subtracted and unsubtracted form of the dispersion integrals would allow us to get some idea of the sensitivity of the answers to the high-energy behavior of the integrals.

Equations (3.8b) and (3.8c) suggest that it will prove convenient to evaluate the residue in $\tilde{M}_{1-}^{(0)}(W)$ and in $\tilde{M}_{1-}^{(+)}(W)+2\tilde{M}_1^{(-)}(W)$, separately. These two independent equations will enable us separately to compute μ_p' and μ_N.

First we compute the residue in $\tilde{M}_{1-}^{(0)}(W)$. Our basic formula (2.2) applied to this amplitude yields

$$R^{(0)}=\frac{1}{D'(M)}\frac{1}{\pi}\int_L\frac{D(W')\,\mathrm{Im}\tilde{M}_{1-}^{(0)}(W')dW'}{W'-M},\qquad(6.1)$$

where $R^{(0)}$ is the residue of $\tilde{M}_{1-}^{(0)}$ at $W=M$:

$$-\tfrac{4}{3}Mg_r(\mu_p'+\mu_n).$$

As indicated earlier, the cuts of $\tilde{M}_{1-}^{(0)}$ consist of the arcs of the circle coming from ρ exchange as well as cuts along the imaginary axis. We will temporarily neglect the latter. The circular cuts (radius of circle is $|W|=M$) extend from $\theta_0=\cos^{-1}[1-(M_\rho^2/2M^2)]$ to $\pi-\theta_0$, and from $\pi+\theta_0$ to $2\pi-\theta_0$. The distance from the nearest endpoints of the circular cuts to the pole at $W=M$ is 2.83μ. We will integrate over the circular cut from θ_0 to an angle θ_{end} which corresponds to a distance approximately M away from the pole. This procedure is motivated by the fact the Born approximations for ρ-exchange result in a integrand in Eq. (6.1) which is very sharply peaked around the nearby part of the cut. We shall discuss the question of convergence shortly.

With these remarks in mind, we write

$$\pi D'(M)R^{(0)}=\int_{\theta_0}^{\theta_{\mathrm{end}}}\frac{D(Me^{i\theta})\,\mathrm{Im}\tilde{M}_{1-}^{(0)}(Me^{i\theta})ie^{i\theta}d\theta}{(e^{i\theta}-1)}+\int_{-\theta_{\mathrm{end}}}^{-\theta_0}\frac{D(Me^{i\theta})\,\mathrm{Im}\tilde{M}_{1-}^{(0)}(Me^{i\theta})ie^{i\theta}d\theta}{(e^{i\theta}-1)}$$
$$=2\int_{\theta_0}^{\theta_{\mathrm{end}}}\mathrm{Re}\left\{\frac{D(Me^{i\theta})\,\mathrm{Im}\tilde{M}_{1-}^{(0)}(Me^{i\theta})ie^{i\theta}}{(e^{i\theta}-1)}\right\}d\theta.\qquad(6.2)$$

It is convenient to split the integral into two parts; one associated with the charge coupling of the ρ to nucleons, the other with the magnetic moment coupling. We then have

$$\pi D'(M)R^{(0)}=\tfrac{1}{6}\pi M(f_{\rho\pi\gamma}F_{1\rho NN})I_1(\theta_{\mathrm{end}})+\tfrac{1}{6}\pi M^2(f_{\rho\pi\gamma}F_{2\rho NN})I_2(\theta_{\mathrm{end}}),\qquad(6.3)$$

where $I_1(\theta_{\mathrm{end}})$ and $I_2(\theta_{\mathrm{end}})$ are the values of the charge

TABLE IV. Values of the charge and moment integrals for the ρ meson using a linear D.

θ_{end}	$I_1(\theta_{end})$	$I_2(\theta_{end})$
0.41	0.00	0.00
0.70	−8.14	−7.37
1.00	−10.49	−8.89
1.30	−11.31	−10.56
1.57	−11.57	−11.99

TABLE V. Values of the charge and moment integrals for the ρ meson using a curved D.

θ_{end}	$I_1(\theta_{end})$	$I_2(\theta_{end})$
0.41	0.00	0.00
0.70	−6.65	−5.30
1.00	−8.24	−6.69
1.30	−8.60	−7.85
1.57	−8.63	−8.52

and magnetic-moment contributions as functions of θ_{end}.

To complete the specification of the problem, we must choose an approximate form for the D function of πN scattering. Two different choices were made.

The first was a simple linear D function:

$$D(W)=W-M.$$

Here we have chosen to normalize D to one at the pole. Note that the normalization constant of a linear D would drop out of our equations anyway. This approximation is extremely crude away from the pole, and in particular has an incorrect high-energy behavior.

