Application of the Faddeev Equations to the Three-Nucleon Problem*

A. C. Phillips†
Physics Department, Imperial College, London, England
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The low-energy three-nucleon system is considered using the multiple-scattering formalism of Faddeev and an approximation scheme that preserves bound-state and three-particle scattering unitarity. The approximations involve the use of nonlocal separable potentials to describe the low-energy nucleon-nucleon interaction and a phenomenological three-body force to represent the effects on the three-nucleon system of the nucleon-nucleon tensor and short-range interactions. The neutron-deuteron scattering problem above and below the three-particle threshold, and the triton binding-energy problem, are solved exactly. The theoretical results are in impressive agreement with the experimental data.

1. INTRODUCTION

The work of Faddeev,[1] who succeeded in formulating a correct theory of nonrelativistic three-particle scattering, has opened the way for calculations on the three-body problem in its full complexity. In contrast to equations of the Lippmann-Schwinger type, the Faddeev equations have kernels that depend upon the solutions of the three two-particle subsystems. Thus, all the properties of the three-body problem may be derived as long as one knows the exact two-body scattering amplitudes off the energy shell.

Solutions to these equations exist in principle for a very wide class of two-body interactions.[2] However, the number of degrees of freedom involved in the three-body problem render impractical attempts at numerical solution without approximation. Approximation schemes that still allow the exact treatment of the three-body aspects of the problem have been presented by several authors.[3–5] In this paper we present numerical results based on the application of Lovelace's[2,5] theory to the three-nucleon system.

The Lovelace formalism takes into account completely the effects due to spin, statistics, bound-state scattering unitarity, and three-particle unitarity. It does so, however, at the expense of ignoring the high-energy behavior of the two-nucleon interaction and the effects of tensor forces. To include these effects would greatly increase the numerical difficulties involved in the solution of the three-nucleon problem. We have therefore chosen to represent these effects by a single adjustable parameter in the hope that with a single choice of this constant a good representation of several three-nucleon observables can be obtained.

In Sec. 2 we describe the two-nucleon interaction, its low-energy representation, and the phenomenological representation of the tensor and high-energy behavior. The bound-state scattering equations and the three-nucleon equations are presented in Secs. 3 and 4. Section 5 contains a discussion of our results. We conclude our remarks in Sec. 6.

Several authors[6–10] have considered the three-nucleon problem with equations similar to ours. Their results show a considerable degree of disagreement. As this is probably due to inaccurate numerical approximations, we present in an Appendix a brief description of the numerical methods used in this work.

2. THE TWO-NUCLEON INTERACTION

The two-nucleon system contains the deuteron bound state (denoted by d) and a singlet antibound state (s). In this work the assumption is made that the two-nucleon transition operator is dominated by these states. This assumption leads to the use of a nonlocal separable two-nucleon interaction.[11] It was shown in L that such an interaction, if adjusted to give a good description of the deuteron and the antibound state, will automatically lead to a T matrix that has the correct behavior in the neighborhood of the bound and antibound state poles and will also satisfy two-particle unitarity. Furthermore, in this approximation the two-nucleon transition operator T can be written as a simple analytic function of the two-nucleon low-energy parameters. Assuming charge independence

$$T(\mathbf{p},\mathbf{p}';s)=g_d(\mathbf{p})t_d(s)g_d(\mathbf{p}')P_d+g_s(\mathbf{p})t_s(s)g_s(\mathbf{p}')P_s, \quad (2.1)$$

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† Present address: Physics Department, University of Sussex, Falmer, Sussex, England.

1 L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. **39**, 1459 (1960) [English transl.: Soviet Phys.—JETP **12**, 1014 (1961)]; Izv. Akad. Nauk. SSSR **138**, 565 (1961); **145**, 301 (1962) [English transls.: Soviet Phys.—Doklady **6**, 384 (1961); **7**, 600 (1963)].

