pion-production mechanisms effective at 600 MeV are also effective at 780 MeV. The backward production of the π^+p isobar suggests the possibility of including ρ exchange in the $D_{3/2}^-$ isobar model. In their development of the ρ-exchange model, Stodolsky and Sakurai[15] predict an angular distribution for the decay pion in the isobar rest system of the form $1+3(\hat{q}\cdot\hat{n})^2$, where $\hat{q}$ is the unit vector along the pion direction and $\hat{n}$ is the unit vector normal to the production plane. In our experiment, the distribution for the decay of the π^+p isobar (Fig. 8) is

$$W(\hat{q}\cdot\hat{n})=1.0\pm0.1-(0.05\pm0.10)(\hat{q}\cdot\hat{n})+(1.37\pm0.46)(\hat{q}\cdot\hat{n})^2.$$

[15] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters **11**, 90 (1963); L. Stodolsky, Phys. Rev. **134**, B1099 (1964).

This result is suggestive of the importance of ρ exchange.

Finally, we remark once more upon the similarity between our results at 780 MeV and those reported by Newcomb[11] at 600 MeV. There are no effects in our data that can be ascribed to the shoulder.

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πp Charge-Exchange Scattering and a "Coherent Droplet" Model of High-Energy Exchange Processes

N. BYERS* AND C. N. YANG
Institute for Advanced Study, Princeton, New Jersey

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A model calculation is presented following a simple viewpoint to approach high-energy two-body processes $A+B\rightarrow C+D$. πp charge-exchange scattering is discussed in detail. A number of further experiments are suggested.

RECENT experiments reveal the following characteristics of high-energy collisions.

(i) The angular distribution of elastic scattering[1] is approximately

$$d\sigma/dt\cong Ae^{\gamma t},\quad -t<0.6(\mathrm{BeV}/c)^2, \tag{1}$$

where $-t$ is the square of the momentum transfer in the center-of-mass system. The constant γ is not drastically different for various types of collisions ($pp,\bar{p}p,\pi p,Kp$) or for different energies (5 BeV/c lab momentum and up). It ranges in value from 6 to 13 $(\mathrm{BeV}/c)^{-2}$. For pp and πp collisions, the range[2] is even narrower;

$$8.5<\gamma<10.5,\quad \text{in } (\mathrm{BeV}/c)^{-2}. \tag{2}$$

(The constancy of γ with energy strongly suggests a "size" of the interaction volume. This size has remarkably similar values in the different types of collisions.)

(ii) Recent measurements[3] of the charge-exchange scattering $\pi^-p\rightarrow\pi^0n$ indicates an angular distribution similar to Eq. (1) for an important range of momentum transfer

$$0.1<-t<0.6.$$

(iii) Data[4] on backward elastic π^+p and π^-p scattering indicate an angular distribution similar to Eq. (1) if $-t$ is taken to be the invariant square of the 4-momentum difference between the incoming π and the outgoing p. Again, the constant γ seems to be in the range indicated by Eq. (2).

It is remarkable that in exchange processes

(a) γ is not very much dependent upon the quantum numbers exchanged. [Traditionally, it is expected that the exchange of a heavy (light) particle entails a close (distant) collision, yielding a wide (narrow) angular distribution.]

(b) The value of γ is similar to that for elastic

* On leave of absence from the University of California, Los Angeles, California.

[1] See, e.g., K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 425 (1963); **11**, 503 (1963); O. Czyzewski, B. Escoubes, Y. Goldschmidt-Clermont, M. Guinea-Moorhead, D. R. O. Morrison, and S. DeUnamuno-Escoubes, Phys. Letters **15**, 188 (1965), and the work of many other groups quoted in these papers.

[2] All momenta are in units of (BeV/c) in this paper, and length in $\hbar(\mathrm{BeV}/c)^{-1}$.

[3] I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters **14**, 408 (1965); A. V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, and P. Borgeaud, *ibid.* **14**, 763 (1965).