An alternative form for $D(W)$, which has a high-energy behavior consistent with our initial assumptions, was obtained by Dashen[3] as a fit to the D function obtained by Balázs[19] in the course of obtaining an approximate solution to the πN-scattering problem. This is

$$D(W)=(W-M)(W_0-M)/(W_0-W), \qquad (6.4)$$

with $W_0-M=9\mu$.

In Tables IV and V the values of $I_1(\theta_{end})$ and $I_2(\theta_{end})$ are given as θ_{end} varies from $\pi/3$ (a distance M from the pole), to $\pi/2$ (a distance $\sqrt{2}M$ from the pole) for both the curved and linear D functions.

A short survey of Tables IV and V will reveal that the "charge" integral I_1 seems to be converging rather well as we move away from the pole, especially when the curved D function is used. The magnetic integral I_2, on the other hand, is probably not converging. That this is really the case is confirmed by an inspection of $\tilde{M}_{1-}^{(0)}(W)$, where the coefficients of $f_{\rho\pi\gamma}F_{2\rho NN}$ are seen to diverge linearly at infinity, if a linear D function is used. The use of a curved D function results in a constant behavior at infinity. This divergent behavior is due, of course, to our having used the Born amplitude to describe ρ exchange, neglecting the momentum-transfer dependence of the form factors at the ρNN and $\rho\pi\gamma$ vertices. Almost any reasonable behavior of these form factors at large t, say $1/t$ behavior, would result in a convergent integral. By keeping the ρ-exchange cuts only along the circular arcs, we have effectively placed a cutoff on our integrals, which approximates the effect of the form factors.

An alternative procedure would be to make a subtraction in the dispersion integrals. This possibility will be considered later. One should bear in mind that the magnetic integrals associated with ω and N^* exchange have similar diseases, and analogous procedures will be used there.

The reader will note that we have ignored the contributions from the cuts along the imaginary axis. These contributions have been systematically dropped from the calculation. This has been done for the following reasons: (i). We do not claim to know a reasonable approximation for $\tilde{M}_{1-}(W)$ any further than M away from the pole at $W=M$, and this is, of course, just where this contribution would begin. The Born approximation is probably a very poor one along the imaginary axis, and would result in an incorrect estimate for the value of the integral: (ii). The contributions from the integrals which were kept are seen to come primarily from the parts of the cut nearest $W=M$. (See Tables IV–VIII and Fig. 6): (iii). An explicit evaluation of the contributions of the cuts along the imaginary axis, using the Born approximation for the amplitudes, was made in the case of nucleon exchange in the u channel and π exchange in the t channel. The nucleon exchange cut, both with and without a subtraction at $W=-(M+\mu)$, contributed $\sim 10\%$ to the answer in a direction which would improve the values of μ_p' and μ_N which we shall subsequently determine. The pion-exchange contributions gave a term of order $(\mu/M)^2$ compared to the other contributions to the residue $R^{(-)}$. (iv). Any reasonable attempts to include these cuts would demand a knowledge of the form factors at the various vertices involved for large momentum transfers. (v). To do these integrals properly would require a knowledge of the D function in an unphysical region rather far from the pole. This is simply not known at present.

Now let us complete our evaluation of the isoscalar moment. Eqs. (4.2a) and (6.2) give

$$-\tfrac{4}{3}Mg_r(\mu_p'+\mu_n)=\tfrac{1}{6}M(f_{\rho\pi\gamma}F_{1\rho NN})I_1(\theta_{end}) + \tfrac{1}{6}M^2(f_{\rho\pi\gamma}F_{2\rho NN})I_2(\theta_{end}). \qquad (6.5)$$

Using the values of the coupling constants given in Table III, the curved D function of Eq. (6.4), and taking θ_{end} to be $\pi/3$ (corresponding to a distance M from the pole), we find

$$\mu_p+\mu_n=1.65(e_r/2M), \qquad (6.6)$$

compared to the experimental value of $0.88(e_r/2M)$. If our estimates for the coupling constants of the ρ meson

[19] L. Balázs, Phys. Rev. 128, 1935 (1962).

TABLE VI. Values of the charge and moment integrals for the ω meson using a linear D.

θ_{end}	$I_1'(\theta_{end})$	$I_2'(\theta_{end})$
0.43	0.00	0.00
0.70	−7.53	−6.97
1.00	−10.08	−8.68
1.30	−11.00	−10.41
1.57	−11.33	−11.88

TABLE VII. Values of the charge and moment integrals for the ω meson using a curved D.