2 C. Lovelace, *Strong Interactions and High Energy Physics*, edited by R. G. Moorhouse (Oliver and Boyd, London, 1964).

3 L. Rosenberg, Phys. Rev. **131**, 874 (1963); **135**, B715 (1964).

4 R. D. Amado, Phys. Rev. **132**, 485 (1963).

5 C. Lovelace, Phys. Rev. **135**, B1225 (1964). (Hereafter referred to as L.)

6 R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. **136**, B650 (1964).

7 R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. Letters **13**, 574 (1964).

8 A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. **49**, 15 (1963).

9 A. N. Mitra, Nucl. Phys. **32**, 529 (1962); A. N. Mitra and V. S. Bhasin, Phys. Rev. **131**, 1264 (1963).

10 M. Bander, Stanford University, Report SLAC-PUB-64, 1964 (unpublished); Phys. Rev. **138**, B303 (1965).

11 Y. Yamaguchi, Phys. Rev. **95**, 1628, 1635 (1954).

where

$$t_n(s)=\left[\frac{1}{\lambda_n}+\int\frac{d^3p\, g_n^2(p)}{(p^2-s-i\epsilon)}\right]^{-1}$$

and P_d and P_s are the appropriate spin-isospin projection operators. The s and d form factors $g_n(p)$ are taken as

$$g_n(\mathbf{p})=N_d/(p^2+\mu_n^2).$$

The parameters μ_d, λ_d, μ_s, λ_s, and N_d are determined by the deuteron binding energy, the triplet and singlet scattering lengths, and the singlet effective range.[11] It should be noted that no physical significance is meant to be attached to the nonlocal separable nature of this interaction.

We pause here to consider the data for the low-energy two-nucleon system. The triplet parameters are accurately known.[12] The singlet parameters are less well determined. This situation is aggravated by our assumption of charge independence which forces us to take an average neutron-neutron neutron-proton interaction. If we take an average of $1/k\cot\delta_0$, the singlet scattering length and the effective range are given by

$$a_s=\tfrac{1}{2}[a_s^{np}+a_s^{nn}],$$

$$r_s=\frac{2}{a_s^{np}+a_s^{nn}}\left[a_s^{np}a_s^{np}+a_s^{nn}a_s^{nn}-\frac{a_s^{np}a_s^{nn}(r_s^{np}+r_s^{nn})}{a_s^{np}+a_s^{nn}}\right].$$

Experimentally the singlet neutron-proton scattering length is accurately determined to be -23.68 F.[12] Two recent measurements[13–15] of the neutron-neutron scattering length give a value of -17 ± 1 F. This value is adopted in this present work as it is consistent with the charge symmetric value of -16.96 F.[16]

The situation with regard to the effective ranges is less certain. There are no experimental measurements of the neutron-neutron effective range. We have adopted the charge symmetric value of $r_s^{nn}=2.8455$ F calculated by Signell, Yoder, and Heller.[16] Recently, Noyes[17] has pointed out that the accepted value for the singlet neutron-proton effective range of 2.51 ± 0.11 F was incompatible with charge independence. Using a model constrained to fit the experimental nucleon-nucleon scattering lengths, he predicted a charge-independent value of 2.73 ± 0.03 F; a new analysis of *np* total cross sections below 5 MeV showed that all but two measurements were consistent with this prediction.

As yet no attempt has been made to take into account the effects due to tensor forces and to the high-energy behavior of the two-nucleon interaction. Amado *et al.*[6,7] attempted to simulate these effects by weakening the residue of the deuteron pole of the T matrix. This resulted in the use of a T matrix which did not satisfy two-particle unitarity; the resulting three-body equations did not satisfy unitarity. In our work an attempt is made to represent the effect on the three-nucleon system of both the nucleon-nucleon tensor and the short-range interactions by a phenomenological three-body force. This force is taken to be separable in the initial and final variables. That is

$$V_4=\lambda_4|4\rangle\langle4|. \tag{2.2}$$

The three-particle equations, in the presence of this force, are formulated in the next section. They are found to sarisfy bound-state scattering unitarity and three-particle unitarity.[18]