[4] W. R. Frisken, A. L. Read, H. Ruderman, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, and D. H. White, Phys. Rev. Letters **15**, 313 (1965); C. T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams (unpublished report).

scattering. (Notice that the peak in the latter is mainly shadow scattering which is absent in the exchange processes. In other words, the partial-wave amplitude in elastic processes is proportional to $e^{2i\delta}-1$, where the -1 contributes predominantly to the forward peak. In exchange processes this term is absent.[5])

(c) The forward peak rises above the differential cross sections at $-t\sim 0.6$ by factors of 100 to 1000. Furthermore the shape of the peak in $d\sigma/dt$ versus t is largely independent of the incoming energy. Both of these features are usually characteristics of elastic "diffraction" peaks, but not of inelastic processes.

There have been a number of attempts[6,7] to understand narrow peaks like (1) in exchange processes. Here we make a new attempt which is based on a very simple picture of high-energy two-body processes. Our discussions *are confined* to the forward and backward peaks where the momentum transfer $-t$ (or $-u$) is <0.6. Collisions with larger momentum transfers are, we believe, due to a physically different mechanism, and other complementary pictures must be used to describe them.

The essential independence of (1) of the incoming energy is indicative of an impact parameter picture of elastic processes. We therefore take the "eikonal" viewpoint of elastic high-energy collisions,[8] and extend it to exchange processes. We argue that, for exchange processes, the existence at small angles of enormous peaks rising above the small value of the large-angle differential cross sections, irrespective of the quantum numbers exchanged and with a shape independent of energy, is indicative of the *great difficulty in transferring large momenta, but relative ease,* to *varying degrees, in coherently transferring quantum numbers:* charge, spin, strangeness, *nucleon number*, etc. Because of the similarity of these peaks to the elastic one, we choose an impact parameter, or eikonal, description of such coherent transfers. Elastic and exchange processes are thus pictured as very much similar to the passage of a particle through an absorptive medium with or without coherent excitation of the medium.

In more precise terms, we express for πp scattering the spin-nonflip and spin-flip amplitudes in the usual partial-wave expansion:

$$A(\theta)=\tfrac{1}{2}\lambda\!\!\!^{-} i\sum_{l=0}^{\infty}(2l+1)\alpha_l P_l(\cos\theta)\,, \tag{3}$$

$$B(\theta)=\lambda\!\!\!^{-} i\sin\theta\sum_{l=0}^{\infty}\beta_l\frac{d}{d\cos\theta}P_l(\cos\theta)\,, \tag{4}$$

$$\alpha_l=\frac{l+1}{2l+1}a_l^{+}+\frac{l}{2l+1}a_l^{-},\quad \beta_l=-(a_l^{+}-a_l^{-})/2\,,$$

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_{\text{nonflip}}+\left(\frac{d\sigma}{dt}\right)_{\text{flip}},$$

$$\left(\frac{d\sigma}{dt}\right)_{\text{nonflip}}=\pi\lambda\!\!\!^{-2}|A(\theta)|^2, \tag{5}$$

$$\left(\frac{d\sigma}{dt}\right)_{\text{flip}}=\pi\lambda\!\!\!^{-2}|B(\theta)|^2, \tag{6}$$

$$-t=2k^2(1-\cos\theta),\quad k=1/\lambda\!\!\!^{-}=\text{c.m. momentum.} \tag{7}$$

The amplitudes a_l^+ and a_l^- are related to (complex) phase shifts $\delta^\pm$ by

$$a^\pm=1-\exp(2i\delta^\pm)\,. \tag{8}$$

It is clear from experimental evidence (Ref. 5; also remark H below) that all exchange processes (charge exchange or spin exchange, etc.) are small compared with the elastic forward peak at the same t. Thus $a_l^+\cong a_l^-$, and we shall write

$$\alpha=1-\exp(2i\delta)\,, \tag{9}$$

where $\delta\cong\delta^+\cong\delta^-$.