θ_{end}	$I_1'(\theta_{end})$	$I_2'(\theta_{end})$
0.43	0.00	0.00
0.70	−5.99	−4.83
1.00	−7.66	−6.24
1.30	−8.04	−7.38
1.57	−8.10	−8.05

are too large by a factor of 2, we would find

$$\mu_p+\mu_n=0.83(e_r/2M)\,, \qquad (6.7)$$

which is certainly much better, but commands no greater respect in view of our ignorance of the ρ coupling constants. It is of interest, however, that the ρ-exchange contribution appears to be of the right sign and order of magnitude to account for the isoscalar moment.

Now let us compute the isovector moment. This will come from the residue of $\tilde{M}_{1-}^{(+)}(W)+2\tilde{M}_{1-}^{(-)}(W)$;

$$R^{(+)}+2R^{(-)}=\frac{1}{D'(M)}\frac{1}{\pi}\times\int_L\frac{D(W')[\mathrm{Im}\tilde{M}_{1-}^{(+)}(W')+2\,\mathrm{Im}\tilde{M}_{1-}^{(-)}(W')]dW'}{W'-M}, \qquad (6.8)$$

where $R^{(+)}$ and $R^{(-)}$ are given by Eqs. (4.2b) and (4.2c).

The contribution from the "pi-exchange" cut [so-called because it arises from a term $1/(t-\mu^2)$ in the Born amplitudes] can be calculated quite simply. The only term in the one-particle exchange amplitude having this cut is

$$\tilde{M}_{1-\pi}^{(-)}=-\frac{[(W+M)^2-\mu^2]}{3q}\frac{e_rg_r}{W^2-M^2}\times[Q_0(\omega/q)-Q_2(\omega/q)]. \qquad (6.9)$$

This amplitude goes to zero at infinity. Using the approximate D function of equation (6.4), the integral to be done is

$$I_\pi=\frac{1}{\pi}\int_L\frac{W_0-M}{W_0-W}\mathrm{Im}[2\tilde{M}_{1-\pi}^{(-)}(W)]dW. \qquad (6.10)$$

The "nearby" part of L is a circle of radius $(M^2-\mu^2)^{1/2}$. The quantity $\mathrm{Im}(\tilde{M}_{1-\pi})$ is of order $(\mu/M)^2$ on the whole of L except that part of the circle which passes near the pole at $W=+M$. In fact, to order $(\mu/M)^2$, the whole integral over L comes from the half of the circle nearest $W=M$.

Since $\tilde{M}_{1-\pi}$ goes to zero at infinity, Cauchy's theorem says that the integral of the left-hand cut of the integrand equals the integral around all other singularities of the integrand. The other singularities in this case are poles at $W=\pm M$ and $W=W_0$. The integral around $W=+M$ is just the residue of $2\tilde{M}_{1-\pi}$ at the nucleon pole, or $2(\frac{1}{2}\pi M/\mu-1)e_rg_r$. This cancels an identical term[20] in $2R^{(-)}$, Eqs. (4.2c). The contribution from the pole at $W=-M$ is of order $(\mu/M)^2$ and is neglected. The integral around the pole at W_0 gives

$$(2)(W_0-M)\frac{[(W_0+M)^2\mu^2]}{3q(W_0)}\frac{e_rg_r}{W_0^2-M^2}\times(Q_0(\omega/q)-Q_2(\omega/q))=-3.40e_rg_r. \qquad (6.11)$$

(Increasing W_0 from $M+9\mu$ to ∞ changes the contribution by about 50%). Therefore, the contribution of the "pi-exchange" cut to Eq. (6.8) is $-3.40e_rg_r$ on the right-hand side, providing we drop the term $[(\pi/2)(M/\mu)-1]e_rg_r$ in $2R^{(-)}$.

The discussion of the ω-exchange contribution proceeds along lines identical with those given for the ρ above. The only changes are to note that the ω-meson

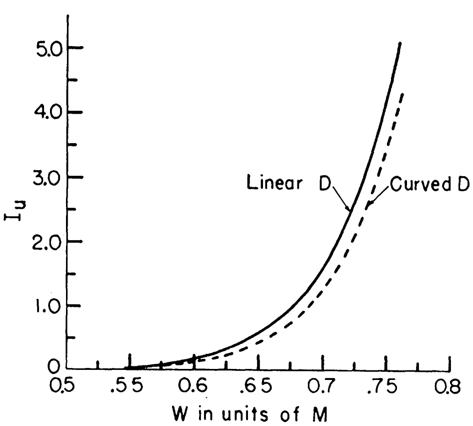

FIG. 6. Value of the u-channel integral. The abscissas give the position of the right end of the N^* cut as we vary it from 0.51 to 0.76 M.