3. THE SEPARABLE APPROXIMATION WITH THREE-BODY FORCES

In this section we generalize the equations of Sec. 3 of L to include the possibility of a three-body force. Our formulation is as in L with the exception of the choice of momentum variables. These variables are such that the kinetic energy of three free particles is given by

$$E=q_1^2/2\mu_1+p_1^2/2m_1,$$

where μ_1 and m_1 are the reduced masses for the 1-(2,3) and the 2-3 systems, respectively. The scattering amplitudes for bound states and resonances may be defined as

$$X_{\alpha n,\beta m}(s)=\langle\alpha n|G_0(s)U_{\alpha\beta}^+(s)G_0(s)|\beta m\rangle-Z_{\alpha n,\beta m}(s)[1+\lambda_{\beta m}\langle\beta m|G_0(s)|\beta m\rangle], \tag{3.1}$$

where in the equation for $U_{\alpha\beta}$,

$$U_{\alpha\beta}^+(s)=\sum_{\gamma\neq\alpha}V_\gamma-\sum_{\delta\neq\beta}U_{\alpha\delta}^+(s)G_0(s)T_\delta(s),$$

and for $Z_{\alpha n,\beta m}(s)$,

$$Z_{\alpha n,\beta m}(s)=(1-\delta_{\alpha\beta})\langle\alpha n|G_0(s)|\beta n\rangle,$$

α, β, δ, γ now run from 0 to 4. $X_{\alpha n,\beta m}(s)$ now satisfies the equation

$$X_{\alpha n,\beta m}(s)=-Z_{\alpha n,\beta m}(s)-\sum_{\gamma r}X_{\alpha n,\gamma r}(s)\times\tau_{\gamma r}(s)Z_{\gamma r,\beta m}(s). \tag{3.2}$$

In particular,

$$X_{\alpha n,4}=-Z_{\alpha n,0}(s)-\sum_{\gamma r}X_{\alpha n,\gamma r}(s)\tau_{\gamma r}(s)Z_{\gamma r,4}(s).$$

[12] H. Pierre Noyes, Phys. Rev. **130**, 2025 (1963).
[13] J. W. Ryan, Phys. Rev. Letters **12**, 564 (1964).
[14] R. P. Haddock, R. Slater, M. Zeller, J. B. Czirr, D. R. Nygren, and T. Maung, Bull. Am. Phys. Soc. **9**, 443 (1964).
[15] See also M. Bander, Phys. Rev. **134**, 1052 (1964) for a discussion of theoretical error involved in the determination of a_s^{nn}.
[16] L. Heller, P. Signell, and N. R. Yoder, Phys. Rev. Letters **13**, 557 (1964).
[17] H. Pierre Noyes, Stanford University, Report SLAC-PUB-59, 1964 (unpublished).
[18] For similar work on the relativistic Faddeev equations see C. Lovelace, D. Z. Freedman, and J. M. Namyslowski, CERN report, 1965 (unpublished).

It is possible to solve completely for the three-body force. Inserting $X_{\alpha n,4}(s)$ into Eq. (3.2) we get an equation for $X_{\alpha n,\beta m}(s)$ with α and β running from 1 to 3.

$$X_{\alpha n,\beta m}(s)=-\{Z_{\alpha n,\beta m}(s)+Z_{\alpha n,\beta m}'(s)\}-\sum_{\gamma r} X_{\alpha n,\gamma r}(s)\times\tau_{\gamma r}(s)\{Z_{\gamma r,\beta m}(s)+Z_{\gamma r,\beta m}'(s)\}\,, \quad (3.3)$$

where

$$Z_{\alpha n,\beta m}'(s)=-Z_{\alpha n,0}(s)\tau_0(s)Z_{0,\beta m}(s)\,.$$

The amplitude for a bound state, $|\alpha n\rangle$, to disintegrate to give three final particles is given by

$$-\langle\alpha n|G_0(s)U_{\alpha 0}^+(s)=-Z_{\alpha n,4}(s)\tau_4(s)\langle 4| + \sum_{\substack{\beta m\\ \beta\neq 4}} X_{\alpha n,\beta m}(s)\tau_{\beta m}(s)\{\langle\beta m|-Z_{\beta m,4}(s)\tau_4(s)\langle 4|\}\,. \quad (3.4)$$