Although not necessary, we shall use in this paper, consistent with the eikonal picture, the approximations, valid for small angles,[9] of replacing $P_l(\cos\theta)$ by $J_0(b\sqrt{(-t)})$, $[b=\lambda\!\!\!^{-}(l+\frac{1}{2})=\text{impact parameter}]$ and $\sin\theta dP_l(\cos\theta)/d\cos\theta$ by $(l+\frac{1}{2})J_1(b\sqrt{(-t)})$, and the sums in (3) and (4) by integrals:

$$A(\theta)=ik\int_0^\infty \alpha(b)J_0(b\sqrt{(-t)})b\,db\,, \tag{3'}$$

$$B(\theta)=ik\int_0^\infty \beta(b)J_1(b\sqrt{(-t)})b\,db\,. \tag{4'}$$

[5] The difference of the amplitudes for $I=\frac{1}{2}$ and $I=\frac{3}{2}$ scattering is the charge-exchange amplitude. Pure I-spin $\frac{1}{2}$ and $\frac{3}{2}$ πp elastic scattering differential cross sections are essentially the same since the charge-exchange process is $\sim 1\%$ of elastic cross sections at the same energy and angle. The experimental data indicate that the t dependence of the difference of the amplitudes ($I=\frac{1}{2}$ and $\frac{3}{2}$) is similar to that of each amplitude. This is remarkable, since whatever causes the *difference* has no *a priori* reason to give an amplitude similar to that for elastic scattering. Notice also that the t dependence is so sharp that the charge-exchange differential cross section at $t=0$ is larger than the elastic differential cross section at $-t>1$.

[6] N. J. Sopkovich, Nuovo Cimento **26**, 186 (1962); A. Dar, M. Kuyler, Y. Dothan, and S. Nussinov, Phys. Rev. Letters **12**, 82 (1964); A. Dar and W. Tobocman, *ibid.* **12**, 511 (1964); A. Dar, *ibid.* **13**, 91 (1964); L. Durand and Y. T. Chiu, *ibid.* **12**, 399 (1964); **13**, 45 (1964); Phys. Rev. **137**, B1530 (1965); M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 627 (1964); R. C. Arnold, Phys. Rev. **136**, B1388 (1964); K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964); J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. **139**, B428 (1965).

[7] See the review article of J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965).

[8] S. Fernbach, R. Serber, and T. B. Taylor, Phys. Rev. **75**, 1352 (1949). See also R. Serber, Rev. Mod. Phys. **36**, 649 (1964); R. J. Glauber, *High Energy Collision Theory Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959).

[9] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, New York, 1944), Sec. 5.72.

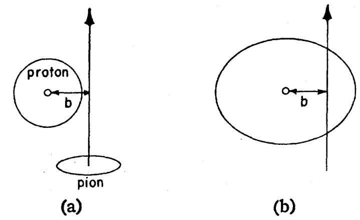

FIG. 1. (a) Schematic representation of π (ellipse) and p (circle) in rest frame of proton. (b) Eikonal description collision.

For charge-exchange scattering these formulas hold with α and β replaced by

α^{ce} = charge-exchange spin-nonflip amplitude,
β^{ce} = charge-exchange spin-flip amplitude.

Our extension of the eikonal picture to charge-exchange scattering depicts α^{ce} as being the total coherent amplitude of a charge exchange $\pi^- p\rightarrow\pi^0 n$ during a passage of a Lorentz-contracted extended object (π) through another extended object (p) with impact parameter b. [See Fig. 1(a).] For simplicity, we picture the process as a coherent excitation in the passage of a point particle in Fig. 1(b) through an extended object, and use the language one would customarily use for describing such a process (see remarks D and I below).

$$\alpha^{ce}\cong \text{(absorption factor) (probability amplitude of exchange per g/cm}^2\text{ of path) (total material thickness of path in g/cm}^2\text{)}. \qquad (10)$$

Now the absorption factor is the product of that before the charge exchange with that after the charge exchange. Both factors are essentially the same for each g/cm^2 of path since the value of $e^{2i\delta}$ is essentially the same in $\pi^- p$ and $\pi^0 n$ scattering. The material thickness of the path is approximately proportional to δ in the eikonal picture. Thus α^{ce} is proportional to $e^{2i\delta}\delta$, or

$$\alpha^{ce}\cong -K^{ce}(1-\alpha)\ln(1-\alpha), \qquad (11)$$

where K^{ce} is a numerical constant (*in general complex*) independent of the impact parameter b, but dependent on the incoming momentum.