[20] The cancellation of the pion contribution to the residue of the pole at $W=+M$ in the $\tilde{M}_{1-}$ amplitude by the pion-exchange cut is an exact result, and does not involve an approximate cancellation of two large numbers or uncertainties about the high-energy behavior of the amplitudes. This point has been studied by one of the authors (DHS) and R. F. Dashen in the case of pion production of *massive* photons, using the partial-wave expansions of Liu and Singer [Phys. Rev. **135**, B1017 (1964)]. Here one can see explicitly that there is no pion pole contribution to the residue of the $\tilde{M}_{1-}$ amplitude at $W=M$ for any finite photon mass, and that none appears in the limit when the photon mass goes to zero. The part of the pion-cut contribution which is *not* cancelled by the pole has an analog in the massive photon case.

TABLE VIII. Values of $\mu_p+\mu_n$, $\mu_p'-\mu_n$, μ_p', and μ_n for various choices of the coupling constants listed in Table III. Column I is the answer accepted here. $d_1=f_{\rho\pi\gamma}F_{1\rho NN}$, $d_2=f_{\rho\pi\gamma}F_{2\rho NN}$, and $d_1'=f_{\omega\pi\gamma}F_{1\omega NN}$.

Value of d_1,d_2,d_1'	$d_1=3.0$ $4\pi(e_r/2M)=a$; $d_2=(5.6)$ $4\pi(e_r/2M^2)=b$; $d_1'=1.34$ $4\pi(e_r/\mu)=c$	$d_1=a/2$; $d_2=b/2$; $d_1'=c$	$d_1=3a/2$; $d_2=3b/2$; $d_1'=3c/2$	$d_1=a/2$; $d_2=b/2$; $d'=2c/3$	Experimental value
$\mu_p+\mu_n$	$+1.65(e_r/2M)$	$+0.83(e_r/2M)$	$+2.48(e_r/2M)$	$+0.83(e_r/2M)$	$+0.88(e_r/2M)$
$\mu_p'-\mu_n$	$+6.64(e_r/2M)$	$+6.64(e_r/2M)$	$+7.57(e_r/2M)$	$+6.03(e_r/2M)$	$+3.70(e_r/2M)$
μ_p'	$+3.65(e_r/2M)$	$+3.23(e_r/2M)$	$+4.03(e_r/2M)$	$+2.93(e_r/2M)$	$+1.79(e_r/2M)$
μ_n	$-3.00(e_r/2M)$	$-3.40(e_r/2M)$	$-3.04(e_r/2M)$	$-3.20(e_r/2M)$	$-1.91(e_r/2M)$

cut begins at $\theta_0=0.43$, and that it contributes only to $\tilde{M}_{1-}^{(+)}$. The values of the charge integrals $I_1'(\theta_{\text{end}})$ and magnetic integrals $I_2'(\theta_{\text{end}})$ for the curved and linear D function are shown in Tables VI and VII. The remarks about the convergence of $I_1(\theta_{\text{end}})$ and $I_2(\theta_{\text{end}})$ apply here.

N^* exchange in the u channel has been shown to lead to three cuts in the W plane: The cut along the imaginary axis, the cut from $W=-M^2/M^*$ to $-M[2-(M^{*2}/M^2)]^{1/2}$, and the cut from $W=+M[2-(M^{*2}/M^2)]^{1/2}\approx 0.51M$ to $W=(M^2/M^*)\approx 0.76M$. Of these we keep only the third cut as the other two are "faraway" singularities. In Fig. 6 we plot the contribution of this cut to the dispersion integral as a function of the endpoint nearer the pole for our two choices of $D(W)$.

With these remarks and numbers in mind we may write for Eq. (6.8):

$$\pi(R^{(+)}+2R^{(-)})$$
$$=2\int_{\theta_0}^{\theta_{\text{end}}}\text{Re}\left\{\frac{D(Me^{i\theta})\,\text{Im}\tilde{M}_{1-}^{(+)}(Me^{i\theta})ie^{i\theta}}{(e^{i\theta}-1)}\right\}d\theta$$
$$+\int_{0.51M}^{0.76M}D(x)\left\{\frac{\text{Im}\tilde{M}_{1-}^{(+)}(x)+2\,\text{Im}\tilde{M}_{1-}^{(-)}(x)}{x-M}\right\}dx$$
$$-3.40\pi e_r g_r\,, \qquad (6.12)$$

and therefore

$$\frac{-8Mg_r}{3}(\mu_p'-\mu_n)-2e_rg_r=\frac{-4cM^2}{3\pi}I_u$$
$$+\frac{M}{6}(f_{\omega\pi\gamma}F_{1\omega NN})I_1'(\theta_{\text{end}})$$
$$+\frac{M^2}{6}(f_{\omega\pi\gamma}F_{2\omega NN})I_2'(\theta_{\text{end}})-3.40e_rg_r\,, \qquad (6.13)$$