The three-particle and bound-state scattering unitarity relation is

$$\begin{aligned}X_{\alpha n,\beta m}(s+i\epsilon)-X_{\alpha n,\beta m}(s-i\epsilon) &=2\pi i\sum_{\gamma r} X_{\alpha n,\gamma r}(s+i\epsilon)\,|\mathbf{q}_\gamma\rangle d^3q_\gamma\delta(q^2/2\mu_\gamma-E_{\gamma r}-s)\\ &\times\langle\mathbf{q}_\gamma|X_{\gamma r,\beta m}(s-i\epsilon)+2\pi i\langle\alpha n|G_0(s+i\epsilon)\\ &\times U_{\alpha 0}^+(s+i\epsilon)\,|\mathbf{q}_\gamma,\mathbf{p}_\gamma\rangle d^3p_\gamma d^3q_\gamma\delta(q^2/2\mu_\gamma+p^2/2m_\gamma-s)\\ &\times\langle\mathbf{p}_\gamma,\mathbf{q}_\gamma|U_{0\beta}^-(s-i\epsilon)G_0(s-i\epsilon)|\beta m\rangle\,, \quad (3.5)\end{aligned}$$

where $E_{\gamma r}$ is the binding energy of the γr bound state. The proof of Eq. (3.5) is straightforward but tedious.

4. THE THREE-NUCLEON EQUATIONS

(a) Neutron-Deuteron Scattering

We may generalize Eq. (3.38) of L to give the scattering equation for identical particles in the presence of a three-body force. We have

$$\begin{aligned}X_{nm}(s)=-\{2Z_{nm}(s)+2Z_{nm}'^N(s)+Z_{nm}'^D(s)\}\\ -\textstyle\sum_r X_{nr}(s)\tau_r(s)\{2Z_{rm}(s)\\ +2Z_{rm}'^N(s)+Z_{rm}'^D(s)\}\,, \quad (4.1)\end{aligned}$$

where $Z_{nm}'^N(s)$ and $Z_{nm}'^D(s)$ are the nondiagonal and diagonal elements of $Z_{\alpha n,\beta m}'(s)$, respectively.

For the three-nucleon problem, in addition to being an operator in the Hilbert space $L_2(\mathbf{q})$, $Z_{nm}(s)$ also depends on spin and isospin. The spin angular-momentum analysis is given in L. There are two states of interest; the quartet state with $I=\frac{1}{2}$, $S=\frac{3}{2}$, which is described by a single-channel Lippmann-Schwinger equation (i.e., n, m take on one value corresponding to d-N scattering), and the doublet state with $I=\frac{1}{2}$, $S=\frac{1}{2}$, consisting of two coupled channels corresponding to d-N and s-N scattering.

(b) The Triton

The triton is taken as a pure $I=S=\frac{1}{2}$ state of three nucleons. The triton wave function satisfies the equation

$$|\Psi\rangle=-G_0(-E_t)\sum_{\alpha=1}^{4} V_\alpha|\Psi\rangle\,, \quad (4.2)$$

where E_t is the triton binding energy. Following Faddeev,[1] we write

$$|\Psi\rangle=\sum_{\alpha=1}^{4}|\psi^\alpha\rangle\,, \quad (4.3)$$

where

$$|\psi^\alpha\rangle=\sum_{\beta\neq\alpha}-G_0(-E_t)T_\alpha(-E_t)|\psi^\beta\rangle\,. \quad (4.4)$$

In the separable approximation for T, $|\psi^\alpha\rangle$ has the form

$$|\psi^\alpha\rangle=G_0(-E_t)|\alpha m\rangle|Q_{\alpha m}\rangle\,, \quad (4.5)$$

where

$$|Q_{\alpha m}\rangle=-\tau_{\alpha m}(s)\sum_{\gamma r} Z_{\alpha m,\gamma r}(-E_t)|Q_{\gamma r}\rangle\,. \quad (4.6)$$

Again we can solve completely for the three-body force and thus eliminate $|Q_4\rangle$. We have

$$|Q_{\alpha m}\rangle=-\tau_{\alpha m}(-E_t)\sum_{\gamma r}\{Z_{\alpha m,\gamma r}(-E_t)+Z_{\alpha m,\gamma r}'(-E_t)\}|Q_{\gamma r}\rangle\,. \quad (4.7)$$

Finally, using the identity of the particles, the set of integral equations (4.7) reduces to

$$|Q_m\rangle=-\tau_m(-E_t)\sum_{r=d,s}\{2Z_{mr}(-E_t)+2Z_{mr}'^N(-E_t)+Z_{nr}'^D(-E_t)\}|Q_r\rangle\,. \quad (4.8)$$

Thus we have a set of homogeneous integral equations that has a solution only at an energy equal to the binding energy of the three-nucleon system.