Equation (11) above was suggested by a "geometrical picture" of high-energy exchange processes. If one adopts a "potential model" of such processes, one could also arrive at Eq. (11) in the following way. Let

$$V=\mathcal{V}\rho(r),\quad U=\mathcal{U}\rho(r), \qquad (12)$$

be the elastic and charge-exchange scattering potentials, respectively. Assume $\mathcal{V}$ and $\mathcal{U}$ to be constants independent of r. Since U is off diagonal, $\mathcal{U}$ is in general complex. To lowest order in U, taking V into account in all orders, one has

$$\alpha_l^{ce}=-e^{2i\delta_l}\int_0^\infty dr|g_l(r)|^2U.$$

In the WKB approximation, when $k\gg V$,

$$2i\delta_l\cong -i\mathcal{V}\int_0^\infty dz\,\rho((z^2+b^2)^{1/2}),$$

and

$$g_l(r)\cong\frac{1}{(1-b^2/r^2)^{1/4}}\cos\left\{k\int_b^r(1-b^2/r^2)^{1/2}dr-\tfrac{1}{4}\pi\right\}$$

for $r>b$. For $r<b$,

$$g_l(r)\cong\frac{1}{(|1-b^2/r^2|)^{1/4}}\exp-k\int_r^b(b^2/r^2-1)^{1/2}\,dr,$$

so as $k\rightarrow\infty$, if ρ is a smooth function, one obtains (11). [The above expression for δ may be inverted and a $V(r)$ obtained from experiment; see remark I below. Using Eqs. (15) and (23), one has $V(0)=-i0.13$ BeV.]

For the spin-flip amplitudes β and β^{ce}, we adopt the standard idea that spin flip is proportional to $\boldsymbol{\sigma}\cdot\mathbf{L}$, so that β contains a factor proportional to the orbital angular momentum, or to b. (The assumption is made here that whatever factor $\boldsymbol{\sigma}\cdot\mathbf{L}$ multiplies into has no singularity at $b=0$. This assumption is in agreement with the general picture of the nucleon and pion as structures without a central localized singularity. See remark J below.) We find, however, that experimental data[3] indicate further dominance of high b in spin flip[10]; therefore, we take

$$\beta^{ce}\cong -K_f^{ce}(1-\alpha)b^2\ln(1-\alpha). \qquad (13)$$

The constant K_f^{ce} here has similar properties as K^{ce}.

To test these ideas against experiments, we take the elastic πp differential cross section as

$$(d\sigma/dt)_{el}=41e^{10t}\,\text{mb}(\text{BeV}/c)^{-2}. \qquad (14)$$

Neglecting the spin flip-term in the elastic cross sections, and neglecting the real part of $A(\theta)$, one obtains from (14)

$$\alpha(b)=0.58e^{-b^2/20},\quad b\text{ in }(\text{BeV}/c)^{-1}, \qquad (15)$$

which can be verified by substitution into (3′) and (5). Substitution of (15) into (11) and (13), and then into

[10] To be precise, if one takes $\beta^{ce}=-K_f^{ce}(1-\alpha)b\ln(1-\alpha)$ the decrease of the theoretical $[\ln(d\sigma/dt)]_{\rm flip}$ value from $-t=0.2$ to $-t=0.4$ is then not enough to give as good a fit with experiments as that exhibited in Fig. 2, although a passable fit can be produced.

In this connection if one takes a β^{ce} that does not vanish in the limit $b\rightarrow 0$, as, e.g., $\beta^{ce}=(\text{const})e^{-b^2c}$, (i.e., a Gaussian), the spin-flip charge-exchange amplitude $B(\theta)$ would be $\sim 1/t$ for large $-t$, which is in violent disagreement with the principal general features of the experimental data.

In nuclear physics, the dependence of the spin-flip term on the impact parameter heavily emphasizes the edge of the nucleus. See, e.g., recent data and analysis in P. G. McManigal, R. D. Eandi, S. N. Kaplan, and B. J. Moyer, Phys. Rev. **137**, B620 (1965).