I_u is the N^* contribution[21] and is ≈ 4.19; the values of the ω-coupling constants are given in Table III (and Appendix A). We again choose $\theta_{\text{end}}=\pi/3$ and use the curved D. Then Eq. (6.13) gives

$$\mu_p'-\mu_n=6.64(e_r/2M)\,. \qquad (6.14)$$

Combining Eqs. (6.6) and (6.14) allows us to determine μ_p' and μ_n, separately. We find

$$\mu_p'=+3.65(e_r/2M)\,, \qquad (6.15)$$

$$\mu_n=-3.00(e_r/2M)\,. \qquad (6.16)$$

These values differ in magnitude from the experimental values by nearly a factor of two. It is interesting, however, that the isovector moment is much larger than the isoscalar moment. Using our calculated value, Eq. (6.6), for the total isoscalar moment μ_s and Eq. (6.14) to compute the total isovector moment μ_v, we find that the ratio of isovector moment to isoscalar moment is ~ 4.6, compared to an experimental value for this ratio of ~ 5.4. The values for $\mu_p\pm\mu_n$, μ_p', and μ_n are collected in Table VIII where they are compared with the experimental results and with the values one obtains for different choices of the ρ- and ω-coupling constants.

It is clear that our answer for the magnetic moments, and especially the isoscalar moment, depends critically on the values of the ρ-coupling constants. Ball[8] has considered the effect of the two-pion intermediate state on the photoproduction process. He summarizes this effect in a parameter Λ. If we approximate the two pion state by the ρ resonance, we can compute Λ. One finds

$$(\Lambda/e)=\frac{F_\pi(1)}{1+a}\frac{8\sqrt{2}}{3}\frac{f_{\rho\pi\gamma}}{\gamma_{\rho\gamma}}(a+m_\rho^2)\,, \qquad (6.17)$$

where $F_\pi(1)$ is the pion form factor at $t=\mu^2$ and $a=5\mu^2$ is a constant. Ball gives[8] $F_\pi(1)=1.08$ which, together with the values for $f_{\rho\pi\gamma}$ and $\gamma_{\rho\gamma}=-e_rm_\rho^2/f_\rho$ given in the Appendix, yields

$$\Lambda=-0.64e\,. \qquad (6.18)$$

Ball shows that the experimental data constrain Λ to lie between $\pm 1.8e$. Our value, Eq. (6.18), certainly satisfies this constraint. Increasing $f_{\rho\pi\gamma}$ and f_ρ would tend to move Λ out of the allowed region, while decreasing them by a factor of 1.5, for example, would keep us in the allowed region and improve our answer for the isoscalar moments.

[21] We see that an important contribution to the isovector moment comes from N^* exchange. That this would be the case was suggested by R. Dashen, Phys. Letters 11, 89 (1964), who studied the relationship between the total nucleon isovector moment and the magnetic dipole γNN^* coupling from the reciprocal bootstrap point of view.

We have pointed out that the Born amplitudes which we have used to approximate the true amplitudes do not converge rapidly enough to permit one to write the unsubtracted dispersion relations which were assumed in Sec. II. Of course, there is no reason to believe that the Born approximation gives a reasonable description of the high-energy behavior of the amplitudes. Our point of view has been to suppose that the bulk of the contribution comes from a region near the pole (within a circle of radius M), and that a correct description of inelastic processes would give the amplitude an appropriately convergent behavior.

One way to check this conjecture would be to make a subtraction at some point where we know the amplitude, and see if our answers are greatly affected.

For reasons to be made clear in a moment, we will subtract at $W=-(M+\mu)$. The dispersion relation for $D(W)\tilde{M}_{1-}(W)$ then reads

$$D(W)\tilde{M}_{1-}(W)=D\tilde{M}_{1-}(W=-M-\mu)$$
$$+\frac{W+(M+\mu)}{\pi}\int_L \frac{D(W')\,\mathrm{Im}\tilde{M}_{1-}(W')dW'}{(W'-W)(W'+M+\mu)}. \quad (6.19)$$