5. DISCUSSION OF RESULTS

The aim of this research is to predict a considerable number of three-nucleon observables in terms of the low-energy two-nucleon data and one phenomenological parameter. No attempt is made to include Coulomb effects, and therefore we restrict ourselves to the neutron-neutron-proton (nnp) system. In our approximation, the total spin and orbital angular momentum of the system are conserved. One effect of tensor forces, which cannot be simulated by the three-body force, is the mixing of states of different spin and orbital angular momentum. This mixing of states plays an important role in polarization effects, but it is less important in the prediction of the triton binding energy and the neutron-deuteron cross sections. Consequently, we confine our discussion to the latter aspects of the nnp system.

The experimental values for the triton binding energy and the neutron-deuteron scattering lengths are given in the first row of Table I. Note that there are two sets of scattering lengths, set 1 and set 2, which satisfy the data.

First let us consider the results obtained when only the low-energy nucleon-nucleon interaction is used. For the quartet scattering length, which is free from the uncertainties of the singlet nucleon-nucleon interaction, our calculated value is 6.28 F. This value agrees with previous theoretical values[7–9] and with the experimental result associated with set 1 (Table I). We have calculated the triton binding energy and the doublet scattering length for several values of the singlet effective range. In particular, for $r_s^{np}=2.51$ F, the binding energy is 11.5 MeV and the scattering length is -1.06 F. The corresponding results for the charge symmetric effective range of 2.73 F are 11.1 MeV and -0.79 F.

In brief, the low-energy two-nucleon amplitude (2.1), while giving rise to a reasonable description of the quartet state, gives too much attraction in the doublet state. However, it should be noted that the hitherto neglected tensor forces and nucleon-nucleon short-range interaction will play a more significant role in the doublet state. In this state the over-all attractive interaction weakens the centrifugal barrier which normally prevents tensor forces, etc., from being effective at low energies.

Now we consider the results of the calculations when the effects of the nucleon-nucleon tensor and short-range forces on the *nnp* system are represented by the three-body force, V_4 (see Sec. 2). The form factor in Eq. (2.2) is taken to be

$$\langle \mathbf{p}_\alpha,\mathbf{q}_\alpha|4\rangle=(p_\alpha^2+\tfrac{3}{4}q_\alpha^2+\mu_d^2)^{-1}$$

and λ_4 is taken to be an adjustable parameter. For the quartet state, the terms $2Z_{nm}'^N$ and $Z_{nm}'^D$ in Eq. (4.1) cancel. This state, therefore, is independent of the three-body force. The doublet state, on the other hand, is quite sensitive to V_4. However, no reasonable value of V_4 gives rise to the set 2 doublet scattering length. Nevertheless, we do obtain the set 1 value and a good value for the triton binding energy. In particular, with a singlet effective range, $r_s^{np}=2.73$ F, the value of V_4 which gives a doublet scattering length of 0.7 F is repulsive and corresponds to a 4.3% effect; i.e., $\langle\mathbf{q}|Z_{nm}'(s)|\mathbf{q}'\rangle$ in Eq. (3.3) is 0.043 of $\langle\mathbf{q}|Z_{nm}(s)|\mathbf{q}'\rangle$ for $q=q'=0$ at the elastic threshold. The corresponding result for $r_s^{np}=2.51$ F is 5.7%. We believe that 4 to 5% is a reasonable measure for effects due to the two-nucleon tensor and short-range behavior. We present in detail the results obtained with $r_s^{np}=2.73$ F.

The scattering lengths and triton binding energy obtained with $V_4=0$ and $V_4\neq0$ are shown in Table I. Other calculations very similar to ours have been performed by Mitra and Bhasin (MB),[9] Sitenko and Kharenchenko (SK),[8] and Aaron, Amado, and Yam (AAY).[7] Their results are also shown in Table I. Our $V_4=0$ results agree moderately well with the results of AAY obtained with the deuteron wave-function renormalization $Z=0$.[19] Bander has also considered the three-nucleon problem in a separable approximation.[10] He found that the three-body scattering amplitude had two poles.[20] One of these poles was found to be a ghost. This result prompted us to search for additional singularities of our three-body amplitude. For the $V_4\neq0$ amplitude, we found one pole only. This pole corresponds to the triton.