(3′) and (4′) leads to a calculation[11] of the charge-exchange spin-flip and spin-nonflip amplitudes. The corresponding differential cross sections are plotted in Fig. 2, where we have chosen the constants K to fit the experimental points:

$$|K^{\mathrm{ce}}|=0.31(p_L)^{-1/2}, \tag{16}$$

$$|K_f^{\mathrm{ce}}|=0.18(p_L)^{-1/2}\gamma^{-1}, \tag{17}$$

$$p_L=\text{lab pion momentum in (BeV}/c),$$

$$\gamma=10\ (\mathrm{BeV}/c)^{-2}.$$

We shall now make a few remarks:

(A) The fit with experiments exhibited in Fig. 2 has besides one normalizing constant [which is energy-dependent, as shown in Eqs. (16) and (17)] one adjustable constant, namely $K_f^{\mathrm{ce}}/K^{\mathrm{ce}}$. The energy independence[3] of the shape of the $d\sigma/dt$ versus t curve is reflected in the energy independence of $K_f^{\mathrm{ce}}/K^{\mathrm{ce}}$.

(B) For $-t>0.6$, the experimental points[3] for $\pi^-p\to\pi^0n$ show a wide peak. The total cross section under this peak is, however, very small. Its existence depends on the exact details of α^{ce} and β^{ce}. [Many crude assumptions are implicit in Eqs. (11) and (13); see, e.g., remark I below.] We have made no attempt to fit the data for $-t\gtrsim 0.6$, although assumptions (11) and (13) do lead to secondary peaks. It is interesting to speculate as to whether the "shoulder" in the elastic scattering data[4] for $-t\sim 1$ is due to a spin-flip term like (13a). (See remark E below.)

(C) For spin zero-zero elastic scattering, one has

$$\frac{d}{d(-t)}\left(\frac{d\sigma}{dt}\right)\bigg|_{t=0}=-\tfrac{1}{2}\lambda\!\!\!^{-2}\left(\frac{d\sigma}{dt}\right)_{t=0}$$
$$\times\mathrm{Re}\{[\sum l(l+1)(l+\tfrac{1}{2})\alpha_l][\sum(l+\tfrac{1}{2})\alpha_l]^{-1}\}. \tag{18}$$

If α is real, this is always negative. For spin $\frac{1}{2}$-spin 0 scattering one has, for the spin-nonflip part, similarly

$$\left[\frac{d}{d(-t)}\left(\frac{d\sigma}{dt}\right)_{\mathrm{nonflip}}\right]_{t=0}=-\tfrac{1}{2}\lambda\!\!\!^{-2}\left(\frac{d\sigma}{dt}\right)_{t=0}$$
$$\times\mathrm{Re}\{[\sum l(l+1)(l+\tfrac{1}{2})\alpha_l][\sum(l+\tfrac{1}{2})\alpha_l]^{-1}\}. \tag{19}$$

These two formulas hold for charge-exchange processes as well if we replace α by α^{ce}. If $\arg(\alpha^{\mathrm{ce}})$ does not depend on l, (19) is always negative. Thus, the observed positive slope of the charge-exchange differential cross section near $-t=0$ means either that the spin-flip part contributes importantly in this region, or $\arg(\alpha^{\mathrm{ce}})$ has large variation as l increases. The former alternative is the one chosen in our model.

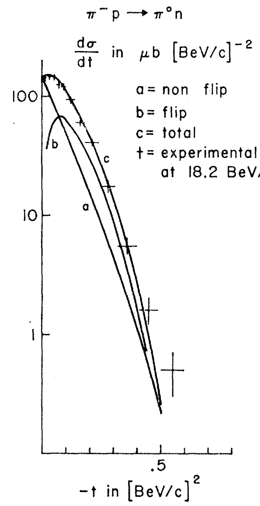

FIG. 2. $d\sigma/dt$ versus $-t$ for $\pi^-p\to\pi^0n$. The smooth curves are calculated values. Experimental data of Stirling *et al.* in Ref. 3 are indicated. Note that the data in Ref. 3 give very nearly parallel curves for $d\sigma/dt$ versus t, $0\le -t\le 0.4$, for lab momenta from 5.9 to 18.2 BeV/c. For $-t>0.4$, the data show faster decrease of $d\sigma/dt$ (with increasing $-t$) for higher energies. This trend suggests that our fit at $-t\approx 0.5$ will be better at higher energies.