We see that we must know $D(W)\tilde{M}_{1-}(W)$ at $W=-(M+\mu)$. By the reflection symmetries of Eq. (3.12), we observe that $M_{1-}(-W)=E_{0+}(W)$. If we can determine E_{0+}, the $s_{1/2}$ electric dipole amplitude, at threshold we will have our subtraction constant. Now, according to the Kroll-Ruderman theorem,[22] the E_{0+} amplitude is given correctly (to order μ/M) at threshold by the Born approximation. This gives

$$E_{0+}^{(+)}=0, \quad (6.20a)$$

$$E_{0+}^{(-)}=\frac{e_r}{2M}\frac{g_r}{4\pi}\left(1+\frac{\mu}{M}\right)^{-1}, \quad (6.20b)$$

$$E_{0+}^{(0)}=-(\mu_p+\mu_n)\frac{\mu}{2M}\frac{g_r}{4\pi}. \quad (6.20c)$$

We are stopped at this point by two practical problems. (i). We have no idea of how to get a reliable approximation for the $\tilde{M}_{1-}$ amplitudes in the left-hand W plane. Moreover, subtracting at $W=-(M+\mu)$ will make the integrals over the cuts in this part of the plane as important as their counterparts in the right-hand plane. (ii). We do not know a sensible approximation for the D function in this faraway unphysical region. If these difficulties could be surmounted, this method would provide an attractive alternative to the unsubtracted approach as the questions of convergence for large W are not so severe.[23]

[22] N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954).

[23] If one ignores these warnings and proceeds to compute $\mu_p'\pm\mu_n$, using the curved D function of Eq. (6.4) to the left of $W=0$, a "good" answer is found for the isoscalar moment and a fair answer for the isovector moment.

VII. SUMMARY AND CONCLUSIONS

In this final section we will briefly review the calculations explained above and try to comment on them.

We began, after the tedious but straightforward kinematics of Sec. III, with the construction of the invariant amplitudes $A_i(s,t)$. These were evaluated under the assumptions: (i) that only a few low-mass intermediate states (N, $N+\pi$, π, 2π, 3π) need be kept; (ii) that few particle intermediate states could be adequately approximated as one-particle resonant states (N^*, ρ, ω), and (iii) that the Born approximation could be used to compute the discontinuity of each of these amplitudes.

Next, we performed a partial-wave projection and found that the resulting partial-wave amplitudes $M_{1-}(W)$ had kinematic singularities. These were eliminated in Sec. IV and we were instructed to consider $\tilde{M}_{1-}(W)$ instead.

Finally, we wished to evaluate the integrals of Eq. (2.2) which determine the residues $R^{(+)}+2R^{(-)}$ and $R^{(0)}$ and consequently the magnetic moments. To carry out these integrations necessitated a number of further approximations. The first of these pertains to the use and form of the D function. (i). It was necessary to assume that the denominator function for elastic pion-nucleon scattering in the $P_{1/2}$ state cancels out the right-hand cuts in $\tilde{M}_{1-}(W)$. This it does up to inelastic threshold. After that we were forced to assume that states like $N+2\pi$ and $N+3\pi$ would not severely affect the phase on the unitarity cuts in the region of interest. This amounts to the neglect of intermediate states $N+\rho$, $N+\omega$, $N+\pi$ (in the $T=\frac{1}{2}$ state) in the direct channel. (ii). In addition to this, we had to approximate D by simple expressions in the nearby unphysical region.

The second approximation made in evaluating the integrals was to keep contributions only from those cuts within a circle of radius M around the pole. The reasons for this have been discussed in detail in Sec. VI. We shall let it suffice here to repeat that this approximation was motivated not only by our ignorance of $\tilde{M}_{1-}(W)$ and $D(W)$ outside this region, but also by the fact that those contributions outside this region which we *were* able to evaluate turned out to be small.

With these approximations one finds, upon evaluation of the dispersion integrals, the values for the proton and neutron magnetic moments given in Sec. VI and Table VIII. Our results, while producing values for the magnetic moments that are uniformly too large by a factor of about two, do suggest that an approximately correct model for the magnetic moments has been achieved. It is difficult to isolate which of the many approximations made are responsible for the lack of a closer numerical agreement of the calculated moments with experiment. The fact that the calculated moments are all somewhat too large indicates that the use of the

Born approximation, which is notorious for overestimating things, may be the source of the problem.

The values for the moments, especially the isoscalar moment, depend rather sensitively on the ρ- and ω-coupling constants. The methods used to estimate these coupling constants are outlined in the Appendix. It is far from our intention to claim that our estimates for these coupling constants represent anything like a "best value." On the contrary, our estimates must be regarded as uncertain perhaps to within a factor of two. In Table VIII, we have indicated how the values of the isovector, isoscalar, proton and neutron moments depend on the choice of these coupling constants. We feel that the values given in Eqs. (6.6), (6.14), (6.15), and (6.16) (see the first column of Table VIII) represent a conservative choice for these constants.