Inspection of Table I shows that our $V_4\neq0$ amplitude gives rise to the set 1 scattering lengths and to a reasonable value for the triton binding energy. These results are encouraging. However, they are crucially dependent on the assumption that the singlet antibound state and the deuteron bound state dominate the two-nucleon amplitude. Whether or not this assumption is justified can only be resolved by investigating its consequences for other three-nucleon observables.

In Figs. 1 to 7 we plot the calculated ($V_4\neq0$) and

TABLE I. Theoretical and experimental observables for the *nnp* system.

	Triton binding energy (MeV)	Doublet scattering length (F)	Quartet scattering length (F)
Experimental[a]	8.49	Set 1, 0.7 ±0.3 Set 2, 8.26±0.12	6.38±0.06 2.6 ±0.2
Phillips (this work)			
$V_4=0$	11.1	−0.79	6.28
$V_4\neq0$	9.1	0.7	6.28
MB		9.25	5.91
SK	12.5	−2.76	6.28
AAY $Z=0$	11.0	−1.01	6.28
$Z=0.0488$	8.76	0.7	6.14

[a] D. Hurst and N. Alcock, Can. J. Phys. 29, 36 (1951).

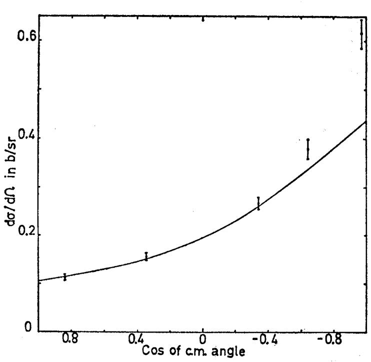

FIG. 1. Angular distribution for neutron-deuteron scattering at 0.5 MeV. Experimental points: Adair *et al.*

[19] This is the separable potential limit of the Aaron-Amado-Yam (AAY) model.

[20] A second pole was also found by AAY for the scalar three-nucleon system. This pole disappeared when the nucleon-nucleon interaction was weakened. See Ref. 6.

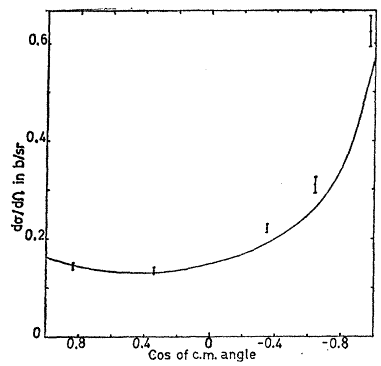

FIG. 2. Angular distribution for neutron-deuteron scattering at 1.0 MeV. Experimental points: Adair *et. al.*

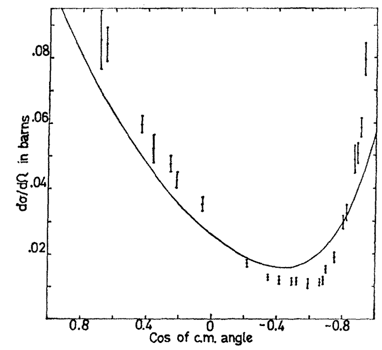

FIG. 7. Angular distribution for neutron-deuteron scattering at 14 MeV. Experimental points: Allred *et al.*

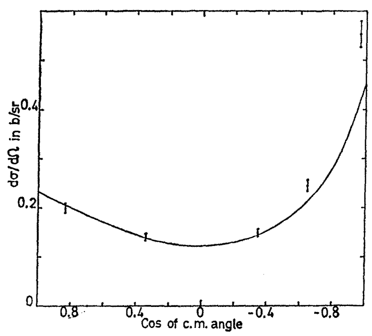

FIG. 3. Angular distribution for neutron-deuteron scattering at 1.75 MeV. Experimental points: Elwyn *et al.*

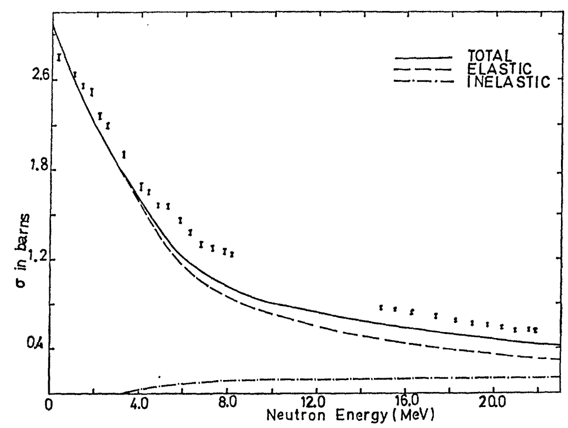

FIG. 8. Total inelastic and elastic cross sections for neutron-deuteron scattering as a function of energy. Experimental points: Seagrave and Henkel.