Equation (19) for elastic scattering can be cast into the following form:

$$2\frac{d}{d(-t)}\ln\left(\frac{d\sigma}{dt}\right)_{\mathrm{nonflip}}=-\mathrm{Re}\langle b^2\rangle,$$

where $\langle b^2\rangle$ is the average of b^2 with weights $(2l+1)\alpha_l$. Thus, the logarithmic derivative with respect to t at $t=0$ of $d\sigma/dt$ for elastic scattering *gives a measure of the size of the interaction volume.* Using (1), one obtains

$$\langle b^2\rangle\approx 2\gamma.$$

(D) Equation (10) is applicable if the wave number difference before and after the charge exchange Δk is small. (If Δk is too large, it will destroy the coherence of phase over the interaction volume.) In the center-of-mass system, the value of

$$\Delta k \text{ is } \cong(m_1^2+m_2^2-m_3^2-m_4^2)/(4k).$$

With the Lorentz contraction taken into consideration one obtains, for high energies, the following condition for the preservation of coherence:

$$|m_1^2+m_2^2-m_3^2-m_4^2|Rp_L^{-1}\ll\pi,$$

where R is the radius of the interaction volume $[\sim 4(\mathrm{BeV}/c)^{-1}]$ and p_L is the laboratory momentum of the incident particle. This condition is well satisfied for $\pi^-p\to\pi^0n$ even at relatively low energies.

(E) In our model, the polarization perpendicular to the scattering plane of the neutron from an unpolarized target is easily computed:

$$P=\frac{2\,\mathrm{Im}(A^*B)}{AA^*+BB^*}=\eta\xi(t), \tag{20}$$

[11] These calculations are easily effected numerically by first expanding (11) and (13) in powers of α. Each power α^n leads to an integral in (1′) and (2′) of the form

$$\int_0^\infty J_\nu(xb)\exp(-p^2b^2)b^{\mu-1}db$$
$$=\frac{(x/2p)^\nu\Gamma[(\mu+\nu)/2]}{2p^\mu\Gamma(\nu+1)}\,{}_1F_1\left(\frac{\mu+\nu}{2},\nu+1,\frac{-x^2}{4p^2}\right)$$

and, since ${}_1F_1(\alpha,\gamma-z)=e^{-z}\,{}_1F_1(\gamma-\alpha,\gamma,z)$, it is easily evaluated. [See G. N. Watson, Ref. 9, Sec. 13.3.]

where

$$\eta=\sin[\arg(K_f^{ce}/K^{ce})],$$

$$\xi(t)=2\left[\left(\frac{d\sigma}{dt}\right)_{\text{flip}}\left(\frac{d\sigma}{dt}\right)_{\text{nonflip}}\right]^{1/2}\left(\frac{d\sigma}{dt}\right)^{-1}. \tag{21}$$

Thus, if K_f^{ce} and K^{ce} are not relatively real, the polarization is proportional to $\xi(t)$ which, for our model, is quite large for $0.02<-t<0.6$. Its values are tabulated below:

$-t$	0.02	0.06	0.1	0.2	0.3	0.4	0.5
ξ	0.86	1.0	0.99	0.94	0.93	0.98	0.72.

It should be emphasized that while the diffraction peak in elastic scattering has predominantly an imaginary amplitude, the charge-exchange amplitudes, in our model, have complex phases equal to those of K^{ce} and K_f^{ce}. In general, these are neither real nor imaginary. For K^{ce}, experiments[3] indicate an appreciable phase since

$$\text{Re}A(0)/\text{Im}A(0)\cong 1.$$

There seems to be no *a priori* reason why η of Eq. (21) should be small. *A measurement of the neutron polarization in $\pi^-p\rightarrow\pi^0n$, or of the right-left asymmetry in this process using a polarized target, is thus very interesting.*