ACKNOWLEDGMENTS

We would like to thank S. L. Adler, R. Blankenbecler, S. C. Frautschi, F. J. Gilman, and M. L. Goldberger for their advice and comments throughout the course of this calculation. One of the authors (DHS) would like to give special thanks to Dr. R. F. Dashen for numerous interesting and helpful suggestions and for a very careful reading of the manuscript. S. B. Treiman was especially helpful in clarifying Sec. II. This work made use of computer facilities supported in part by National Science Foundation Grant NSF-GP-579.

APPENDIX A: DETERMINATION OF THE ρ AND ω COUPLING CONSTANTS

In this Appendix we shall briefly discuss the determination of the various ρ and ω coupling constants used in this calculation. Unfortunately, these cannot be determined directly from experiment in a model-independent way at the present time. Here, we shall employ the arguments introduced by Gell-Mann and Zachariasen,[24] and extended by a number of workers,[25,26] who discussed the dispersion-theoretic basis for the approximation of the 2π and 3π resonances as vector mesons coupled to conserved currents.

We shall consider first the $T=1$, $J=1^{-+}$ vector meson ρ and the $T=0$, $J=1^{--}$ vector meson Y, which we suppose are members of a unitary symmetry octet. The ρ meson is furthermore assumed to couple universally, with strength $f\rho$, to the conserved isospin current, while the Y meson couples universally, with strength fY, to the conserved hypercharge current. That is, the ρ and Y mesons are assumed to couple universally to the same currents as the isovector and isoscalar parts of the photon, respectively. Then one may write[24]

$$\langle\alpha|j_\mu{}^s|\beta\rangle=-\frac{e_r}{2f_Y}\frac{m_Y{}^2}{t-m_Y{}^2}\langle\alpha|j_\mu{}^y|\beta\rangle, \tag{A1}$$

$$\langle\alpha|j_\mu{}^v|\beta\rangle=-\frac{e_r}{f_\rho}\frac{m_\rho{}^2}{t-m_\rho{}^2}\langle\alpha|j_\mu{}^\rho|\beta\rangle. \tag{A2}$$

At this point we may introduce a third vector meson, B, which is coupled universally to the conserved baryonic current, and is a singlet in unitary symmetry. When unitary symmetry is broken, the $|Y\rangle$ and $|B\rangle$ states may mix [26] to give the two states

$$|\varphi\rangle=a|Y\rangle+b|B\rangle, \tag{A3}$$

$$|\omega\rangle=a|B\rangle-b|Y\rangle. \tag{A4}$$

The $|\omega\rangle$ is identified with the $T=0$, $J=1^{--}$, 3π resonance at 780 MeV, and the $|\varphi\rangle$ with the $T=0$, $J=1^{--}$ $K\bar{K}$ resonance at 1020 MeV. The mixing parameters have been roughly estimated to have the values $a\approx0.78$, $b\approx0.62$.

We may now take matrix elements of Eq. (A1) between a ρ and a π state, and of Eq. (A2) between an ω and a π state, to find at $t=0$

$$f_{\rho\pi\gamma}=\frac{e_r}{2f_Y}f_{Y\rho\pi}, \tag{A5}$$

$$f_{\omega\pi\gamma}=\frac{e_r}{f_\rho}f_{\omega\rho\pi}. \tag{A6}$$

As a consequence of Eq. (A3) and (A4), one may write

$$f_{Y\rho\pi}=af_{\varphi\rho\pi}-bf_{\omega\rho\pi}. \tag{A7}$$

A study of the available data on the decays of the φ and ω mesons strongly suggest that[26] $|f_{\varphi\rho\pi}|\ll|f_{\omega\rho\pi}|$, so we have

$$f_{Y\rho\pi}\approx-bf_{\omega\rho\pi} \tag{A8}$$

and

$$f_{\rho\pi\gamma}\approx-b(e_r/2f_Y)f_{\omega\rho\pi}. \tag{A9}$$

In unitary symmetry, $f_Y=(\sqrt{\tfrac{3}{4}})f_\rho$, and from universality we find $f_\rho\approx f_{\rho\pi\pi}$. Estimating f_ρ from the ρ width (assuming 100 MeV for this width) gives $f_\rho^2/4\pi\approx2.0$. We see, therefore, that we could evaluate $f_{\rho\pi\gamma}$ and $f_{\omega\pi\gamma}$, if we knew $f_{\omega\rho\pi}$. This may be related to the ω width,[25] if one assumes that the decay $\omega\to3\pi$ takes place through the process $\omega\to\rho+\pi$, followed by $\rho\to2\pi$. One finds, for a width of $\sim$9.5 MeV,[26,27]

$$f_{\omega\rho\pi}{}^2/4\pi\approx(0.4)/m_\pi{}^2. \tag{A10}$$

The corresponding values of $f_{\omega\pi\gamma}$ and $f_{\rho\pi\gamma}$ are then

$$\begin{aligned}f_{\rho\pi\gamma}{}^2/4\pi&\approx0.024(e_r{}^2/m_\pi{}^2);\\ f_{\omega\pi\gamma}{}^2/4\pi&\approx8.4(f_{\rho\pi\gamma}{}^2/4\pi).\end{aligned} \tag{A11}$$

---

[24] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 965 (1961).
[25] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 251 (1962).
[26] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964), and references there.
[27] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **36**, 977 (1964).