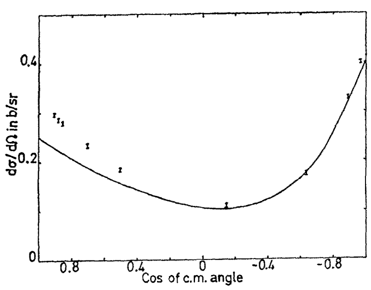

FIG. 4. Angular distribution for neutron-deuteron scattering at 2.45 MeV. Experimental points: Elwyn *et al.*

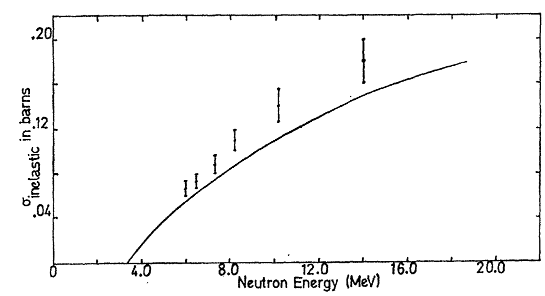

FIG. 9. Inelastic cross section as a function of energy. Experimental points: Catron *et al.*

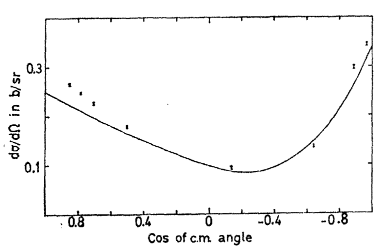

FIG. 5. Angular distribution for neutron-deuteron scattering at 3.27 MeV. Experimental points: Cranberg and Seagrave.

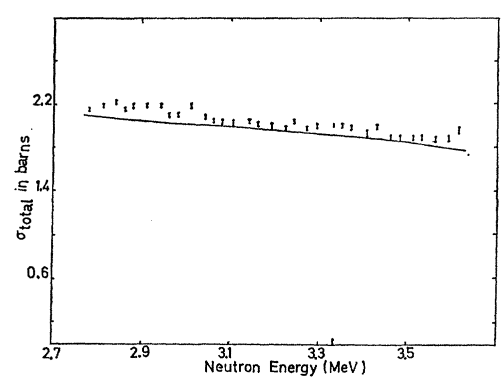

FIG. 10. Total cross sections in the neighborhood of the three-particle threshold. Experimental points: Willard *et al.*

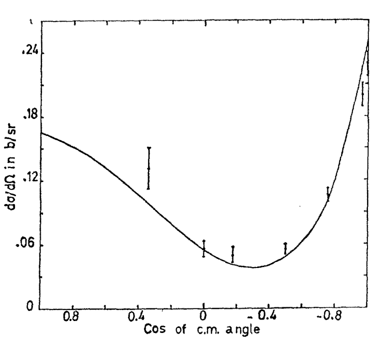

FIG. 6. Angular distribution for neutron-deuteron scattering at 5.5 MeV. Experimental points: Wantuch.

experimental[21–25] angular distributions for neutron-deuteron elastic scattering at various energies below and above the three-particle threshold of 3.334 MeV. Except at large scattering angles the agreement is excellent for center-of-mass energies from 0.5 to 14 MeV.

In Fig. 8 the calculated total, elastic and inelastic cross sections are plotted. The experimental points are for the total cross section[26] and are in reasonable agreement with the calculated values. In Fig. 9 the total cross section for disintegration is compared with the experimental data of Catron *et al.*[27]

Finally we compare our results for the total cross section in the neighborhood of the inelastic threshold with the experiment of Willard *et al.*[28] This experiment was designed to search for the violation of charge independence which would be caused by the existence of a dineutron. In comparing their results with the calculations of Alzetta *et al.*,[29] it was concluded that the dineutron may or may not exist. Inspection of Fig. 10 shows that Willard's results are consistent with our results and hence with the assumption of charge independence.