In our picture, the spin-flip amplitude β in elastic scattering is small compared with α (this is generally assumed to be true); its value is given by an equation like (13); e.g.,

$$\beta=-K(1-\alpha)b^2\ln(1-\alpha). \tag{13a}$$

If the spin-flip differential cross section is about 1% of the non-spin flip, our model could give a polarization as large as 20% when $0.02<-t<0.6$ [if $\arg K\approx\frac{1}{2}\pi$ $(\text{mod}\pi)$]. *It seems worthwhile to measure this polarization.*

Similarly, measurements of the polarization of the scattered proton in the backward peak in π^+p and π^-p scattering would be interesting. The π^-p backward peak[4] has a flattened top, resembling that of the forward $\pi^-p\rightarrow\pi^0n$ process. In our model, this behavior indicates a large spin-flip contribution (see remark C above) possibly giving rise to a large polarization for the recoil proton.

(F) One can compare (11) with the assumption of the absorptive peripheral model[6,7] which is

$$\alpha^{ce}=(1-\alpha)\ \text{(one-particle exchange amplitude in lowest order perturbation calculation)}.$$

The absorption factor $1-\alpha=e^{2i\delta}$ is essential here since the one-particle exchange amplitude always gives too large values at small impact parameters. In our model, the factor $1-\alpha$ is not as crucial since "the material thickness" factor $\delta\propto\ln(1-\alpha)$ already quite effectively limits the value of α^{ce} for small impact parameters.

For extremely large energies, the collision time is short because of Lorentz contraction [see Fig. 1(a)]. During such short times the exchange of energy, momentum, and quantum numbers is an instantaneous process, and therefore not much related to the lowest mass state with those quantum numbers. In other words, it takes time for the low-mass effects to dominate, and there is not sufficient time for that in very high-energy collisions. This may account for the difficulties encountered by the absorptive peripheral model.

(G) It would be interesting to know whether for large angles, say 60°∼90° in the center-of-mass system, the ratio of charge-exchange to elastic scattering becomes of the order of unity.[12] If so, one has a clear indication of the difference of the physical mechanisms for large- and for small-angle scattering.

(H) In our view, the amplitudes $a_l^{\pm}$ with specific quantum numbers dominantly follow a smooth function of b independent of parity, isotopic spin, and energy. However at energies $<\infty$, there are finite deviations from this smooth function. One can say that the difference between the spin-orbit alignment (parallel and antiparallel) gives rise to the small spin-flip amplitude β, the difference between $I=\frac{3}{2}$ and $I=\frac{1}{2}$ gives rise to the small charge-exchange scattering, and the difference between even and odd l gives rise to the small backward peaks in πp scattering (small compared to the dominant term which gives rise to the elastic forward peak). The differences all seem to $\rightarrow 0$ as $E\rightarrow\infty$. But it is perhaps more illuminating to view the situation the other way around: A spin-flip mechanism causes a small spin-flip amplitude β which can always be written as $a_l^+-a_l^-$; a neutron-exchange mechanism causes a small backward scattering in π^-p collisions which can be described as an even l–odd l difference; etc.

From this viewpoint, it is not surprising that the difference of a difference, for example, the charge-exchange spin-flip process, is not a second-order process which is much smaller than the charge exchange without spin flip. It would be interesting in this connection to *measure the backward $\pi^-p\rightarrow\pi^0p$ cross section* and compare it with the forward $\pi^-p\rightarrow\pi^0n$ and backward elastic π^-p cross sections.

(I) Our model is essentially a droplet model of elementary particles, which are pictured as very much similar to nuclei. (It is interesting to compare the angular distributions discussed here with those in "high-energy" $\alpha-\alpha$ and α-nuclei scattering.[13]) Corresponding to the concept of the density of nuclear matter we now have a "density distribution" $\rho(r)$ which is related to α in the eikonal picture through

$$\ln(1-\alpha)=2i\delta(b)=(\text{const})\int_{-\infty}^{\infty}\rho((b^2+x^2)^{1/2})\,dx. \tag{22}$$

[12] See, e.g., T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965).