Approximately the same value for $f_{\omega\pi\gamma}$ is obtained directly from the radiative width of the ω. Now let us turn to the couplings of the nucleons to a ρ and ω. The charge couplings may be roughly estimated if one assumes that the ρ and ω dominate the isovector and isoscalar charge form factors of the nucleons. A simple analysis, made without the introduction of a soft core, gives

$$F_{1\rho NN}\approx f_\rho \tag{A12}$$

and[26]

$$4\leq F_{1\omega NN}{}^2/4\pi\leq 16.$$

In Sec. VI, we used $F_{1\rho NN}/4\pi\approx 2$ and $F_{1\omega NN}{}^2/4\pi\approx 9$.

To arrive at a crude idea of the magnetic couplings, we proceed as follows: If *both* the electric and magnetic isoscalar form factors are dominated by the ω meson, then

$$F_{2\omega NN}/F_{1\omega NN}\approx (\mu_p'+\mu_n)/e_r=F_2{}^s(0)/F_1{}^s(0). \tag{A13}$$

This would mean that the magnetic coupling of the ω is small compared to the charge coupling. This result is probably not greatly affected by φ-ω mixing, because the couplings of the φ to nucleons and to $\pi\gamma$ are substantially weaker than the ω couplings.[26]

A similar analysis of the electric and magnetic isovector form factors, on the assumption that these are *both* dominated by the ρ, leads to the result:

$$F_{2\rho NN}\approx +3F_{1\rho NN}. \tag{A14}$$

In our Eqs. (6.5) and (6.10), we need $f_{\rho\pi\gamma}F_{1\rho NN}$, $f_{\rho\pi\gamma}F_{2\rho NN}$, $f_{\omega\pi\gamma}F_{1\omega NN}$, and $f_{\omega\pi\gamma}F_{2\omega NN}$. We shall neglect the last of these, on the basis of Eq. (A13).

Our equations require us to know the relative sign of $f_{\rho\pi\gamma}$ and $F_{1\rho NN}$. (Given these, the relative signs of $f_{\omega\pi\gamma}F_{1\omega NN}$ and of the magnetic couplings are determined, assuming universality and unitary symmetry.) This relative sign is physically meaningful and could be determined, for example, by looking at the photoproduction of ρ's at low momentum transfers (but not from decay rates). In the absence of such information, we must rely on an indirect argument, or a guess.

One such argument is provided by some recent work by Adler and Drell,[28] who study the ρ-π contribution to the exchange current which affects the magnetic moment of the deuteron. The contribution of this current is proportional to $f_{\rho\pi\gamma}F_{1\rho NN}$. They find that *if* this relative sign is $+$, then this contribution is of the right *sign* and order of magnitude to account for the discrepancy between the observed moment $\mu_d=0.857$ nuclear magnetons, and that obtained using a wave function with a 7% D-state probability for the deuteron, $\mu_{\rm TH}=0.840$ nuclear magnetons.

The relative $+$ sign is the same as we have chosen, and if we were to choose a $-$ sign, our values for the moments would be much worse.[29] This calculation and the Adler-Drell calculation[28] support each other in this respect then, although (of course) neither is a substitute for a direct measurement of the relative sign.

Combining all the above statements, we arrive at the values for the coupling constants given in Table III.

In closing, we wish to emphasize that in this Appendix we do not in any way purport to have determined the "best" values of these coupling constants. Other methods of estimating the coupling constants, for example using photoproduction data in the case of $f_{\rho\pi\gamma}$ or $f_{\omega\pi\gamma}$ or pion-nucleon scattering data in the case of $F_{1\rho NN}$, have an equal claim to validity. These estimates produce values of the coupling constants that may differ by as much as a factor of two from ours. This range of uncertainty in our knowledge of the coupling constants is reflected in the range of values we quote for the magnetic moments in Table VIII. We feel that the values we have used in the text and give in Table III represent a conservative value for the couplings, but by no means a definitive or best value.

---

[28] R. J. Adler and S. D. Drell, Phys. Rev. Letters **13**, 349 (1964).
[29] See Table VIII.