6. CONCLUSIONS

We have considered three aspects of the *nnp* system. They are (1) the binding energy of the triton, (2) the differential and total cross sections for the elastic scattering of neutrons by deuterons, (3) the total cross section for breakup of deuterons by neutron impact. In all three cases our results are in favorable agreement with the experimental data. We conclude with confidence that the Faddeev theory in the separable approximation gives a good representation of the low-energy *nnp* system. In particular, the well-known ambiguity existing in the experimental scattering lengths is certainly removed.

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APPENDIX

The momentum-space integral equations for the quartet and doublet partial-wave amplitudes and for the triton wave function, were cast into matrix equations by approximating the integrals by summations of the integrand at discrete values of the momentum. A 100×100 mesh was taken. Integrations over τ_d were approximated using a method that took into account the analytic structure implied by the d-N bound-state scattering cut.[30] The required integral is of the form

$$f(t)=\int_{t_1}^{\infty}\frac{\operatorname{Im}f(t')}{t'-t-i\epsilon}dt'. \tag{A1}$$

We make the conformal mapping

$$\eta=\frac{t-t_0}{[(t_1-t_0)^{1/2}+(t_1-t)^{1/2}]^2},$$

which takes the cut t plane into the interior of the unit circle in the η plane. The upper lip of the cut becomes the upper half of the unit circle; the lower lip becomes the lower half. Under quite weak restrictions the power series in η_s for $f(t)$ converges inside and on the unit circle. If this power series is truncated, it is possible to relate $f(t)$ to $\operatorname{Im}f(t)$, and hence evaluate integrals of the form (A1). The program was arranged such that the integrations over τ_s, there being no s-N scattering cut, could be approximated by any one of several Newton-Cotes formulas.

Above the three-particle threshold the kernel of the integral equations has logarithmic singularities. In evaluating integrals of the form

$$\int f(x)\ln|x|dx,$$

$\ln|x|$ was replaced by $\ln\epsilon-1$ in range $-\epsilon$ to $+\epsilon$. If $f(x)$ is quadratic in x, the error incurred is of the order ϵ^2. In our work $\epsilon=10^{-4}$ was taken. An alternative method,[31] involving distortion of the contour of integration into the complex momentum plane, was considered but not used since the method is only applicable in the determination of elastic amplitudes.

The K-matrix integral equations were solved using standard matrix routines. As a check on the numerical accuracy, the corresponding T-matrix equations were also solved at selected energies.

For the triton binding energy the 100×100 matrix equation corresponding to Eq. (4.8) has the form

$$K_{ij}(E_t)X_j(E_t)=\eta X_j(E_t).$$

An iterative method for finding the largest eigenvalue, $\eta_{\max}$, of $K_{ij}(E_t)$ was used. If E_t is the binding energy of the triton, $\eta_{\max}=1$. A search for zeros of the Fredholm determinant of the three-nucleon amplitude provided an independent but less accurate determination of the triton binding energy.

[21] R. K. Adair, A. Okazaki, and M. Walt, Phys. Rev. **89**, 1165 (1953).
[22] A. J. Elwyn, R. O. Lane, and A. Langsdoef, Jr., Phys. Rev. **128**, 779 (1962).
[23] J. D. Seagrave and L. Cranberg, Phys. Rev. **105**, 1816 (1957).
[24] E. Wantuch, Phys. Rev. **86**, 679 (1952).
[25] J. C. Allred, A. H. Armstrong, and L. Rosen, Phys. Rev. **91**, 90 (1953).
[26] J. D. Seagrave and R. L. Henkel, Phys. Rev. **98**, 666 (1955).
[27] H. C. Catron, M. D. Goldberg, R. W. Hill, J. M. Leblanc, J. B. Stoering, C. J. Taylor, and M. A. Williamson, Phys. Rev. **123**, 218 (1961).
[28] H. B. Willard, J. K. Bair, and C. M. Jones, Phys. Letters **9**, 339 (1964).
[29] R. Alzetta, G. C. Ghirardi, and A. Rimini, Phys. Rev. **131**, 740 (1963).

[30] C. Lovelace (unpublished).
[31] J. H. Hetherington and L. H. Schick, Phys. Rev. **137**, B935 (1965).