[13] For recent data, see P. Darriulat, G. Igo, H. G. Pugh, J. M. Meriwether, and S. Yambe, Phys. Rev. **134**, B42 (1964); P. Darriulat, G. Igo, H. G. Pugh, and H. D. Holmgren, *ibid.* **137**, B315 (1965) and footnote 10. We are indebted to G. E. Brown for informing us of these papers. Notice that the presence of a sharp edge in the nucleus accounts for the existence of many diffraction minima in α-Fe scattering.

If α is Gaussian as in (15), we can solve for ρ from (22), obtaining

$$\rho(r)=(\text{const})g_{1/2}(\alpha), \quad (23)$$

where $g_{1/2}$ is a familiar function in the theory of Bose-Einstein condensation of free particles:

$$g_{1/2}(\alpha)=\alpha+\alpha^2/\sqrt{2}+\alpha^3/\sqrt{3}+\cdots. \quad (24)$$

In presenting our model, we are suggesting that the same density function ρ determines the elastic and the charge-exchange scattering. [In other words, we assume the colliding particles to have no variation of "composition"; or in the language of potentials, in Eq. (12) we assume the same radial dependence for U and V.] This is a simple possibility, but at best only a crude approximation.

(J) It seems reasonable to us to describe the nucleon and pion as extended structures *without a localized central singularity*. Earlier fits to higher t scattering seem to demand such a singularity.[14] It seems to us that large t processes are due to a different physical mechanism and probably should not be fitted with a "coherent" model.

(K) Equation (15) indicates that in $\pi^- p$ collisions the interaction is not represented by a totally "black" region. If the spatial extension of higher meson resonances (say ρ) is not larger than that of the pion, there should be more interaction "per unit path length" in ρp scattering than in πp scattering. Consequently, ρp interaction should be "blacker" and should also contain more charge exchange. (We have implicitly assumed here that the "composition" of ρ is roughly the same as that of π.) We wonder whether it is possible to check these conclusions in reactions such as $\pi p \to \rho p$.

The question of the *spatial extensions* of resonances is an important one and has not, to our knowledge, been discussed in the literature. It seems to us unlikely that the resonances are much larger in size than the pion or the proton. Larger sizes lead usually to higher density of states. Experimentally, there seems to be very little increase in the density of resonant states per unit energy interval as one goes to higher resonance energies.

(L) One can ask, of course, what is ρ the density of? In other words, if p and π are extended structures, what are they made of? Clearly they are made of "stuff" which when isolated (if possible) and observed for long times would separate into mesons and hyperons, particles with extended structures themselves. In this respect, nucleons and pions are very different from a liquid drop or a nucleus.

The concept of ρ perhaps resembles more that of the probability density of the electrons in an atom. The charge-exchange scattering then resembles the scattering of x rays by an atom (except for the fact that a single atom is quite transparent to x rays).

Now, in the case of atomic physics, if one wants to study the interaction of x rays with the constituent parts of the atom (i.e., the electron), one does not study the *elastic* scattering of x rays by an atom. Instead, one studies the scattering of x rays by atoms with large momentum transfers to a single electron and very little recoil for the atom. One wonders whether similar but less clean-cut considerations should be applied to high-energy scatterings.

Is the concept of ρ useful only insofar as it enters into a coherent contribution to the amplitude of some process [with $\rho(r)d^3r$ not representing the density of any "stuff"]? In which way is it related to the possibility of "measuring" structures in spatial dimensions much less than one fermi by strongly interacting particles, which themselves have spatial extensions of $\sim 0.7 \to 1$ F? In what way is ρ related to the electromagnetic form factors? We do not know the answers to these questions.

(M) We have mentioned before the usefulness of experiments on large-angle exchange processes [remark (G)]. We also mentioned the interest in backward cross sections [remark (H)] and polarizations [remark (E)] in forward and backward directions. Many experiments of such types can be envisaged, involving $\pi p \to \Lambda K$, $pp \to pp^*$, $\pi p \to \rho p$, $pp \to p^* p^*$, etc. In those cases where the outgoing particles undergo decay processes, as, e.g., in $\pi p \to \Lambda K$, polarization measurements are, of course, relatively simple.

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[14] R. Serber, Rev. Mod. Phys. **36**, 649 (1964).